



Tài liệu vật lý

**Lý thuyết hệ
nhiều hạt**

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Chương 1: Tính chất chung của hệ nhiều hạt

0- Khái niệm về hệ nhiều hạt

0.1- *Nhiều* : $N \geq 2$: Vấn đề kỹ thuật : số biến ; tương tác ; thay đổi về chất

0.2- *Nhiều* ($N \gg 1$) : không làm thay đổi chất

0.3- *Nhiều* ($N \gg 1$) : làm thay đổi chất

0.4- *Hệ nhiều hạt ở $T=0K$.*

0.5- *Hệ kín.*

0.6- *Hệ ở $T \neq 0K$.*

Quan hệ giữa Cơ học và Vật lý thống kê (bao gồm cả cổ điển và lượng tử)

1- Hệ hạt đồng nhất:

1.1- Nguyên lý không phân biệt các hạt đồng nhất trong cơ học lượng tử

1.2- Hàm sóng của hệ các hạt đồng nhất

1.2.1- Tính đối xứng của hàm sóng

$$\hat{P}_{ij} \psi(q_1, \dots, q_i, \dots, q_j, \dots, q_N) = \psi(q_1, \dots, q_j, \dots, q_i, \dots, q_N) \quad (1.1)$$

$$\psi_+(q_1, \dots, q_i, \dots, q_j, \dots, q_N) = \psi_+(q_1, \dots, q_j, \dots, q_i, \dots, q_N) \quad (1.2)$$

$$\psi_-(q_1, \dots, q_i, \dots, q_j, \dots, q_N) = -\psi_-(q_1, \dots, q_j, \dots, q_i, \dots, q_N) \quad (1.3)$$

1.2.2- Đặc điểm của tính đối xứng của hàm sóng

1.2.2.1- Tính đối xứng là như nhau đối với tất cả các cặp biến :

1.2.2.2- Tính đối xứng của hàm sóng phụ thuộc vào spin :

$$\begin{cases} \text{Spin nguyên (0 ; 1 ; 2 ;)} \\ \text{Spin bán nguyên (1/2 ; 3/2 ; 5/2 ;)} \end{cases}$$

1.2.2.3- Tính đối xứng của hàm sóng là vĩnh cửu :

1.2.3- Dạng của hàm sóng của hệ hạt đồng nhất không tương tác

$$\varphi_{p_i}(q_i) = \phi_{n_i}(\vec{r}_i) \chi_{\alpha}(s_i) ; \quad q_i = (\vec{r}_i, s_i) ; \quad p_i = (n_i, \alpha) \quad (1.4)$$

$$\int \varphi_{p_i}^*(q_i) \varphi_{p_k}(q_i) dq_i = \int d\vec{r}_i \sum_{S_i} \phi_{n_i}^*(\vec{r}_i) \chi_{\alpha}^*(s_i) \phi_{n_k}(\vec{r}_i) \chi_{\beta}(s_i) = \delta_{n_i, n_k} \delta_{\alpha\beta} = \delta_{p_i p_k} \quad (1.5)$$

$$d\vec{r}_i = dx_i dy_i dz_i .$$

$$\psi_+(q_1, q_2, \dots, q_N) = c \sum_{(q)} \varphi_{p_1}(q_1) \varphi_{p_2}(q_2) \dots \varphi_{p_N}(q_N) \quad (1.6a)$$

$$\psi_-(q_1, q_2, \dots, q_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \varphi_{p_1}(q_1) & \varphi_{p_1}(q_2) & \dots & \varphi_{p_1}(q_N) \\ \varphi_{p_2}(q_1) & \varphi_{p_2}(q_2) & \dots & \varphi_{p_2}(q_N) \\ \dots & \dots & \dots & \dots \\ \varphi_{p_N}(q_1) & \varphi_{p_N}(q_2) & \dots & \varphi_{p_N}(q_N) \end{vmatrix} \quad (1.7a)$$

Định thức Slater chứa đựng Nguyên lý loại trừ Pauli .

2- Các đại lượng bảo toàn của hệ nhiều hạt.

2.1-Hamiltonian của hệ nhiều hạt.

$$H = -(\hbar^2 / 2) \sum_{i=1}^N (\Delta_i / m_i) + V(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) \quad (2.1a)$$

$$H = -(\hbar^2 / 2) \sum_{i=1}^N \frac{1}{r_i^2} \frac{\partial}{\partial r_i} \left(r_i^2 \frac{\partial}{\partial r_i} \right) + \frac{1}{r_i^2 \sin \vartheta_i} \frac{\partial}{\partial \vartheta_i} \left(\sin \vartheta_i \frac{\partial}{\partial \vartheta_i} \right) + \frac{1}{r_i^2 \sin^2 \vartheta_i} \frac{\partial^2}{\partial \varphi^2} + V(r, \vartheta, \varphi) \quad (2.1b)$$

$$(r \equiv (r_1, r_2, \dots, r_N) ; \vartheta \equiv (\vartheta_1, \vartheta_2, \dots, \vartheta_N) ; \varphi \equiv (\varphi_1, \varphi_2, \dots, \varphi_N))$$

2.2- Bảo toàn động lượng của hệ nhiều hạt.

$$\hat{P} = -i\hbar \sum_{k=1}^N \vec{\nabla}_k \quad (2.2)$$

Do đó:
$$\hat{P} = \frac{1}{i\hbar} (\hat{P}H - H\hat{P}) = \frac{1}{i\hbar} (\hat{P}V - V\hat{P}) = - \sum_{k=1}^N (\vec{\nabla}_k V - V\vec{\nabla}_k) = - \sum_{k=1}^N \vec{\nabla}_k V = \sum_{k=1}^N \vec{F}_k = \vec{F}_{int} + \vec{F}_{ext} = \vec{F}_{ext} \quad (2.3)$$

$$\vec{F}_{int} = \sum_i \vec{F}_i = \sum_i \sum_j \vec{F}_{ij} = 0. \text{ Nếu: } \vec{F}_{ext} = 0$$

2.3- Bảo toàn mô men động lượng của hệ nhiều hạt.

$$\hat{L} = \sum_{k=1}^N \hat{\ell}_k ; \hat{L}_z = -i\hbar \sum_{k=1}^N \hat{\ell}_{kz} ; \text{ thay } \hat{\ell}_{kz} = -i\hbar \partial / \partial \varphi_k, \quad \hat{L}_z = -i\hbar \sum_{k=1}^N \frac{\partial}{\partial \varphi_k} \quad (2.4)$$

$$\hat{L}_z = \frac{1}{i\hbar} (\hat{L}_z H - H \hat{L}_z) = - \sum_{k=1}^N \frac{\partial V}{\partial \varphi_k} = \sum_{k=1}^N M_{kz} \quad (2.5a)$$

$$\sum_{k=1}^N M_{kz} = M_{z,int} + M_{z,ext} = M_z \quad (2.5b)$$

CM : $M_{z,int} = 0$

Với L_z và L^2 bảo toàn.

3- Biểu diễn tương tác

Biểu diễn Shrodinger :
$$i\hbar \frac{\partial \Phi_S(t)}{\partial t} = H \Phi_S(t) \quad (3.1)$$

$$\Phi_S(t) = [\exp(-iHt/\hbar)] \Phi_H \quad (3.2)$$

Biểu diễn Heisenberg :
$$\hat{F}_H(t) = e^{iHt/\hbar} \hat{F}_S e^{-iHt/\hbar} \quad (3.3)$$

$$\Phi_H = [\exp(iHt/\hbar)] \Phi_S(t) \quad (3.4)$$

Biểu diễn tương tác :
$$H = H_0 + \hat{V} \quad (3.5)$$

$$\hat{F}_i(t) = e^{iH_0 t/\hbar} \hat{F}_S e^{-iH_0 t/\hbar} \quad (3.6)$$

$$\Phi_i(t) = [\exp(iH_0 t/\hbar)] \Phi_S(t) \quad (3.7)$$

$$i\hbar \frac{\partial \Phi_i(t)}{\partial t} = \hat{V}_i(t) \Phi_i(t) \quad (3.8)$$

$$\hat{V}_i(t) = e^{iH_0 t/\hbar} \hat{V}_S e^{-iH_0 t/\hbar} \quad (3.9)$$

$$\Phi_i(t) = \Phi_i(t_0) - (i/\hbar) \int_{t_0}^t \hat{V}_i(t') \Phi_i(t') dt' \quad (3.10)$$

$$\Phi_i(t) = \Phi_i^{(0)}(t) + \Phi_i^{(1)}(t) + \Phi_i^{(2)}(t) + \dots \quad (3.11)$$

$$\Phi_i(t) = \hat{S}(t, t_0) \Phi_i(t_0) \quad (3.16)$$

$$\hat{S}(t, t_0) = 1 - (i/\hbar) \int_{t_0}^t \hat{V}_i(t_1) dt_1 - (1/\hbar)^2 \int_{t_0}^t \hat{V}_i(t_1) dt_1 \int_{t_0}^{t_1} \hat{V}_i(t_2) dt_2 + \dots + (-i/\hbar)^n \int_{t_0}^t \hat{V}_i(t_1) dt_1 \int_{t_0}^{t_1} \hat{V}_i(t_2) dt_2 \dots \int_{t_0}^{t_{n-1}} \hat{V}_i(t_n) dt_n + \dots \quad (3.17)$$

$$\hat{S}(t, t_0) = \hat{T} \exp \left\{ (-i/\hbar) \int_{t_0}^t \hat{V}_i(t') dt' \right\} \quad (3.18)$$

$$\hat{S}(t_2, t_1) \hat{S}(t_1, t_0) = \hat{S}(t_2, t_0) ; t_2 > t_1 > t_0 \quad (3.19)$$

Coi : $V(t_V) = 0$ Ký hiệu : $\hat{S}(t) = \hat{S}(t, t_V)$ (3.20)

$$\Rightarrow \hat{S}(t_2, t_1) = \hat{S}(t_2) \hat{S}^{-1}(t_1) \quad (3.21)$$

Từ (3.2) : $\Phi_S(t) = [\exp(-iHt/\hbar)] \Phi_H$. Trong đó : $\Phi_i(t) = \hat{S}(t) \Phi_i(t_V)$ (3.22)

$\Phi_i(t) = [\exp(iH_0 t/\hbar)] \cdot [\exp(-iHt/\hbar)] \Phi_H$; thay $t = t_V \Rightarrow \Phi_i(t_V) = \Phi_H$

$$\Rightarrow \Phi_i(t) = \hat{S}(t) \Phi_H \quad (3.23)$$

$$\hat{F}_i(t) = \hat{S}(t) \hat{F}_H(t) \hat{S}^{-1}(t) \quad (3.24)$$

$$M = \langle \Phi_H^{0*} \hat{T} [\hat{A}_H(t) \hat{B}_H(t') \hat{C}_H(t'') \dots] \Phi_H^0 \rangle \quad (3.25)$$

Giả thiết $t > t' > t'' > \dots$

$$M = \langle \Phi_H^{0*} \hat{S}^{-1}(\infty) \hat{T} [\hat{A}_i(t) \hat{B}_i(t') \hat{C}_i(t'') \dots \hat{S}(\infty)] \Phi_H^0 \rangle \quad (3.26)$$

$$\hat{S}(\infty) \Phi_H^0 = e^{i\alpha} \Phi_H^0 \quad (3.27)$$

Cuối cùng :

$$M = \frac{\langle \Phi_H^{0*} \hat{T} [\hat{A}_i(t) \hat{B}_i(t') \hat{C}_i(t'') \dots \hat{S}(\infty)] \Phi_H^0 \rangle}{\langle \Phi_H^{0*} \hat{S}(\infty) \Phi_H^0 \rangle} \quad (3.28)$$

Chương 2 : Một số phương pháp giải bài toán hệ nhiều hạt

4- Phương pháp tách chuyển động khối tâm của hệ :

4.1- Đặc điểm của thế tương tác:

$$H = -(\hbar^2/2) \sum_{i=1}^N (\Delta_i / m_i) + V(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) \quad (4.1a)$$

$$V(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = V(\vec{r}_1 - \vec{r}_2, \vec{r}_1 - \vec{r}_3, \dots, \vec{r}_{N-1} - \vec{r}_N) \quad (4.1b)$$

Sự phụ thuộc này dẫn đến kết quả là

Décartes $\vec{r}(x, y, z) \Rightarrow$ Jacobi $\vec{\rho}(\xi, \eta, \zeta)$:

$$\xi_1 = (m_1 x_1 : m_1) - x_2 \quad ; \quad \xi_2 = [(m_1 x_1 + m_2 x_2) : (m_1 + m_2)] - x_3 \quad ; \quad \dots$$

$$\xi_k = \left[\left(\sum_{j=1}^k m_j x_j \right) : \left(\sum_{j=1}^k m_j \right) \right] - x_{k+1} \quad , \quad \text{với } k = 1, 2, \dots, N-1 \quad (4.2a)$$

$$\xi_N = \left(\sum_{i=1}^N m_i x_i \right) : \left(\sum_{i=1}^N m_i \right) \quad (4.2b)$$

Tương tự cho các tọa độ η_i và ζ_i .

Có thể chứng minh được :

$$\sum_{i=1}^N (\Delta_{\vec{r}, i} / m_i) = \sum_{i=1}^N (\Delta_{\vec{\rho}, i} / \mu_i) \quad (4.3a)$$

trong đó :

$$\Delta_{\vec{r}, i} = \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2} \quad ; \quad \Delta_{\vec{\rho}, i} = \frac{\partial^2}{\partial \xi_i^2} + \frac{\partial^2}{\partial \eta_i^2} + \frac{\partial^2}{\partial \zeta_i^2} \quad (4.3b)$$

và :

$$(\mu_k)^{-1} = \left(\sum_{j=1}^k m_j \right)^{-1} + (m_{k+1})^{-1} \quad \text{với } k = 1, 2, \dots, N-1 \quad (4.3c)$$

$$\mu_N = \sum_{i=1}^N m_i \quad (4.3d)$$

Khi đó

$$H(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = -(\hbar^2/2) \sum_{i=1}^N (\Delta_{\vec{r}, i} / m_i) + V(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N)$$

$$= H'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) = -(\hbar^2/2) \sum_{i=1}^N (\Delta_{\vec{\rho}, i} / \mu_i) + V'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) \quad (4.4)$$

$$\vec{r} \Rightarrow \vec{r} + \vec{a} \quad ; \quad H(\vec{r}) = H(\vec{r} + \vec{a}), \Rightarrow V(\vec{r}) = V(\vec{r} + \vec{a}) :$$

$$V(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = V(x_1 + a_x, y_1 + a_y, z_1 + a_z, x_2 + a_x, y_2 + a_y, z_2 + a_z, \dots, x_N + a_x, y_N + a_y, z_N + a_z)$$

$$= V'(\xi_1, \eta_1, \zeta_1, \xi_2, \eta_2, \zeta_2, \dots, \xi_N + a_x, \eta_N + a_y, \zeta_N + a_z)$$

$$\Rightarrow V'(\xi_1, \eta_1, \zeta_1, \xi_2, \eta_2, \zeta_2, \dots, \xi_N, \eta_N, \zeta_N) = V'(\xi_1, \eta_1, \zeta_1, \xi_2, \eta_2, \zeta_2, \dots, \xi_N, \eta_N, \zeta_N),$$

Kết quả là
$$H'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) = -(\hbar^2/2) \sum_{i=1}^N (\Delta_{\vec{\rho}_i} / \mu_i) + V'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1}) \quad (4.5)$$

4.2- Phương trình Shrodinger cho hệ đã tách chuyển động khối tâm:

$$\psi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) = \phi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1}) G(\vec{\rho}_N) \quad (4.6)$$

$$[-(\hbar^2/2) \sum_{i=1}^N (\Delta_{\vec{\rho}_i} / \mu_i) + V'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1})] \phi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) G(\vec{\rho}_N) = E \phi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) G(\vec{\rho}_N)$$

$$\frac{1}{\phi} [-(\hbar^2/2) \sum_{i=1}^{N-1} (\Delta_{\vec{\rho}_i} / \mu_i) + V'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1})] \phi - \frac{\hbar^2}{2G} [\Delta_{\vec{\rho}_N} / \mu_N] G = E \quad (4.7)$$

$$[-(\hbar^2/2) \sum_{i=1}^{N-1} (\Delta_{\vec{\rho}_i} / \mu_i) + V'(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1})] \phi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1}) = E_1 \phi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_{N-1}) \quad (4.8a)$$

$$-\frac{\hbar^2}{2} [\Delta_{\vec{\rho}_N} / \mu_N] G(\vec{\rho}_N) = E_2 G(\vec{\rho}_N) \quad (4.8b)$$

Với
$$E_1 + E_2 = E \quad (4.8c)$$

Ví dụ : Xét hệ gồm 2 hạt (N = 2): \Rightarrow Bài tập

5- Phương pháp trường trung bình

5.1- Ý tưởng của phương pháp trường trung bình

$$H\Psi = E\Psi \quad (5.1)$$

$$H = \sum_{i=1}^N H_i(\vec{r}_i) + (1/2) \sum_{i,j} V_{ij}(\vec{r}_i, \vec{r}_j) \quad H_i(\vec{r}_i) = -\frac{\hbar^2}{2m_i} \Delta_i + u_i(\vec{r}_i) \quad (5.2)$$

$$\Rightarrow H = \sum_{i=1}^N H'_i(\vec{r}_i) \quad \text{với} \quad H'_i(\vec{r}_i) = -\frac{\hbar^2}{2m_i} \Delta_i + u_i(\vec{r}_i) + V_{ef}(\vec{r}_i) \quad (5.3)$$

$$\psi = \psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = \prod_{i=1}^N \varphi_{p_i}(\vec{r}_i) \quad (5.4)$$

$$[-\frac{\hbar^2}{2m_i} \Delta_i + u_i(\vec{r}_i) + V_{ef}(\vec{r}_i)] \varphi_{p_i}(\vec{r}_i) = \varepsilon_i \varphi_{p_i}(\vec{r}_i) \quad \sum_{i=1}^N \varepsilon_i = E \quad (5.5)$$

$$Q[\psi] = \int \psi^* [H - E] \psi dq \quad (5.6)$$

$$dq = \prod_{i=1}^N dq_i, \text{ còn } q_i = (\vec{r}_i, s_i); \int \dots dq = \int \prod_{i=1}^N d\vec{r}_i \sum_{s_i} \dots \quad (5.7)$$

$$\delta Q[\psi] = \delta \int \psi^* [H - E] \psi dq = 0 \quad (5.8)$$

5.2- Thế hiệu dụng V_{ef} đối với hệ các hạt boson

$$\int \prod_{i \neq k} \varphi_{p_i}^*(q_i) \delta \varphi_{p_k}^*(q_k) [\sum_{i=1}^N H_i(\vec{r}_i) + (1/2) \sum_{i,j} V_{ij}(\vec{r}_i, \vec{r}_j) - E] \psi dq$$

$$+ \int \psi^* [\sum_{i=1}^N H_i(\vec{r}_i) + (1/2) \sum_{i,j} V_{ij}(\vec{r}_i, \vec{r}_j) - E] \prod_{i \neq k} \varphi_{p_i}(q_i) \delta \varphi_{p_k}(q_k) dq = 0, \quad (5.9)$$

$$\int dq_k \delta \varphi_{p_k}^*(q_k) \int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [\sum_{i=1}^N H_i(\vec{r}_i) + (1/2) \sum_{i,j} V_{ij}(\vec{r}_i, \vec{r}_j) - E] \psi \prod_{i \neq k} dq_i$$

$$+ \int dq_k \delta \varphi_{p_k}(q_k) \int \prod_{i \neq k} \varphi_{p_i}(q_i) [\sum_{i=1}^N H_i^*(\vec{r}_i) + (1/2) \sum_{i,j} V_{ij}^*(\vec{r}_i, \vec{r}_j) - E] \psi^* \prod_{i \neq k} dq_i = 0 \quad (5.10)$$

$$\Rightarrow \int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [\sum_{i=1}^N H_i(\vec{r}_i) + (1/2) \sum_{i,j} V_{ij}(\vec{r}_i, \vec{r}_j) - E] \psi \prod_{i \neq k} dq_i = 0 \quad (5.11)$$

$$\sum_{i,j} V_{ij}(\vec{r}_i, \vec{r}_j) = 2 \sum_i V_{ik} + \sum_{\substack{i \neq k \\ j \neq k}} V_{ij} \quad (5.12)$$

$$c_1 = \int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [\sum_{i \neq k} H_i(\vec{r}_i)] \prod_{i \neq k} \varphi_{p_i}(q_i) \prod_{i \neq k} dq_i \quad (5.13a)$$

$$c_2 = \int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [(1/2) \sum_{\substack{i \neq k \\ j \neq k}} V_{ij}(\vec{r}_i, \vec{r}_j)] \prod_{i \neq k} \varphi_{p_i}(q_i) \prod_{i \neq k} dq_i \quad (5.13b)$$

$$V_{ef}(\vec{r}_k) = \int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [\sum_i V_{ik}(\vec{r}_i, \vec{r}_k)] \prod_{i \neq k} \varphi_{p_i}(q_i) \prod_{i \neq k} dq_i \quad (5.14)$$

$$\int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [\sum_i H_i(\vec{r}_i)] \psi \prod_{i \neq k} dq_i = [c_1 + H_k(\vec{r}_k)] \varphi_{p_k}(q_k) \quad (5.15a)$$

$$\int \prod_{i \neq k} \varphi_{p_i}^*(q_i) [(1/2) \sum_{i,j} V_{ij}(\vec{r}_i)] \psi \prod_{i \neq k} dq_i = [c_2 + V_{ef}(\vec{r}_k)] \varphi_{p_k}(q_k) \quad (5.15b)$$

$$\int \prod_{i \neq k} \varphi_{p_i}^*(q_i) E \psi \prod_{i \neq k} dq_i = E \varphi_{p_k}(q_k) \quad (5.15c)$$

$$[H_k(\vec{r}_k) + V_{ef}(\vec{r}_k)] \varphi_{p_k}(q_k) = \varepsilon_k \varphi_{p_k}(q_k) \quad (5.17a)$$

$$\varepsilon_k = E - c_1 - c_2 \quad (5.17b)$$

5.3- Thế hiệu dụng đối với hệ các hạt fermion

$$\psi(q_1, q_2) = \frac{1}{\sqrt{2}} [\varphi_1(q_1) \varphi_2(q_2) - \varphi_1(q_2) \varphi_2(q_1)] \quad (1.7b)$$

$$\begin{aligned} & \int [\delta \varphi_1^*(q_1) \varphi_2^*(q_2) - \delta \varphi_1^*(q_2) \varphi_2^*(q_1)] (H_1 + H_2 + V_{12} - E) [\varphi_1(q_1) \varphi_2(q_2) - \varphi_1(q_2) \varphi_2(q_1)] dq_1 dq_2 \\ & + \int [\varphi_1^*(q_1) \varphi_2^*(q_2) - \varphi_1^*(q_2) \varphi_2^*(q_1)] (H_1 + H_2 + V_{12} - E) [\delta \varphi_1(q_1) \varphi_2(q_2) - \delta \varphi_1(q_2) \varphi_2(q_1)] dq_1 dq_2 = 0 \end{aligned} \quad (5.18)$$

$$\begin{aligned} & \int \delta \varphi_1^*(q_2) \varphi_2^*(q_1) (H_1 + H_2 + V_{12} - E) \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 = \\ & = \int \delta \varphi_1^*(q_1) \varphi_2^*(q_2) (H_1 + H_2 + V_{12} - E) \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 \\ & \int \varphi_1^*(q_2) \varphi_2^*(q_1) (H_1 + H_2 + V_{12} - E) \delta \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 = \int \varphi_1^*(q_1) \varphi_2^*(q_2) (H_1 + H_2 + V_{12} - E) \delta \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 \\ & \int \delta \varphi_1^*(q_1) \varphi_2^*(q_2) V_{12} \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 = \int \delta \varphi_1^*(q_2) \varphi_2^*(q_1) V_{12} \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 \\ & \int \varphi_1^*(q_1) \varphi_2^*(q_2) V_{12} \delta \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 = \int \varphi_1^*(q_2) \varphi_2^*(q_1) V_{12} \delta \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 \\ & \int \delta \varphi_1^*(q_1) \varphi_2^*(q_2) (H_1 + H_2 - E) \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 = 0 \\ & \int \delta \varphi_1^*(q_2) \varphi_2^*(q_1) (H_1 + H_2 - E) \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 = 0 \\ & \int \varphi_1^*(q_1) \varphi_2^*(q_2) (H_1 + H_2 - E) \delta \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 = 0 \\ & \int \varphi_1^*(q_2) \varphi_2^*(q_1) (H_1 + H_2 - E) \delta \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 = 0 \end{aligned}$$

$$\begin{aligned} & \int \delta \varphi_1^*(q_1) \varphi_2^*(q_2) (H_1 + H_2 + V_{12} - E) \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 - \int \delta \varphi_1^*(q_1) \varphi_2^*(q_2) V_{12} \varphi_1(q_2) \varphi_2(q_1) dq_1 dq_2 + \\ & + \int \varphi_1^*(q_1) \varphi_2^*(q_2) (H_1 + H_2 + V_{12} - E) \delta \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 - \int \varphi_1^*(q_2) \varphi_2^*(q_1) V_{12} \delta \varphi_1(q_1) \varphi_2(q_2) dq_1 dq_2 = 0 \\ & \text{và } \int dq_1 \delta \varphi_1^*(q_1) \{ \int \varphi_2^*(q_2) (H_1 + H_2 + V_{12} - E) \varphi_2(q_2) dq_2 \} \varphi_1(q_1) - \int \varphi_2^*(q_2) V_{12} \varphi_1(q_2) \varphi_2(q_1) dq_2 \} + \\ & + \int dq_1 \delta \varphi_1(q_1) \{ \int \varphi_2(q_2) (H_1 + H_2 + V_{12} - E) \varphi_2^*(q_2) dq_2 \} \varphi_1^*(q_1) - \int \varphi_2(q_2) V_{12} \varphi_1^*(q_2) \varphi_2^*(q_1) dq_2 \} = 0 \\ & \Rightarrow [H_1 + V_{ef1}(\vec{r}_1)] \phi_1(q_1) = \varepsilon_1 \phi_1(q_1) \end{aligned} \quad (5.19a)$$

trong đó $\varepsilon_1 = E - \varepsilon_{02}$, $H_2 \varphi_2(q_2) = \varepsilon_{02} \varphi_2(q_2)$

$$V_{ef1}(\vec{r}_1) = \int \varphi_2^*(q_2) V_{12}(\vec{r}_1, \vec{r}_2) \varphi_2(q_2) dq_2 - \frac{\varphi_2(q_1)}{\varphi_1(q_1)} \int \varphi_2^*(q_2) V_{12}(\vec{r}_1, \vec{r}_2) \varphi_1(q_2) dq_2 \quad (5.20a)$$

$$[H_2 + V_{ef}(\vec{r}_2)]\varphi_2(q_2) = \varepsilon_2 \varphi_2(q_2) \quad (5.19b)$$

$$\varepsilon_2 = E - \varepsilon_{01} \quad ; \quad H_1 \varphi_1(q_1) = \varepsilon_{01} \varphi_1(q_1)$$

$$V_{ef}(\vec{r}_2) = \int \varphi_1^*(q_1) V_{12}(\vec{r}_1, \vec{r}_2) \varphi_1(q_1) dq_1 - \frac{\varphi_1(q_2)}{\varphi_1(q_2)} \int \varphi_1^*(q_1) V_{12}(\vec{r}_1, \vec{r}_2) \varphi_2(q_1) dq_1 \quad (5.20b)$$

$$V_{efi}(\vec{r}_i) = \sum_j \int \varphi_{p_j}^*(q_j) V_{ij}(\vec{r}_i, \vec{r}_j) \varphi_{p_j}(q_j) dq_j - \sum_j \frac{\varphi_j(q_i)}{\varphi_j(q_i)} \int \varphi_{p_j}^*(q_j) V_{ij}(\vec{r}_i, \vec{r}_j) \varphi_{p_j}(q_j) dq_j \quad (5.20c)$$

6- Phương pháp lượng tử hoá lần thứ hai.

6.1- Ý tưởng của phương pháp

$$\psi(q_1, q_2, \dots, q_N) = c \sum_{(q)} \varphi_{p_1}(q_1) \varphi_{p_2}(q_2) \dots \varphi_{p_N}(q_N) \quad (6.1)$$

6.2- Toán tử sinh hạt, toán tử huỷ hạt và toán tử số hạt cho hệ hạt boson:

$$\hat{a}_i \psi_{\dots, N_i, \dots} = \sqrt{N_i} \psi_{\dots, N_i-1, \dots} \quad (6.2)$$

$$\hat{a}_i^+ \psi_{\dots, N_i, \dots} = \sqrt{N_i+1} \psi_{\dots, N_i+1, \dots} \quad (6.3)$$

$$\hat{a}_i^+ \hat{a}_i \psi_{\dots, N_i, \dots} = \sqrt{N_i} \hat{a}_i^+ \psi_{\dots, N_i-1, \dots} = \sqrt{N_i} \sqrt{N_i} \psi_{\dots, N_i, \dots} = N_i \psi_{\dots, N_i, \dots} \quad (6.4)$$

Ký hiệu $\hat{N}_i = \hat{a}_i^+ \hat{a}_i \quad (6.5)$

chúng ta được : $\hat{N}_i \psi_{\dots, N_i, \dots} = N_i \psi_{\dots, N_i, \dots} \quad (6.6)$

Do đó : $\hat{a}_i \hat{a}_k^+ - \hat{a}_k^+ \hat{a}_i = \delta_{ik} \quad (6.7)$

$$\hat{a}_i \hat{a}_k - \hat{a}_k \hat{a}_i = 0 \quad \text{và} \quad \hat{a}_i^+ \hat{a}_k^+ - \hat{a}_k^+ \hat{a}_i^+ = 0 \quad (6.8)$$

6.3- Toán tử sinh hạt, toán tử huỷ hạt và toán tử số hạt cho hệ hạt fermion:

$N_i = 0$ hoặc 1 :

$$\begin{aligned} \hat{a}_i \psi_{\dots, N_i=0, \dots} &= 0 \quad ; \quad \hat{a}_i \psi_{\dots, N_i=1, \dots} = \psi_{\dots, N_i=0, \dots} \\ \hat{a}_i^+ \psi_{\dots, N_i=1, \dots} &= 0 \quad ; \quad \hat{a}_i^+ \psi_{\dots, N_i=0, \dots} = \psi_{\dots, N_i=1, \dots} \\ \hat{a}_i \psi_{\dots, N_i, \dots} &= \sqrt{N_i} \psi_{\dots, N_i-1, \dots} \quad ; \quad \hat{a}_i \psi_{\dots, N_i+1, \dots} = \sqrt{1-N_i} \psi_{\dots, N_i, \dots} \end{aligned} \quad (6.9)$$

$$\hat{a}_i^+ \psi_{\dots, N_i, \dots} = \sqrt{1-N_i} \psi_{\dots, N_i+1, \dots} \quad ; \quad \hat{a}_i^+ \psi_{\dots, N_i-1, \dots} = \sqrt{N_i} \psi_{\dots, N_i, \dots} \quad (6.10)$$

$$\hat{N}_i \psi_{\dots, N_i, \dots} = \hat{a}_i^+ \hat{a}_i \psi_{\dots, N_i, \dots} = \sqrt{N_i} \hat{a}_i^+ \psi_{\dots, N_i-1, \dots} = N_i \psi_{\dots, N_i, \dots} \quad (6.11a)$$

$$\hat{N}_i = \hat{a}_i^+ \hat{a}_i \quad (6.11b)$$

$$\hat{a}_k \hat{a}_i^+ + \hat{a}_i^+ \hat{a}_k = \delta_{ik} \quad (6.12)$$

$$\hat{a}_i \hat{a}_k + \hat{a}_k \hat{a}_i = 0 \quad \text{và} \quad \hat{a}_i^+ \hat{a}_k^+ + \hat{a}_k^+ \hat{a}_i^+ = 0 \quad (6.13)$$

6.4- Hamilton trong phương pháp lượng tử hoá lần thứ hai

$$H = \sum_a H_a + \sum_{a,b} V_{a,b} + \sum_{a,b,c} V_{a,b,c} + \dots \quad (6.14)$$

$$H_a = -\frac{\hbar^2}{2m_a} \Delta_a + u(\vec{r}_a) \quad (6.15)$$

$$\langle \sum_a H_a \rangle = \sum_i \varepsilon_i N_i \quad (6.16)$$

$$\varepsilon_i = \langle i | H_a | i \rangle = \int \varphi_i^*(q_a) H_a \varphi_i(q_a) dq_a \quad (6.17)$$

$$\sum_a H_a = \sum_i \varepsilon_i \hat{N}_i = \sum_i \varepsilon_i \hat{a}_i^+ \hat{a}_i \quad (6.18)$$

$$V_{ik} = \int \varphi_i^*(q_a) V(q_a, q_b) \varphi_k(q_b) dq_a dq_b \quad (6.19)$$

$$V_{ik} \Rightarrow V_{ik} N_k \Rightarrow V_{ik} N_k N_i \Rightarrow \frac{1}{2} \sum_{i,k} V_{ik} N_i N_k$$

$$\langle \sum_{a,b} V_{a,b} \rangle = \frac{1}{2} \sum_{i,k} V_{ik} N_i N_k \Rightarrow \sum_{a,b} V_{a,b} = \frac{1}{2} \sum_{i,k} V_{ik} \hat{N}_i \hat{N}_k = \frac{1}{2} \sum_{i,k} V_{ik} \hat{a}_i^+ \hat{a}_i \hat{a}_k^+ \hat{a}_k \quad (6.21)$$

$$H = \sum_i \varepsilon_i \hat{a}_i^+ \hat{a}_i + \frac{1}{2} \sum_{i,k} V_{ik} \hat{a}_i^+ \hat{a}_i \hat{a}_k^+ \hat{a}_k + \dots \quad (6.22)$$

$$H = \sum_{i,k} (H_a)_{i,k} \hat{a}_i^+ \hat{a}_k + \frac{1}{2} \sum_{i,k,\ell,m} (V_{a,b})_{i,k,\ell,m} \hat{a}_i^+ \hat{a}_k \hat{a}_\ell^+ \hat{a}_m \quad (6.23a)$$

Trong đó: $(H_a)_{i,k} = \varepsilon_{i,k} = \int \varphi_i^*(q_a) H_a \varphi_k(q_a) dq_a \quad (6.23b)$

$$V_{i,k,\ell,m} = \iint \varphi_i^*(q_a) \varphi_k^*(q_b) V(q_a, q_b) \varphi_\ell(q_a) \varphi_m(q_b) dq_a dq_b \quad (6.23c)$$

$$\hat{\psi}(q_a) = \sum_i \varphi_i(q_a) \hat{a}_i \quad (6.24a)$$

$$\hat{\psi}^+(q_a) = \sum_i \varphi_i^*(q_a) \hat{a}_i^+ \quad (6.24b)$$

$$\hat{\psi}(q_a) \hat{\psi}^+(q_b) \mp \hat{\psi}^+(q_a) \hat{\psi}(q_b) = \delta(q_a - q_b) \delta_{ab} \quad (6.25a)$$

$$\hat{\psi}(q_a) \hat{\psi}(q_a') \mp \hat{\psi}(q_a') \hat{\psi}(q_a) = 0 \quad (6.25b)$$

$$\hat{\psi}^+(q_a) \hat{\psi}^+(q_a') \mp \hat{\psi}^+(q_a') \hat{\psi}^+(q_a) = 0 \quad (6.25c)$$

$$\hat{F}^{(1)} = \sum_a \hat{f}(q_a) \quad (6.26)$$

$$\Rightarrow \hat{F}^{(1)} = \int \hat{\psi}^+(q_a) \hat{f}(q_a) \hat{\psi}(q_a) dq_a \quad (6.27)$$

$$H = \int \hat{\psi}^+(q_a) H_a \hat{\psi}(q_a) dq_a + \frac{1}{2} \iint \hat{\psi}^+(q_a) \hat{\psi}^+(q_b) V(q_a, q_b) \hat{\psi}(q_b) \hat{\psi}(q_a) dq_a dq_b \quad (6.28)$$

Chương 3: Hamiltonian và phương trình Shrodinger cho một số hệ nhiều hạt

7- Phương trình Shrodinger cho hệ các electron và các ion trong tinh thể

7.1- Phương trình Shrodinger tổng quát cho hệ các electron và các ion

$$H \Phi(\vec{r}, \vec{R}) = E \Phi(\vec{r}, \vec{R}) \quad (7.1)$$

$$H = - \sum_i \frac{\hbar^2}{2m} \Delta_{\vec{r}_i} - \sum_j \frac{\hbar^2}{2M_j} \Delta_{\vec{R}_j} + V(\vec{r}, \vec{R}) \quad (7.4)$$

$$V(\vec{r}, \vec{R}) = V_1(\vec{r}, \vec{R}) + V_2(\vec{R}) \quad (7.5)$$

$$\begin{cases} V_1(\vec{r}, \vec{R}) = V_{e-e}(\vec{r}) + V_{e-l}(\vec{r}, \vec{R}) \\ V_2(\vec{R}) = V_{l-l}(\vec{R}) \end{cases} \quad (7.6)$$

7.2- Gần đúng đoạn nhiệt và các phương trình Shrodinger cho hệ các electron và cho hệ các ion

$$\Phi(\vec{r}, \vec{R}) = \Phi_1(\vec{r}, \vec{R}) \Phi_2(\vec{R}) \quad (7.7)$$

$$[- \sum_i \frac{\hbar^2}{2m} \Delta_{\vec{r}_i} + V_1(\vec{r}, \vec{R})] \Phi_1(\vec{r}, \vec{R}) \Phi_2(\vec{R}) - [\sum_j \frac{\hbar^2}{2M_j} \Delta_{\vec{R}_j} - V_2(\vec{R})] \Phi_1(\vec{r}, \vec{R}) \Phi_2(\vec{R}) = E \Phi_1(\vec{r}, \vec{R}) \Phi_2(\vec{R})$$

$$\frac{\partial}{\partial X_{Ja}} [\Phi_1(\vec{r}, \vec{R})] \approx 0, \frac{1}{\Phi_1(\vec{r}, \vec{R})} [- \sum_i \frac{\hbar^2}{2m} \Delta_{\vec{r}_i} + V_1(\vec{r}, \vec{R})] \Phi_1(\vec{r}, \vec{R}) + \frac{1}{\Phi_2(\vec{R})} [- \sum_j \frac{\hbar^2}{2M_j} \Delta_{\vec{R}_j} + V_2(\vec{R})] \Phi_2(\vec{R}) = E$$

$$[- \sum_i \frac{\hbar^2}{2m} \Delta_{\vec{r}_i} + V_1(\vec{r})] \Phi_1(\vec{r}) = \varepsilon \Phi_1(\vec{r}) \quad V_1(\vec{r}) = V_1(\vec{r}, \vec{R})$$

$$[- \sum_j \frac{\hbar^2}{2M_j} \Delta_{\vec{R}_j} + V_2(\vec{R}) + V_{ef}(\vec{R})] \Phi_2(\vec{R}) = W \Phi_2(\vec{R})$$

Với: $W = E - E_e \quad (7.13)$

8- Trạng thái và năng lượng của electron trong mạng tinh thể

$$\sum_i \left[-\frac{\hbar^2}{2m} \Delta_{\vec{r}_i} + V_{1ef}(\vec{r}_i) \right] \Phi_1(\vec{r}) = \varepsilon \Phi_1(\vec{r}) \quad (8.1a)$$

$$\left[-\frac{\hbar^2}{2m} \Delta_{\vec{r}_i} + V_{1ef}(\vec{r}_i) \right] \phi_{ni}(\vec{r}_i) = \varepsilon_i \phi_{ni}(\vec{r}_i) \quad (8.1b)$$

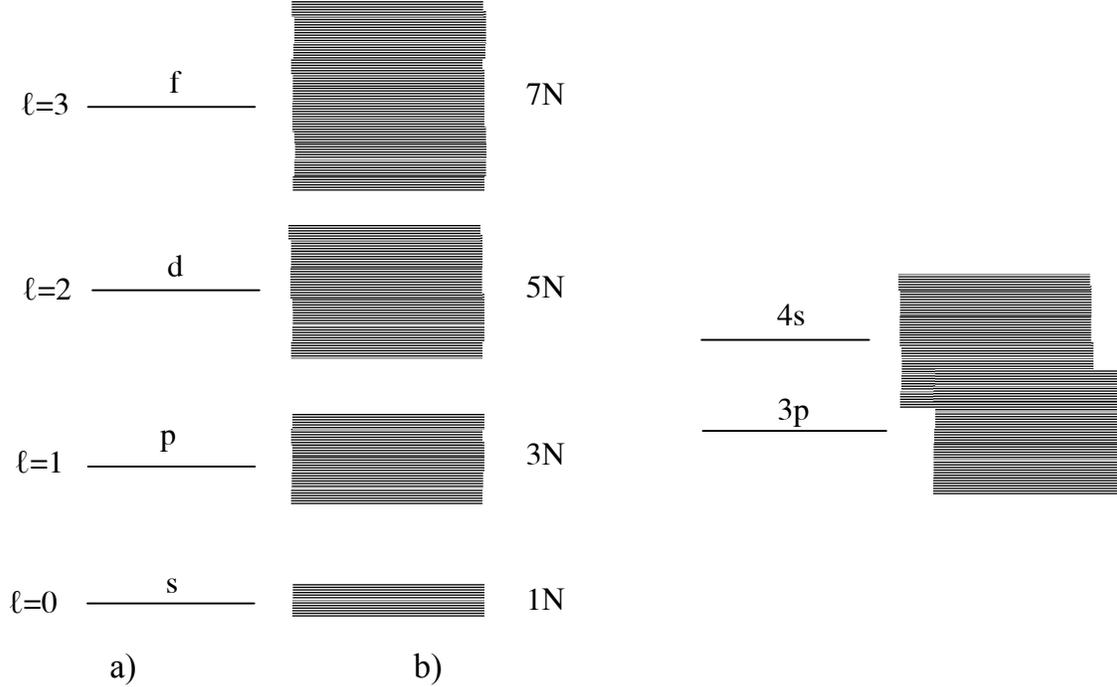
$$(\varepsilon = \sum_i \varepsilon_i) ; V_{1ef}(\vec{r}_i) = V_{ef-e}(\vec{r}_i) + V_{i-1}(\vec{r}_i, \vec{R}_1) + V_{i-J}(\vec{r}_i, \vec{R}_J) \quad (8.2)$$

$$V_{i-1}(\vec{r}_i, \vec{R}_1) = -\frac{z_1 e^2}{4\pi\epsilon_0 |\vec{r}_i - \vec{R}_1|} \quad V_{i-J}(\vec{r}_i, \vec{R}) = -\sum_{J \neq i} \frac{z_J e^2}{4\pi\epsilon_0 |\vec{r}_i - \vec{R}_J|}$$

8.1- Phương trình Shrodinger cho electron trong trường hợp liên kết mạnh

Nguyên tử cô lập

Tinh thể



Hình 8.1 : Các mức năng lượng của electron

a) trong nguyên tử cô lập

b) trong tinh thể

Hình 8.2 : Hiện tượng chồng miền

8.2- Phương trình Shrodinger cho electron trong trường hợp liên kết yếu

$$V_{ef-e}(\vec{r}_i) = \sum_j \int \phi_{nj}^*(\vec{r}_j) V_{ij}(\vec{r}_i, \vec{r}_j) \phi_{nj}(\vec{r}_j) d\vec{r}_j - \delta_{\alpha\beta} \sum_j \frac{\phi_{nj}(\vec{r}_i)}{\phi_{ni}(\vec{r}_i)} \int \phi_{nj}^*(\vec{r}_j) V_{ij}(\vec{r}_i, \vec{r}_j) \phi_{ni}(\vec{r}_j) d\vec{r}_j \quad (8.5)$$

$$\phi_{\vec{k}}(\vec{r}) = v_{\vec{k}}(\vec{r}) \exp(i\vec{k}\vec{r}) \quad (8.6) \quad ; \quad v_{\vec{k}}(\vec{r} + \vec{a}) = v_{\vec{k}}(\vec{r}) \quad (8.7)$$

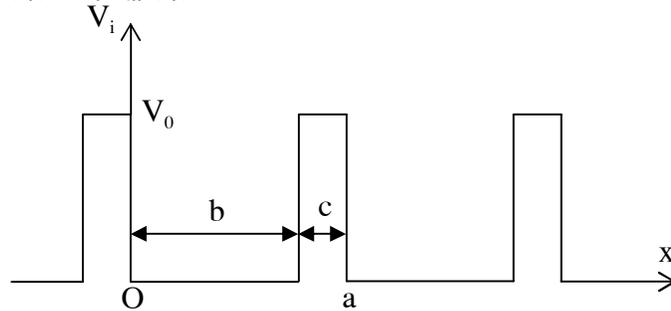
$$\left[-\frac{\hbar^2}{2m} \Delta_{\vec{r}_i} + V_{ef-e}(\vec{r}_i) \right] \phi_{ni}(\vec{r}_i) = \varepsilon_i \phi_{ni}(\vec{r}_i) \quad (8.1b)$$

Mô hình Kronig-Penney :

$$V_{ef-e}(X_J + a) = V_i(X_J) \quad (8.8)$$

$$V_{ef-e}(X_J) = \lambda \sum_n \delta(X_J - na)$$

$$\lambda = \lim_{\substack{c \rightarrow 0 \\ V_0 \rightarrow \infty}} (cV_0) = const$$



Hình 8.3

Sơ đồ thế năng của mô hình Kronig-Penney

9- Dao động mạng tinh thể

9.1- Phương trình Shrodinger cho các dao động mạng tinh thể trong biểu diễn tọa độ

$$V_J(\vec{R}) = V_2(\vec{R}) + V_{ef}(\vec{R}) \quad (9.1)$$

$\vec{u}_n = (u_{n,x}, u_{n,y}, u_{n,z})$ = độ lệch của nguyên tử khối vị trí cân bằng ở nút mạng thứ n

$$V_J(\vec{R}) = V_J(\vec{u}) = V_J(0) + \sum_{n,\alpha} (\partial V_J / \partial u_{n,\alpha})_0 u_{n,\alpha} + \\ + (1/2) \sum_{n,n',\alpha,\beta} (\partial^2 V_J / \partial u_{n,\alpha} \partial u_{n',\beta})_0 u_{n,\alpha} u_{n',\beta} + \dots + (1/6) \sum_{n,n',n'',\alpha,\beta,\gamma} (\partial^3 V_J / \partial u_{n,\alpha} \partial u_{n',\beta} \partial u_{n'',\gamma})_0 u_{n,\alpha} u_{n',\beta} u_{n'',\gamma} + \dots \\ (\partial V_J / \partial u_{n,\alpha})_0 = 0$$

9.2- Phương trình Shrodinger cho các phonon trong biểu diễn lượng tử hoá lần thứ hai

$$V_J(\vec{R}) = \sum_{n,n'} A_{n,n'} x_n x_{n'} \quad (9.4)$$

$$V_J(\vec{R}) = \sum_n A_n x_n^2 \quad (9.5)$$

$$H_{ph} = \sum_n \left[\hat{p}_n^2 / (2M_n) + M_n \omega_n^2 \hat{x}_n^2 / 2 \right] = \sum_n H_n \quad (9.6)$$

trong đó

$$H_n = \hat{p}_n^2 / (2M_n) + M_n \omega_n^2 \hat{x}_n^2 / 2 \quad (9.7)$$

$$\hat{A}^+ = \sqrt{M\omega/2} \hat{x} - i(1/\sqrt{2M\omega}) \hat{p} \quad (9.8a)$$

$$\hat{A} = \sqrt{M\omega/2} \hat{x} + i(1/\sqrt{2M\omega}) \hat{p} \quad (9.8b)$$

$$\hat{A} \hat{A}^+ - \hat{A}^+ \hat{A} = \hbar \quad (9.9)$$

$$H = H_n = (\hbar\omega/2) + \omega \hat{A}^+ \hat{A} \quad (9.10)$$

$$H \phi_E = E \phi_E \quad (9.11)$$

$$\Rightarrow H \hat{A} \phi_E = (E - \hbar\omega) \hat{A} \phi_E \quad ; \quad H \hat{A}^+ \phi_E = (E + \hbar\omega) \hat{A}^+ \phi_E \quad (9.13)$$

$$\hat{A} \phi_0 = 0 \quad \Rightarrow \quad E_0 = \hbar\omega/2 \quad (9.16)$$

$$\text{Từ (9.13)} \quad \Rightarrow \quad E_n = \left(\frac{1}{2} + n\right) \hbar\omega \quad ; \quad n = 0, 1, 2, 3, \dots \quad (9.21)$$

$$C_n \phi_n = (\hat{A}^+)^n \phi_0 \quad (9.22)$$

$$\Rightarrow \quad |C_n|^2 = \langle (\hat{A}^+)^n \phi_0 | (\hat{A}^+)^n \phi_0 \rangle = \langle \phi_0 | \hat{A}^n (\hat{A}^+)^n | \phi_0 \rangle \quad (9.23)$$

$$\hat{A}^n (\hat{A}^+)^n = \hat{A}^{n-1} n \hbar (\hat{A}^+)^{n-1} + \hat{A}^{n-1} (\hat{A}^+)^n \hat{A}$$

$$|C_n|^2 = n \hbar \langle \phi_0 | \hat{A}^{n-1} (\hat{A}^+)^{n-1} | \phi_0 \rangle = n \hbar |C_{n-1}|^2 = n! \hbar^n |C_0|^2$$

$$C_0 = 1 \quad ; \quad \text{do đó } |C_n|^2 = n! \hbar^n \quad \text{và} \quad |C_n| = \sqrt{n! \hbar^n}$$

$$\text{Cuối cùng :} \quad \phi_n = \frac{1}{\sqrt{n!}} \left(\frac{\hat{A}^+}{\sqrt{\hbar}} \right)^n \phi_0 \quad (9.24)$$

$$\hat{A} \phi_0 = 0 \quad \Rightarrow \quad [\sqrt{M\omega/2} x + \hbar(1/\sqrt{2M\omega})(\partial/\partial x)] \phi_0(x) = 0 \quad (9.25)$$

$$\phi_0(x) = C \cdot \exp[-m\omega x^2 / (2\hbar)] \quad (9.26)$$

$$\int_{-\infty}^{\infty} |\phi_0(x)|^2 dx = 1 \quad \Rightarrow \quad C = \left(\frac{m\omega}{\pi \hbar} \right)^{1/4}, \quad \text{do đó} \quad \phi_0(x) = \left(\frac{m\omega}{\pi \hbar} \right)^{1/4} \exp[-m\omega x^2 / (2\hbar)] \quad (9.27)$$

10- Hamiltonian cho hệ các spin

10.1- Trường hợp hệ các electron linh động

$$M = -g \mu_B \frac{N_\uparrow - N_\downarrow}{V} \quad (10.1)$$

$$N_\uparrow + N_\downarrow = N \quad (10.2)$$

$$H = H_1 + H_2 \quad (10.3a)$$

$$H_1 = -\sum_{i=1}^N \frac{\hbar^2}{2m} \Delta_{\vec{r}_i} \quad H_2 = \sum_i V_{1ef}(\vec{r}_i) \quad (10.3b)$$

$$H_2 = -\delta_{\alpha\beta} \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^N \frac{\phi_{ij}(\vec{r}_i)}{\phi_{ni}(\vec{r}_i)} \int \phi_{nj}^*(\vec{r}_j) V_{ij}(\vec{r}_i, \vec{r}_j) \phi_{ni}(\vec{r}_j) d\vec{r}_j \quad (10.3d)$$

$$E = \langle \psi | H | \psi \rangle \quad (10.4)$$

$$\psi = \psi(q_1, q_2, \dots, q_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \varphi_{p_1}(q_1) & \varphi_{p_1}(q_2) & \dots & \varphi_{p_1}(q_N) \\ \varphi_{p_2}(q_1) & \varphi_{p_2}(q_2) & \dots & \varphi_{p_2}(q_N) \\ \dots & \dots & \dots & \dots \\ \varphi_{p_N}(q_1) & \varphi_{p_N}(q_2) & \dots & \varphi_{p_N}(q_N) \end{vmatrix} \quad (10.5)$$

$$\varphi_{p_j}(q_j) = \phi_{\vec{k}_j}(\vec{r}_j) \cdot \chi_{\alpha}(s_j) = \frac{1}{\sqrt{V}} \exp(i\vec{k}_j \vec{r}_j) \cdot \chi_{\alpha}(s_j) \quad (10.6)$$

$$E_d = \langle \psi | H_1 | \psi \rangle = \int \psi^*(q_1, q_2, \dots, q_N) \sum_{i=1}^N \left(-\frac{\hbar^2}{2m} \Delta_{\vec{r}_i}\right) \psi(q_1, q_2, \dots, q_N) dq_1, dq_2, \dots, dq_N \quad (10.7)$$

$$\int \dots dq_i = \int d\vec{r}_i \sum_{s_i} \dots \quad (10.8a)$$

$$d\vec{r}_i = dx_i dy_i dz_i \quad (10.8b)$$

$$E_d = \langle \psi | H_1 | \psi \rangle = \sum_{j=1}^N \frac{\hbar^2 k_j^2}{2m} = 2 \sum_{k < k_F} \frac{\hbar^2 k^2}{2m} \quad (10.9)$$

$$\sum_{\vec{k}} \dots = \frac{V}{8\pi^3} \int d\vec{k} \dots \quad d\vec{k} = dk_x dk_y dk_z$$

$$E_d = \frac{V\hbar^2}{8m\pi^3} \int_{k < k_F} k^2 d\vec{k} = \frac{V\hbar^2}{2m\pi^2} \int_0^{k_F} k^4 dk = \frac{V\hbar^2 k_F^5}{10m\pi^2} \quad (10.10)$$

$$N = 2 \sum_{k < k_F} 1 = \frac{V}{4\pi^3} \int d\vec{k} = \frac{V}{4\pi^3} 4\pi \int_0^{k_F} k^2 dk = \frac{V k_F^3}{3\pi^2} \quad (10.11)$$

$$E_d = \frac{V\hbar^2 k_F^5}{10m\pi^2} = \frac{3\hbar^2 N k_F^2}{10m} = \frac{3}{5} N \varepsilon_F \quad (\text{Trong đó: } \varepsilon_F = \frac{\hbar^2 k_F^2}{2m})$$

$$N_{\uparrow} = \sum_{k < k_{\uparrow F}} 1 = \frac{V}{8\pi^3} \int d\vec{k} = \frac{V}{8\pi^3} 4\pi \int_0^{k_{\uparrow F}} k^2 dk = \frac{V k_{\uparrow F}^3}{6\pi^2} \quad (10.14a)$$

$$N_{\downarrow} = \sum_{k < k_{\downarrow F}} 1 = \frac{V k_{\downarrow F}^3}{6\pi^2} \quad (10.14b)$$

$$N = N_{\uparrow} + N_{\downarrow} = \frac{V k_{\uparrow F}^3}{6\pi^2} + \frac{V k_{\downarrow F}^3}{6\pi^2} = \frac{V k_F^3}{3\pi^2} \quad \frac{k_{\uparrow F}^3}{2} + \frac{k_{\downarrow F}^3}{2} = k_F^3$$

$$E_i = \langle \psi | H_2 | \psi \rangle \quad (10.16)$$

$$H_2 = -\frac{1}{2V} \sum_{\substack{i,j=1 \\ i \neq j}}^N \int \exp[-i(\vec{k}_j - \vec{k}_i)(\vec{r}_j - \vec{r}_i)] V_{ij}(\vec{r}_i, \vec{r}_j) d\vec{r}_j \quad (10.17)$$

$$V_{i,j}(\vec{r}_i, \vec{r}_j) = \frac{e^2}{4\pi\epsilon_0 |\vec{r}_i - \vec{r}_j|} \quad (10.18)$$

$$H_2 = -\frac{1}{2V} \sum_{\substack{i,j=1 \\ i \neq j}}^N \int \exp[-i(\vec{k}_j - \vec{k}_i)\vec{r}] V_{ij}(\vec{r}) d\vec{r} = -\frac{1}{2V} \sum_{\substack{i,j=1 \\ i \neq j}} V(\vec{k}_j - \vec{k}_i) \quad (10.19)$$

$$V(\vec{k}_j - \vec{k}_i) = \frac{e^2}{\epsilon_0 |\vec{k}_j - \vec{k}_i|^2} \quad (10.20)$$

$$E_t = H_2 = -\frac{e^2}{V\epsilon_0} \sum_{k,k' < k_F} \frac{1}{|\vec{k} - \vec{k}'|^2} \quad (10.21)$$

$$E_t = -\frac{e^2}{V\epsilon_0} \left(\frac{V}{8\pi^3} \right)^2 \int_{k < k_F} \int_{k' < k_F} \frac{d\vec{k}' d\vec{k}}{|\vec{k} - \vec{k}'|^2} = -\frac{e^2}{4\pi^2 \epsilon_0} \left(\frac{V}{8\pi^3} \right) \int_{k < k_F} k_F F\left(\frac{k}{k_F}\right) d\vec{k} \quad (10.22)$$

$$F(x) = 1 + \frac{1-x^2}{2x} \ln \left| \frac{1+x}{1-x} \right| \quad (10.23)$$

$$E_t = -N \frac{3}{2\pi} k_F a_0 R_y \quad (10.24)$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} \quad (10.25) \quad ; \quad R_y = \frac{me^4}{2\hbar^2 (4\pi\epsilon_0)^2} \quad (10.26)$$

$$E_t = -N_{\uparrow} \frac{3}{2\pi} k_{\uparrow F} a_0 R_y - N_{\downarrow} \frac{3}{2\pi} k_{\downarrow F} a_0 R_y \quad (10.27)$$

$$E_d = N_{\uparrow} \frac{3}{5} (k_{\uparrow F} a_0)^2 R_y + N_{\downarrow} \frac{3}{5} (k_{\downarrow F} a_0)^2 R_y \quad (10.28)$$

$$E = E_d + E_t = N_{\uparrow} \left[\frac{3}{5} (k_{\uparrow F} a_0)^2 - \frac{3}{2\pi} (k_{\uparrow F} a_0) \right] R_y + N_{\downarrow} \left[\frac{3}{5} (k_{\downarrow F} a_0)^2 - \frac{3}{2\pi} (k_{\downarrow F} a_0) \right] R_y \quad (10.29)$$

$$N_{\uparrow} = N_{\downarrow} = N/2, \quad k_{\uparrow F} = k_{\downarrow F} = k_F : E_N = E_{dN} + E_{tN} = N \left[\frac{3}{5} (k_F a_0)^2 - \frac{3}{2\pi} (k_F a_0) \right] R_y \quad (10.30)$$

$$N_{\uparrow} = N \text{ và } N_{\downarrow} = 0, \quad k_{\uparrow F} = 2^{1/3} k_F; k_{\downarrow F} = 0.$$

$$E_M = E_{dM} + E_{tM} = N \left[\frac{3}{5} \cdot 2^{2/3} (k_F a_0)^2 - \frac{3}{2\pi} \cdot 2^{1/3} (k_F a_0) \right] R_y \quad (10.31)$$

$$E_M < E_N \iff k_F a_0 < \frac{5}{2\pi} \frac{1}{2^{1/3} + 1} = 0,352125 \quad (10.32) \iff |E_t| > E_d \quad (10.33)$$

Ý nghĩa của điều kiện (10.32)

$$H = \sum_{\sigma, k < k_F} \frac{\hbar k^2}{2m} a_{\vec{k},\sigma}^+ a_{\vec{k},\sigma} - \frac{e^2}{2V\epsilon_0} \sum_{\sigma, k, k' < k_F} \frac{1}{|\vec{k} - \vec{k}'|^2} a_{\vec{k},\sigma}^+ a_{\vec{k},\sigma} a_{\vec{k}',\sigma}^+ a_{\vec{k}',\sigma} \quad (10.34)$$

10.2- Mô hình Heisenberg

$$H \phi(\vec{r}_1, \vec{r}_2) = E \phi(\vec{r}_1, \vec{r}_2) \quad (10.35)$$

$$H = H_1 + H_2 + V(\vec{r}_1, \vec{r}_2) \quad (10.36a)$$

$$H_i = -\frac{\hbar^2}{2m} \nabla_i^2 - \frac{e^2}{4\pi\epsilon_0} \frac{1}{|\vec{r}_i - \vec{R}_1|} - \frac{e^2}{4\pi\epsilon_0} \frac{1}{|\vec{r}_i - \vec{R}_2|}, \quad i = 1, 2 \quad (10.36b)$$

$$V(\vec{r}_1, \vec{r}_2) = \frac{e^2}{4\pi\epsilon_0} \frac{1}{|\vec{r}_1 - \vec{r}_2|} \quad (10.36c)$$

$$\psi(\vec{r}_1, \vec{r}_2, \vec{s}_1, \vec{s}_2) = \phi(\vec{r}_1, \vec{r}_2) \chi(\vec{s}_1, \vec{s}_2) \quad (10.37)$$

$$a = |s_1, \uparrow\rangle \otimes |s_2, \uparrow\rangle; b = |s_1, \uparrow\rangle \otimes |s_2, \downarrow\rangle; c = |s_1, \downarrow\rangle \otimes |s_2, \uparrow\rangle; d = |s_1, \downarrow\rangle \otimes |s_2, \downarrow\rangle \quad (10.38)$$

$$\vec{S} = \vec{s}_1 + \vec{s}_2, \quad s_1 = s_2 = (1/2)\hbar \quad (10.39)$$

$$\chi(S, S_z) = \sum_{s_{1z}, s_{2z}, S_z} C_{s_{1z}, s_{2z}, S_z}^{s_1, s_2, S} |s_1, s_{1z}\rangle \otimes |s_2, s_{2z}\rangle \quad (10.40a)$$

(Với $C_{s_{1z}, s_{2z}, S}^{s_1, s_2, S}$ là các hệ số Clebsch-Gordan, và $s_{1z}, s_{2z} = \pm(1/2)\hbar$).

$$\chi(0,0) = (1/2)[|(1/2)\hbar, \uparrow\rangle \otimes |(1/2)\hbar, \downarrow\rangle - |(1/2)\hbar, \downarrow\rangle \otimes |(1/2)\hbar, \uparrow\rangle] \quad (10.40b)$$

$$\chi(1,0) = (1/2)[|(1/2)\hbar, \uparrow\rangle \otimes |(1/2)\hbar, \downarrow\rangle + |(1/2)\hbar, \downarrow\rangle \otimes |(1/2)\hbar, \uparrow\rangle] \quad (10.40c)$$

$$\chi(1,1) = |(1/2)\hbar, \uparrow\rangle \otimes |(1/2)\hbar, \uparrow\rangle \quad (10.40d)$$

$$\chi(1,-1) = |(1/2)\hbar, \downarrow\rangle \otimes |(1/2)\hbar, \downarrow\rangle \quad (10.40e)$$

$$E_s = \langle \phi_s(\vec{r}_1, \vec{r}_2) | H | \phi_s(\vec{r}_1, \vec{r}_2) \rangle \quad (10.41a)$$

$$E_t = \langle \phi_a(\vec{r}_1, \vec{r}_2) | H | \phi_a(\vec{r}_1, \vec{r}_2) \rangle \quad (10.41b)$$

$$\vec{S}^2 = (\vec{s}_1 + \vec{s}_2)^2 = \vec{s}_1^2 + \vec{s}_2^2 + 2\vec{s}_1 \vec{s}_2 \quad (10.42)$$

$$H_{e-spin} = [(E_s + 3E_t)/4] - J_{12} \vec{s}_1 \vec{s}_2 \quad (10.43)$$

$$J_{12} = E_s - E_t \quad (10.44)$$

$$H_{e-spin} = -J_{12} \vec{s}_1 \vec{s}_2 \quad (10.45)$$

$$H_{spin} = -\sum_{i \neq j} J_{ij} \vec{s}_i \vec{s}_j \quad (10.46)$$

10.3- Mô hình Hubbard

$$H = H_h + H_p \quad (10.47)$$

$$H_h = \sum_{x,y,\sigma} t_{xy} \hat{a}_{y,\sigma}^+ \hat{a}_{x,\sigma} \quad (10.48)$$

$$H_p = \sum_x U_x \hat{N}_{x,\uparrow} \hat{N}_{x,\downarrow} \quad (10.49)$$

$$\hat{N}_{x,\sigma} = \hat{a}_{x,\sigma}^+ \hat{a}_{x,\sigma}$$

$$\hat{a}_x \Rightarrow \hat{c}_i \quad \hat{c}_{i,\sigma} = \sum_x \varphi_{i,\sigma}(x) \hat{a}_{x,\sigma} \quad \hat{a}_{x,\sigma} = \sum_i \varphi_{i,\sigma}^*(x) \hat{c}_{i,\sigma}$$

$$H_h = \sum_{x,y,\sigma} t_{xy} \sum_{i,j} \varphi_{i,\sigma}(y) \hat{c}_{i,\sigma}^+ \varphi_{j,\sigma}^*(x) \hat{c}_{j,\sigma} = \sum_{i,j,\sigma} \varepsilon_{ij} \hat{c}_{i,\sigma}^+ \hat{c}_{j,\sigma}$$

$$\varepsilon_{ij} = \sum_{x,y} t_{xy} \varphi_{i,\sigma}(y) \varphi_{j,\sigma}^*(x) \quad \varphi_{i,\sigma}(x) \equiv \varphi_{k,\sigma}(x) \sim \exp(ikx)$$

$$\varepsilon_{ij} = \sum_{x,y} t_{xy} \varphi_{i,\sigma}(y) \varphi_{j,\sigma}^*(x) \sim \sum_x \exp[ik(x+1)] \cdot \exp(-ik'x) \sim \sum_x \exp[i(k-k')x] \sim \delta(k-k') = \delta_{ij}$$

$$H_h = \sum_{i,\sigma} \varepsilon_i \hat{c}_{i,\sigma}^+ \hat{c}_{i,\sigma} \quad \varepsilon_i = \varepsilon_{ii} \quad (10.52)$$

$$H_p = \sum_{i,j,\ell,m} \lambda_{i\ell,mj} \hat{c}_{i\uparrow}^+ \hat{c}_{\ell\downarrow}^+ \hat{c}_{m\downarrow} \hat{c}_{j\uparrow} \quad (10.53)$$

$$\lambda_{i\ell,mj} = \sum_x U_x \varphi_{j\uparrow}^*(x) \varphi_{m\downarrow}^*(x) \varphi_{\ell\downarrow}(x) \varphi_{i\uparrow}(x) \quad (10.54)$$

$$H = \sum_{i,\sigma} \varepsilon_i \hat{c}_{i,\sigma}^+ \hat{c}_{i,\sigma} + \sum_{i,j,\ell,m} \lambda_{i\ell,mj} \hat{c}_{i\uparrow}^+ \hat{c}_{\ell\downarrow}^+ \hat{c}_{m\downarrow} \hat{c}_{j\uparrow} \quad (10.47b)$$

11- Phương trình Shrodinger cho cặp Cooper

11.1- Trạng thái liên kết hai electron trong lý thuyết BCS

Cặp Cooper

$$\vec{S}_1 + \vec{S}_2 = 0 \quad \vec{p}_1 + \vec{p}_2 = 0$$

$$H\psi = E\psi$$

$$H = H_0 + V$$

$$\vec{p} = \hbar \vec{k} \quad H_0 \varphi_{\vec{k}} = \varepsilon_{\vec{k}} \varphi_{\vec{k}}$$

$$(H_0 + V - E) \sum_{\vec{k}' > k_F} a_{\vec{k}', \varphi_{\vec{k}'}} = 0 \quad \psi = \sum_{\vec{k}} a_{\vec{k}} \varphi_{\vec{k}}$$

$$(\varepsilon_{\vec{k}} - E) a_{\vec{k}} + \sum_{\vec{k}' > k_F} a_{\vec{k}'} V_{\vec{k}\vec{k}'} = 0$$

$$V_{\vec{k}\vec{k}'} = \int \varphi_{\vec{k}}^* V \varphi_{\vec{k}'} d\vec{r}$$

$$V_{\vec{k}\vec{k}'} = \begin{cases} -V_0 < 0 & \text{ khi } \varepsilon_F < \varepsilon_{\vec{k}} ; \varepsilon_{\vec{k}'} < \varepsilon_F + \hbar\omega_D \\ 0 & \text{ khi } \varepsilon_{\vec{k}'} > \varepsilon_F + \hbar\omega_D \end{cases}$$

$$(\varepsilon_{\vec{k}} - E) a_{\vec{k}} - \frac{1}{V_0} \sum_{\vec{k}' > k_F} \frac{1}{(\varepsilon_{\vec{k}'} - E)} a_{\vec{k}'} = 0$$

$$(11.11) \quad -\frac{1}{V_0} = \sum_{\vec{k}' > k_F} \frac{1}{(E - \varepsilon_{\vec{k}'})}$$

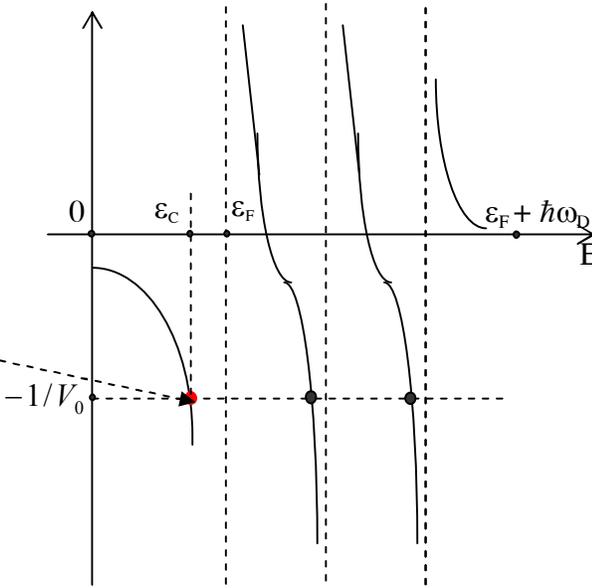
$\varepsilon_{\vec{k}} > \varepsilon_F$, $E > 0$
 Trạng thái liên kết

$$\varepsilon_{lk} = \varepsilon_F - \varepsilon_C$$

$$E = \varepsilon_C < \varepsilon_F$$

(11.12)

$$\frac{1}{V_0} = \sum_{\vec{k}' > k_F} \frac{1}{(\varepsilon_{\vec{k}'} - E_C)}$$



$$g(\varepsilon) \approx g(\varepsilon_F) \quad \frac{1}{V_0} = \int_{\varepsilon_F}^{\varepsilon_F + \hbar\omega_D} \frac{g(\varepsilon) d\varepsilon}{(\varepsilon - E_C)} \quad \frac{1}{V_0} = g(\varepsilon_F) \int_{\varepsilon_F}^{\varepsilon_F + \hbar\omega_D} \frac{d\varepsilon}{(\varepsilon - E_C)} = g(\varepsilon_F) \ln \frac{\varepsilon_F - E_C + \hbar\omega_D}{\varepsilon_F - E_C}$$

$$\frac{1}{g(\varepsilon_F) V_0} = \ln \frac{\varepsilon_{lk} + \hbar\omega_D}{\varepsilon_{lk}} \quad \lambda = g(\varepsilon_F) V_0 \quad \varepsilon_{lk} = \varepsilon_F - \varepsilon_C \quad \varepsilon_{lk} = \hbar\omega_D \exp(-1/\lambda)$$

$$kT_C \approx \varepsilon_{lk} \quad T_C \approx (\hbar\omega_D / k) \exp(-1/\lambda)$$

11.2- Toán tử hai hạt và trạng thái chân không của hệ siêu dẫn

$$\hat{c}_{\vec{k}}^+ = \hat{a}_{\vec{k}}^+ \hat{a}_{-\vec{k}}^+ ; \quad \hat{c}_{\vec{k}} = \hat{a}_{-\vec{k}} \hat{a}_{\vec{k}}$$

$$H = \sum_{\vec{k}} (\varepsilon_{\vec{k}} / 2) (\hat{a}_{\vec{k}}^+ \hat{a}_{\vec{k}} + \hat{a}_{-\vec{k}}^+ \hat{a}_{-\vec{k}}) + (1/2) \sum_{\vec{k}, \vec{k}'} V_{\vec{k}\vec{k}'} \hat{a}_{\vec{k}}^+ \hat{a}_{-\vec{k}}^+ \hat{a}_{-\vec{k}'} \hat{a}_{\vec{k}'}$$

$$\hat{n}_{\vec{k}} \varphi_{n_{\vec{k}}=0} = 0 ; \quad \hat{n}_{\vec{k}} \varphi_{n_{\vec{k}}=1} = \varphi_{n_{\vec{k}}=1} \quad \hat{n}_{-\vec{k}\downarrow} = \hat{n}_{\vec{k}\uparrow} = \hat{n}_{\vec{k}\uparrow} \hat{n}_{-\vec{k}\downarrow}$$

$$\hat{c}_{\vec{k}}^+ \hat{c}_{\vec{k}} = \hat{a}_{\vec{k}}^+ \hat{a}_{-\vec{k}}^+ \hat{a}_{-\vec{k}} \hat{a}_{\vec{k}} = \hat{a}_{\vec{k}}^+ \hat{a}_{-\vec{k}} \hat{a}_{-\vec{k}} \hat{a}_{\vec{k}} = \hat{n}_{\vec{k}} \hat{n}_{-\vec{k}} = \hat{n}_{\vec{k}} = \hat{n}_{-\vec{k}}$$

$$H = \sum_{\vec{k}} \varepsilon_{\vec{k}} \hat{c}_{\vec{k}}^+ \hat{c}_{\vec{k}} + (1/2) \sum_{\vec{k}, \vec{k}'} V_{\vec{k}\vec{k}'} \hat{c}_{\vec{k}}^+ \hat{c}_{\vec{k}'}$$

$$\Phi_{\vec{k}}(0) = u_{\vec{k}} \varphi_{\vec{k}}(0) + v_{\vec{k}} \hat{c}_{\vec{k}}^+ \varphi_{\vec{k}}(0) = (u_{\vec{k}} + v_{\vec{k}} \hat{c}_{\vec{k}}^+) \varphi_{\vec{k}}(0)$$

$$u_{\vec{k}}^2 + v_{\vec{k}}^2 = 1$$

$$\Phi(0) = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} \hat{c}_{\vec{k}}^+) \varphi_{\vec{k}}(0)$$

và : $\hat{c}_{\vec{k}} \Phi(0) = 0$

$$\begin{aligned} \hat{c}_{\bar{k}} \Phi(0) &= \hat{c}_{\bar{k}} \prod_{\bar{k}'} [u_{\bar{k}'} + v_{\bar{k}'} \hat{c}_{\bar{k}'}^+] \varphi_{n_{\bar{k}}=0} = \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} \prod_{\bar{k}'} [u_{\bar{k}'} + v_{\bar{k}'} \hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+] \varphi_{n_{\bar{k}}=0} \\ &= \prod_{\substack{\bar{k}' \neq \bar{k} \\ \bar{k}' \neq -\bar{k}}} [u_{\bar{k}'} + v_{\bar{k}'} \hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+] \varphi_{n_{\bar{k}}=0} \cdot \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} [u_{\bar{k}} + v_{\bar{k}} \hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+] \varphi_{n_{\bar{k}}=0} [u_{-\bar{k}} + v_{-\bar{k}} \hat{a}_{-\bar{k}\downarrow}^+ \hat{a}_{\bar{k}\uparrow}^+] \varphi_{n_{\bar{k}}=0} \\ &\quad \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} u_{\bar{k}} \varphi_{n_{\bar{k}}=0} v_{-\bar{k}} \hat{a}_{-\bar{k}\downarrow}^+ \hat{a}_{\bar{k}\uparrow}^+ \varphi_{n_{\bar{k}}=0} = -u_{\bar{k}} v_{-\bar{k}} \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} \varphi_{n_{\bar{k}}=0} [\hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+] \varphi_{n_{\bar{k}}=0} \\ &= -u_{\bar{k}} v_{-\bar{k}} \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} [\hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+] \varphi_{n_{\bar{k}}=0} \varphi_{n_{\bar{k}}=0} = -u_{\bar{k}} v_{-\bar{k}} \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} \hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+ \varphi_{n_{\bar{k}}=0} \varphi_{n_{\bar{k}}=0} \\ &\quad u_{-\bar{k}} v_{\bar{k}} \hat{a}_{-\bar{k}\downarrow} \hat{a}_{\bar{k}\uparrow} \hat{a}_{\bar{k}\uparrow}^+ \hat{a}_{-\bar{k}\downarrow}^+ \varphi_{n_{\bar{k}}=0} \varphi_{n_{\bar{k}}=0} \\ \hat{a}_{-\bar{k}} \hat{a}_{\bar{k}} v_{\bar{k}} \hat{a}_{\bar{k}}^+ \hat{a}_{-\bar{k}}^+ \varphi_{\bar{k}}(0) v_{-\bar{k}} \hat{a}_{-\bar{k}}^+ \hat{a}_{\bar{k}}^+ \varphi_{-\bar{k}}(0) &\sim v_{\bar{k}} \hat{a}_{\bar{k}} \hat{a}_{\bar{k}}^+ \varphi_{\bar{k}}(0) \hat{a}_{\bar{k}} v_{-\bar{k}} \hat{a}_{-\bar{k}} \hat{a}_{-\bar{k}}^+ \varphi_{-\bar{k}}(0) = 0 \end{aligned}$$

Chương 4: Phương pháp hàm Green lượng tử

Ý tưởng của phương pháp

12- Phương pháp hàm Green lượng tử ở nhiệt độ T=0K

12.1- Định nghĩa hàm Green lượng tử ở nhiệt độ T=0K

$$G_{\alpha\beta}(x, x') = -i \langle \hat{T} [\hat{\psi}_{H\alpha}(x) \hat{\psi}_{H\beta}^+(x')] \rangle \quad (12.1a)$$

$$G_{\alpha\beta}(x, x') = -i \frac{\langle \hat{T} [\hat{\psi}_{i\alpha}(x) \hat{\psi}_{i\beta}^+(x') \hat{S}(\infty)] \rangle}{\langle \hat{S}(\infty) \rangle} \quad (12.1b)$$

$$i \int_{\substack{t' \rightarrow t+0 \\ \vec{r}' \rightarrow \vec{r}}} \lim G_{\alpha\alpha}(x, x') d\vec{r} = \pm \sum_i \langle \hat{a}_i^+ \hat{a}_i \rangle = \pm N$$

$$\text{Vì: } N = \int n(\vec{r}) d\vec{r} \Rightarrow n(x) = \pm i \lim_{\substack{t' \rightarrow t+0 \\ \vec{r}' \rightarrow \vec{r}}} G_{\alpha\alpha}(x, x') \quad (12.2)$$

$$\overline{F_H^{(1)}(t)} = \pm i \sum_{\alpha, \beta} \int [\lim_{\substack{t' \rightarrow t+0 \\ \vec{r}' \rightarrow \vec{r}}} f_{\alpha\beta}(x) G_{\alpha\beta}(x, x')] d\vec{r} \quad (12.3)$$

12.2- Hàm Green cho hệ hạt fermion

$$iG(x, x') = \langle T [e^{iHt/\hbar} \hat{\psi}_S(q) e^{-iHt'/\hbar} e^{iHt'/\hbar} \hat{\psi}_S^+(q') e^{-iHt'/\hbar}] \rangle =$$

$$= \langle T [e^{iE_0(t-t')/\hbar} \sum_i \varphi_i(\vec{r}) \sum_k \varphi_k^*(\vec{r}') \hat{a}_i e^{-iH(t-t')/\hbar} \hat{a}_k^+] \rangle$$

$$iG(x, x') = \langle T [e^{iE_0(t-t')/\hbar} \sum_i \varphi_i(\vec{r}) \varphi_i^*(\vec{r}') \hat{a}_i e^{-iH(t-t')/\hbar} \hat{a}_i^+] \rangle$$

$$G(x - x') = \int \frac{d^4 p}{(2\pi)^4} G(\vec{p}, \omega) e^{i[\vec{p}(\vec{r}-\vec{r}') - \omega t]} \quad ; \quad d^4 p = d\vec{p} d\omega \quad (12.4)$$

$$\hat{\psi}_H(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} e^{i\vec{p}\vec{r}/\hbar} e^{iH_0 t/\hbar} \hat{a}_{\vec{p}} e^{-iH_0 t/\hbar}$$

$$\hat{a}_{\vec{p}} e^{-i \sum_{\vec{p}''} \varepsilon_0(\vec{p}'') \hat{a}_{\vec{p}''}^+ \hat{a}_{\vec{p}''} t/\hbar} = e^{-i \left[\left(\sum_{\vec{p}''} \varepsilon_0(\vec{p}'') \hat{a}_{\vec{p}''}^+ \hat{a}_{\vec{p}''} \right) + \varepsilon_0(\vec{p}) \right] t/\hbar} \hat{a}_{\vec{p}}$$

$$\hat{a}_{\vec{p}}^+ e^{-i \sum_{\vec{p}''} \varepsilon_0(\vec{p}'') \hat{a}_{\vec{p}''}^+ \hat{a}_{\vec{p}''} t/\hbar} = e^{-i \left[\left(\sum_{\vec{p}''} \varepsilon_0(\vec{p}'') \hat{a}_{\vec{p}''}^+ \hat{a}_{\vec{p}''} \right) - \varepsilon_0(\vec{p}) \right] t/\hbar} \hat{a}_{\vec{p}}^+$$

$$\hat{\psi}_H(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} e^{i[\vec{p}\vec{r} - \varepsilon_0(\vec{p})t]/\hbar} \hat{a}_{\vec{p}}$$

$$G^{(0)}(x, x') = -i \langle \hat{T} [\hat{\psi}_H(x) \hat{\psi}_H^+(x')] \rangle = -i \begin{cases} \langle \hat{\psi}_H(x) \hat{\psi}_H^+(x') \rangle, & t > t' \\ -\langle \hat{\psi}_H^+(x') \hat{\psi}_H(x) \rangle, & t' > t \end{cases}$$

$$G^{(0)}(x, x') = \frac{-i}{V} \sum_{\vec{p}} e^{i[\vec{p}(\vec{r}-\vec{r}')-\varepsilon_0(\vec{p})(t-t')]/\hbar} \begin{cases} 1 - N_{\vec{p}}, & t' > t \\ -N_{\vec{p}}, & t' < t \end{cases}$$

$$N_{\vec{p}} = \langle \hat{a}_{\vec{p}}^+ \hat{a}_{\vec{p}} \rangle = \begin{cases} 1, & |\vec{p}| < p_0 \\ 0, & |\vec{p}| > p_0 \end{cases}$$

$$G^{(0)}(x) = \frac{-i}{V} \sum_{\vec{p}} e^{i[\vec{p}\vec{r}-\varepsilon_0(\vec{p})t]/\hbar} \begin{cases} 1 - N_{\vec{p}}, & t > 0 \\ -N_{\vec{p}}, & t < 0 \end{cases}$$

$$G^{(0)}(\vec{p}, \omega) = \int_{-\infty}^{\infty} \int G^{(0)}(x) e^{-i[(\vec{p}\vec{r}/\hbar)-\omega t]} d\vec{r} dt = -i \int_{-\infty}^{\infty} dt e^{i[\omega-\varepsilon_0(\vec{p})/\hbar]t} \begin{cases} 1 - N_{\vec{p}}, & t > 0 \\ -N_{\vec{p}}, & t < 0 \end{cases} =$$

$$= -i\theta(p-p_0) \int_0^{\infty} dt e^{i[\omega-\varepsilon_0(\vec{p})/\hbar]t} + i\theta(p_0-p) \int_0^{\infty} dt e^{-i[\omega-\varepsilon_0(\vec{p})/\hbar]t} \quad (\text{Vóí: } \theta(z) = \begin{cases} 1, & z > 0 \\ 0, & z < 0 \end{cases})$$

$$\int_0^{\infty} e^{ist} dt = \lim_{\delta \rightarrow +0} \int_0^{\infty} e^{ist-\delta t} dt = i \lim_{\delta \rightarrow +0} \frac{1}{s+i\delta} \Rightarrow \int F(s) \frac{ds}{s+i\delta} = \int \frac{F(s)}{s} ds - i\pi F(0)$$

$$G^{(0)}(\vec{p}, \omega) = \frac{\theta(p-p_0)}{\hbar\omega - \varepsilon_0(\vec{p}) + i\delta} + \frac{\theta(p_0-p)}{\hbar\omega - \varepsilon_0(\vec{p}) - i\delta} = \frac{1}{\hbar\omega - \varepsilon_0(\vec{p}) + i\delta \text{ sign}(p-p_0)}$$

12.3- Hàm Green phonon

$$D(x, x') = -i \langle \hat{T}[\hat{\phi}_H(x) \hat{\phi}_H(x')] \rangle$$

$$\hat{u}(\vec{r}, t) = \sum_{\vec{k}} \frac{\vec{k}}{k} \left\{ \hat{u}_{\vec{k}}(\vec{r}, t) e^{i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} + \hat{u}_{\vec{k}}^+(\vec{r}, t) e^{-i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} \right\}$$

$$\rho[\hat{u}_i(\vec{r}, t), \hat{u}_j(\vec{r}', t)] = -i\hbar \delta(\vec{r}-\vec{r}') \delta_{ij}$$

$$\hat{b}_{\vec{k}} = \sqrt{2\rho\omega_0(\vec{k})/\hbar} \hat{u}_{\vec{k}}$$

$$\hat{b}_{\vec{k}}^+ = \sqrt{2\rho\omega_0(\vec{k})/\hbar} \hat{u}_{\vec{k}}^+$$

$$\hat{K} = \frac{\rho}{2} \int [\hat{u}(\vec{r}, t)]^2 d\vec{r}$$

$$E = \langle H \rangle = 2 \langle \hat{K} \rangle = \frac{1}{2} \left\langle \sum_{\vec{k}, \vec{k}'} \frac{\vec{k}\vec{k}'}{kk'} \hbar \sqrt{\omega_0(\vec{k})\omega_0(\vec{k}')} \left\{ \hat{b}_{\vec{k}} \hat{b}_{\vec{k}'}^+ e^{i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} e^{-i[\vec{k}'\vec{r}-\omega_0(\vec{k}')t]} + \hat{b}_{\vec{k}}^+ \hat{b}_{\vec{k}'} e^{-i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} e^{i[\vec{k}'\vec{r}-\omega_0(\vec{k}')t]} \right\} \right\rangle$$

$$E = (\hbar/2) \sum_{\vec{k}} \omega_0(\vec{k}) [\langle \hat{b}_{\vec{k}} \hat{b}_{\vec{k}}^+ + \hat{b}_{\vec{k}}^+ \hat{b}_{\vec{k}} \rangle] = \sum_{\vec{k}} \hbar\omega_0(\vec{k}) [N_{\vec{k}} + (1/2)]$$

$$H = \sum_{\vec{k}} \hbar\omega_0(\vec{k}) [\hat{N}_{\vec{k}} + (1/2)]$$

$$\hat{\phi}_H(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \sqrt{\omega_0(\vec{k})/2} \left\{ \hat{b}_{\vec{k}} e^{i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} + \hat{b}_{\vec{k}}^+ e^{-i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} \right\}$$

$$D^{(0)}(x) = \frac{-i}{2V} \sum_{\vec{k}} \omega_0(\vec{k}) \left\{ \theta(t) e^{i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} + \theta(-t) e^{-i[\vec{k}\vec{r}-\omega_0(\vec{k})t]} \right\}$$

$$D^{(0)}(\vec{k}, \omega) = \int_{-\infty}^{\infty} \int D^{(0)}(x) e^{-i[\vec{k}\vec{r}-\omega t]} d\vec{r} dt = \frac{-i\omega_0(\vec{k})}{2} \int_{-\infty}^{\infty} dt \begin{cases} e^{i[\omega-\omega_0(\vec{k})]t}, & t > 0 \\ e^{i[\omega+\omega_0(\vec{k})]t}, & t < 0 \end{cases}$$

$$= \frac{-i\omega_0(\vec{k})}{2} \left\{ \int_0^{\infty} dt e^{i[\omega-\omega_0(\vec{k})]t} + \int_0^{\infty} dt e^{-i[\omega+\omega_0(\vec{k})]t} \right\}$$

$$D^{(0)}(\vec{k}, \omega) = \frac{\omega_0(\vec{k})}{2} \left\{ \frac{1}{\omega - \omega_0(\vec{k}) + i\delta} - \frac{1}{\omega + \omega_0(\vec{k}) - i\delta} \right\} = \frac{\omega_0^2(\vec{k})}{\omega^2 - \omega_0^2(\vec{k}) + i\delta}$$

12.4- Định lý Wick

$$\langle \Phi_H^{(0)*} | \hat{T}[\hat{A}(q_1, t_1) \hat{B}(q_2, t_2) \hat{C}(q_3, t_3) \dots \hat{X}(q_{m-1}, t_{m-1}) \hat{Y}(q_m, t_m)] | \Phi_H^{(0)} \rangle$$

$$\hat{\psi}(q_n, t_\ell) \hat{\psi}^+(q_n, t_n) \implies \langle \hat{T}[\hat{\psi}(q_n, t_\ell) \hat{\psi}^+(q_n, t_n)] \rangle$$

$$\langle \hat{T}[\hat{A}(q_1, t_1) \hat{B}(q_2, t_2) \hat{C}(q_3, t_3) \hat{D}(q_4, t_4) \dots \hat{X}(q_{m-1}, t_{m-1}) \hat{Y}(q_m, t_m)] \rangle =$$

$$= \langle \hat{T}[\hat{A}(q_1, t_1) \hat{B}(q_2, t_2)] \rangle \langle \hat{T}[\hat{C}(q_3, t_3) \hat{D}(q_4, t_4)] \rangle \dots \langle \hat{T}[\hat{X}(q_{m-1}, t_{m-1}) \hat{Y}(q_m, t_m)] \rangle \pm$$

$$\pm \langle \hat{T}[\hat{A}(q_1, t_1) \hat{C}(q_3, t_3)] \rangle \langle \hat{T}[\hat{B}(q_2, t_2) \hat{D}(q_4, t_4)] \rangle \dots \langle \hat{T}[\hat{X}(q_{m-1}, t_{m-1}) \hat{Y}(q_m, t_m)] \rangle \pm \dots$$

$$G(x, x') = -i \frac{\langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{S}(\infty)] \rangle}{\langle \hat{S}(\infty) \rangle}$$

$$\begin{aligned} \hat{S}(\infty) = & 1 - (i/\hbar) \int_{-\infty}^{\infty} \hat{V}(t_1) dt_1 - (1/\hbar)^2 \int_{-\infty}^{\infty} \hat{V}(t_1) dt_1 \int_{-\infty}^{t_1} \hat{V}(t_2) dt_2 + \dots + \\ & + (-i/\hbar)^n \int_{-\infty}^{\infty} \hat{V}(t_1) dt_1 \int_{-\infty}^{t_1} \hat{V}(t_2) dt_2 \dots \int_{-\infty}^{t_{n-1}} \hat{V}(t_n) dt_n + \dots \end{aligned}$$

$$\hat{V}(t) = g \int \hat{\psi}^+(x) \hat{\psi}(x) \hat{\phi}(x) d\vec{r}$$

$$g^2 = \frac{2\pi^2 \zeta}{p_0 m}$$

$$\delta G^{(1)}(x, x') = \frac{-(g/\hbar)}{\langle \hat{S}(\infty) \rangle} \int dx_1 \langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{\psi}^+(x_1) \hat{\psi}(x_1) \hat{\phi}(x_1)] \rangle$$

$$\langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x')] \rangle \langle \hat{T}[\hat{\psi}^+(x_1) \hat{\psi}(x_1)] \rangle \langle [\hat{\phi}(x_1)] \rangle \pm \dots \langle [\hat{\phi}(x_1)] \rangle = 0$$

$$\langle [\hat{\phi}(x_1)] \rangle = 0 \quad \delta G^{(1)}(x, x') = 0.$$

Dễ dàng chứng tỏ rằng tất cả các bậc lẻ của gia số hàm Green $\delta G^{(2n+1)}(x, x')$ cũng bằng không.

$$\begin{aligned} \delta G^{(2)}(x, x') = & \frac{i(g/\hbar)^2}{\langle \hat{S}(\infty) \rangle} \iint dx_1 dx_2 \langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{\psi}^+(x_1) \hat{\psi}(x_1) \hat{\phi}(x_1) \hat{\psi}^+(x_2) \hat{\psi}(x_2) \hat{\phi}(x_2)] \rangle \\ & \langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{S}(\infty)] \rangle = \langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{S}(\infty)] \rangle_{\ell k} \langle \hat{S}(\infty) \rangle \\ & G(x, x') = -i \langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{S}(\infty)] \rangle_{\ell k} \end{aligned} \quad (12.1c)$$

\implies Hàm Green $G(x, x')$ có thể biểu thị qua các hàm Green của hệ các hạt không tương tác $G^{(0)}(x, x')$.

13- Phương pháp hàm Green lượng tử ở nhiệt độ $T \neq 0K$. Không học vì trong môn Phương pháp hàm Green có một chương về hàm Green nhiệt độ $T \neq 0K$

14- Giải đồ Feynman.

14.1- Giải đồ Feynman trong trường hợp $T=0K$

14.1.1- Những quy tắc chủ yếu của kỹ thuật giải đồ

$$G(x, x') = -i \frac{\langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{S}(\infty)] \rangle}{\langle \hat{S}(\infty) \rangle}$$

$$\begin{aligned} \hat{S}(\infty) = \hat{S}(\infty, -\infty) = & 1 - (i/\hbar) \int_{-\infty}^{\infty} \hat{V}(t_1) dt_1 - \frac{1}{2\hbar^2} \int_{-\infty}^{\infty} \hat{V}(t_1) dt_1 \int_{-\infty}^{\infty} \hat{V}(t_2) dt_2 + \dots + \\ & + \frac{(-i)^n}{n! \hbar^n} \int_{-\infty}^{\infty} \hat{V}(t_1) dt_1 \int_{-\infty}^{\infty} \hat{V}(t_2) dt_2 \dots \int_{-\infty}^{\infty} \hat{V}(t_n) dt_n + \dots \end{aligned}$$

$$\implies G(x, x') = \frac{-i}{\langle \hat{S}(\infty) \rangle} \sum_{n=0}^{\infty} \frac{(-i)^n}{n! \hbar^n} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_n \langle \hat{T}[\hat{\psi}(x) \hat{\psi}^+(x') \hat{V}(t_1) \dots \hat{V}(t_n)] \rangle$$

$$\hat{V}_S = \frac{1}{2} \iint \hat{\psi}_\alpha^+(\vec{r}_1) \hat{\psi}_\beta^+(\vec{r}_2) U_S(\vec{r}_1 - \vec{r}_2) \hat{\psi}_\beta(\vec{r}_2) \hat{\psi}_\alpha(\vec{r}_1) d\vec{r}_1 d\vec{r}_2$$

Ký hiệu: $\mathfrak{I}(x_1 - x_2) = U(\vec{r}_1 - \vec{r}_2) \delta(t_1 - t_2)$

$$\int \hat{V}(t_1) dt_1 = \frac{1}{2} \iint \hat{\psi}_\alpha^+(x_1) \hat{\psi}_\beta^+(x_2) \mathfrak{I}(x_1 - x_2) \hat{\psi}_\beta(x_2) \hat{\psi}_\alpha(x_1) d^4x_1 d^4x_2$$

Khi : n=1.

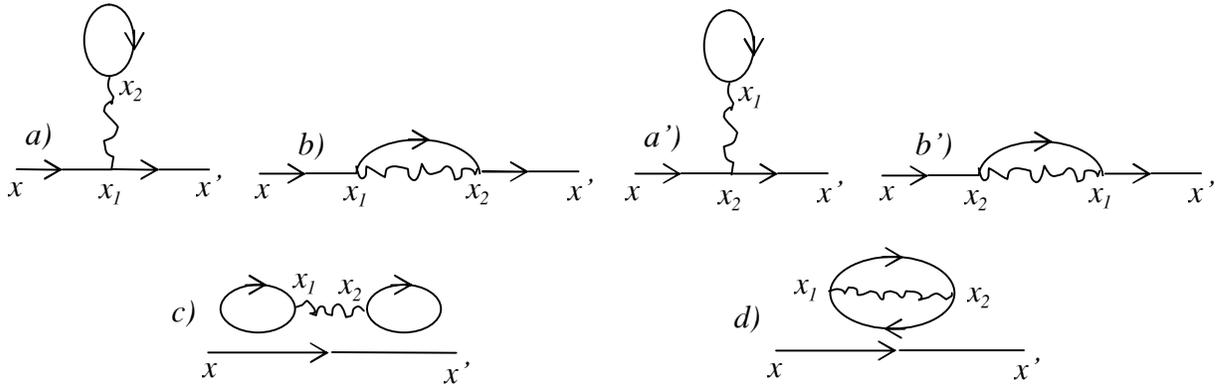
$$G^{(1)} = \frac{-1}{2 \langle \hat{S}(\infty) \rangle} \int d^4x_1 d^4x_2 \langle T[\hat{\psi}_\alpha(x) \hat{\psi}_\beta^+(x') \hat{\psi}_\gamma^+(x_1) \hat{\psi}_\delta^+(x_2) \hat{\psi}_\delta(x_2) \hat{\psi}_\gamma(x_1)] \rangle \mathfrak{I}(x_1 - x_2)$$

$$\begin{aligned} \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\beta^+(x') \hat{\psi}_\gamma^+(x_1) \hat{\psi}_\delta^+(x_2) \hat{\psi}_\delta(x_2) \hat{\psi}_\gamma(x_1)] \rangle = & \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\gamma^+(x_1)] \rangle \langle T[\hat{\psi}_\delta^+(x_2) \hat{\psi}_\delta(x_2)] \rangle \langle T[\hat{\psi}_\gamma(x_1) \hat{\psi}_\beta^+(x')] \rangle - \\ & - \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\gamma^+(x_1)] \rangle \langle T[\hat{\psi}_\delta^+(x_2) \hat{\psi}_\gamma(x_1)] \rangle \langle T[\hat{\psi}_\delta(x_2) \hat{\psi}_\beta^+(x')] \rangle + \\ & + \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\delta^+(x_2)] \rangle \langle T[\hat{\psi}_\gamma^+(x_1) \hat{\psi}_\gamma(x_1)] \rangle \langle T[\hat{\psi}_\delta(x_2) \hat{\psi}_\beta^+(x')] \rangle - \\ & - \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\delta^+(x_2)] \rangle \langle T[\hat{\psi}_\gamma^+(x_1) \hat{\psi}_\delta(x_2)] \rangle \langle T[\hat{\psi}_\gamma(x_1) \hat{\psi}_\beta^+(x')] \rangle + \\ & + \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\beta^+(x')] \rangle \langle T[\hat{\psi}_\gamma^+(x_1) \hat{\psi}_\gamma(x_1)] \rangle \langle T[\hat{\psi}_\delta^+(x_2) \hat{\psi}_\delta(x_2)] \rangle - \\ & + \langle \hat{T}[\hat{\psi}_\alpha(x) \hat{\psi}_\beta^+(x')] \rangle \langle T[\hat{\psi}_\gamma^+(x_1) \hat{\psi}_\delta(x_2)] \rangle \langle T[\hat{\psi}_\delta^+(x_2) \hat{\psi}_\gamma(x_1)] \rangle \end{aligned}$$

Thay $G^{(0)}$:

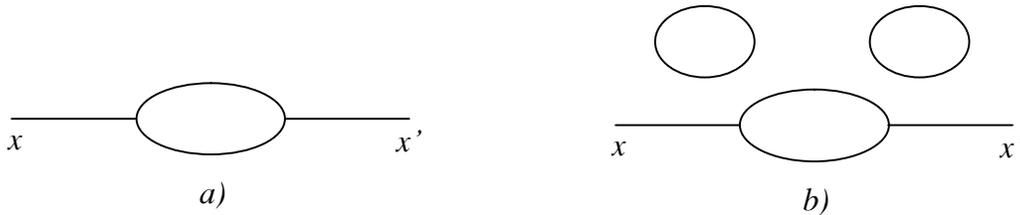
$$\begin{aligned} i G_{\alpha\gamma}^{(0)}(x, x_1) G_{\delta\delta}^{(0)}(x_2, x_2) G_{\gamma\beta}^{(0)}(x_1, x') - i G_{\alpha\gamma}^{(0)}(x, x_1) G_{\gamma\delta}^{(0)}(x_1, x_2) G_{\delta\beta}^{(0)}(x_2, x') + i G_{\alpha\delta}^{(0)}(x, x_2) G_{\gamma\gamma}^{(0)}(x_1, x_1) G_{\delta\beta}^{(0)}(x_2, x') - \\ - i G_{\alpha\delta}^{(0)}(x, x_2) G_{\delta\gamma}^{(0)}(x_2, x_1) G_{\gamma\beta}^{(0)}(x_1, x') - i G_{\alpha\beta}^{(0)}(x, x') G_{\gamma\gamma}^{(0)}(x_1, x_1) G_{\delta\delta}^{(0)}(x_2, x_2) + \\ + i G_{\alpha\beta}^{(0)}(x, x') G_{\delta\gamma}^{(0)}(x_2, x_1) G_{\gamma\delta}^{(0)}(x_1, x_2) \end{aligned}$$

Giản đồ Feynman: $G^{(1)}$ phù hợp với 6 giản đồ trên hình 14.1



Hình 14.1

Giản đồ liên kết:



Hình 14.2

$$\begin{aligned} \langle T[\hat{\psi}(x) \hat{\psi}^+(x') \hat{S}(\infty)] \rangle = \sum_{n=0}^{\infty} \frac{(-i)^n}{n! \hbar^n} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_n \langle T[\hat{\psi}(x) \hat{\psi}^+(x') \hat{V}(t_1) \dots \hat{V}(t_n)] \rangle = \\ = \sum_{n=0}^{\infty} \sum_{m=0}^n \left\{ \frac{(-i)^n}{n! \hbar^n} A(n, m) \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_m \langle T[\hat{\psi}(x) \hat{\psi}^+(x') \hat{V}(t_1) \dots \hat{V}(t_m)] \rangle \right\}_{\text{lk}} \end{aligned}$$

$$\left. \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_{m+1} \dots dt_n < T[\hat{V}(t_{m+1}) \dots \hat{V}(t_n)] > \right\}$$

$$A(n, m) = \frac{n!}{m! (n-m)!}.$$

$$\begin{aligned} < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{S}(\infty)] > = \sum_{n=0}^{\infty} \sum_{m=0}^n \left\{ \frac{(-i)^m}{m! \hbar^m} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_m < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{V}(t_1) \dots \hat{V}(t_m)] >_{\ell k} \cdot \right. \\ & \left. \frac{(-i)^{n-m}}{(n-m)! \hbar^{n-m}} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_{m+1} \dots dt_n < T[\hat{V}(t_{m+1}) \dots \hat{V}(t_n)] > \right\} \\ = \sum_{m=0}^{\infty} \frac{(-i)^m}{m! \hbar^m} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_m < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{V}(t_1) \dots \hat{V}(t_m)] >_{\ell k} \cdot \\ & \cdot \sum_{k=0}^{\infty} \frac{(-i)^k}{k! \hbar^k} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_{m+1} \dots dt_{m+k} < T[\hat{V}(t_{m+1}) \dots \hat{V}(t_{m+k})] > \quad (14.7) \end{aligned}$$

$$\begin{aligned} \text{Vi} \quad \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_{m+1} \dots dt_{m+k} < T[\hat{V}(t_{m+1}) \dots \hat{V}(t_{m+k})] > = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_k < T[\hat{V}(t_1) \dots \hat{V}(t_k)] > \\ \sum_{k=0}^{\infty} \frac{(-i)^k}{k! \hbar^k} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_{m+1} \dots dt_{m+k} < T[\hat{V}(t_{m+1}) \dots \hat{V}(t_{m+k})] > = < \hat{S}(\infty) > \quad (14.8a) \end{aligned}$$

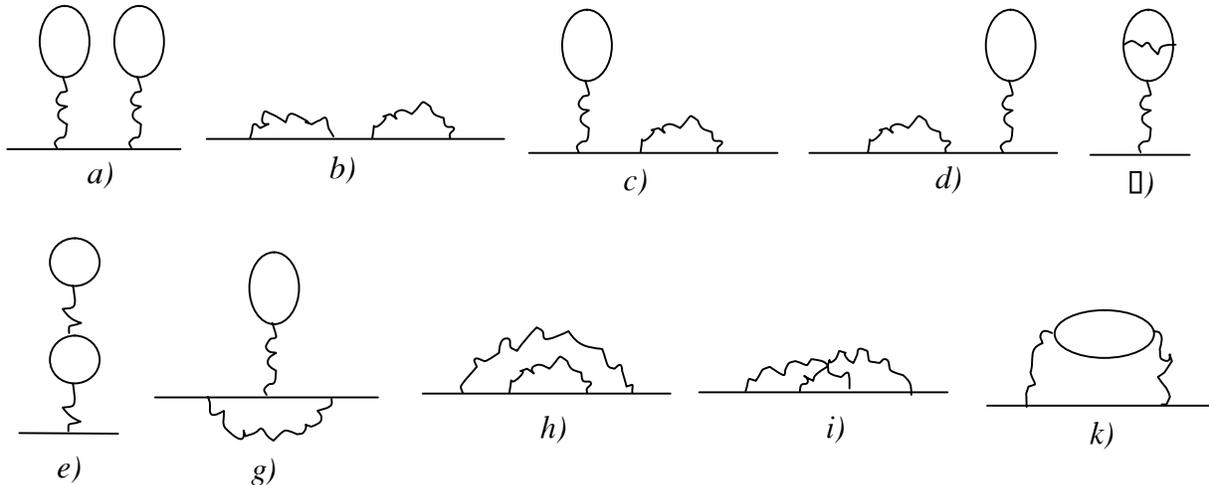
$$\sum_{m=0}^{\infty} \frac{(-i)^m}{m! \hbar^m} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dt_1 \dots dt_m < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{V}(t_1) \dots \hat{V}(t_m)] >_{\ell k} = < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{S}(\infty)] >_{\ell k} \quad (14.8b)$$

$$< T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{S}(\infty)] > = < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{S}(\infty)] >_{\ell k} < \hat{S}(\infty) > \quad (14.9)$$

$$G(x, x') = -i < T[\hat{\psi}(x)\hat{\psi}^+(x')\hat{S}(\infty)] >_{\ell k} \quad (14.10)$$

14.1.2- Kỹ thuật giản đồ trong không gian tọa độ.

Xét Tương tác hai hạt:



Hình 14.3

$$a) - \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma}^{(0)}(x-x_1) G_{\gamma\beta}^{(0)}(x_1-x_2) G_{\beta\gamma}^{(0)}(x_2-x_3) G_{\gamma\delta}^{(0)}(x_3-x_4) G_{\delta\alpha}^{(0)}(x_4-x_1) V(x_1-x_3) V(x_2-x_4)$$

$$b) - \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma}^{(0)}(x-x_1) G_{\gamma\beta}^{(0)}(x_1-x_2) G_{\beta\gamma}^{(0)}(x_2-x_3) G_{\gamma\delta}^{(0)}(x_3-x_4) G_{\delta\alpha}^{(0)}(x_4-x_1) V(x_1-x_2) V(x_3-x_4)$$

$$c) \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma}^{(0)}(x-x_1) G_{\gamma\beta}^{(0)}(x_1-x_2) G_{\beta\gamma}^{(0)}(x_2-x_3) G_{\gamma\delta}^{(0)}(x_3-x_4) G_{\delta\alpha}^{(0)}(x_4-x_1) V(x_1-x_4) V(x_2-x_3)$$

$$\begin{aligned}
 d) & \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\gamma_2}^{(0)}(x_1-x_2) G_{\gamma_2\gamma_3}^{(0)}(x_2-x_3) G_{\gamma_3\beta}^{(0)}(x_3-x') G_{\gamma_4\gamma_4}^{(0)}(0) V(x_1-x_2) V(x_3-x_4) \\
 đ) & \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\beta}^{(0)}(x_1-x') G_{\gamma_2\gamma_3}^{(0)}(x_2-x_3) G_{\gamma_3\gamma_4}^{(0)}(x_3-x_4) G_{\gamma_4\gamma_2}^{(0)}(x_4-x_2) V(x_1-x_2) V(x_3-x_4) \\
 e) & - \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\beta}^{(0)}(x_1-x') G_{\gamma_2\gamma_3}^{(0)}(x_2-x_3) G_{\gamma_3\gamma_2}^{(0)}(x_3-x_2) G_{\gamma_4\gamma_4}^{(0)}(0) V(x_1-x_2) V(x_3-x_4) \\
 g) & \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\gamma_2}^{(0)}(x_1-x_2) G_{\gamma_2\gamma_3}^{(0)}(x_2-x_3) G_{\gamma_3\beta}^{(0)}(x_3-x') G_{\gamma_4\gamma_4}^{(0)}(0) V(x_1-x_3) V(x_2-x_4) \\
 h) & - \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\gamma_2}^{(0)}(x_1-x_2) G_{\gamma_2\gamma_3}^{(0)}(x_2-x_3) G_{\gamma_3\gamma_4}^{(0)}(x_3-x_4) G_{\gamma_4\beta}^{(0)}(x_4-x') V(x_1-x_4) V(x_2-x_3) \\
 i) & - \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\gamma_2}^{(0)}(x_1-x_2) G_{\gamma_2\gamma_3}^{(0)}(x_2-x_3) G_{\gamma_3\gamma_4}^{(0)}(x_3-x_4) G_{\gamma_4\beta}^{(0)}(x_4-x') V(x_1-x_3) V(x_2-x_4) \\
 k) & \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_{\alpha\gamma_1}^{(0)}(x-x_1) G_{\gamma_1\gamma_2}^{(0)}(x_1-x_2) G_{\gamma_2\beta}^{(0)}(x_2-x') G_{\gamma_3\gamma_4}^{(0)}(x_3-x_4) G_{\gamma_4\gamma_3}^{(0)}(x_4-x_3) V(x_1-x_3) V(x_2-x_4)
 \end{aligned}$$

$$\begin{aligned}
 & \int \hat{V}(t_1) dt_1 = \frac{1}{2} \iint \hat{\psi}_\alpha^+(x_1) \hat{\psi}_\beta^+(x_2) \mathfrak{I}(x_1-x_2) \hat{\psi}_\beta(x_2) \hat{\psi}_\alpha(x_1) dx_1 dx_2 = \\
 & = \frac{1}{2} \int \dots \int \hat{\psi}_{\gamma_1}^+(x_1) \hat{\psi}_{\gamma_2}^+(x_2) \mathfrak{I}(x_1-x_2) \delta(x_1-x_3) \delta(x_2-x_4) \delta_{\gamma_1\gamma_3} \delta_{\gamma_2\gamma_4} \hat{\psi}_{\gamma_4}(x_4) \hat{\psi}_{\gamma_3}(x_3) dx_1 dx_2 dx_3 dx_4 = \\
 & = -\frac{1}{2} \int \dots \int \hat{\psi}_{\gamma_1}^+(x_1) \hat{\psi}_{\gamma_2}^+(x_2) \mathfrak{I}(x_1-x_2) \delta(x_1-x_4) \delta(x_2-x_3) \delta_{\gamma_1\gamma_4} \delta_{\gamma_2\gamma_3} \hat{\psi}_{\gamma_4}(x_4) \hat{\psi}_{\gamma_3}(x_3) dx_1 dx_2 dx_3 dx_4 = \\
 & = \frac{1}{4} \int \dots \int \hat{\psi}_{\gamma_1}^+(x_1) \hat{\psi}_{\gamma_2}^+(x_2) \Gamma_{\gamma_1\gamma_2 \gamma_3\gamma_4}^{(0)}(x_1 x_2, x_3 x_4) \hat{\psi}_{\gamma_4}(x_4) \hat{\psi}_{\gamma_3}(x_3) dx_1 dx_2 dx_3 dx_4
 \end{aligned}$$

$$\Gamma_{\gamma_1\gamma_3 \gamma_2\gamma_4}^{(0)}(x_1 x_2, x_3 x_4) = \mathfrak{I}(x_1-x_2) [\delta(x_1-x_3) \delta(x_2-x_4) \delta_{\gamma_1\gamma_3} \delta_{\gamma_2\gamma_4} - \delta(x_1-x_4) \delta(x_2-x_3) \delta_{\gamma_1\gamma_4} \delta_{\gamma_2\gamma_3}]$$

$$\begin{aligned}
 G^{(1)} & = -\frac{1}{4} \int \dots \int d^4x_1 \dots d^4x_4 \Gamma_{\gamma_1\gamma_2 \gamma_3\gamma_4}^{(0)}(x_1 x_2, x_3 x_4) \langle T[\hat{\psi}_\alpha(x) \hat{\psi}_\beta^+(x') \hat{\psi}_{\gamma_1}^+(x_1) \hat{\psi}_{\gamma_2}^+(x_2) \hat{\psi}_{\gamma_4}(x_4) \hat{\psi}_{\gamma_3}(x_3)] \rangle = \\
 & = i \int \dots \int d^4x_1 \dots d^4x_4 \Gamma_{\gamma_1\gamma_2 \gamma_3\gamma_4}^{(0)}(x_1 x_2, x_3 x_4) G^{(0)}(x-x_1) G^{(0)}(x_3-x_2) G^{(0)}(x_4-x')
 \end{aligned}$$

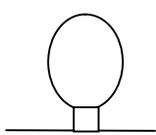
$\Gamma_{\gamma_1\gamma_3 \gamma_2\gamma_4}^{(0)}(x_1 x_2, x_3 x_4)$ = một hình vuông

(14.13) = giản đồ 14.4.

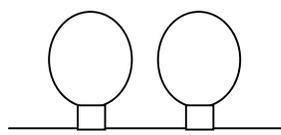
các giản đồ 14.3 a, b, c, d \implies 14.5 a ;

các giản đồ 14.3 đ, e, g, h \implies 14.5 b ;

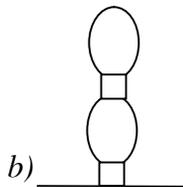
các giản đồ 14.3 i, k \implies 14.5 c.



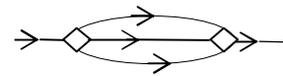
Hình 14.4



a)



b)



c)

Hình 14.5

$$a) - \int \dots \int d^4x_1 \dots d^4x_8 \Gamma_{\gamma_1\gamma_2 \gamma_3\gamma_4}^{(0)}(x_1 x_2, x_3 x_4) \Gamma_{\gamma_5\gamma_6 \gamma_7\gamma_8}^{(0)}(x_5 x_6, x_7 x_8).$$

$$.G^{(0)}(x-x_1) G^{(0)}(x_3-x_5) G^{(0)}(x_7-x') G^{(0)}(x_4-x_2) G^{(0)}(x_8-x_6)$$

$$b) - \int \dots \int d^4x_1 \dots d^4x_8 \Gamma_{\gamma_1\gamma_2 \gamma_3\gamma_4}^{(0)}(x_1 x_2, x_3 x_4) \Gamma_{\gamma_5\gamma_6 \gamma_7\gamma_8}^{(0)}(x_5 x_6, x_7 x_8).$$

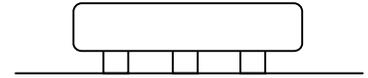
$$.G^{(0)}(x-x_1) G^{(0)}(x_3-x') G^{(0)}(x_4-x_5) G^{(0)}(x_7-x_2) G^{(0)}(x_8-x_6)$$

$$c) -\frac{1}{2} \int \dots \int d^4 x_1 \dots d^4 x_8 \Gamma_{\gamma_1 \gamma_2 \gamma_3 \gamma_4}^{(0)}(x_1 x_2, x_3 x_4) \Gamma_{\gamma_5 \gamma_6 \gamma_7 \gamma_8}^{(0)}(x_5 x_6, x_7 x_8).$$

$$.G^{(0)}(x-x_1)G^{(0)}(x_3-x_5)G^{(0)}(x_7-x_2)G^{(0)}(x_4-x_6)G^{(0)}(x_8-x')$$

Quy tắc tính phần bổ sung bậc n của hàm Green :

- 1) Vẽ tất cả các giản đồ liên kết có cấu trúc tô pô không tương đương nhau;
 - 2) Mỗi đường cho tương ứng với một hàm Green $G_{\alpha\beta}^{(0)}(x_i - x_j)$;
 - 3) Mỗi tứ giác cho tương ứng với một hàm $\Gamma_{\gamma_1 \gamma_3 \gamma_2 \gamma_4}^{(0)}(x_1 x_2, x_3 x_4)$;
 - 4) Lấy tích phân theo các tọa độ của tất cả các đỉnh và lấy tổng theo các biến spin ;
 - 5) Biểu thức nhận được nhân với hệ số $(i)^n (1/2)^{n-(m/2)}$, trong đó m là số giản đồ trong biểu diễn không đối xứng tương ứng với giản đồ đối xứng đang xét. Dấu của giản đồ cũng tương ứng với dấu trong trường hợp không đối xứng.
- Giản đồ bậc ba vẽ trên hình 14.6



Hình 14.6

$$-\frac{(i)^3}{2} \int \dots \int d^4 x_1 \dots d^4 x_{12} \Gamma_{\gamma_1 \gamma_2 \gamma_3 \gamma_4}^{(0)}(x_1 x_2, x_3 x_4) \Gamma_{\gamma_5 \gamma_6 \gamma_7 \gamma_8}^{(0)}(x_5 x_6, x_7 x_8) \Gamma_{\gamma_9 \gamma_{10} \gamma_{11} \gamma_{12}}^{(0)}(x_9 x_{10}, x_{11} x_{12}).$$

$$.G^{(0)}(x-x_1)G^{(0)}(x_3-x_5)G^{(0)}(x_7-x_9)G^{(0)}(x_4-x_{10})G^{(0)}(x_{12}-x_6)G^{(0)}(x_8-x_2)G^{(0)}(x_{11}-x')$$

14.1.3- Kỹ thuật giản đồ trong không gian xung lượng

Xét Tương tác hai hạt

$$G_{\alpha\beta}^{(1b)}(x, x') = -i \int \int G_{\alpha\gamma}^{(0)}(x - x_1) G_{\gamma\delta}^{(0)}(x_1 - x_2) G_{\delta\beta}^{(0)}(x_2 - x') \mathfrak{I}(x_1 - x_2) d^4 x_1 d^4 x_2$$

$$G_{\alpha\beta}^{(0)}(x_1 - x_2) = \int \frac{d^4 p}{(2\pi\hbar)^4} G_{\alpha\beta}^{(0)}(p) e^{ip(x_1 - x_2)/\hbar}$$

$$\mathfrak{I}(x_1 - x_2) = \int \frac{d^4 q}{(2\pi\hbar)^4} \mathfrak{I}(q) e^{iq(x_1 - x_2)/\hbar}$$

$$G_{\alpha\beta}^{(0)}(p) = \int G_{\alpha\beta}^{(0)}(x) e^{-ipx/\hbar} d^4 x$$

$$p = (\vec{p}, \hbar\omega), \quad q = (\vec{q}, \hbar\omega), \quad p(x_1 - x_2) = \vec{p}(\vec{r}_1 - \vec{r}_2) - \hbar\omega(t_1 - t_2).$$

$$G_{\alpha\beta}^{(1b)}(x, x') = -i(2\pi\hbar)^{-16} \int \dots \int d^4 p_1 d^4 p_2 d^4 p_3 d^4 q d^4 x_1 d^4 x_2 e^{i[p_1(x-x_1)+p_1(x_1-x_2)+p_1(x_2-x')+q(x_1-x_2)]/\hbar}.$$

$$.G_{\alpha\gamma}^{(0)}(p_1) G_{\gamma\delta}^{(0)}(p_2) G_{\delta\beta}^{(0)}(p_3) \mathfrak{I}(q) =$$

$$= -i(2\pi\hbar)^{-8} \int \dots \int d^4 p_1 d^4 p_2 d^4 p_3 d^4 q e^{i[p_1 x - p_2 x']/\hbar} \delta(p_1 - p_2 - q) \delta(p_2 - p_3 + q).$$

$$.G_{\alpha\gamma}^{(0)}(p_1) G_{\gamma\delta}^{(0)}(p_2) G_{\delta\beta}^{(0)}(p_3) \mathfrak{I}(q)$$

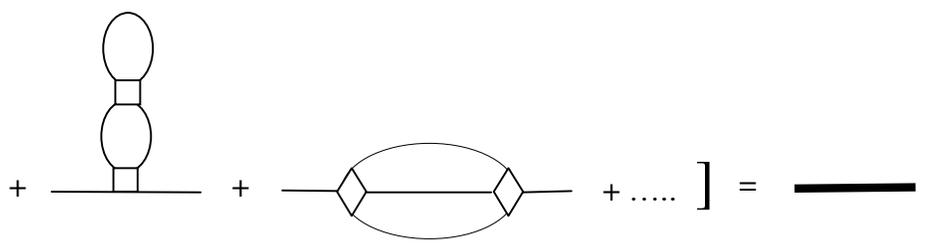
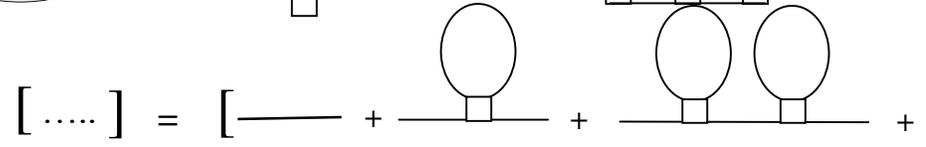
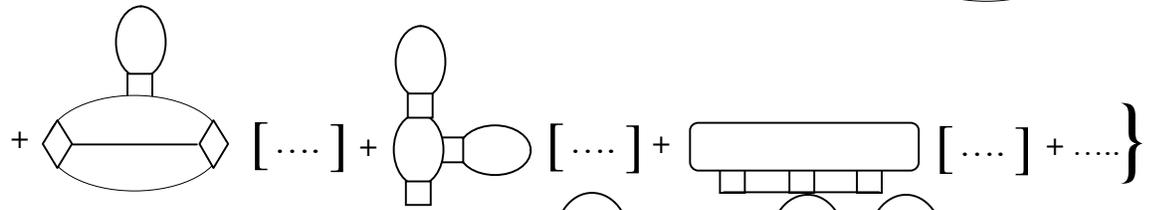
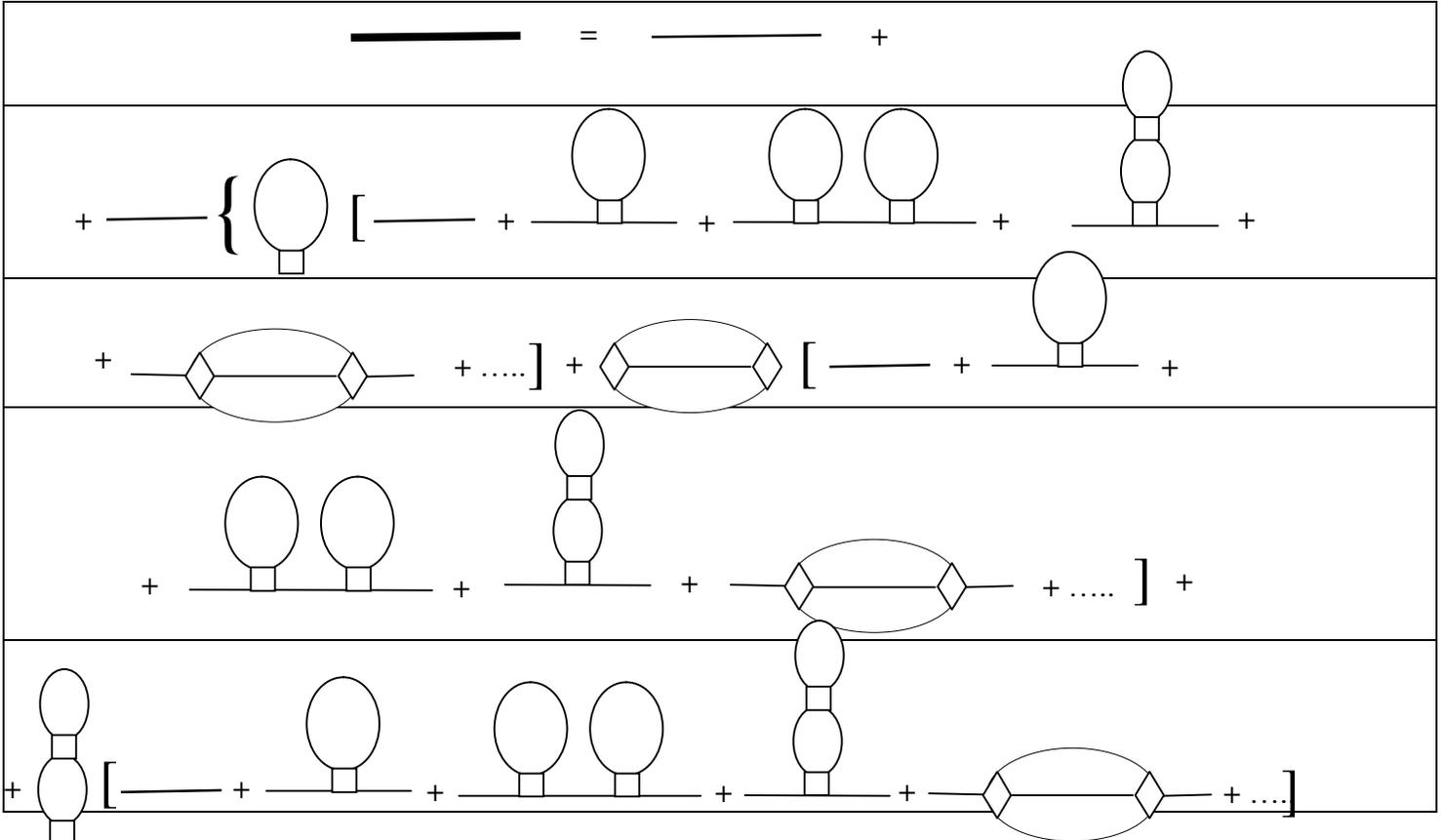
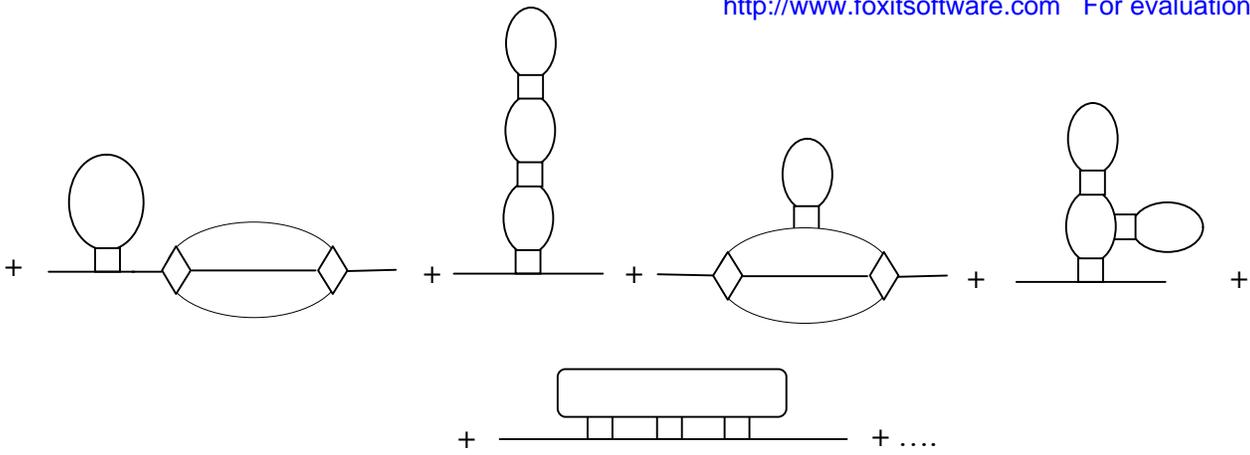
$$= i(2\pi\hbar)^{-8} \int \dots \int d^4 p_1 d^4 p_2 d^4 p_3 d^4 q e^{i[p_1 x - p_2 x']/\hbar} \delta(p_1 - p_2 - q) \delta(p_2 - p_3 + q).$$

$$.G_{\alpha\gamma}^{(0)}(p_1) G_{\gamma\delta}^{(0)}(p_2) G_{\delta\beta}^{(0)}(p_3) \mathfrak{I}(q)$$

$$G_{\alpha\beta}^{(1b)}(p, p') = i \int \dots \int d^4 p_2 d^4 q e^{i[p_1 x - p_2 x']/\hbar} \delta(p - p_2 - q) \delta(p_2 - p' + q).$$

$$.G_{\alpha\gamma}^{(0)}(p) G_{\gamma\delta}^{(0)}(p_2) G_{\delta\beta}^{(0)}(p') \mathfrak{I}(q)$$

$$G_{\alpha\beta}^{(1b)}(p, p') = G^{(1b)}(p) \delta(p - p') (2\pi)^4 \delta_{\alpha\beta}$$



$$G = G^{(0)} + G^{(0)}\Omega G$$

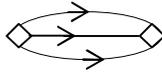
$$\Omega = \text{tadpole} + \text{loop} + \text{tadpole} + \dots$$

$$\Omega = \Omega_1 + \Omega_2 + \Omega_3 + \dots$$

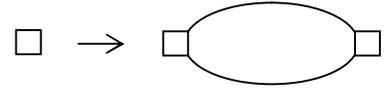
$$\Omega_1 = \Omega_{\alpha\beta}^{(1)}(p) = i \int \frac{d^4 p_1}{(2\pi)^4} \Gamma_{\alpha\gamma, \delta\beta}^{(0)}(p, p_1; p_1, p) G_{\delta\gamma}(p_1)$$



Hình 14.16



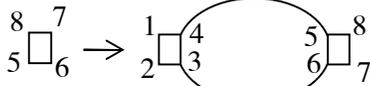
Hình 14.17



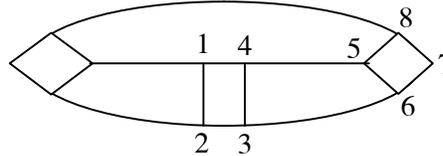
Hình 7.5 c



Hình 7.6



Hình 7.17



Hình 7.6

$\Omega_{\alpha\beta}^{(2)}(p) = \Omega_2 =$ giản đồ vẽ trên hình 14.18

$\Omega_{\alpha\beta}^{(2)}(p) = -\frac{1}{2} \int \Gamma_{\alpha\xi, \eta\delta}^{(0)}(p, p_1; p_2, p + p_1 - p_2) \cdot \Gamma_{\mu\gamma, \nu\beta}(p_2, p + p_1 - p_2; p_1, p) \cdot G_{\eta\mu}(p_2) G_{\nu\xi}(p_1) G_{\delta\gamma}(p + p_1 - p_2) \frac{d^4 p_1 d^4 p_2}{(2\pi)^8}$	<p>Hình 14.18</p>
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Thay $\Omega = \Omega^{(1)} + \Omega^{(2)}$ vào phương trình (14.19) (chú ý $G^{(0)} = \frac{1}{\hbar\omega - E_0(\vec{p})}$):

$$[\hbar\omega - E_0(\vec{p})]G_{\alpha\beta}(p) - i \int \frac{d^4 p_1}{(2\pi)^4} \Gamma_{\alpha\xi, \eta\gamma}^{(0)}(p, p_1; p_1, p) G_{\eta\xi}(p_1) G_{\gamma\beta}(p) +$$

$$+ \frac{1}{2} \int \Gamma_{\alpha\xi, \eta\delta}^{(0)}(p, p_1; p_2, p + p_1 - p_2) \Gamma_{\mu\gamma, \nu\beta}(p_2, p + p_1 - p_2; p_1, p) \cdot$$

$$G_{\eta\mu}(p_2) G_{\nu\xi}(p_1) G_{\delta\gamma}(p + p_1 - p_2) \frac{d^4 p_1 d^4 p_2}{(2\pi)^8} G_{\rho\beta}(p) = \delta_{\alpha\beta}$$