Chapter 1: The time value of money*

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Chapter contents

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Overview

This chapter deals with the most basic concepts in finance: future value, present value, and internal rate of return. These concepts tell you how much your money will grow if deposited in a bank (future value), how much promised future payments are worth today (present value), and what percentage rate of return you're getting on your investments (internal rate of return).

Financial assets and financial planning always have a time dimension. Here are some simple examples:

- You put $100 in the bank today in a savings account. How much will you have in 3 years?
- You put $100 in the bank today in a savings account and plan to add $100 every year for the next 10 years. How much will you have in the account in 20 years?
- XYZ Corporation just sold a bond to your mother for $860. The bond will pay her $20 per year for the next 5 years. In 6 years she gets a payment of $1020. Has she paid a fair price for the bond?
- Your Aunt Sara is considering making an investment. The investment costs $1,000 and will pay back $50 per month in each of the next 36 months. Should she do this or should she leave her money in the bank, where it earns 5%?

This chapter discusses these and similar issues, all of which fall under the general topic of *time value of money*. You will learn how compound interest causes invested income to grow (*future value*), and how money to be received at future dates can be related to money in hand today (*present value*). You will also learn how to calculate the compound rate of return earned by an investment (*internal rate of return*). The concepts of future value, present value, and internal rate of return underlie much of the financial analysis which will appear in the following chapters.

Finance concepts discussed

- Future value
- Present value
- Net present value
- Internal rate of return
- Pension and savings plans and other accumulation problems

Excel functions used

- Excel functions: **PV, NPV, IRR, PMT, NPer**
- Goal seek

1.1. Future value

Future value (FV) tells you the value in the future of money deposited in a bank account today and left in the account to draw interest. The *future value $X* deposited today in an account paying *r%* interest annually and left in the account for *n* years is $X*(1+r)^n$. Future value is our first illustration of *compound interest*—it incorporates the principle that you earn interest on interest. If this sounds confusing, read on.

Suppose you put \$100 in a savings account in your bank today and that the bank pays you 6% interest at the end of every year. If you leave the money in the bank for one year, you will have \$106 after one year: \$100 of the original savings balance + \$6 in interest.

Now suppose you leave the money in the account for a second year: At the end of this year, you will have:

\$106	The savings account balance at the end of the first year
+	
6%*\$106 = \$6.36	The interest in on this balance for the second year
= \$112.36	Total in account after two years

A little manipulation will show you that the future value of the \$100 after 2 years is $\$100*(1+6\%)^2$.

$$\underbrace{\underbrace{\underbrace{\$100}_{\text{Initial deposit}} * \underbrace{1.06}_{\text{Year 1's future value factor at 6\%}}}_{\text{Future value of \$100 after one year = \$100*1.06}} * \underbrace{1.06}_{\text{Year 2's future value factor}}}_{\text{Future value of \$100 after two years}} = \$100*\left(1+6\%\right)^2 = \$112.36$$

Notice that the future value uses the concept of *compound interest*: The interest earned in the first year (\$6) itself earns interest in the second year. To sum up:

The value of \$X deposited today in an account paying r% interest annually and left in the account for n years is its *__future value__* $FV = X*\left(1+r\right)^n$.

Notation

In this book we will often match our mathematical notation to that used by Excel. Since in Excel multiplication is indicated by a star "*", we will generally write 6%*$106 = $6.36, even though this is not necessary. Similarly we will sometimes write $(1.10)^3$ as $1.10\wedge 3$.

In order to confuse you, we make no promises about consistency!

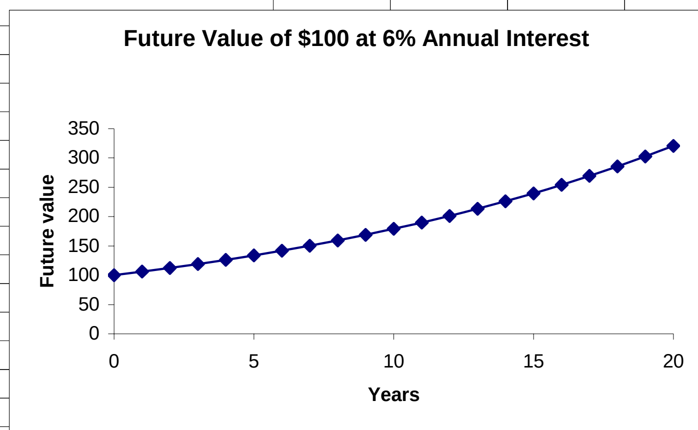
Future value calculations are easily done in Excel:

	A	B	C
1	**CALCULATING FUTURE VALUES WITH EXCEL**		
2	Initial deposit	100	
3	Interest rate	6%	
4	Number of years, n	2	
5			
6	Account balance after n years	112.36	<-- =B2*(1+B3)^B4

Notice the use of the carat (^) to denote the exponent: In Excel $(1+6\%)^2$ is written as (1+B3)^B4, where cell B3 contains the interest rate and cell B4 the number of years.

We can use Excel to make a table of how the future value grows with the years and then use Excel's graphing abilities to graph this growth:

	A	B	C	D	E	F	G
1	**THE FUTURE VALUE OF A SINGLE $100 DEPOSIT**						
2	Initial deposit	100					
3	Interest rate	6%					
4	Number of years, n	2					
5							
6	Account balance after n years	112.36	<-- =B2*(1+B3)^B4				
7							
8	**Year**	**Future value**					
9	0	100.00	<-- =$B$2*(1+$B$3)^A9				
10	1	106.00	<-- =$B$2*(1+$B$3)^A10				
11	2	112.36	<-- =$B$2*(1+$B$3)^A11				
12	3	119.10	<-- =$B$2*(1+$B$3)^A12				
13	4	126.25	<-- =$B$2*(1+$B$3)^A13				
14	5	133.82					
15	6	141.85					
16	7	150.36					
17	8	159.38					
18	9	168.95					
19	10	179.08					
20	11	189.83					
21	12	201.22					
22	13	213.29					
23	14	226.09					
24	15	239.66					
25	16	254.04					
26	17	269.28					
27	18	285.43					
28	19	302.56					
29	20	320.71					

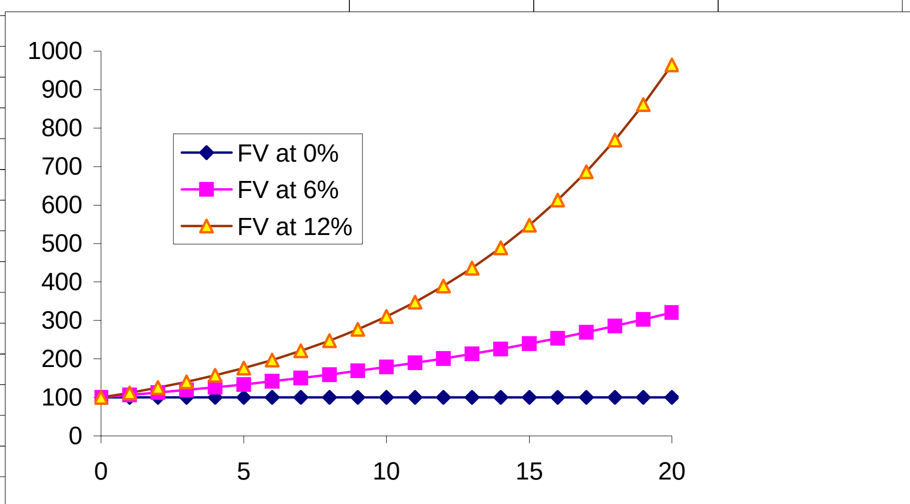


Excel note

Notice that the formula in cells B9:B29 in the table has $ signs on the cell references (for example: **=$B$2*(1+$B$3)^A9**). This use of the *absolute copying* feature of Excel is explained in Chapter 000.

In the spreadsheet below, we present a table and graph that shows the future value of $100 for 3 different interest rates: 0%, 6%, and 12%. As the spreadsheet shows, future value is *very* sensitive to the interest rate! Note that when the interest rate is 0%, the future value doesn't grow.

	A	B	C	D	E
1	**FUTURE VALUE OF A SINGLE PAYMENT AT DIFFERENT INTEREST RATES** **How $100 at time 0 grows at 0%, 6%, 12%**				
2	Initial deposit	100			
3	Interest rate	0%	6%	12%	
4					
5	**Year**	**FV at 0%**	**FV at 6%**	**FV at 12%**	
6	0	100.00	100.00	100.00	<-- =$B$2*(1+D$3)^$A6
7	1	100.00	106.00	112.00	<-- =$B$2*(1+D$3)^$A7
8	2	100.00	112.36	125.44	
9	3	100.00	119.10	140.49	
10	4	100.00	126.25	157.35	
11	5	100.00	133.82	176.23	
12	6	100.00	141.85	197.38	
13	7	100.00	150.36	221.07	
14	8	100.00	159.38	247.60	
15	9	100.00	168.95	277.31	
16	10	100.00	179.08	310.58	
17	11	100.00	189.83	347.85	
18	12	100.00	201.22	389.60	
19	13	100.00	213.29	436.35	
20	14	100.00	226.09	488.71	
21	15	100.00	239.66	547.36	
22	16	100.00	254.04	613.04	
23	17	100.00	269.28	686.60	
24	18	100.00	285.43	769.00	
25	19	100.00	302.56	861.28	
26	20	100.00	320.71	964.63	



Nomenclature: What's a year? When does it begin?

This is a boring but necessary discussion. Throughout this book we will use the following synonyms:

Year 0	**Year 1**	**Year 2**	
Today	**End of year 1**	**End of year 2**	
Beginning of year 1	**Beginning of year 2**	**Beginning of year 3**	
0	1	2	3

We use the words "Year 0," "Today," and "Beginning of year 1" as synonyms. This often causes confusion in finance. For example, "$100 at the beginning of year 2" is the same as "$100 at the end of year 1." Note that we often use "in year 1" to mean "end of year 1": For example: "An investment costs $300 today and pays off $600 in year 1."

There's a lot of confusion on this subject in finance courses and texts. If you're at loss to understand what someone means, ask for a drawing; better yet, ask for an Excel spreadsheet.

Accumulation—savings plans and future value

In the previous example you deposited $100 and left it in your bank. Suppose that you intend to make 10 annual deposits of $100, with the first deposit made in year 0 (today) and each succeeding deposit made at the end of years 1, 2, ..., 9. The *future value* of all these deposits at the end of year 10 tells you how much you will have accumulated in the account. If you are saving for the future (whether to buy a car at the end of your college years or to finance a pension at the end of your working life), this is obviously an important and interesting calculation.

So how much will you have accumulated at the end of year 10? There's an Excel function for calculating this answer which we will discuss later; for the moment we will set this problem up in Excel and do our calculation the long way, by showing how much we will have at the end of each year:

	A	B	C	D	E	F
1	**FUTURE VALUE WITH ANNUAL DEPOSITS at beginning of year**					
2	Interest	6%				
3				=$B$2*(C6+B6)		
4	**Year** =E5	**Account balance, beg. year**	**Deposit at beginning of year**	**Interest earned during year**	**Total in account at end of year**	
5	1	0.00	100.00	6.00	106.00	<-- =B5+C5+D5
6	2	106.00	100.00	12.36	218.36	<-- =B6+C6+D6
7	3	218.36	100.00	19.10	337.46	
8	4	337.46	100.00	26.25	463.71	
9	5	463.71	100.00	33.82	597.53	
10	6	597.53	100.00	41.85	739.38	
11	7	739.38	100.00	50.36	889.75	
12	8	889.75	100.00	59.38	1,049.13	
13	9	1,049.13	100.00	68.95	1,218.08	
14	10	1,218.08	100.00	79.08	1,397.16	
15						
16		Future value using Excel's FV function	$1,397.16	<-- =FV(B2,A14,-100,,1)		

For clarity, let's analyze a specific year: At the end of year 1 (cell E5) you've got $106 in the account. This is also the amount in the account at the beginning of year 2 (cell B6). If you now deposit another $100 and let the whole amount of $206 draw interest during the year, it will earn $12.36 interest. You will have $218.36 = (106+100)*1.06 at the end of year 2.

	A	B	C	D	E
6	2	106.00	100.00	12.36	218.36

Finally, look at rows 13 and 14: At the end of year 9 (cell E13) you have $1,218.08 in the account; this is also the amount in the account at the beginning of year 10 (cell B14). You then deposited $100 and the resulting $1,318.08 earns $79.08 interest during the year, accumulating to $1,397.16 by the end of year 10.

	A	B	C	D	E
13	9	1,049.13	100.00	68.95	1,218.08
14	10	1,218.08	100.00	79.08	1,397.16

The Excel FV (future value) formula

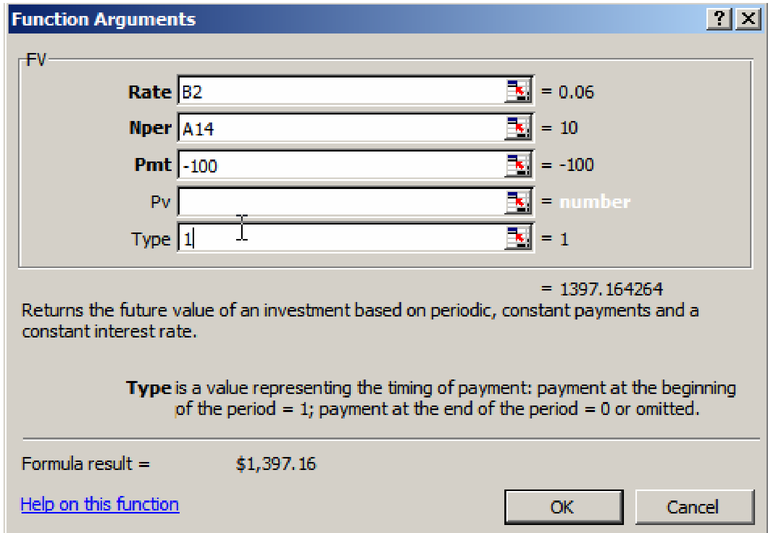
The spreadsheet of the previous subsection illustrates in a step-by-step manner how money accumulates in a typical savings plan. To simplify this series of calculations, Excel has a **FV** formula which computes the future value of any series of constant payments. This formula is illustrated in cell C16:

	B	C	D	E
16	Future value using Excel's FV function	$1,397.16	<-- =FV(B2,A14,-100,,1)	

The **FV** function requires as inputs the **Rate** of interest, the number of periods **Nper**, and the annual payment **Pmt**. You can also indicate the **Type**, which tells Excel whether payments are made at the beginning of the period (type **1** as in our example) or at the end of the period (type **0**).[1]

[1] Exercises 2 and 3 at the end of the chapter illustrate both cases.

Dialog box for FV function



Excel's function dialog boxes have room for two types of arguments.

- **Bold faced** parameters must be filled in—in the **FV** dialog box these are the interest **Rate**, the number of periods **Nper**, and the payment **Pmt**. (Read on to see why we wrote a negative payment.)
- Arguments which are not bold faced are optional. In the example above we've indicated a 1 for the **Type**; this indicates (as shown in the dialog box itself) that the future value is calculated for payments made at the beginning of the period. Had we omitted this argument or put in 0, Excel would compute the future value for a series of payments made at the end of the period; see the next example for an illustration.

Notice that the dialog box already tells us (even before we click on **OK**) that the future value of $100 per year for 10 years compounded at 6% is $1397.16.

Excel note—a peculiarity of the FV function

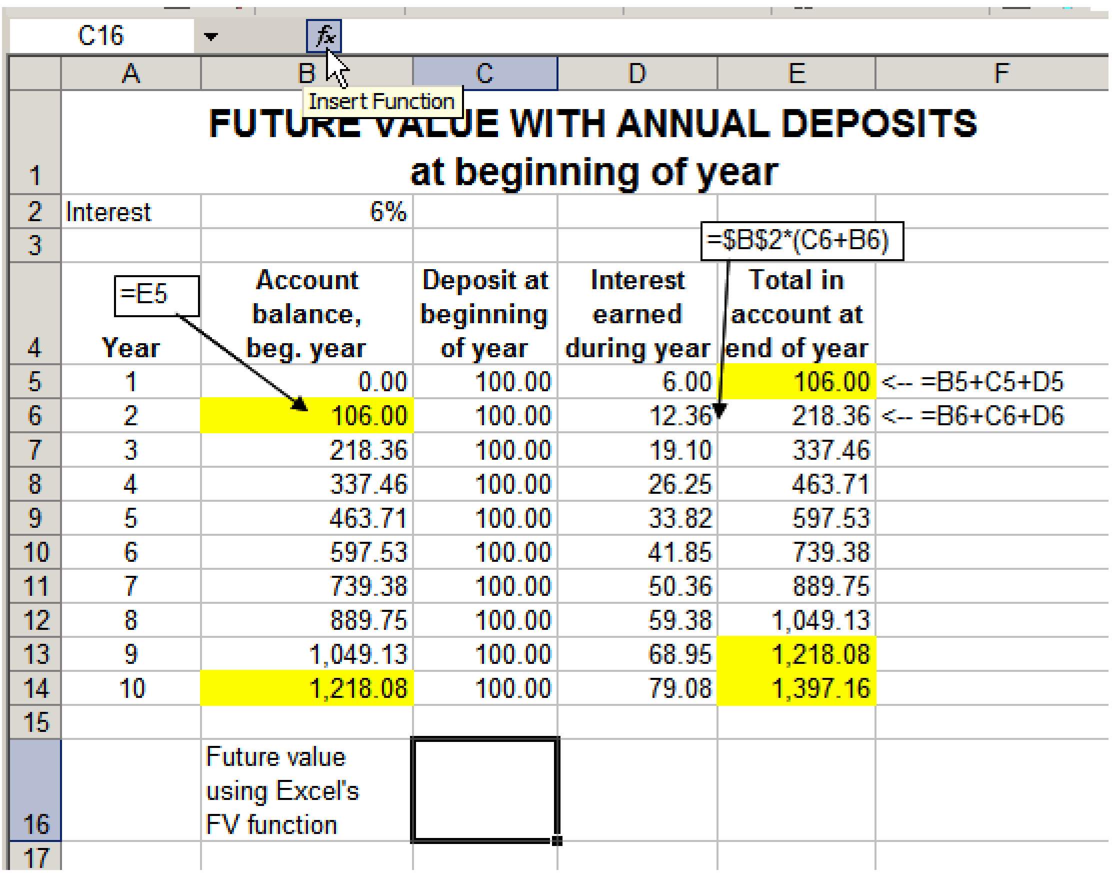
In the **FV** dialog box we've entered in the payment **Pmt** in as a negative number, as -100. The **FV** function has the peculiarity (shared by some other Excel financial functions) that a *positive* deposit generates a *negative* answer. We won't go into the (strange?) logic that produced this thinking; whenever we encounter it we just put in a negative deposit.

Sidebar: Functions and Dialog Boxes

The dialog box which comes with an Excel function is a handy way to utilize the function. There are several ways to get to a dialog box. We'll illustrate with the example of the **FV** function in Section 1.1.

Going through the function wizard

Suppose you're in cell B16 and you want to put the Excel function for future value in the cell. With the cursor in B16, you move your mouse to the fx icon on the tool bar:

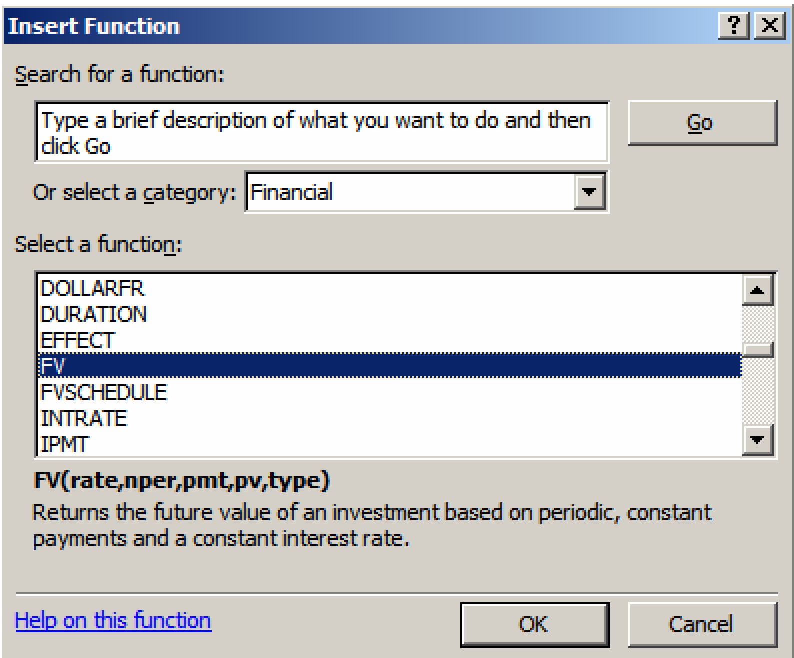


C16 | fx | Insert Function

	A	B	C	D	E	F
1	FUTURE VALUE WITH ANNUAL DEPOSITS at beginning of year					
2	Interest	6%				
3					=$B$2*(C6+B6)	
4	Year	Account balance, beg. year	Deposit at beginning of year	Interest earned during year	Total in account at end of year	
5	1	0.00	100.00	6.00	106.00	<-- =B5+C5+D5
6	2	106.00	100.00	12.36	218.36	<-- =B6+C6+D6
7	3	218.36	100.00	19.10	337.46	
8	4	337.46	100.00	26.25	463.71	
9	5	463.71	100.00	33.82	597.53	
10	6	597.53	100.00	41.85	739.38	
11	7	739.38	100.00	50.36	889.75	
12	8	889.75	100.00	59.38	1,049.13	
13	9	1,049.13	100.00	68.95	1,218.08	
14	10	1,218.08	100.00	79.08	1,397.16	
15						
16		Future value using Excel's FV function				
17						

=E5

Clicking on the icon brings up the dialog box below. We've already chosen the **category** to be the **Financial** functions, and we've scrolled down in the next section of the dialog box to put the cursor on the **FV** function.



Clicking OK brings up the dialog box for the **FV** function.

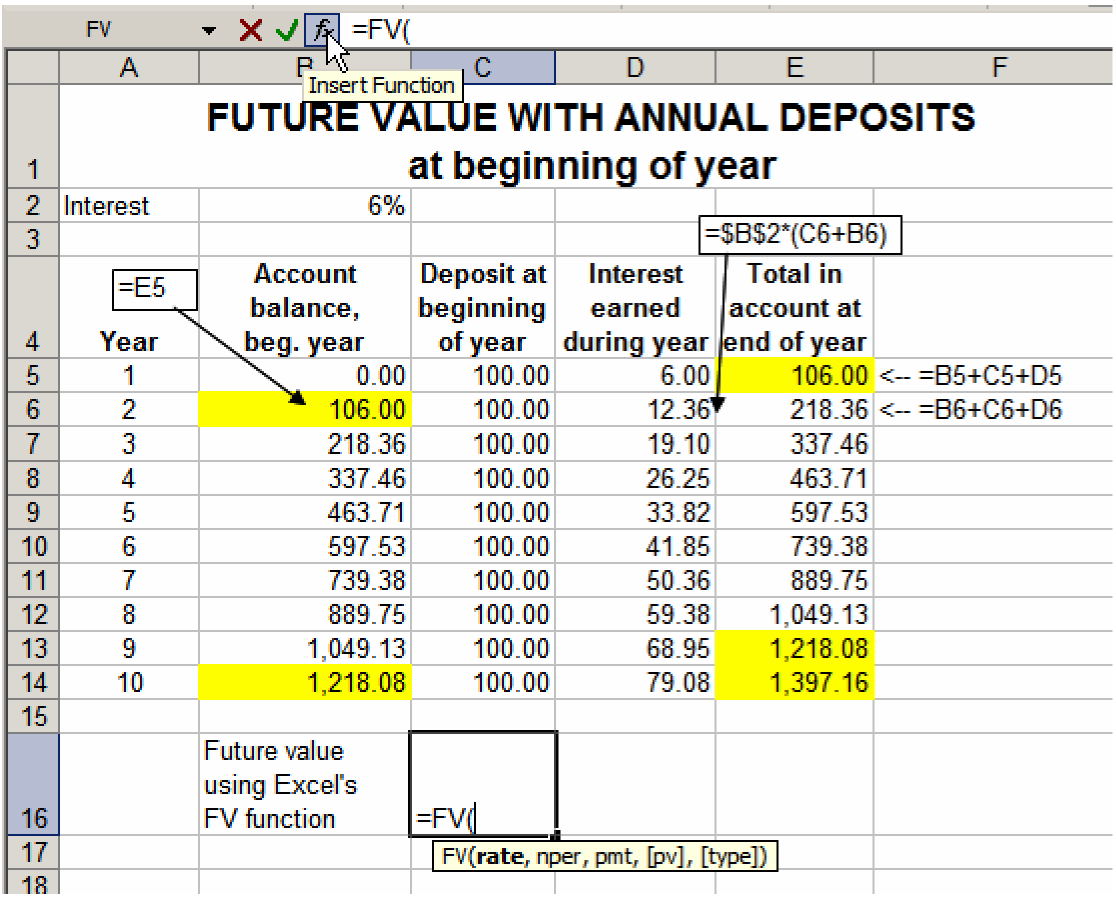
A short way to get to the dialog box

If you know the name of the function you want, you can just write it in the cell and then click the icon on the tool bar. As illustrated below, you have to write

=FV(

and then click on the icon—note that we've written an **equal sign**, the **name of the function**, and the **opening parenthesis**.

Here's how the spreadsheet looks in this case:


FV =FV(

Insert Function

	A	B	C	D	E	F
1	FUTURE VALUE WITH ANNUAL DEPOSITS at beginning of year					
2	Interest	6%				
3					=$B$2*(C6+B6)	
4	Year	Account balance, beg. year	Deposit at beginning of year	Interest earned during year	Total in account at end of year	
5	1	0.00	100.00	6.00	106.00	<-- =B5+C5+D5
6	2	106.00	100.00	12.36	218.36	<-- =B6+C6+D6
7	3	218.36	100.00	19.10	337.46	
8	4	337.46	100.00	26.25	463.71	
9	5	463.71	100.00	33.82	597.53	
10	6	597.53	100.00	41.85	739.38	
11	7	739.38	100.00	50.36	889.75	
12	8	889.75	100.00	59.38	1,049.13	
13	9	1,049.13	100.00	68.95	1,218.08	
14	10	1,218.08	100.00	79.08	1,397.16	
15						
16		Future value using Excel's FV function	=FV(			
17			FV(**rate**, nper, pmt, [pv], [type])			
18						

=E5

Look in the text displayed by Excel below cell C16: Some versions of Excel show the format of the function when you type it in a cell.

One further option

You don't have to use a dialog box! If you know the format of the function then just type in its arguments and you're all set. In the example of Section 1.1 you could just type **=FV(B2,A14,-100,,1)** in the cell. Hitting [Enter] would give the answer.

[END OF SIDEBAR]

Beginning versus end of period

In the example above you make deposits of $100 at the *beginning* of each year. In terms of timing, your deposits are made at dates 0, 1, 2, 3, ..., 9. Here's a schematic way of looking at this, showing the future value of each deposit at the end of year 10:

DEPOSITS AT BEGINNING OF YEAR

Beginning of year 1	Beginning of year 2	Beginning of year 3	Beginning of year 4	Beginning of year 5	Beginning of year 6	Beginning of year 7	Beginning of year 8	Beginning of year 9	Beginning of year 10	End of year 10
0	1	2	3	4	5	6	7	8	9	10
$100	$100	$100	$100	$100	$100	$100	$100	$100	$100	

	179.08	<-- =100*(1.06)^10
	168.95	<-- =100*(1.06)^9
	159.38	<-- =100*(1.06)^8
	150.36	<-- =100*(1.06)^7
	141.85	<-- =100*(1.06)^6
	133.82	<-- =100*(1.06)^5
	126.25	<-- =100*(1.06)^4
	119.10	<-- =100*(1.06)^3
	112.36	<-- =100*(1.06)^2
	106.00	<-- =100*(1.06)^1
Total	1397.16	<-- Sum of the above

In the above example and in the previous spreadsheet you made deposits of $100 at the beginning of each year. Suppose you made 10 deposits of $100 at the *end of each year*. How would this affect the accumulation in the account at the end of 10 years?

DEPOSITS AT END OF YEAR

Beginning of year 1	Beginning of year 2	Beginning of year 3	Beginning of year 4	Beginning of year 5	Beginning of year 6	Beginning of year 7	Beginning of year 8	Beginning of year 9	Beginning of year 10	End of year 10
0	1	2	3	4	5	6	7	8	9	10
	$100	$100	$100	$100	$100	$100	$100	$100	$100	$100

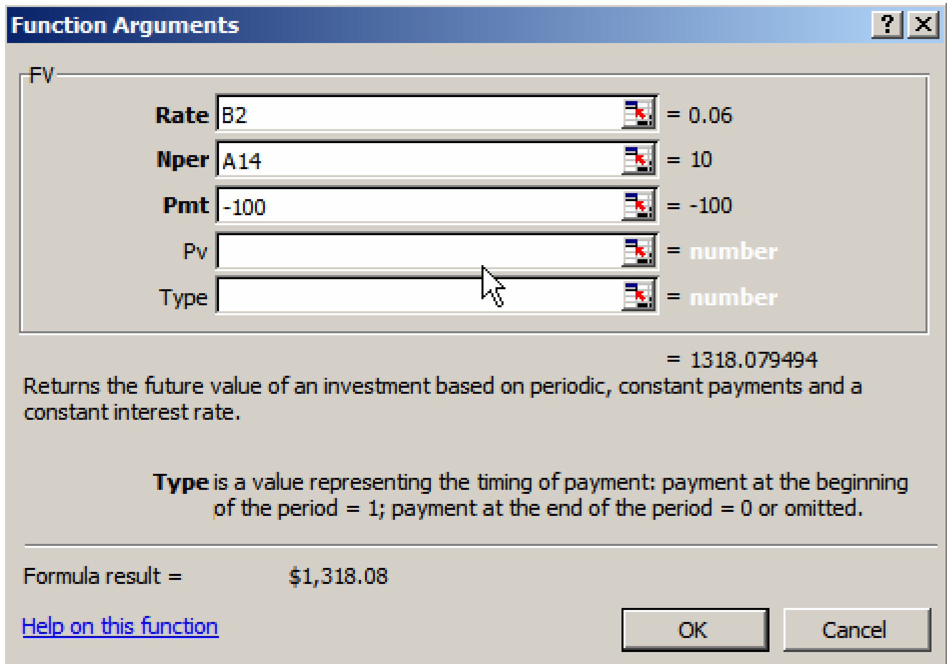
	168.95	<-- =100*(1.06)^9
	159.38	<-- =100*(1.06)^8
	150.36	<-- =100*(1.06)^7
	141.85	<-- =100*(1.06)^6
	133.82	<-- =100*(1.06)^5
	126.25	<-- =100*(1.06)^4
	119.10	<-- =100*(1.06)^3
	112.36	<-- =100*(1.06)^2
	106.00	<-- =100*(1.06)^1
	100.00	<-- =100*(1.06)^0
Total	1318.08	<-- Sum of the above

The account accumulation is less in this case (where you deposit at the end of each year) than in the previous case, where you deposit at the beginning of the year. In the second example, each deposit is in the account one year less and consequently earns one year's less interest. In a spreadsheet, this looks like:

	A	B	C	D	E	F
1	**FUTURE VALUE WITH ANNUAL DEPOSITS at end of year**					
2	Interest	6%				
3					=$B$2*B6	
4	=E5 **Year**	**Account balance, beg. year**	**Deposit at end of year**	**Interest earned during year**	**Total in account at end of year**	
5	1	0.00	100.00	0.00	100.00	<-- =B5+C5+D5
6	2	100.00	100.00	6.00	206.00	<-- =B6+C6+D6
7	3	206.00	100.00	12.36	318.36	
8	4	318.36	100.00	19.10	437.46	
9	5	437.46	100.00	26.25	563.71	
10	6	563.71	100.00	33.82	697.53	
11	7	697.53	100.00	41.85	839.38	
12	8	839.38	100.00	50.36	989.75	
13	9	989.75	100.00	59.38	1,149.13	
14	10	1,149.13	100.00	68.95	1,318.08	
15						
16		Future value	$1,318.08	<-- =FV(B2,A14,-100)		

Cell C16 illustrates the use of the Excel **FV** formula for this case. In the dialog box for this formula, we have put in a zero under **Type**, which indicates that the payments are made at the end of each year:

Dialog box for FV with end-period payments



In the example above we've omitted any entry in the **Type** box. As indicated on the dialog box itself, we could have also put a 0 in the **Type** box. Meaning: Excel's default for the **FV** function is a deposit at the end of the year.

Some finance jargon and the Excel FV function

An annuity with payments at the end of each period is often called a *regular annuity*. As you've seen in this section, the value of a regular annuity is calculated with **=FV(B2,A14,-100)**. An annuity with payments at the beginning of each period is often called an *annuity due* and its value is calculated with the Excel function **=FV(B2,A14,-100,,1)**.

1.2. Present value

In this section we discuss *present value*.

The present value is the value today of a payment (or payments) that will be made in the future.

Here's a simple example: Suppose that you anticipate getting $100 in 3 years from your Uncle Simon, whose word is as good as a bank's. Suppose that the bank pays 6% interest on savings accounts. *How much is the anticipated future payment worth today?* The answer is $\$83.96 = \frac{100}{(1.06)^3}$; if you put $83.96 in the bank today at 6 percent annual interest, then in 3 years you would have $100 (see the "proof" in rows 9 and 10).[2] $83.96 is also called the *discounted or present value of $100 in 3 years at 6 percent interest*.

	A	B	C
1	**SIMPLE PRESENT VALUE CALCULATION**		
2	X, future payment	100	
3	n, time of future payment	3	
4	r, interest rate	6%	
5	Present value, $X/(1+r)^n$	83.96	<-- =B2/(1+B4)^B3
6			
7	**Proof**		
8	Payment today	83.96	
9	Future value in *n* years	100	<-- =B8*(1+B4)^B3

To summarize:

The present value of $X to be received in n years when the appropriate interest rate is r% is $\frac{X}{(1+r)^n}$.

The interest rate *r* is also called the *discount rate*. We can use Excel to make a table of how the present value decreases with the discount rate. As you can see—higher discount rates make for lower present values:

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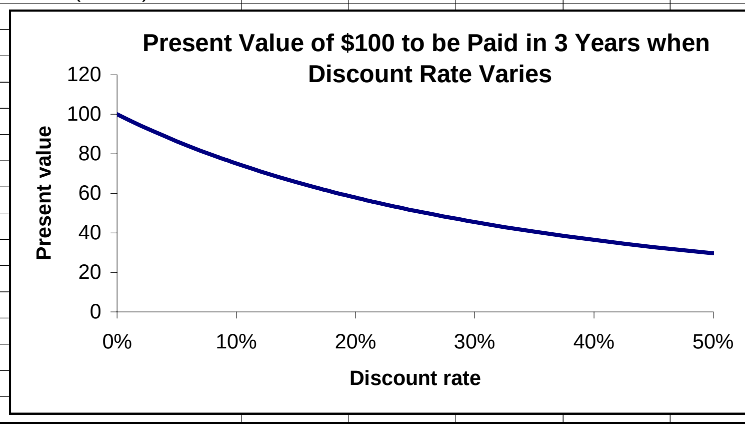
[2] Actually, $\frac{100}{(1.06)^3} = 83.96193$, but we've used **Format|Cells|Number** to show only 2 decimals.

	A	B	C	D	E	F	G	H
1	**THE PRESENT VALUE OF $100 IN 3 YEARS in this example we vary the discount rate *r***							
2	X, future payment	100						
3	n, time of future payment	3						
4	r, interest rate	6%						
5	Present value, $X/(1+r)^n$	83.96	<-- =B2/(1+B4)^B3					
6								
7	**Discount rate**	**Present value**						
8	0%	100.00	<-- =100/(1+A8)^3					
9	1%	97.06	<-- =100/(1+A9)^3					
10	2%	94.23	<-- =100/(1+A10)^3					
11	3%	91.51	<-- =100/(1+A11)^3					
12	4%	88.90	<-- =100/(1+A12)^3					
13	5%	86.38						
14	6%	83.96						
15	7%	81.63						
16	8%	79.38						
17	9%	77.22						
18	12%	71.18						
19	15%	65.75						
20	18%	60.86						
21	20%	57.87						
22	22%	55.07						
23	25%	51.20						
24	30%	45.52						
25	35%	40.64						
26	40%	36.44						
27	45%	32.80						
28	50%	29.63						



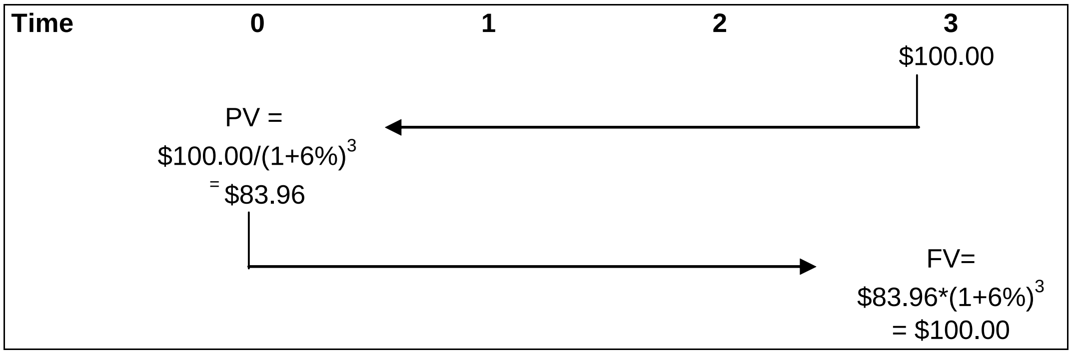
Why does PV decrease as the discount rate increases?

The Excel table above shows that the $100 Uncle Simon promises you in 3 years is worth $83.96 today if the discount rate is 6% but worth only $40.64 if the discount rate is 35%. The mechanical reason for this is that taking the present value at 6% means dividing by a smaller denominator than taking the present value at 35%:

$$83.96 = \frac{100}{(1.06)^3} > \frac{100}{(1.35)^3} = 40.64$$

The economic reason relates to future values: If the bank is paying you 6% interest on your savings account, you would have to deposit $83.96 today in order to have $100 in 3 years. If the bank pays 35% interest, then $\$40.64 * (1.35)^3 = \$100$.

What this short discussion shows is that the *present value is the inverse of the future value*:



Present value of an annuity

In the jargon of finance, an *annuity* is a series of equal periodic payments. Examples of annuities are widespread:

- The allowance your parents give you ($1000 per month, for your next 4 years of college) is a monthly annuity with 48 payments.
- Pension plans often give the retiree a fixed annual payment for as long as he lives. This is a bit more complicated annuity, since the number of payments is uncertain.
- Certain kinds of loans are paid off in fixed periodic (usually monthly, sometimes annual) installments. Mortgages and student loans are two examples.

The *present value* of an annuity tells you *the value today* of all the future payments on the annuity. Here's an example that relates to your generous Uncle Simon. Suppose he has promised you $100 at the end of each of the next 5 years. Assuming that you can get 6% at the bank, this promise is worth $421.24 today:

	A	B	C	D
1	**CALCULATING PRESENT VALUES WITH EXCEL**			
2	Annual payment	100		
3	r, interest rate	6%		
4				
5	**Year**	**Payment at end of year**	**Present value of payment**	
6	1	100	94.34	<-- =B6/(1+$B$3)^A6
7	2	100	89.00	<-- =B7/(1+$B$3)^A7
8	3	100	83.96	
9	4	100	79.21	
10	5	100	74.73	
11				
12	**Present value of all payments**			
13	Summing the present values		421.24	<-- =SUM(C6:C10)
14	Using Excel's PV function		421.24	<-- =PV(B3,5,-100)
15	Using Excel's NPV function		421.24	<-- =NPV(B3,B6:B10)

The example above shows three ways of getting the present value of $421.24:

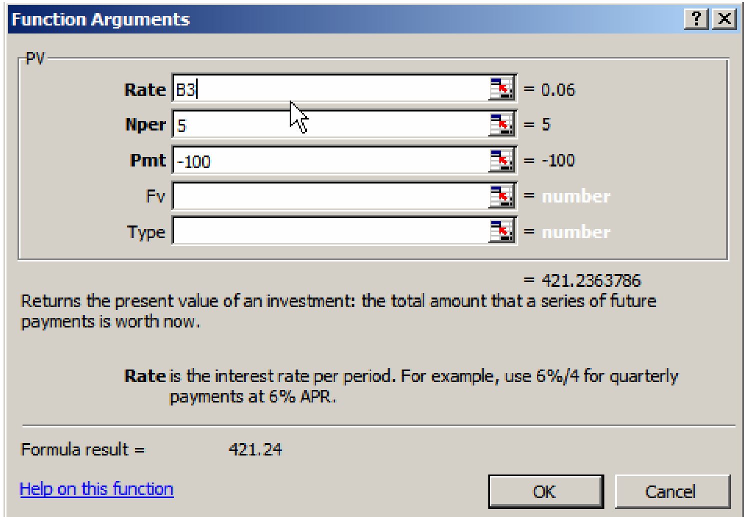
- You can sum the individual discounted values. This is done in cell C13.
- You can use Excel's **PV** function, which calculates the present value of an annuity (cell C14).
- You can use Excel's **NPV** function (cell C16). This function calculates the present value of any series of periodic payments (whether they're flat payments, as in an annuity, or non-equal payments).

  We devote separate subsections to the **PV** function and to the **NPV** function.

The Excel PV function

The **PV** function calculates the present value of an *annuity* (a series of equal payments). It looks a lot like the **FV** discussed above, and like **FV**, it also suffers from the peculiarity that positive payments give negative results (which is why we set **Pmt** equal to –100). As in the case of the **FV** function, **Type** denotes whether the payments are made at the beginning or the end of the year. Because end-year is the default, can either enter "0" or leave the **Type** entry blank:

Dialog box for the PV function

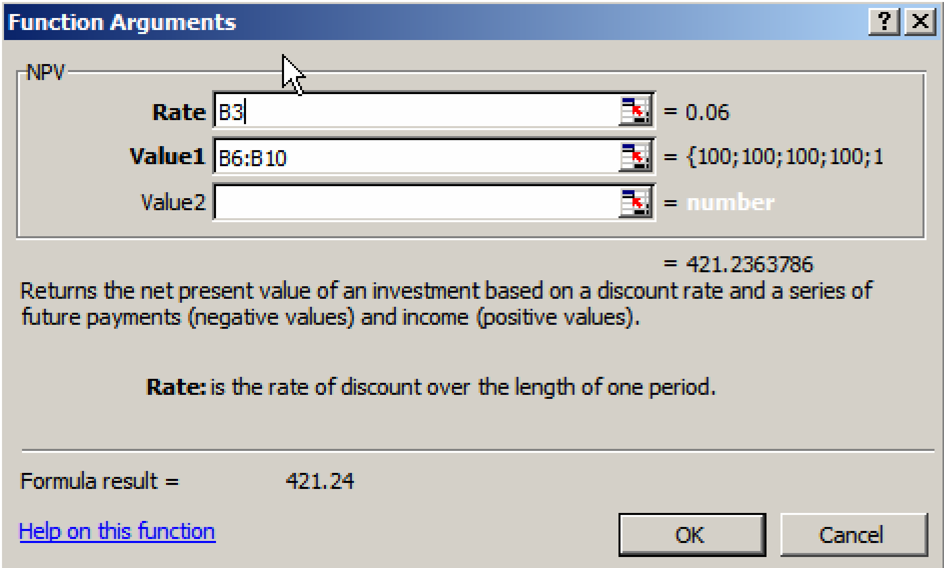


The "Formula result" in the dialogue box shows that the answer is $421.24.

The Excel NPV function

The NPV function computes the present value of a series of payments. The payments need not be equal, though in the present example they are. The ability of the **NPV** function to handle non-equal payments makes it one of the most useful of all Excel's financial functions. We will make extensive use of this function throughout this book.

Dialog box for the NPV function



Important note: Finance professionals use "NPV" to mean "net present value," a concept we explain in the next section. Excel's **NPV** function actually calculates the *present value* of a series of payments. Almost all finance professionals and textbooks would call the number computed by the Excel **NPV** function "PV." Thus the Excel use of "NPV" differs from the standard usage in finance.

Choosing a discount rate

We've defined the present value of *$X* to be received in *n* years as $\frac{X}{(1+r)^n}$. The interest rate *r* in the denominator of this expression is also known as the *discount rate*. Why is 6% an appropriate discount rate for the money promised you by Uncle Simon? The basic principle is to choose a discount rate that is appropriate to the *riskiness* and the duration of the cash flows being discounted. Uncle Simon's promise of $100 per year for 5 years is assumed as good as the promise of your local bank, which pays 6% on its savings accounts. Therefore 6% is an appropriate discount rate.[3]

The present value of non-annuity (meaning: non-constant) cash flows

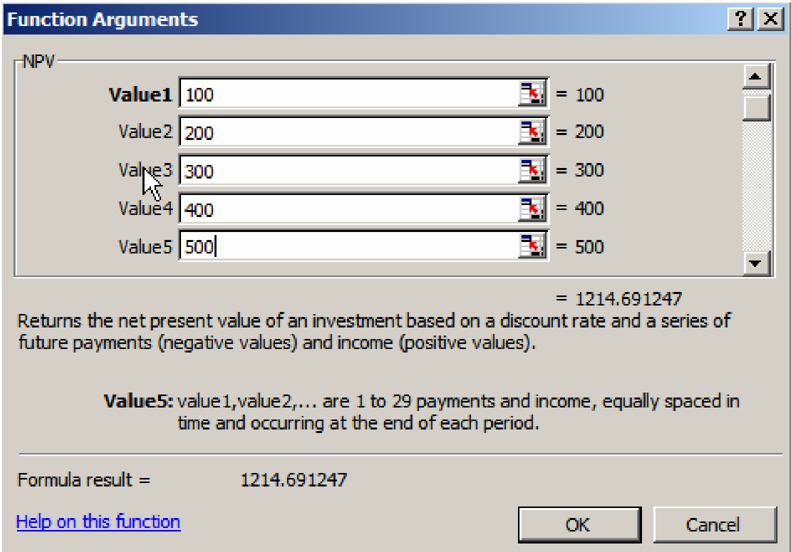
The present value concept can also be applied to non-annuity cash flow streams, meaning cash flows that are not the same every period. Suppose, for example, that your Aunt Terry has promised to pay you $100 at the end of year 1, $200 at the end of year 2, $300 at the end of year 3, $400 at the end of year 4 and $500 at the end of year 5. This is not an annuity, and so it cannot be accommodated by the **PV** function. But we can find the present value of this promise by using the **NPV** function:

	A	B	C	D
1	**CALCULATING PRESENT VALUES WITH EXCEL**			
2	r, interest rate	6%		
3				
4	**Year**	**Payment at end of year**	**Present value**	
5	1	100	94.34	<-- =B5/(1+$B$2)^A5
6	2	200	178.00	<-- =B6/(1+$B$2)^A6
7	3	300	251.89	
8	4	400	316.84	
9	5	500	373.63	
10				
11	**Present value of all payments**			
12	Summing the present values		1,214.69	<-- =SUM(C5:C9)
13	Using Excel's NPV function		1,214.69	<-- =NPV($B$2,B5:B9)

[3] There's more to be said on the choice of a discount rate, but we postpone the discussion until Chapters 5 and 6.

Excel note

Excel's **NPV** function allows you to input up to 29 payments directly in the function dialogue box. Here's an illustration for the example above:



1.3. Net present value

In this section we discuss *net present value.*

The net present value (NPV) of a series of future cash flows is their present value minus the initial investment required to obtain the future cash flows. The NPV = PV(future cash flows) – initial investment. The NPV of an investment represents the increase in wealth which you get if you make the investment.

Here's an example based on the spreadsheet on page000. Would you pay $1500 today to get the series of future cash flows in cells B6:B10? Certainly not—they're worth only $1214.69, so why pay $1500? If asked to pay $1500, the NPV of the investment would be

$$NPV = \underbrace{-\$1,500}_{\substack{\uparrow \\ \text{Cost of the} \\ \text{investment}}} + \underbrace{\$1,214.69}_{\substack{\uparrow \\ \text{Present value of} \\ \text{investment's future} \\ \text{cash flows at discount} \\ \text{rate of 6\%}}} = \underbrace{-\$285.31}_{\substack{\uparrow \\ \text{Net present value}}}.$$

If you paid $1,500 for this investment, you would be overpaying $285.31 for the investment, and you would be poorer by the same amount. That's a bad deal!

On the other hand, if you were offered the same future cash flows for $1,000, you'd snap up the offer, you would be paying $214.69 less for the investment than its worth:

$$NPV = \underbrace{-\$1,000}_{\substack{\uparrow \\ \text{Cost of the} \\ \text{investment}}} + \underbrace{\$1,214.69}_{\substack{\uparrow \\ \text{Present value of} \\ \text{investment's future} \\ \text{cash flows at discount} \\ \text{rate of 6\%}}} = \underbrace{\$214.69}_{\substack{\uparrow \\ \text{Net present value}}}$$

In this case the investment would make you $214.69 richer. As we said before, the NPV of an investment represents the increase in your wealth if you make the investment.

To summarize:

The net present value (NPV) of a series of cash flows is used to make investment decisions: An investment with a positive NPV is a good investment and an investment with a negative NPV is a bad investment. You should be indifferent to making in a zero-NPV investment. An investment with a zero NPV is a "fair game"—the future cash flows of the investment exactly compensate you for the investment's initial cost.

Net present value (NPV) is a basic tool of financial analysis. It is used to determine whether a particular investment ought to be undertaken; in cases where we can make only one of several investments, it is the tool-of-choice to determine which investment to undertake.

Here's another example: You've found an interesting investment—If you pay $800 today to your local pawnshop, the owner promises to pay you $100 at the end of year 1, $150 at the end of year 2, $200 at the end of year 3, ... , $300 at the end of year 5. In your eyes, the pawnshop owner is as reliable as your local bank, which is currently paying 5% interest. The following spreadsheet shows the NPV of this $800 investment:

	A	B	C	D
1	**CALCULATING NET PRESENT VALUE (NPV) WITH EXCEL**			
2	r, interest rate	5%		
3				
4	**Year**	**Payment**	**Present value**	
5	0	-800	-800.00	
6	1	100	95.24	<-- =B6/(1+$B$2)^A6
7	2	150	136.05	<-- =B7/(1+$B$2)^A7
8	3	200	172.77	
9	4	250	205.68	
10	5	300	235.06	
11				
12	**NPV**			
13	Summing the present values		44.79	<-- =SUM(C5:C10)
14	Using Excel's NPV function		44.79	<-- =NPV($B$2,B6:B10)+C5

The spreadsheet shows that the value of the investment—the *net present value(NPV)* of its payments, including the initial payment of -$800—is $44.79:

$$NPV = -800 + \underbrace{\frac{100}{(1.05)} + \frac{150}{(1.05)^2} + \frac{200}{(1.05)^3} + \frac{250}{(1.05)^4} + \frac{300}{(1.05)^5}}_{\substack{\text{The } \textit{present value} \text{ of the future payments:} \\ \text{Calculated with Excel NPV function} = 844.79}} = 44.97$$

At a 5% discount rate, you should make the investment, since its NPV is $44.79, which is positive.

An Excel Note

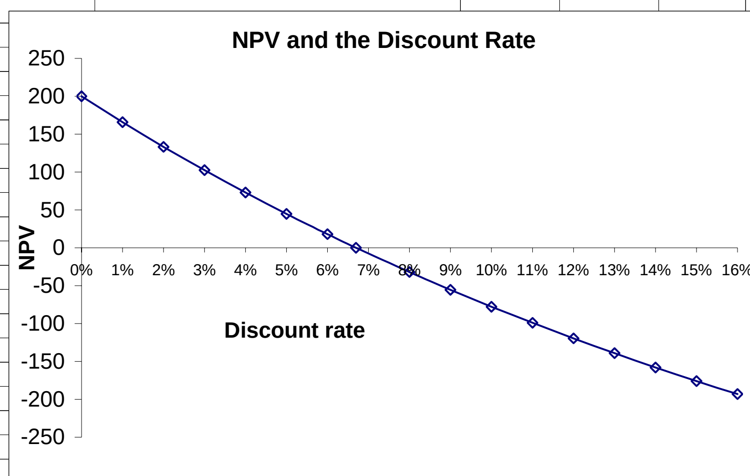
As mentioned earlier, the Excel **NPV** function's name does **not correspond** to the standard finance use of the term "net present value."[4] In finance, "present value" usually refers to the value today of future payments (in the example, this is $\frac{100}{(1.05)}+\frac{150}{(1.05)^2}+\frac{200}{(1.05)^3}+\frac{250}{(1.05)^4}+\frac{300}{(1.05)^5}=844.79$). Finance professionals use *net present value* (NPV) to mean the *present value* of future payments *minus the cost of the initial payment*; in the previous example this is \$844.79 - \$800 = \$44.79. In this book we use the term "net present value" (NPV) to mean its true finance sense. The Excel function **NPV** will always appear in boldface. We trust that you will rarely be confused

NPV depends on the discount rate

Let's revisit the pawnshop example on page000, and use Excel to create a table which shows the relation between the discount rate and the NPV. As the graph below shows, the higher the discount rate, the lower the net present value of the investment:

[4] There's a long history to this confusion, and it doesn't start with Microsoft. The original spreadsheet—Visicalc—(mistakenly) used "NPV" in the sense which Excel still uses today; this misnomer has been copied every since by all other spreadsheets: Lotus, Quattro, and Excel.

	A	B	C	D	E	F	G	H
1	**CALCULATING NET PRESENT VALUE (NPV) WITH EXCEL**							
2	r, interest rate	5%						
3								
4	**Year**	**Payment**	**Present value**					
5	0	-800	-800.00					
6	1	100	95.24	<-- =B6/(1+$B$2)^A6				
7	2	150	136.05	<-- =B7/(1+$B$2)^A7				
8	3	200	172.77					
9	4	250	205.68					
10	5	300	235.06					
11								
12	**NPV**							
13	Summing the present values		44.79	<-- =SUM(C5:C10)				
14	Using Excel's NPV function		44.79	<-- =NPV($B$2,B6:B10)+C5				
15								
16	**Discount rate**	**NPV**						
17	0%	200.00	<-- =NPV(A17,$B$6:$B$10)+$B$5					
18	1%	165.86	<-- =NPV(A18,$B$6:$B$10)+$B$5					
19	2%	133.36	<-- =NPV(A19,$B$6:$B$10)+$B$5					
20	3%	102.41						
21	4%	72.92						
22	5%	44.79						
23	6%	17.96						
24	6.6965%	0.00						
25	8%	-32.11						
26	9%	-55.48						
27	10%	-77.83						
28	11%	-99.21						
29	12%	-119.67						
30	13%	-139.26						
31	14%	-158.04						
32	15%	-176.03						
33	16%	-193.28						
34								
35								
36								
37								
38								
39								
40								



Note that we've indicated a special discount rate: When the discount rate is 6.6965%, the net present value of the investment is zero. This rate is referred to as the *internal rate of return (IRR)*, and we'll return to it in Section 000. For discount rates less than the IRR, the net present value is positive, and for discount rates greater than the IRR the net present value is negative.

Using NPV to choose between investments

In the examples discussed thus far, we've used NPV only to choose whether to undertake a particular investment or not. But NPV can also be used to choose between investments. Look at the following spreadsheet: You have $800 to invest, and you've been offered the choice between Investment A and Investment B. The spreadsheet below shows that at an interest rate of 15%, you should choose Investment B because it has a higher net present value. Investment A will increase your wealth by $219.06, whereas Investment B increases your wealth by $373.75.

	A	B	C	D
1	**USING NPV TO CHOOSE BETWEEN INVESTMENTS**			
2	Discount rate	15%		
3				
4	**Year**	**Investment A**	**Investment B**	
5	0	-800	-800	
6	1	250	600	
7	2	500	200	
8	3	200	100	
9	4	250	500	
10	5	300	300	
11				
12	NPV	219.06	373.75	<-- =NPV(B2,C6:C10)+C5

To summarize:

In using the NPV to choose between two positive-NPV investments, we choose the investment with the higher NPV.[5]

---

[5] There's a possible exception to this rule: If we neither have the cash nor can borrow the money to make the investment (the jargon is *cash constrained*), we may want to use the *profitability index* to choose between investments. The profitability index is defined as the ratio of the PV(future cash flows) to the investment's cost. See Chapter 3 for a discussion of this topic.

Nomenclature—Is it a *discount rate* or an *interest rate?*

In some of the examples above we've used *discount rate* instead of *interest rate* to describe the rate used in the net present value calculation. As you will see in further chapters of this book, the rate used in the NPV has several synonyms: Discount rate, interest rate, cost of capital, opportunity cost—these are but a few of the names for the rate that appears in the denominator of the NPV:

$$\frac{Cash\ flow\,in\ year\ t}{\underbrace{(1+r)^t}_{\substack{\uparrow \\ Discount\,rate \\ Interest\,rate \\ Cost\,of\ capital \\ Opportunity\ cost}}}$$

1.4. The internal rate of return (IRR)

In this section we discuss the *internal rate of return* (IRR):

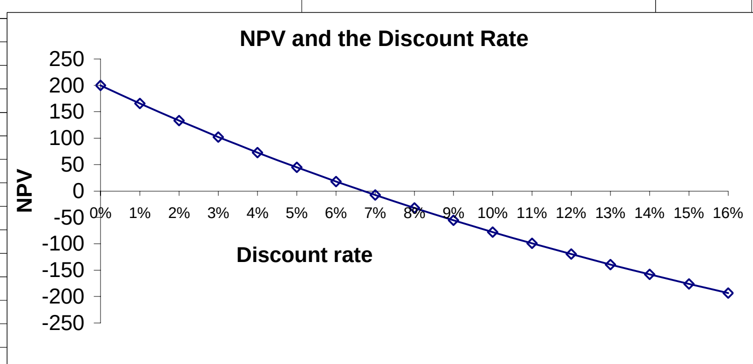
The IRR of a series of cash flows is the discount rate that sets the net present value of the cash flows equal to zero.

Before we explain in depth (in the next section) why you want to know the IRR, we explain how to compute it. Let's go back to the example on page000: If you pay \$800 today to your local pawnshop, the owner promises to pay you \$100 at the end of year 1, \$150 at the end of year 2, \$200 at the end of year 3, \$250 at the end of year 4 , and \$300 at the end of year 5. Discounting these cash flows at rate *r*, the NPV can be written:

$$NPV = -800 + \frac{100}{(1+r)} + \frac{150}{(1+r)^2} + \frac{200}{(1+r)^3} + \frac{250}{(1+r)^4} + \frac{300}{(1+r)^5}$$

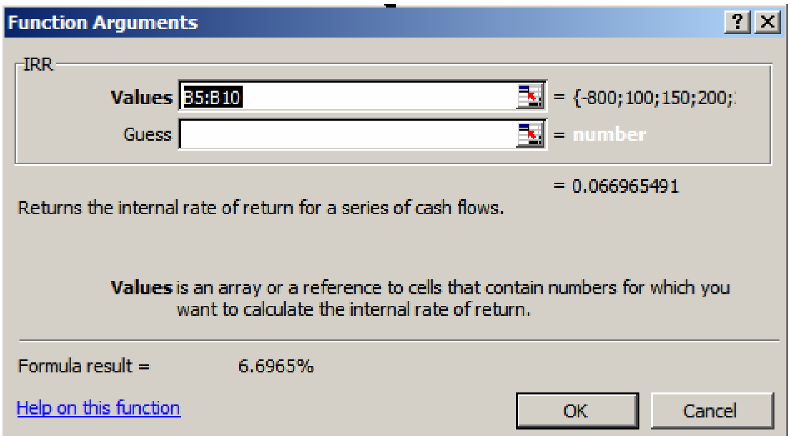
In cells B16:B32 of the spreadsheet below, we calculate the NPV for various discount rates. As you can see, somewhere between $r = 6\%$ and $r = 7\%$, the NPV becomes negative.

	A	B	C	D	E	F
1	**CALCULATING THE IRR WITH EXCEL**					
2	r, interest rate	6.6965%				
3						
4	**Year**	**Payment**				
5	0	-800				
6	1	100				
7	2	150				
8	3	200				
9	4	250				
10	5	300				
11						
12	NPV	0.00	<-- =NPV(B2,B6:B10)+B5			
13	IRR	6.6965%	<-- =IRR(B5:B10)			
14						
15	**Discount rate**	**NPV**				
16	0%	200.00	<-- =NPV(A16,$B$6:$B$10)+$B$5			
17	1%	165.86	<-- =NPV(A17,$B$6:$B$10)+$B$5			
18	2%	133.36	<-- =NPV(A18,$B$6:$B$10)+$B$5			
19	3%	102.41				
20	4%	72.92				
21	5%	44.79				
22	6%	17.96				
23	7%	-7.65				
24	8%	-32.11				
25	9%	-55.48				
26	10%	-77.83				
27	11%	-99.21				
28	12%	-119.67				
29	13%	-139.26				
30	14%	-158.04				
31	15%	-176.03				
32	16%	-193.28				
33						
34						
35						



In cell B13, we use Excel's **IRR** function to calculate the exact discount rate at which the NPV becomes 0. The answer is 6.6965%; at this interest rate, the NPV of the cash flows equals zero (look at cell B12). Using the dialog box for the Excel **IRR** function:

Dialog box for IRR function



Notice that we haven't used the second option ("Guess") to calculate our IRR. We discuss this option in Chapter 4.

What does the IRR mean?

Suppose you could get 6.6965% interest at the bank and suppose you wanted to save today to provide yourself with the future cash flows of the example on page000:

- To get \$100 at the end of year 1, you would have to put the present value $\frac{100}{1.06965} = 93.72$ in the bank today.
- To get \$150 at the end of year 2, you would have to put its present value $\frac{150}{(1.06965)^2} = 131.76$ in the bank today.
- And so on … (see the picture below)

The total amount you would have to save is \$800, exactly the cost of this investment opportunity. This is what we mean when we say that:

The internal rate of return is the compound interest rate you earn on an investment.

Time	0	1	2	3	4	5
Save for time 1's $100 $100/(1+6.6965%)	93.72	FV=93.72*(1+6.6965%) =$100.00				
Save for time 1's $150 $150/(1+6.6965%)^2	131.76		FV=131.76*(1+6.6965%)^2 = $150.00			
Save for time 3's $200 $200/(1+6.6965%)^3	164.66			FV=164.66*(1+6.6965%)^3 = $200.00		
Save for time 4's $250 $250/(1+6.6965%)^4	192.90				FV=192.90*(1+6.6965%)^4 =$250.00	
Save for time 5's $300 $300/(1+6.6965%)^5	216.95					FV=216.95*(1+6.6965%)^5 = $300.00
Total saving at time 0	**800.00**					

Using IRR to make investment decisions

The IRR is often used to make investment decisions. Suppose your Aunt Sara has been offered the following investment by her broker: For a payment of $1,000, a reputable finance company will pay her $300 at the end of each of the next four years. Aunt Sara is currently getting 5% on her bank savings account. Should she withdraw her money from the bank to make the investment? To answer the question, we compute the IRR of the investment and compare it to the bank interest rate:

	A	B	C
1	**USING IRR TO MAKE INVESTMENT DECISIONS**		
2	**Year**	**Cash flow**	
3	0	-1,000	
4	1	300	
5	2	300	
6	3	300	
7	4	300	
8			
9	IRR	7.71%	<-- =IRR(B3:B7)

The IRR of the investment, 7.71%, is greater than 5% Sara can earn on her alternative investment (the bank account). Thus she should make the investment.

Summarizing:

In using the IRR to make investment decisions, an investment with an IRR greater than the alternative rate of return is a good investment and an investment with an IRR less than the alternative rate of return is a bad investment.

Using IRR to choose between two investments

We can also use the internal rate of return to choose between two investments. Suppose you've been offered two investments. Both Investment A and Investment B cost $1,000, but they have different cash flows. If you're using the IRR to make the investment decision, then you would choose the investment with the *higher* IRR. Here's an example:

	A	B	C	D
1	**USING IRR TO CHOOSE BETWEEN INVESTMENTS**			
2	**Year**	**Investment A cash flows**	**Investment B cash flows**	
3	0	-1,000.00	-1,000.00	
4	1	450.00	550.00	
5	2	425.00	300.00	
6	3	350.00	475.00	
7	4	450.00	200.00	
8				
9	IRR	24.74%	22.26%	<-- =IRR(C3:C7)

We would choose Investment A, which has the higher IRR.

To summarize:

In using the IRR to choose between two comparable investments, we choose the investment which has the higher IRR. [This assumes that: 1) Both investments have IRR greater than the alternative rate. 2) The investments are of comparable risk.]

Using NPV and IRR to make investment decisions

In this chapter we have now developed two tools, NPV and IRR, for making investment decisions. We've also discussed two kinds of investment decisions. Here's a summary:

	"Yes or No": Choosing whether to undertake a single investment	**"Investment ranking": Comparing two investments which are mutually exclusive**
NPV criterion	The investment should be undertaken if its NPV > 0:	Investment *A* is preferred to investment *B* if NPV(A) > NPV(B)
IRR criterion	The investment should be undertaken if its IRR > *r*, where *r* is the appropriate discount rate.	Investment *A* is preferred to investment *B* if IRR(A) > IRR(B).

In Chapter 3 we discuss further implementation of these two rules and two decision problems.

1.5. What does IRR mean? Loan tables and investment amortization

In the previous section we gave a simple illustration of what we meant when we said that *the internal rate of return (IRR) is the compound interest rate that you earn on an asset*. This simple sentence—which is not easy to understand—underlies a slew of finance applications: When finance professionals discuss the "rate of return" on an investment or the "effective interest rate" on a loan, they are almost always refering to the IRR. In this section we explore some meanings of the IRR. Almost the whole of Chapter 2 is devoted to this topic.

A simple example

Suppose you buy an asset for \$200 today and that the asset has a promised payment of \$300 in one year. The IRR is 50%; to see this recall that the IRR is the interest rate which makes the NPV zero. Since the investment NPV $= -200 + \frac{300}{1+r}$, this means that the NPV is zero when $1+r = \frac{300}{200} = 1.5$. Solving this simple equation gives *r = 50%*.

Here's another way to think about this investment and its 50% IRR:

- At time zero you pay \$200 for the investment.
- At time one, the \$300 investment cash flow repays the initial \$200. The remaining \$100 represents a 50% return on the initial \$200 investment. This is the IRR.

  The IRR is the rate of return on an investment; it is the rate that repays, over the life of the asset, the initial investment in the asset and that pays interest on the outstanding investment balances.

A more complicated example

We now give a more complicated example, which illustrates the same point. This time, you buy an asset costing \$200. The asset's cash flow are \$130.91 at the end of year 1 and \$130.91 at the end of year 2. Here's our IRR analysis of this investment:

	A	B	C	D	E	F
1	**THE IRR AS A RATE OF RETURN ON AN INVESTMENT**					
2	IRR	20.00%	<-- =IRR({-200,130.91,130.91})			
3	**Year**	**Investment at beginning of year**	**Payment at end of year**	**Part of payment which is interest**	**Part of payment which is repayment of principal**	
4	1	200.00	130.91	40.00	90.91	
5	2	109.09	130.91	21.82	109.09	
6	3	0.00				
7						
8	=B4-E4			=$B$2*B4	=C4-D4	
9						
10		=B5-E5		=$B$2*B5		=C5-D5
11						

- The IRR for the investment is 20.00%. Note how we calculated this—we simply typed into cell B2 the formula **=IRR({-200,130.91,130.91})** (if you're going to use this method of calculating the IRR in Excel, you have to put the cash flows in the curly brackets).
- Using the 20% IRR, $40.00 (=20%*$200) of the first year's payment is interest, and the remainder—$90.91—is repayment of principal. Another way to think of the $40.00 is to consider that to buy the asset, you gave the seller the $200 cost of the asset. When he pays you $130.91 at the end of the year, $40 (=20%*$200) is interest—your payment for allowing someone else to use your money. The remainder, $90.91, is a partial repayment of the money lent out.
- This leaves the outstanding principal at the beginning of year 2 as $109.09. Of the $130.91 paid out by the investment at the end of year 2, $21.82 (=20%*109.09) is interest, and the rest (*exactly* $109.09) is repayment of principal.
- The outstanding principal at the beginning of year 3 (the year *after* the investment finishes paying out) is *zero*.

As in the first example of this section, the IRR is the rate of return on the investment—defined as the rate that repays, over the life of the asset, the initial investment in the asset and that pays interest on the outstanding investment balances.

Using future value, net present value, and internal rate of return—several problems

In the remaining sections we apply the concepts learned in the chapter to solve several common problems:

1.7 and 1.8. Saving for the future

1.9. Paying off a loan with "flat" payments of interest and principal

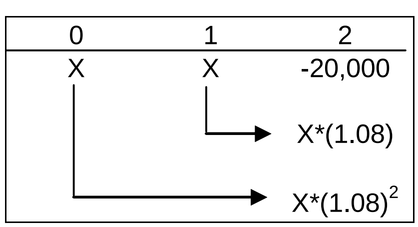
1.10. How long does it take to pay off a loan?

1.7. Saving for the future—buying a car for Mario

Mario wants to buy a car in 2 years. He wants to open a bank account and to deposit \$X today and \$X in one year. Balances in the account will earn 8%. How much should Mario deposit so that he has \$20,000 in 2 years? In this section we'll show you that:

In order to finance future consumption with a savings plan, the net present value of all the cash flows has to be zero. In the jargon of finance—the future consumption plan is *<u>fully funded</u>* *if the net present value of all the cash flows is zero.*

In order to see this, start with a graphical representation of what happens:



In year 2 Mario will have accumulated $X*(1.08)+X*(1.08)^2$. This should finance the \$20,000 car, so that:

$$\underbrace{X*(1.08)+X*(1.08)^2}_{\substack{\uparrow \\ \text{Future value of} \\ \text{deposits in 2 years}}} = \underbrace{20,000}_{\substack{\uparrow \\ \text{Desired accumulation}}}$$

Now subtract the \$20,000 from both sides of the equation and divide through by $(1.08)^2$:

$$\underbrace{X+\frac{X}{(1.08)}-\frac{20,000}{(1.08)^2}}_{\substack{\uparrow \\ \text{Net present value} \\ \text{of all cash flows}}}=0$$

We've proved it—in order to fully fund Mario's future purchase of the car, the net present value of all the payments has to be zero.

Excel solution

Of course, we won't leave it at this—we'll show you Mario's problem in Excel:

	A	B	C	D	E
1	**HELPING MARIO SAVE FOR A CAR**				
2	Deposit, X	8,903.13			
3	Interest rate	8.00%			
4	**Year**	**In bank, before deposit**	**Deposit or withdrawal**	**Total at beginning of year**	**End of year with interest**
5	0	0.00	8,903.13	8,903.13	9,615.38
6	1	9,615.38	8,903.13	18,518.52	20,000.00
7	2	20,000.00	(20,000.00)	0.00	0.00
8					
9		NPV of all deposits and payments	\$0.00	<-- =C5+NPV(B3,C6:C7)	

If he deposits \$8,903.13 in years 0 and 1, then the accumulation in the account at the beginning of year 2 will be exactly \$20,000 (cell B7). The NPV of all the payments (cell C9) is zero.

How did we actually arrive at \$8,903.13? We'll postpone this to the next section, where we discuss a somewhat more complicated and realistic version of the same problem.

1.8. Saving for the future—more realistic problems

In this section we present more complicated versions of Mario's problem from section 1.7. We start by trying to determine whether a young girl's parents are putting enough money aside to save for her college education. Here's the problem:

- On her tenth birthday Linda Jones's parents decide to deposit \$4,000 in a savings account for their daughter. They intend to put an additional \$4,000 in the account each year on her 11^{th}, 12^{th}, ..., 17^{th} birthdays.
- All account balances will earn 8% per year.
- On Linda's 18^{th}, 19^{th}, 20^{th}, and 21^{st} birthdays, her parents will withdraw \$20,000 to pay for Linda's college education.

Is the \$4,000 per year sufficient to cover the anticipated college expenses? We can easily solve this problem in a spreadsheet:

	A	B	C	D	E
1	**SAVING FOR COLLEGE**				
2	Interest rate	8%			
3	Annual deposit	4,000.00			
4	Annual cost of college	20,000			
5					
6	**Birthday**	**In bank on birthday, before deposit/withdrawal**	**Deposit or withdrawal at begin. of year**	**Total**	**End of year with interest**
7	10	0.00	4,000.00	4,000.00	4,320.00
8	11	4,320.00	4,000.00	8,320.00	8,985.60
9	12	8,985.60	4,000.00	12,985.60	14,024.45
10	13	14,024.45	4,000.00	18,024.45	19,466.40
11	14	19,466.40	4,000.00	23,466.40	25,343.72
12	15	25,343.72	4,000.00	29,343.72	31,691.21
13	16	31,691.21	4,000.00	35,691.21	38,546.51
14	17	38,546.51	4,000.00	42,546.51	45,950.23
15	18	45,950.23	-20,000.00	25,950.23	28,026.25
16	19	28,026.25	-20,000.00	8,026.25	8,668.35
17	20	8,668.35	-20,000.00	-11,331.65	-12,238.18
18	21	-12,238.18	-20,000.00	-32,238.18	-34,817.24
19					
20		NPV of all payments	-13,826.4037	<-- =NPV(B2,C8:C18)+C7	

By looking at the end-year balances in column E, the $4,000 is *not* enough—Linda and her parents will run out of money somewhere between her 19th and 20th birthdays.[6] By the end of her college career, they will be $34,817 "in the hole." Another way to see this is to look at the net present value calculation in cell C20: As we saw in the previous section, a combination savings/withdrawal plan is fully funded when the NPV of all the payments/withdrawals is zero. In cell C20 we see that the NPV is negative—Linda's plan is *underfunded*.

How much should Linda's parents put aside each year? There are several ways to answer this question, which we explore below.

[6] At the end of Linda's 19 year (row 16), there is $8,668.35 remaining in the account. At the end of the following year, there is a negative amount in the account.

Method 1: Trial and error

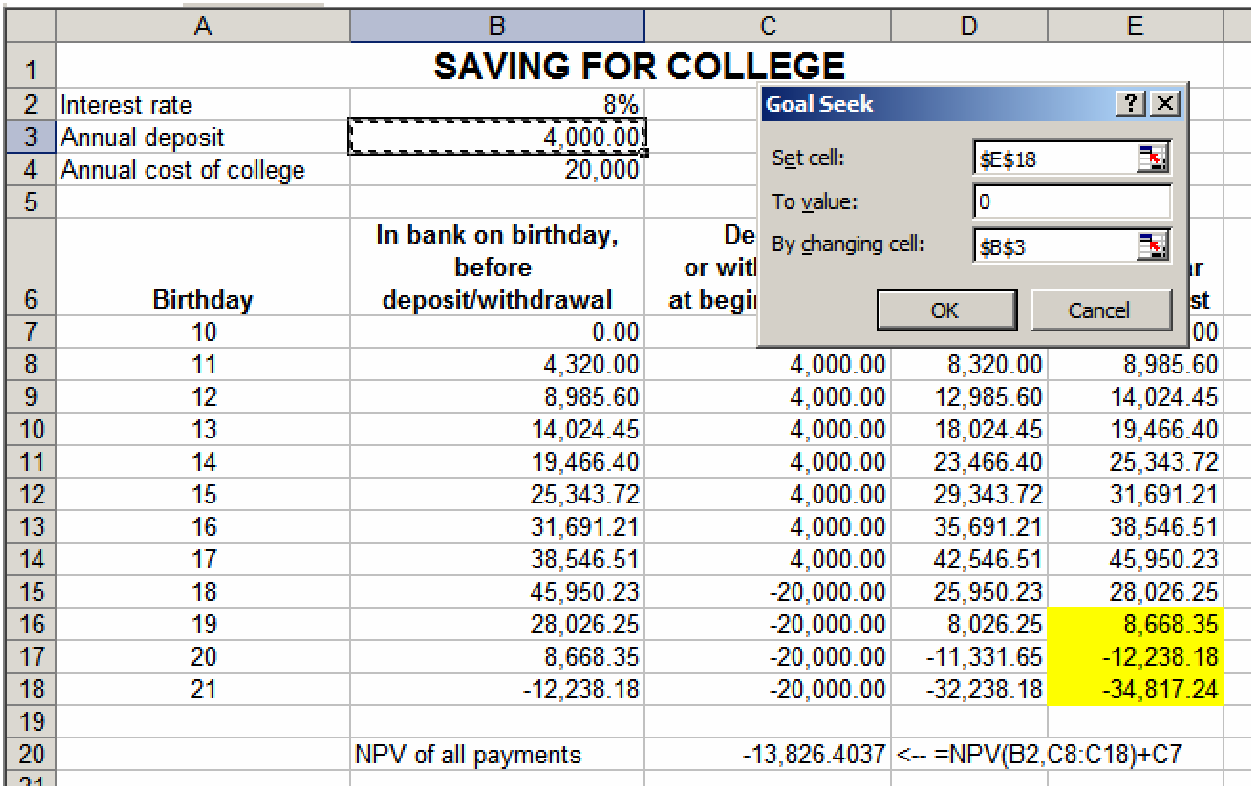
Assuming that you have written the spreadsheet correctly, you can "play" with cell B3 until cell E18 or cell C20 equals zero. Doing this shows that Linda's parents should have planned to deposit $6,227.78 annually:

	A	B	C	D	E
1	**SAVING FOR COLLEGE**				
2	Interest rate	8%			
3	Annual deposit	6,227.78			
4	Annual cost of college	20,000			
5					
6	**Birthday**	**In bank on birthday, before deposit/withdrawal**	**Deposit or withdrawal at begin. of year**	**Total**	**End of year with interest**
7	10	0.00	6,227.78	6,227.78	6,726.00
8	11	6,726.00	6,227.78	12,953.77	13,990.08
9	12	13,990.08	6,227.78	20,217.85	21,835.28
10	13	21,835.28	6,227.78	28,063.06	30,308.10
11	14	30,308.10	6,227.78	36,535.88	39,458.75
12	15	39,458.75	6,227.78	45,686.52	49,341.45
13	16	49,341.45	6,227.78	55,569.22	60,014.76
14	17	60,014.76	6,227.78	66,242.54	71,541.94
15	18	71,541.94	-20,000.00	51,541.94	55,665.29
16	19	55,665.29	-20,000.00	35,665.29	38,518.52
17	20	38,518.52	-20,000.00	18,518.52	20,000.00
18	21	20,000.00	-20,000.00	0.00	0.00
19					
20		NPV of all payments	0.0000	<-- =NPV(B2,C8:C18)+C7	

Notice that the net present value of all the payments (cell C20) is zero when the solution is reached. The future payouts are fully funded when the NPV of all the cash flows is zero.

Method 2: Using Excel's Goal Seek

Goal Seek is an Excel function that looks for a specific number in one cell by adjusting the value of a different cell (for a discussion of how to use **Goal Seek**, see Chapter 000). To solve our problem of how much to save, we can use **Goal Seek** to set E18 equal to zero. After hitting **Tools|Goal Seek**, we fill in the dialog box:



	A	B	C	D	E
1		SAVING FOR COLLEGE			
2	Interest rate	8%			
3	Annual deposit	4,000.00			
4	Annual cost of college	20,000			
5					
6	Birthday	In bank on birthday, before deposit/withdrawal	De or wit at begi		r st
7	10	0.00			00
8	11	4,320.00	4,000.00	8,320.00	8,985.60
9	12	8,985.60	4,000.00	12,985.60	14,024.45
10	13	14,024.45	4,000.00	18,024.45	19,466.40
11	14	19,466.40	4,000.00	23,466.40	25,343.72
12	15	25,343.72	4,000.00	29,343.72	31,691.21
13	16	31,691.21	4,000.00	35,691.21	38,546.51
14	17	38,546.51	4,000.00	42,546.51	45,950.23
15	18	45,950.23	-20,000.00	25,950.23	28,026.25
16	19	28,026.25	-20,000.00	8,026.25	8,668.35
17	20	8,668.35	-20,000.00	-11,331.65	-12,238.18
18	21	-12,238.18	-20,000.00	-32,238.18	-34,817.24
19					
20		NPV of all payments	-13,826.4037	<-- =NPV(B2,C8:C18)+C7	

When we hit "OK," Goal Seek looks for the solution. The result is the same as before: \$6,227.78.

Method 3: Using the Excel NPV formula

The method in this subsection involves the most preparation. Its advantage is that it leads to a very compact solution to the problem—a solution that doesn't require a long Excel table for its implementation. On the other hand, the formulas required are somewhat intricate (if you really hate formulas, skip this method!).

Linda's parents are going to make 8 deposits of $\$X$ each, starting today. The present value of these deposits is

$$X+\frac{X}{(1.08)}+\frac{X}{(1.08)^2}+\ldots+\frac{X}{(1.08)^7}=X\left(1+\frac{1}{(1.08)}+\frac{1}{(1.08)^2}+\ldots+\frac{1}{(1.08)^7}\right).$$

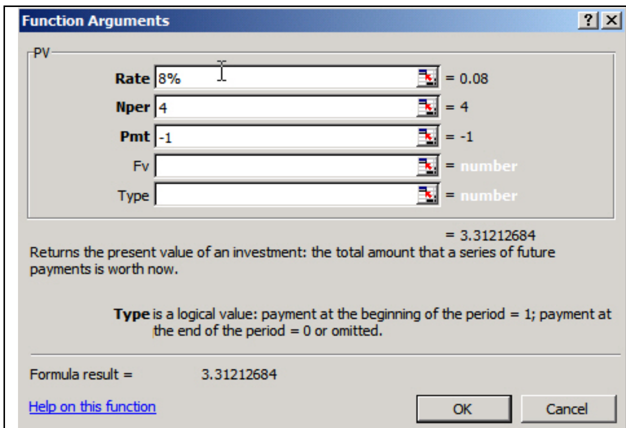
The account created will then have 4 withdrawals of $20,000, starting in year 8. The present value of these withdrawals is:

$$\frac{20,000}{(1.08)^8}+\frac{20,000}{(1.08)^9}+\frac{20,000}{(1.08)^{10}}+\frac{20,000}{(1.08)^{11}}=\frac{20,000}{1.08^7}*\left(\frac{1}{(1.08)}+\frac{1}{(1.08)^2}+\frac{1}{(1.08)^3}+\frac{1}{(1.08)^4}\right)$$

Setting these two equations equal allows us to solve for *X*:

$$X=\frac{\frac{20,000}{1.08^7}*\left(\frac{1}{(1.08)}+\frac{1}{(1.08)^2}+\frac{1}{(1.08)^3}+\frac{1}{(1.08)^4}\right)}{1+\frac{1}{(1.08)}+\frac{1}{(1.08)^2}+\ldots+\frac{1}{(1.08)^7}}$$

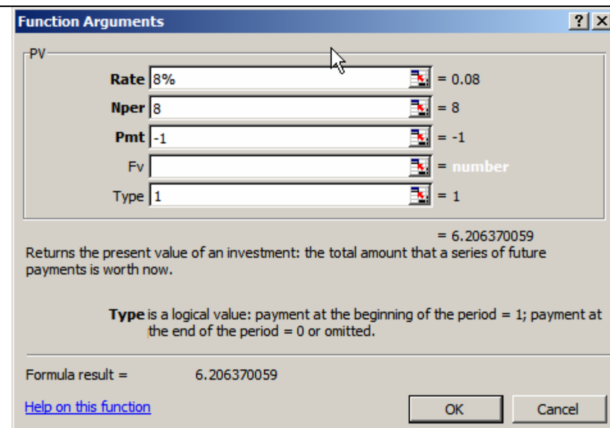
In Excel both the numerator and the denominator are computed by filling in the dialog box for the **PV** function:



The numerator:

$$\frac{1}{(1.08)}+\frac{1}{(1.08)^2}+\frac{1}{(1.08)^3}+\frac{1}{(1.08)^4}=3.1212684$$

to complete the numerator, we have to multiply by $20,000/(1.08)^7$.



The denominator:

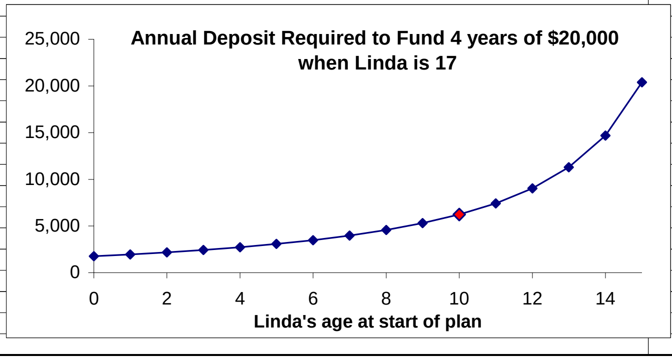
$$1+\frac{1}{(1.08)}+\frac{1}{(1.08)^2}+\ldots+\frac{1}{(1.08)^7}=6.206370059$$

Note that **Type** is 1 (payments at beginning of period).

Note that for both of these dialog boxes we've put in a negative payment **Pmt**. For the reason, refer to our discussion on page000.

Rows 2-9 of the following spreadsheet show how we use these two **PV** functions to solve for the annual deposit required:

	A	B	C	D
1	SAVING FOR COLLEGE--USING EXCEL FORMULAS ONLY			
2	Linda's age when plan started	10		
3	Linda's age at last deposit	17		
4	Number of deposits	8	<-- =B3-B2+1	
5	Number of withdrawals	4		
6	Annual cost of college	20,000		
7	Interest rate	8%		
8				
9	Annual deposit	6,227.78	<-- =(B6/(1+B7)^(B4-1))*PV(B7,4,-1)/PV(B7,B4,-1,,1)	
10				
11	**Linda's age today**	**Annual amount deposited**		
12	0	1,768.81	<-- =($B$6/(1+$B$7)^($B$3-A12))*PV($B$7,4,-1)/PV($B$7,$B$3-A12+1,-1,,1)	
13	1	1,962.73	<-- =($B$6/(1+$B$7)^($B$3-A13))*PV($B$7,4,-1)/PV($B$7,$B$3-A13+1,-1,,1)	
14	2	2,184.47	<-- =($B$6/(1+$B$7)^($B$3-A14))*PV($B$7,4,-1)/PV($B$7,$B$3-A14+1,-1,,1)	
15	3	2,439.68		
16	4	2,735.61		
17	5	3,081.72		
18	6	3,490.65		
19	7	3,979.61		
20	8	4,572.69		
21	9	5,304.68		
22	10	6,227.78		
23	11	7,423.96		
24	12	9,029.88		
25	13	11,291.47		
26	14	14,700.60		
27	15	20,404.92		
28				
29				
30				
31				



The formula in cell B9 is the solution:

$$=\underbrace{\left(B6/(1+B7)^\wedge(B4-1)\right)}_{\frac{20{,}000}{(1.08)^7}} * \underbrace{PV(B7,B5,-1)}_{\frac{1}{1.08}+\frac{1}{(1.08)^2}+\ldots+\frac{1}{(1.08)^4}} \;/\; \underbrace{PV(B7,B4,-1,,1)}_{1+\frac{1}{1.08}+\frac{1}{(1.08)^2}+\ldots+\frac{1}{(1.08)^7}}$$

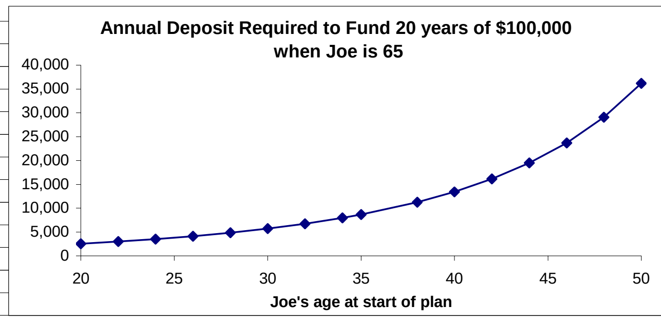
The problem as initially set out assumed that Linda was 10 years old today. The table in rows 12 – 27 shows the problem solution for other starting ages.[7]

[7] The table in rows 12-27 would be simpler to compute if we used **Data Table**. This advanced feature of Excel is explained in Chapter 30. The file **Chapter01.xls** on the disk accompanying *Principles of Finance with Excel* shows how to use **Data Table** to do the calculations in rows 12-27.

Pension plans

The savings problem of Linda's parents is exactly the same as that faced by an individual who wishes to save for his retirement. Suppose that Joe is 20 today and wishes to start saving so that when he's 65 he can have 20 years of $100,000 annual withdrawals. Adapting the previous spreadsheet, we get:

	A	B	C
1	**SAVING FOR RETIREMENT**		
2	Joe's age today	20	
3	Joe's age at last deposit	64	
4	Number of deposits	45	<-- =B3-B2+1
5	Number of withdrawals	20	
6	Annual withdrawal from age 65	100,000	
7	Interest rate	8%	
8			
9	Annual deposit	2,540.23	<-- =(B6/(1+B7)^(B4-1))*PV(B7,B5,-1)/PV(B7,B4,-1,,1)
10			
11	**Joe's age today**	**Annual amount deposited**	
12	20	2,540.23	<-- =($B$6/(1+$B$7)^($B$3-A12))*PV($B$7,$B$5,-1)/PV($B$7,$B$3-A12+1,-1,,1)
13	22	2,978.96	<-- =($B$6/(1+$B$7)^($B$3-A13))*PV($B$7,$B$5,-1)/PV($B$7,$B$3-A13+1,-1,,1)
14	24	3,496.73	<-- =($B$6/(1+$B$7)^($B$3-A14))*PV($B$7,$B$5,-1)/PV($B$7,$B$3-A14+1,-1,,1)
15	26	4,109.02	
16	28	4,834.85	
17	30	5,697.73	
18	32	6,727.03	
19	34	7,959.85	
20	35	8,666.90	
21	38	11,239.91	
22	40	13,430.03	
23	42	16,123.53	
24	44	19,471.60	
25	46	23,688.86	
26	48	29,090.61	
27	50	36,159.79	
28			
29			



In the table in rows 12 – 27 you see the power of compound interest: If Joe starts saving at age 20 for his retirement, an annual deposit of $2,540.23 will grow to provide him with his retirement needs of $100,000 per year for 20 years at age 65. On the other hand, if he starts saving at age 35, it will require $8,666.90 per year.

1.9. Computing annual "flat" payments on a loan—Excel's PMT function

You've just graduated from college and the balance on your student loan is $100,000. You now have to pay the loan off over 10 years at an annual interest rate of 10%. The payment is in "even payments"—meaning that you pay the same amount each year (although—as you'll soon see—the breakdown of each payment between interest and principal is different). How much will you have to pay off?

Suppose we denote the annual payment by *X*. The correct *X* has the property that the present value of all the payments equals the loan principal:

$$100{,}000 = \frac{X}{1.10} + \frac{X}{(1.10)^2} + \frac{X}{(1.10)^3} + \ldots + \frac{X}{(1.10)^{10}}$$

Rewriting the right-hand side slightly, you can see that

$$X = \frac{100{,}000}{\underbrace{\frac{1}{1.10} + \frac{1}{(1.10)^2} + \frac{1}{(1.10)^3} + \ldots + \frac{1}{(1.10)^{10}}}_{\substack{\uparrow \\ \text{This expression can} \\ \text{be calculated using} \\ \text{Excel's PV function}}}}$$

Here's all this in an Excel spreadsheet:

	A	B	C	D	E
1	**LOAN PAYMENT**				
2	Loan principal	100,000			
3	Loan interest	10%			
4	Years to pay off loan	10			
5	Annual payment	16,274.54	<-- =B2/PV(B3,B4,-1)		
6		16,274.54	<-- =PMT(B3,B4,-B2)		
7					
8	**Year**	**Principal at beginning of year**	**Payment at end year**	**Part of payment which is interest**	**Part of payment which is principal**
9	1	100,000.00	16,274.54	10,000.00	6,274.54
10	2	93,725.46	16,274.54	9,372.55	6,901.99
11	3	86,823.47	16,274.54	8,682.35	7,592.19
12	4	79,231.27	16,274.54	7,923.13	8,351.41
13	5	70,879.86	16,274.54	7,087.99	9,186.55
14	6	61,693.31	16,274.54	6,169.33	10,105.21
15	7	51,588.10	16,274.54	5,158.81	11,115.73
16	8	40,472.37	16,274.54	4,047.24	12,227.30
17	9	28,245.07	16,274.54	2,824.51	13,450.03
18	10	14,795.04	16,274.54	1,479.50	14,795.04

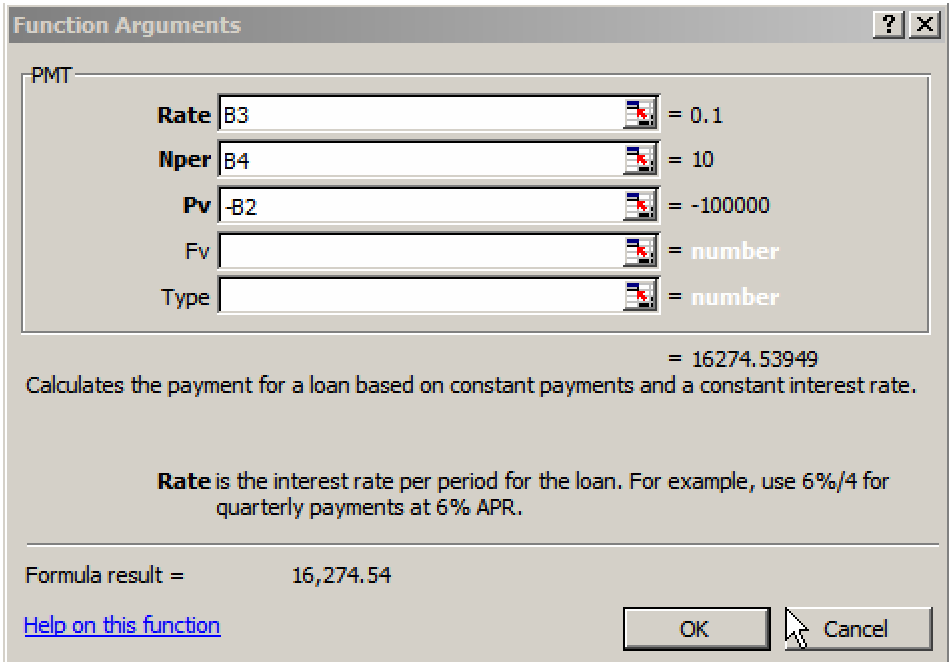
Nomenclature

The table in rows 9-18 above is often called a **loan amortization table** (“amortize”: to pay something off over time).

Here are two things to notice about the computation of this table:

- Excel has a function, **PMT**, which does this calculation directly (cell B6).

Dialog box for PMT function



Like some other Excel financial functions, **PMT** generates positive answers for negative entries in the **Pv** box.

- When we put all the payments in a loan table (rows 9-18 of the above spreadsheet) you can see the split of each end-year payment between interest on the outstanding principal at the beginning of the year and repayment of principal. If you were reporting to the Internal Revenue Service, the interest column (column D) is deductible for tax purposes; the repayment of principal column (column E) is not.

1.10. How long will it take to pay off a loan?

You're getting a $1,000 loan from the bank at 10% interest. The maximum payment you can make is $250 per year. How long will it take you to pay off the loan? There's an Excel function that answers this question, which we'll show you in a bit. But first let's do this the long way so we can understand the question. In the spreadsheet below we look at a loan table like the ones considered in section 1.5:

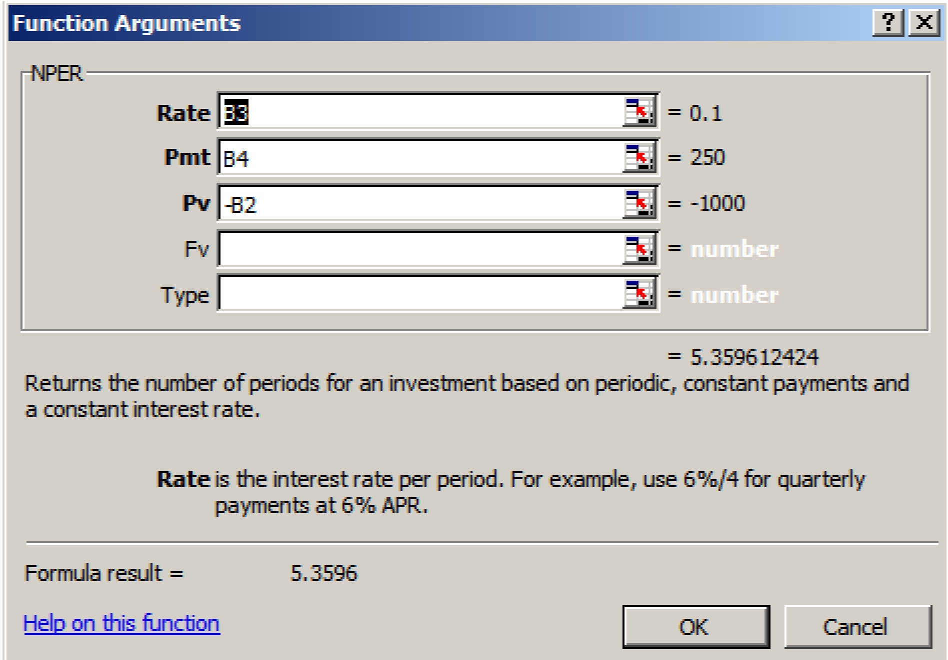
	A	B	C	D	E
1	**HOW LONG TO PAY OFF THIS LOAN?**				
2	Loan amount	1,000			
3	Interest rate	10%			
4	Annual payment	250			
5					
6	**Year**	**Principal beginning of year**	**Payment at end of year**	**Interest**	**Return of principal**
7	1	1,000.00	250.00	100.00	150.00
8	2	850.00	250.00	85.00	165.00
9	3	685.00	250.00	68.50	181.50
10	4	503.50	250.00	50.35	199.65
11	5	303.85	250.00	30.39	219.62
12	6	84.24	250.00	8.42	241.58
13					
14					
15					
16					
17					
18					
19					
20					
21					
22	Excel's **NPER** function	5.3596	<-- =NPER(B3,B4,-B2)		

Year 6 is the first year in which the return of principal at the end of the year is > principal at the beginning of the year. Meaning--sometime during year 6 you will have paid off the loan.

As you can see from row 12, year 6 is the first year in which the return of principal at the end of the year is bigger than the principal at the beginning of the year. Thus, sometime between 5 and 6 years you pay off the loan.

Excel's **NPER** function, illustrated in cell B24, provides an exact answer to this question:

Dialog box for NPER function



Like the functions **PMT**, **PV**, and **FV** discussed elsewhere in this chapter, the **NPER** function requires you to make the amount owed negative in order to get a positive answer.

1.11. An Excel note—building good financial models

If you've gotten this far in Chapter 1, you've probably put together a few basic Excel spreadsheets. You'll be doing lots more in the rest of this book, and you'll be amazed at the insights Excel gives you over even complicated financial problems.

We've chosen this place in the chapter to tell you a bit about financial modeling (that's what you've been doing …).

Here are three important rules for good Excel modeling:

- Put all the variables which are important (the fashionable jargon is "value drivers") at the top of your spreadsheet. In the "Saving for College" spreadsheet of page000, the three value drivers—the interest rate, the annual deposit, and the annual cost of college—are in the top left-hand corner of the spreadsheet:

	A	B	C	D	E
1	**SAVING FOR COLLEGE**				
2	Interest rate	8%			
3	Annual deposit	6,227.78			
4	Annual cost of college	20,000			
5					
6	**Birthday**	**In bank on birthday, before deposit/withdrawal**	**Deposit or withdrawal at begin. of year**	**Total**	**End of year with interest**
7	10	0.00	6,227.78	6,227.78	6,726.00
8	11	6,726.00	6,227.78	12,953.77	13,990.08
9	12	13,990.08	6,227.78	20,217.85	21,835.28
10	13	21,835.28	6,227.78	28,063.06	30,308.10
11	14	30,308.10	6,227.78	36,535.88	39,458.75
12	15	39,458.75	6,227.78	45,686.52	49,341.45
13	16	49,341.45	6,227.78	55,569.22	60,014.76
14	17	60,014.76	6,227.78	66,242.54	71,541.94
15	18	71,541.94	-20,000.00	51,541.94	55,665.29
16	19	55,665.29	-20,000.00	35,665.29	38,518.52
17	20	38,518.52	-20,000.00	18,518.52	20,000.00
18	21	20,000.00	-20,000.00	0.00	0.00
19					
20		NPV of all payments	0.0000	<-- =NPV(B2,C8:C18)+C7	

- Never use a number where a formula will also work. Using formulas instead of "hard-wiring" numbers means that when you change a parameter value, the rest of the spreadsheet changes appropriately. As an example—cell C20 in the above spreadsheet contains the formula **=NPV(B2,C8:C18)+C7**. We could have written this as **=NPV(8%,C8:C18)+C7**. But this means that changing the entry in cell B2 won't go through the whole model.

- Avoid the use of blank columns to accommodate cell "spillovers." Here's an example of a potentially bad model:

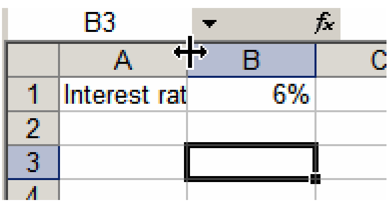
	A	B	C
1	Interest rate		6%

Because “Interest rate” has spilled over to column B, the author of this spreadsheet has decided to put the “6%” in column C. He’s going to end up getting confused (don’t ask why ...); he should have made column A wider and put the 6% in column B:

	A	B
1	Interest rate	6%

An Excel Note: Making a Column Wider

Widening the column is simple: Put the cursor on the break between columns A and B:



Clicking the left mouse button will expand the column to accommodate the widest cell. You can also “stretch” the column by holding the left mouse button down and moving the column width to the right.

Summing up

In this chapter we have covered the basic concepts of the time value of money:

- Future value (FV): The amount you will accumulate at some future date from deposits made in the present.
- Present value (PV): The value today of future anticipated cash flows.
- Net present value (NPV): The value today of a series of future cash flows, including the cost of acquiring these cash flows.

- We've gone to great pains to point out the difference between the finance concept of net present value (NPV) and the Excel **NPV** function. The Excel **NPV** function calculates the present value of the future cash flows, whereas the finance concept of NPV computes the present value of the future cash flows *minus* the initial cash flow.
- Internal rate of return (IRR): The compound interest rate paid by a series of cash flows, including the cost of their acquisition.

We have also showed you the Excel functions (**FV**, **PV**, **NPV**, **IRR**) which do these calculations and discussed some of their peculiarities. Finally, we have showed you how to do these calculations using formulas.

Exercises

1. You just put $600 in the bank and you intend to leave it there for 10 years. If the bank pays you 15% interest per year, how much will you have at the end of 10 years?

2. Your generous grandmother has just announced that she's opened a savings account for you with a deposit of $10,000. Moreover, she intends to make you 9 more similar gifts, at the end of this year, next year, etc. If the savings account pays 8% interest, how much will you have accumulated at the end of 10 years (one year after the last gift)?

Suggestion: Do this problem 2 ways, as shown below: a) take each amount and calculate its future value in year 10 (as illustrated in cells C7:C16) and then sum them; b) use Excel's **FV** function, noting that here the amounts come at the *beginning* of the year (you'll need to enter "1" in the **Type** option as described in Section 1).

	A	B	C	D
3	Interest rate	8.00%		
4				
5				
6	**Year**	**Gift**	**Future value in year 10**	
7	0	10,000	21,589.25	<-- =B7*(1+$B$3)^(10-A7)
8	1	10,000		
9	2	10,000		
10	3	10,000		
11	4	10,000		
12	5	10,000		
13	6	10,000		
14	7	10,000		
15	8	10,000		
16	9	10,000		
17				
18	Total (summing C7:C16)			
19	Using FV function			

3. Your uncle has just announced that he's going to give you $10,000 per year at the end of each of the next 4 years (he's less generous than your grandmother ...). If the relevant interest rate is 7%, what's the value today of this promise? (If you're going to use **PV** to do this problem note that the **Type** option is 0 or omitted.)

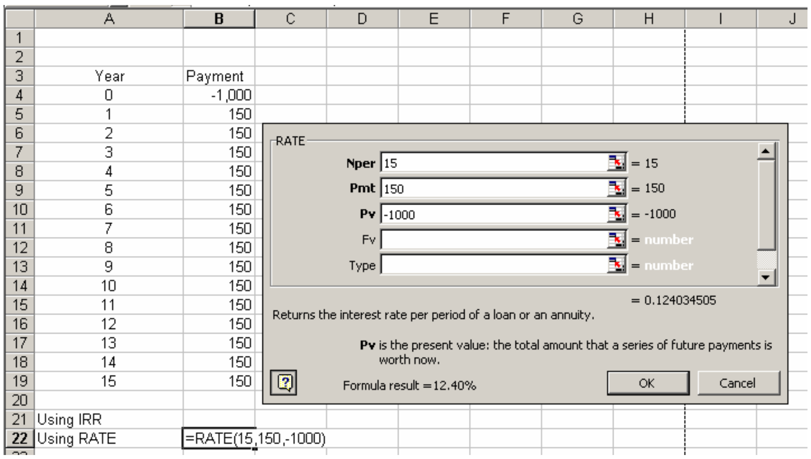
4. What is the present value of a series of 4 payments, each $1,000, to be made at the end of years 1, 2, 3, 4? Assume that the interest rate is 14%.

Suggestion: Do this problem 2 ways, as shown in rows 11 and 12 below.

	A	B	C	D	E
3	Interest rate	14%			
4					
5	Year	Payment	PV		
6	1	1,000	877.19	<-- =B6/(1+$B$3)^A6	
7	2	1,000			
8	3	1,000			
9	4	1,000			
10					
11	Total of C6:C9				
12	Using NPV function				

5. Screw-'Em-Good Corp. has just announced a revolutionary security: If you pay SEG $1,000 now, you will get back $150 at the end of each of the next 15 years. What is the IRR of this investment?

Suggestion: Do this problem two ways—once using Excel's **IRR** function and once using Excel's **RATE** function (illustrated below).



6. Make-'Em-Happy Corp. (MEH) has a different security for sale: You pay MEH $1,000 today and the company will give you back $100 at the end of the first year, $200 at the end of year 2, ... , $1000 at the end of year 10.

a. Calculate the IRR of this investment.

b. Show an amortization table for the investment.

7. You are thinking about buying a $1,000 bond issued by the Appalachian Development Authority (ADA). The bond will pay $120 interest at the end of each of the next 5 years. At the end of year 6, the bond will pay $1,120 (this is its face value of $1,000 plus the interest). If the relevant discount rate is 7%, how much is the present value of the bond's future payments?

8. Look at the pension problem in Section 1.8, page000. Answer the following questions:

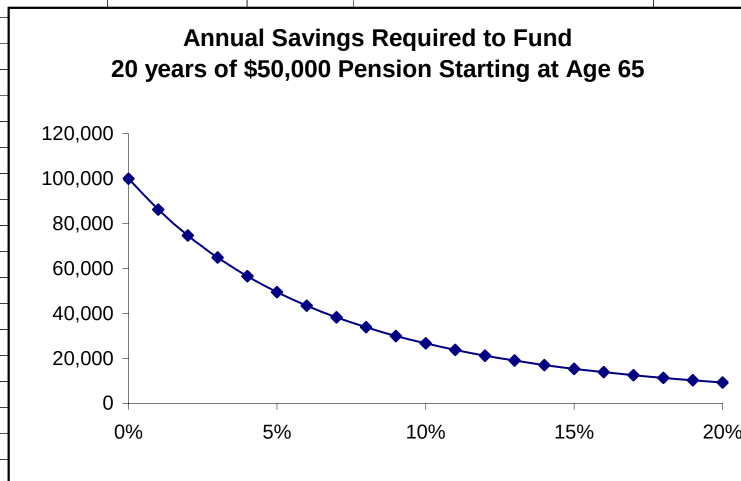
8.a. What if the desired annual pension is $100,000? How much does a 55 year-old have to save annually? The CD-ROM which accompanies the book contains the following template:

	A	B	C	D	E
1	**SAVING FOR THE FUTURE**				
2					
3	Your age today	35			
4	Retirement age	65			
5	Planned age of demise	85			
6					
7	Annual desired pension payout	100,000			
8	Annual payment	???			
9	Interest rate	8%			
10					
11	**Your age**	**Account balance, beg. year**	**Deposit or withdrawal beginning of year**	**Interest earned during year**	**Total in account end of year**
12	55				
13	56				
14	57				
15	58				
16	59				
17	60				
18	61				
19	62				
20	63				
21	64				
22	65				

8.b. Suppose you are 35 years old and you wish to save until you are 65. You wish to withdraw $50,000 per year at the beginning of your 65th, 66th, ..., 89th year. How much would you have to save if the interest rate is 10%?

9. Return to the pension problem discussed in Section 1.8, page000. Use Excel to make a graph showing the relation between the amount saved and the interest rate. Your graph should look like:

	B	C	D	E	F	G	H	I
20	**Interest rate**	**Saving**						
21			<-- Data table header (hidden)					
22	0%	100,000.00						
23	1%	86,241.57						
24	2%	74,665.98						
25	3%	64,888.48						
26	4%	56,597.56						
27	5%	49,540.14						
28	6%	43,509.98						
29	7%	38,338.41						
30	8%	33,887.08						
31	9%	30,042.08						
32	10%	26,709.35						
33	11%	23,810.92						
34	12%	21,282.00						
35	13%	19,068.53						
36	14%	17,125.28						
37	15%	15,414.25						
38	16%	13,903.45						
39	17%	12,565.85						
40	18%	11,378.50						
41	19%	10,321.93						
42	20%	9,379.48						



Note: If you're going to use the formula approach used in the book, you have to modify the formula a bit to make it work for interest = 0%. The existing formula in section 3 is:

$$X = \frac{\begin{pmatrix} annual \\ pension \\ payout \end{pmatrix} * \left(1 - \left(\frac{1}{1+r}\right)^{20}\right)}{(1+r)^{10}\left(1 - \left(\frac{1}{1+r}\right)^{10}\right)},$$

but when $r = 0$, the denominator in this expression becomes 0. On the other hand, when $r = 0$ it is clear that the payout is $X = \frac{\begin{pmatrix} annual \\ pension \\ payout \end{pmatrix} * \begin{pmatrix} number \\ of\ payout \\ years \end{pmatrix}}{\begin{pmatrix} number \\ of\ payment \\ years \end{pmatrix}}$. Use Excel's **If** function to modify the formula in the Section 1.8 spreadsheet.

10. If you deposit $25,000 today, Union Bank offers to pay you $50,000 at the end of 10 years. What is the interest rate?

11. Assuming that the interest rate is 5%, which of the following is more valuable?

11.a. $5,000 today

11.b. $10,000 at the end of 5 years

11.c. $9,000 at the end of 4 years

11.d. $300 a year in perpetuity (meaning: forever), with the first payment at the end of this year

12. You receive a $15,000 signing bonus from your new employer and decide to invest it for two years. Your banker suggests two alternatives, which both require a commitment for the full two years. The first alternative will earn 8% per year for both years. The second alternative earns 6% for the first year, and 10% for the second year. Interest compounds annually.
Which should you choose?

13. Your annual salary is $100,000. You are offered two options for a severance package. Option 1 pays you 6 months salary now. Option 2 pays you and your heirs $6,000 per year forever (first payment at the end of this year). If your required return is 11 percent, which option should you choose?

14. Today is your 40th birthday. You expect to retire at age 65 and actuarial tables suggest that you will live to be 100. You want to move to Hawaii when you retire. You estimate that it will

cost you $200,000 to make the move (on your 65th birthday), and that your annual living expenses will be $25,000 a year after that. You expect to earn an annual return of 7% on your savings.

14.a. How much will you need to have saved by your retirement date?

14.b. You already have $50,000 in savings. How much would you need to save at the end of each of the next 25 years to be able to afford this retirement plan?

14.c. If you did not have any current savings and did not expect to be able to start saving money for the next 5 years (that is, your first savings payment will be made on your 45th birthday), how much would you have to set aside each year after that to be able to afford this retirement plan?

15. You have just invested $10,000 in a new fund that pays $1,500 at the end of the next 10 years. What is the compound rate of interest being offered in the fund? (Suggestion: Do this problem two ways: Using Excel's **IRR** function and using Excel's **Rate** function.)

16. John is turning 13 today. His birthday resolution is to start saving towards the purchase of a car that he wants to buy on his 18th birthday. The car costs $15,000 today, and he expects the price to grow at 2% per year.

John has heard that a local bank offers a savings account which pays an interest rate of 5% per year. He plans to make 6 contributions of $1,000 each to the savings account (the first contribution to be made today); he will use the funds in the account on his 18th birthday as a down payment for the car, financing the balance through the car dealer.

He expects the dealer to offer the following terms for financing: 7 equal yearly payments (with the first payment due one year after he takes possession of the car); an annual interest rate of 7%.

16.a. How much will John need to finance through the dealer?

16.b. What will be the amount of his yearly payment to the dealer?

(Hint: This is like the college savings problem discussed in Section 1.8.)

17. Mary has just completed her undergraduate degree from Northwestern University and is already planning on entering an MBA program four years from today. The tuition will be \$20,000 per year for two years, paid at the beginning of each year. In addition, Mary would like to retire 15 years from today and receive a pension of \$60,000 every year for 20 years and receive the first payment 15 years from today. Mary can borrow and lend as much as she likes at a rate of 7%, compounded annually. In order to fund her expenditures, Mary will save money at the end of years 1-3 and at the end of years 6-14.

- Calculate the constant annual dollar amount that Mary must save at the end of each of these years to cover all of her expenditures (tuition and retirement)?

Note: Just to remove all doubts, here are the cash flows:

	A	B	C
1	MARY		
2			
3	**Year**	**Future expenses**	**Future savings**
4	0		
5	1		$X
6	2		$X
7	3		$X
8	4	20,000	
9	5	20,000	
10	6		$X
11	7		$X
12	8		$X
13	9		$X
14	10		$X
15	11		$X
16	12		$X
17	13		$X
18	14		$X
19	15	60,000	
20	16	60,000	
21	17	60,000	
22	18	60,000	
23	19	60,000	
24	20	60,000	
25	21	60,000	
26	22	60,000	
27	23	60,000	
28	24	60,000	
29	25	60,000	
30	26	60,000	
31	27	60,000	
32	28	60,000	
33	29	60,000	
34	30	60,000	
35	31	60,000	
36	32	60,000	
37	33	60,000	
38	34	60,000	

18. You are the CFO of Termination, Inc. Your company has 40 employees, each earning $40,000 per year. Employee salaries grow at 4% per year. Starting from next year, and every second year thereafter, 8 employees retire and no new employees are recruited. Your company has in place a retirement plan that entitles retired workers to an annual pension which is equal to their annual salary at the moment of retirement. Life expectancy is 20 years after retirement, and the annual pension is paid at year-end. The return on investment is 10% per year. What is the total value of your pension liabilities?

19. You are 30 today and are considering studying for an MBA. You just received your annual salary of $50,000 and expect it to grow by 3% per year. MBAs typically earn $60,000 upon graduation, with salaries growing by 4% per year.

The MBA program you're considering is a full-time, 2-year program that costs $20,000 per year, payable at the end of each study year. You want to retire on your 65th birthday. The relevant discount rate is 8%.[8] Is it worthwhile for you to quit your job in order to do an MBA (ignore income taxes)? What is the internal rate of return of the MBA?

20. You're 55 years old today, and you wish to start saving for your pension. Here are the parameters:

- You intend to make a deposit today and at the beginning of each of the next 9 years (that is, on your 55th, 56th, ... , 64th birthdays).
- Starting from your 65th birthday until your 84th, you would like to withdraw $50,000 per year (no plans for after that).
- The interest rate is 12%

    20.a. How much should you deposit in each of the initial years in order to fully fund the withdrawals?

    20.b. If you start saving at age 45, what is the answer?

[8] Meaning: Your MBA is an investment like any other investment. On other investments you can earn 8% per year; the MBA has to be judged against this standard.

20.c. (More difficult) Set up the formula for the savings amount so that you can solve for various starting ages. Do a sensitivity analysis which shows the amount you need to save as a function of the age at which you start saving.

21. Section 1.8 of this chapter discusses the problem of Linda Jones's parents, who wish to save for Linda's college education. The setup of the problem implicitly assumes that the bank will let the Jones's borrow from their savings account and will charge them the same 8% it was paying on positive balances. This is unlikely!

In this problem you are asked to program the following spreadsheet: In it you will assume that the bank pays Linda's parents 8% on positive account balances but charges them 10% on negative balances.

If Linda's parents can only deposit $4,000 per year in the years preceding college, how much will they owe the bank at the beginning of year 22 (the year after Linda finishes college)?

	A	B	C	D	E
1	**SAVING FOR COLLEGE**				
2	Interest rates				
3	On positive balances	8%			
4	On negative balances	10%			
5	Annual deposit	4,000.00			
6	Annual cost of college	20,000			
7					
8	**Birthday**	**In bank on birthday, before deposit/withdrawal**	**Deposit or withdrawal at begin. of year**	**Total**	**End of year with interest**
9	10		4,000.00		
10	11		4,000.00		
11	12		4,000.00		
12	13		4,000.00		
13	14		4,000.00		
14	15		4,000.00		
15	16		4,000.00		
16	17		4,000.00		
17	18		-20,000.00		
18	19		-20,000.00		
19	20		-20,000.00		
20	21		-20,000.00		
21	22				

Excel note: In order to set up this spreadsheet you will need to use the Excel **If** function (if you are not familiar with this function, see Chapter 28).

22. A fund of $10,000 is set up to pay $250 at the end of each year indefinitely. What is the fund's IRR? (There's no Excel function that answers this question—use some logic!)

23. In the spreadsheet below we calculate the future value of 5 deposits of $100, with the first deposit made at time 0. As shown in Section 1.????, this calculation can also be made using the Excel function **=FV(interest,periods,-amount,,1)**.

23.a. Show that you can also compute this by **=FV(interest,periods,-amount)*(1+interest)**.

23.b. Can you explain why **FV(r,5,-100,,1)=FV(r,5,-100)*(1+r)**?

	A	B	C	D	E	F
1	**FUTURE VALUE**					
2	Interest	6%				
3						
4	**Year**	**Account balance, beg. year**	**Deposit at beginning of year**	**Interest earned during year**	**Total in account at end of year**	
5	1	0.00	100.00	6.00	106.00	<-- =B5+C5+D5
6	2	106.00	100.00	12.36	218.36	<-- =B6+C6+D6
7	3	218.36	100.00	19.10	337.46	
8	4	337.46	100.00	26.25	463.71	
9	5	463.71	100.00	33.82	597.53	

Appendix: Algebraic Present Value Formulas

Most of the computations in the chapter can also be done with one basic bit of high-school algebra relating to the sum of a geometric series. Suppose you want to find the sum of a geometric series of *n* numbers $a + aq + aq^2 + aq^3 + ... + aq^{n-1}$. In the jargon of geometric series:

a is the *first term*

q is the *ratio* between terms (the number by which the previous term is multiplied to get the next term)

n is the *number of terms*

Denote the sum of the series by *S*: $S = a + aq + aq^2 + aq^3 + ... + aq^{n-1}$. In high school you learned a trick to find the value of *S*:

1. Multiply *S* by *q*:

$$qS = \quad aq + aq^2 + aq^3 + \ldots + aq^{n-1} + aq^n$$

2. Subtract *qS* from *S*:

$$\begin{aligned} S &= a + aq + aq^2 + aq^3 + \cdots + aq^{n-1} \\ -qS &= -\left(\quad aq + aq^2 + aq^3 + \ldots + aq^{n-1} + aq^n \right) \end{aligned}$$

$$(1-q)S = a - aq^n \Rightarrow S = \frac{a(1-q^n)}{1-q}$$

In the remainder of this appendix we apply this formula to a variety of situations covered in the chapter.

Future value a constant payment

This topic is covered in section 1.1. The problem there is to find the value of $100 deposited annually over 10 years, with the first payment today:

$S = 100*(1.06)^{10} + 100*(1.06)^{9} + ... + 100*(1.06) = ???$

For this geometric series:

$a = \text{ first term} = 100*1.06^{10}$

$q = ratio = \frac{1}{1.06}$

n = *number of terms = 10*

The formula gives $S = \frac{a\left(1-q^n\right)}{1-q} = \frac{100*1.06^{10}\left(1-\left(\frac{1}{1.06}\right)^{10}\right)}{1-\frac{1}{1.06}} = 1397.16$, where we have done the calculation in Excel:

	A	B	C
1	**FUTURE VALUE FORMULA**		
2	First term, a	179.0848	<-- =100*1.06^10
3	Ratio, q	0.943396	<-- =1/1.06
4	Number of terms, n	10	
5			
6	Sum	1,397.16	<-- =B2*(1-B3^B4)/(1-B3)
7	Excel PV function	1,397.16	<-- =FV(6%,B4,-100,,1)

Substituting symbols for the numerical values we get:

$$\begin{array}{c}\text{Future value of n payments} \\ \text{at end of year n, at interest r} \\ \text{first payment today}\end{array} = \frac{Payment*\left(1+r\right)^n\left(1-\left(\frac{1}{1+r}\right)^n\right)}{1-\frac{1}{1+r}} = \underbrace{\text{FV(r,n,-1,,1)}}_{\uparrow \atop \text{The Excel function}}$$

Present value of an annuity

We can also apply the formula to find the present value of an annuity. Suppose, for example, that we want to calculate the present value of an annuity of \$150 per year for 5 years:

$$\frac{150}{(1.06)}+\frac{150}{(1.06)^2}+\frac{150}{(1.06)^3}+\frac{150}{(1.06)^4}+\frac{150}{(1.06)^5}.$$

For this annuity:

$$a = \textit{first term} = \frac{150}{1.06}.^{9}$$

$$q = ratio = \frac{1}{1.06}$$

n= *number of terms* = *5*.

Thus the present value of the annuity becomes:

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{\frac{150}{1.06}\left(1-\left(\frac{1}{1.06}\right)^5\right)}{1-\frac{1}{1.06}} = 631.85 = \underbrace{\text{PV(6\%,5,-150)}}_{\substack{\uparrow \\ \text{The Excel function}}}.$$

We can work this out in a spreadsheet:

	A	B	C
1	**ANNUITY FORMULAS**		
2	First term, a	141.509434	<-- =150/1.06
3	Ratio, q	0.943396226	<-- =1/1.06
4	Number of terms, n	5	
5			
6	Sum	631.85	<-- =B2*(1-B3^B4)/(1-B3)
7	Excel PV function	631.85	<-- =PV(6%,5,-150)

Cleaning up the formula (a bit)

Standard textbooks often manipulate the annuity formula to make it look "better." Here's an example of something you might see in a textbook:

---

[9] If you're like most of the rest of humanity, you (mistakenly) thought that the first term was $a = 150$. But look at the series—the first term actually is $\frac{150}{1.06}$. So there you are.

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{\dfrac{annual\ \ payment}{(1+r)}\left(1-\left(\dfrac{1}{1+r}\right)^n\right)}{1-\dfrac{1}{1+r}}$$

$$= \frac{annual\ \ payment}{r}\left(1-\left(\frac{1}{1+r}\right)^n\right)$$

This is not a different annuity formula—it's just an algebraic simplification of the formula we've been using. If you put it in Excel you'll get the same answer (and in our opinion, there's no point in the simplification).

The present value of series of growing payments

Suppose we're trying to apply the formula to the following series:

$$\frac{150}{(1.06)} + \frac{150*(1.10)}{(1.06)^2} + \frac{150*(1.10)^2}{(1.06)^3} + \frac{150*(1.10)^3}{(1.06)^4} + \frac{150*(1.10)^4}{(1.06)^5}$$

Here there are five payments, the first of which is \$150; this payment grows at an annual rate of 10%. We can apply the formula:

$$a = first\ \ term = \frac{150}{1.06}.$$

$$q = ratio = \frac{1.10}{1.06}$$

n= number of terms = 5.

In the following spreadsheet, you can see that the formula and the Excel **NPV** function give the same answer for the present value:

	A	B	C
1	**A CONSTANT-GROWTH CASHFLOW**		
2	First term, a	141.5094	<-- =150/1.06
3	Ratio, q	1.037736	<-- =1.1/1.06
4	Number of terms, n	5	
5			
6	Sum	763.00	<-- =B2*(1-B3^B4)/(1-B3)
7			
8	**Year**	**Payment**	
9	1	150.00	
10	2	165.00	<-- =B9*1.1
11	3	181.50	<-- =B10*1.1
12	4	199.65	
13	5	219.62	
14			
15	Present value	763.00	<-- =NPV(6%,B9:B13)

Notice that the formula in cell B6 is more compact than Excel's **NPV** function. **NPV** requires you to list all the payments, whereas the formula in cell B6 requires only several lines (think about finding the present value of a very long series of growing payments—clearly the formula is more efficient).

The present value of a constant growth annuity

An annuity is a series of annual payments; a constant growth annuity is an annuity whose payments grow at a constant rate. Here's an example of such a series:

$$\frac{20}{(1.10)}+\frac{20*(1.05)}{(1.10)^2}+\frac{20*(1.05)^2}{(1.10)^3}+\frac{20*(1.05)^3}{(1.10)^4}+\frac{20*(1.05)^4}{(1.10)^5}+\ldots$$

We can fit this into our formula:

$$a = \textit{first term} = \frac{20}{1.10}$$

$$q = ratio = \frac{1.05}{1.10}$$

n=*number of terms*=∞.

The formula gives:

$$S=\frac{a\left(1-q^n\right)}{1-q}=\frac{\frac{20}{1.10}\left(1-\left(\frac{1.05}{1.10}\right)^n\right)}{1-\frac{1.05}{1.10}}.$$

When $n\to\infty$, $\left(\frac{1.05}{1.10}\right)^n\to 0$, so that:

$$S=\frac{a\left(1-q^n\right)}{1-q}=\frac{\frac{20}{1.10}\left(1-\left(\frac{1.05}{1.10}\right)^n\right)}{1-\frac{1.05}{1.10}}=\frac{\frac{20}{1.10}}{1-\frac{1.05}{1.10}}=\frac{20}{0.10-0.05}=400.$$

Warning: You have to be careful! This version of the formula only works because the growth rate of 5% is smaller than the discount rate of 10%. The discounted sum of an infinite series of constantly-growing payments only exists when the growth rate *g* is less than the discount rate *r*.

Here's a general formula:

$$\begin{array}{c}\text{sum of}\\\text{constant-growth}\\\text{annuity}\end{array}=\frac{CF}{(1+r)}+\frac{CF*(1+g)}{(1+r)^2}+\frac{CF*(1+g)^2}{(1+r)^3}+\ldots=\frac{\frac{CF}{(1+r)}\left(1-\left(\frac{1+g}{1+r}\right)^{\infty}\right)}{1-\frac{1+g}{1+r}}$$

$$=\begin{cases}\dfrac{CF}{r-g} & when\ |g|<|r|\\ undefined & otherwise\end{cases}$$

To summarize:

The present value of a constant-growth annuity—a series of cash flows with first term CF which grows at rate g—that is discounted at rate r is $\frac{CF}{r-g}$, *provided g < r.*

We use this formula in Chapter 6, when we discuss the valuation of stocks using discounted dividends (the "Gordon dividend model").

CHAPTER 3: INTRODUCTION TO CAPITAL BUDGETING*

slight bug fix: September 7, 2003

Chapter contents

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance with Excel* by Simon Benninga, which will be published in 2004. Much more material is posted on the PFE website (http://finance.wharton.upenn.edu/~benninga/pfe.html). Check with the author before distributing this draft (though you will probably get permission). All the material is copyright and the rights belong to the author and MIT Press.

Overview

Capital budgeting is finance jargon for the process of deciding whether to undertake an investment project. There are two standard concepts used in capital budgeting: net present value (NPV) and internal rate of return (IRR). Both of these concepts were introduced in Chapter 1; in this chapter we discuss their application to capital budgeting. Here are some of the topics covered:

- Should you undertake a specific project? We call this the "yes-no" decision, and we show how both NPV and IRR answer this question.
- Ranking projects: If you have several alternative investments, only one of which you can choose, which should you undertake?
- Should you use IRR or NPV? Sometimes the IRR and NPV decision criteria give different answers to the yes-no and the ranking decisions. We discuss why this happens and which criterion should be used for capital budgeting (if there's disagreement).
- Sunk costs. How should we account for costs incurred in the past?
- The cost of foregone opportunities
- Salvage values and terminal values
- Incorporating taxes into the valuation decision. This issue is dealt with briefly in Section 3.7. We return to it at greater length in Chapters 4 – 6.

Finance concepts discussed

- IRR
- NPV
- Project ranking using NPV and IRR
- Terminal value
- Taxation and calculation of cash flows
- Cost of foregone opportunities
- Sunk costs

Excel functions used

- NPV
- IRR
- Data table

3.1. The NPV rule for judging investments and projects

In preceding chapters we introduced the basic NPV and IRR concepts and their application to capital budgeting. We start off this chapter by summarizing each of these rules—the NPV rule in this section and the IRR rule in the following section.

Here's a summary of the decision criteria for investments implied by the net present value:

The NPV rule for deciding whether or not a specific project is worthwhile: Suppose we are considering a project which has cash flows CF_0 , CF_1 , CF_2 , ..., CF_N . Suppose that the appropriate discount rate for this project is r. Then the NPV of the project is

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_N}{(1+r)^N} = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}.$$

Rule: A project is worthwhile by the NPV rule if its NPV > 0.

The NPV rule for deciding between two mutually exclusive projects: Suppose you are trying to decide between two projects *A* and *B*, each of which can achieve the same objective. For example: your company needs a new widget machine, and the choice is between widget machine *A* or machine *B*. You will buy either *A* or *B* (or perhaps neither machine, but you will certainly not buy both machines. In finance jargon these projects are "mutually exclusive."

Suppose project *A* has cash flows $CF_0^A, CF_1^A, CF_2^A, \ldots, CF_N^A$ and that project *B* has cash flows $CF_0^B, CF_1^B, CF_2^B, \ldots, CF_N^B$.

Rule: Project A is preferred to project B if:

$$NPV(A) = CF_0^A + \sum_{t=1}^{N} \frac{CF_t^A}{(1+r)^t} > CF_0^B + \sum_{t=1}^{N} \frac{CF_t^B}{(1+r)^t} = NPV(B)$$

The logic of both the NPV rules presented above is that the *present value* of a project's cash flows—$PV = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}$— is the economic value today of the project. Thus—if we have correctly chosen the discount rate *r* for the project—the PV is what we ought to be able to sell the project for in the market.[1] The net present value is the *wealth increment* produced by the project, so that NPV > 0 means that a project adds to our wealth:

$$NPV = \underbrace{CF_0}_{\substack{\uparrow \\ \text{Initial cash} \\ \text{flow required} \\ \text{to implement} \\ \text{the project.} \\ \text{This is usually} \\ \text{a negative number.}}} + \underbrace{\sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}}_{\substack{\uparrow \\ \text{Market value} \\ \text{of future cash} \\ \text{flows.}}} .$$

An initial example

To set the stage let's assume that you're trying to decide whether to undertake one of two projects. Project A involves buying expensive machinery which produces a better product at a lower cost. The machines for Project A cost \$1000 and if purchased you anticipate that the project will produce cash flows of \$500 per year for the next 5 years. Project B's machines are cheaper, costing \$800, but they produce smaller annual cash flows of \$420 per year for the next 5 years. We'll assume that the correct discount rate is 12 percent.

Suppose we apply the NPV criterion to Projects A and B:

[1] This assumes that the discount rate is "correctly chosen," by which we mean that it is appropriate to the riskiness of the project's cash flows. For the moment we fudge the question of how to choose discount rates; this topic is discussed in Chapter 5.

	A	B	C	D
1	**TWO PROJECTS**			
2	Discount rate	12%		
3				
4	**Year**	**Project A**	**Project B**	
5	0	-1000	-800	
6	1	500	420	
7	2	500	420	
8	3	500	420	
9	4	500	420	
10	5	500	420	
11				
12	NPV	802.39	714.01	<-- =NPV($B$2,C6:C10)+C5

Both projects are worthwhile, since each has a positive NPV. If we have to choose between the projects, then Project A is preferred to Project B because it has the higher NPV.

An Excel Note

We reiterate our Excel note from Chapter 1: Excel's **NPV** function computes the present value of *future* cash flows; this does not correspond to the *finance notion* of NPV, which includes the initial cash flow. In order to calculate the finance NPV concept in the spreadsheet, we have to include the initial cash flow. Hence in cell B12, the NPV is calculated as =NPV($B$2,B6:B10)+B5 and in cell C12 the calculation is =NPV($B$2,C6:C10)+C5.

3.2. The IRR rule for judging investments

An alternative to using the NPV criterion for capital budgeting is to use the internal rate of return (IRR). Recall from Chapter 1 that the IRR is defined as the discount rate for which the NPV equals zero. It is the compound rate of return which you get from a series of cash flows.

Here are the two decision rules for using the IRR in capital budgeting:

The IRR rule for deciding whether a specific investment is worthwhile: Suppose we are considering a project that has cash flows CF_0, CF_1, CF_2, ..., CF_N.

IRR is an interest rate such that :

$$CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N} = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+k)^t} = 0 \cdot$$

Rule: If the appropriate discount rate for a project is *r*, you should accept the project if its IRR > *r* and reject it if its IRR < *r*.

The logic behind the IRR rule is that the IRR is the compound return you get from the project. Since *r* is the project's required rate of return, it follows that if IRR > *r*, you get more than you require.

The IRR rule for deciding between two competing projects: Suppose you are trying to decide between two mutually exclusive projects *A* and *B* (meaning: both projects are ways of achieving the same objective, and you will choose at most one of the projects). Suppose project *A* has cash flows $CF_0^A, CF_1^A, CF_2^A, \ldots, CF_N^A$ and that project *B* has cash flows $CF_0^B, CF_1^B, CF_2^B, \ldots, CF_N^B$.

Rule: Project A is preferred to project B if IRR(A) > IRR(B).

Again the logic is clear: Since the IRR gives a project's compound rate of return, if we choose between two projects using the IRR rule, we prefer the higher compound rate of return.

Applying the IRR rule to our Projects *A* and *B*, we get:

	A	B	C	D
1	**TWO PROJECTS**			
2	Discount rate	12%		
3				
4	**Year**	**Project A**	**Project B**	
5	0	-1000	-800	
6	1	500	420	
7	2	500	420	
8	3	500	420	
9	4	500	420	
10	5	500	420	
11				
12	IRR	41%	44%	<-- =IRR(C5:C10)

Both Project *A* and Project *B* are worthwhile, since each has an IRR > 12%, which is our relevant discount rate. If we have to choose between the two projects by using the IRR rule, Project *B* is preferred to Project *A* because it has a higher IRR.

3.3. NPV or IRR, which to use?

We can sum up the NPV and the IRR rules as follows:

	"Yes or No": Choosing whether to undertake a single project	**"Project ranking": Comparing two mutually exclusive projects**
NPV criterion	The project should be undertaken if its NPV > 0:	Project *A* is preferred to project *B* if NPV(A) > NPV(B)
IRR criterion	The project should be undertaken if its IRR > *r*, where *r* is the appropriate discount rate.	Project *A* is preferred to project *B* if IRR(A) > IRR(B).

Both the NPV rules and the IRR rules look logical. In many cases your investment decision—to undertake a project or not, or which of two competing projects to choose—will be the same whether or not you use NPV or IRR. There are some cases, however (such as that of Projects *A* and *B* illustrated above), where NPV and IRR give different answers. In our present value analysis Project *A* won out because is NPV was greater than Project *B*'s. In our IRR analysis of the same projects, Project *B* was chosen because it had the higher IRR. In such cases we should always use the NPV to decide between projects. The logic is that if individuals are interested in maximizing their wealth, they should use NPV, which measures the incremental wealth from undertaking a project.

3.4. The "Yes-No" criterion: When do IRR and NPV give the same answer?

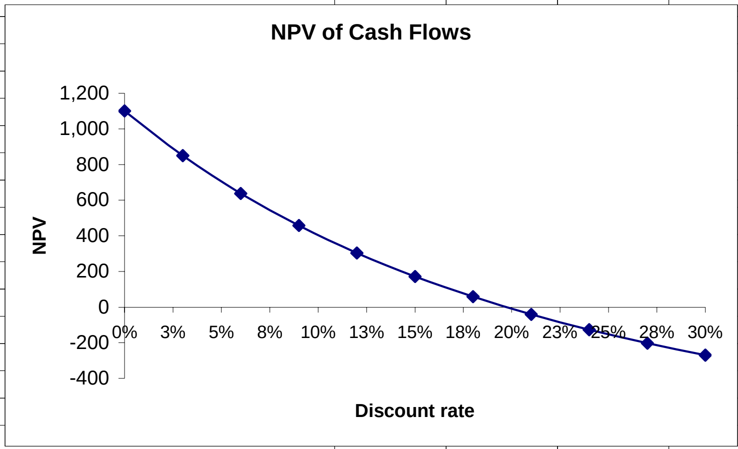
Consider the following project: The initial cash flow of –$1,000 represents the cost of the project today, and the remaining cash flows for years 1-6 are projected future cash flows. The discount rate is 15 percent:

	A	B	C
1	**SIMPLE CAPITAL BUDGETING EXAMPLE**		
2	Discount rate	15%	
3			
4	**Year**	**Cash flow**	
5	0	-1,000	
6	1	100	
7	2	200	
8	3	300	
9	4	400	
10	5	500	
11	6	600	
12			
13	PV of future cash flows	1,172.13	<-- =NPV(B2,B6:B11)
14	NPV	172.13	<-- =B5+NPV(B2,B6:B11)
15	IRR	19.71%	<-- =IRR(B5:B11)

The NPV of the project is $172.13, meaning that the present value of the project's future cash flows ($1,172.13) is greater than the project's cost of $1,000.00. Thus the project is worthwhile.

If we graph the project's NPV we can see that the IRR—the point where the NPV curve crosses the *x*-axis—is very close to 20% As you can see in cell B15, the actual IRR is 19.71%.

	A	B	C	D	E	F	G
18	Discount rate	NPV					
19	0%	1,100.00	<-- =$B$5+NPV(A19,$B$6:$B$11)				
20	3%	849.34	<-- =$B$5+NPV(A20,$B$6:$B$11)				
21	6%	637.67					
22	9%	457.83					
23	12%	304.16					
24	15%	172.13					
25	18%	58.10					
26	21%	-40.86					
27	24%	-127.14					
28	27%	-202.71					
29	30%	-269.16					
30							
31							
32							
33							
34							
35							
36							
37							
38							



Accept or reject? Should we undertake the project?

It is clear that the above project is worthwhile:

- Its NPV > 0, so that by the NPV criterion the project should be accepted.
- Its IRR of 19.71% > the project discount rate of 15%, so that by the IRR criterion the project should be accepted.

A general principle

We can derive a general principle from this example:

For conventional projects, projects with an initial negative cash flow and subsequent non-negative cash flows ($CF_0 < 0, CF_1 \geq 0, CF_2 \geq 0, ..., CF_N \geq 0$ *), the NPV and IRR criteria lead to the same "Yes-No" decision: If the NPV criterion indicates a "Yes" decision, then so will the IRR criterion (and vice versa).*

3.5. Do NPV and IRR produce the same project rankings?

In the previous section we saw that for conventional projects, NPV and IRR give the same "Yes-No" answer about whether to invest in a project. In this section we'll see that NPV and IRR do not necessarily *rank* projects the same, even if the projects are both conventional.

Suppose we have two projects and can choose to invest in only one. The projects are *mutually exclusive*: They are both ways to achieve the same end, and thus we would choose only one. In this section we discuss the use of NPV and IRR to rank the projects. To sum up our results before we start:

- Ranking projects by NPV and IRR can lead to possibly contradictory results. Using the NPV criterion may lead us to prefer one project whereas using the IRR criterion may lead us to prefer the other project.
- Where a conflict exists between NPV and IRR, the project with the larger NPV is preferred. That is, the NPV criterion is the correct criterion to use for capital budgeting. This is not to impugn the IRR criterion, which is often very useful. However, NPV is preferred over IRR because it indicates the *increase in wealth* which the project produces.

An example

Below we show the cash flows for Project A and Project B. Both projects have the same initial cost of $500, but have different cash flow patterns. The relevant discount rate is 15 percent.

	A	B	C	D
1	**RANKING PROJECTS WITH NPV AND IRR**			
2	Discount rate	15%		
3				
4	**Year**	**Project A**	**Project B**	
5	0	-500	-500	
6	1	100	250	
7	2	100	250	
8	3	150	200	
9	4	200	100	
10	5	400	50	
11				
12	NPV	74.42	119.96	<-- =C5+NPV(B2,C6:C10)
13	IRR	19.77%	27.38%	<-- =IRR(C5:C10)

Comparing the projects using IRR: If we use the IRR rule to choose between the projects, then B is preferred to A, since the IRR of Project B is higher than that of Project A.

Comparing the projects using NPV: Here the choice is more complicated. When the discount rate is 15% (as illustrated above), the NPV of Project B is higher than that of Project A. In this case the IRR and the NPV agree: Both indicate that Project B should be chosen. Now suppose that the discount rate is 8%; in this case the NPV and IRR rankings conflict:

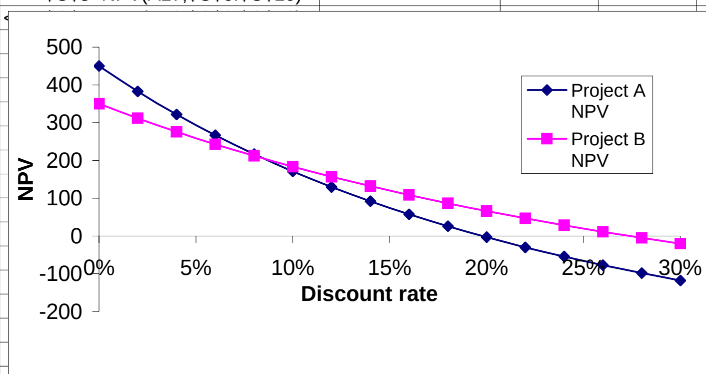
	A	B	C	D
1	**RANKING PROJECTS WITH NPV AND IRR**			
2	Discount rate	8%		
3				
4	**Year**	**Project A**	**Project B**	
5	0	-500	-500	
6	1	100	250	
7	2	100	250	
8	3	150	200	
9	4	200	100	
10	5	400	50	
11				
12	NPV	216.64	212.11	<-- =C5+NPV(B2,C6:C10)
13	IRR	19.77%	27.38%	<-- =IRR(C5:C10)

In this case we have to resolve the conflict between the ranking on the basis of NPV (A is preferred) and ranking on the basis of IRR (B is preferred). As we stated in the introduction to this section, the solution to this question is that you should choose on the basis of NPV. We explore the reasons for this later on, but first we discuss a technical question.

Why do NPV and IRR give different rankings?

Below we build a table and graph that show the NPV for each project as a function of the discount rate:

	A	B	C	D	E	F	G	H
15	TABLE OF NPVs AND DISCOUNT RATES							
16		Project A NPV	Project B NPV					
17	0%	450.00	350.00	<-- =$C$5+NPV(A17,$C$6:$C$10)				
18	2%	382.57	311.53	<				
19	4%	321.69	275.90					
20	6%	266.60	242.84					
21	8%	216.64	212.11					
22	10%	171.22	183.49					
23	12%	129.85	156.79					
24	14%	92.08	131.84					
25	16%	57.53	108.47					
26	18%	25.86	86.57					
27	20%	-3.22	66.00					
28	22%	-29.96	46.66					
29	24%	-54.61	28.45					
30	26%	-77.36	11.28					
31	28%	-98.39	-4.93					
32	30%	-117.87	-20.25					
33								



From the graph you can see why contradictory rankings occur:

- Project B has a higher IRR (27.38%) than project A (19.77%). (Remember that the IRR is the point at which the NPV curve cuts the *x*-axis.)
- When the discount rate is low, Project A has a higher NPV than project B, but when the discount rate is high, project B has a higher NPV. There is a crossover point (in the next subsection you will see that this point is 8.51%) that marks the disagreement/agreement range. Thus:

	Discount rate < 8.51%	Discount rate = 8.51%	Discount rate > 8.51%
NPV criterion	A preferred: NPV(A) > NPV(B)	Indifferent between A and B: NPV(A) = NPV(B)	B preferred: NPV(B) > NPV(A)
IRR criterion	B always preferred to A, since IRR(B) > IRR(A)		

Calculating the crossover point

The crossover point—which we claimed above was 8.51% — is the discount rate at which the NPV of the two projects is equal. A bit of formula manipulation will show you that *the crossover point is the IRR of the differential cash flows*. To see this, suppose that for some rate *r*, NPV(A) = NPV(B):

$$NPV(A) = CF_0^A + \frac{CF_1^A}{(1+r)} + \frac{CF_2^A}{(1+r)^2} \ldots + \frac{CF_N^A}{(1+r)^N}$$
$$= CF_0^B + \frac{CF_1^B}{(1+r)} + \frac{CF_2^B}{(1+r)^2} \ldots + \frac{CF_N^B}{(1+r)^N} = NPV(B)$$

Subtracting and rearranging shows that *r* must be the IRR of the differential cash flows:

$$CF_0^A - CF_0^B + \frac{CF_1^A - CF_1^B}{(1+r)} + \frac{CF_2^A - CF_2^B}{(1+r)^2} \ldots + \frac{CF_N^A - CF_N^B}{(1+r)^N} = 0$$

We can use Excel to calculate this crossover point. To do this, we first set up the *differential cash flows* (you can see them in column D below):

	A	B	C	D	E
34	**Calculating the crossover point**				
35	**Year**	**Project A**	**Project B**	**Cashflow(A) - cashflow(B)**	
36	0	-500	-500	0	<-- =B36-C36
37	1	100	250	-150	<-- =B37-C37
38	2	100	250	-150	
39	3	150	200	-50	
40	4	200	100	100	
41	5	400	50	350	
42					
43	IRR			8.51%	<-- =IRR(D36:D41)

What to use? NPV or IRR?

Let's go back to the initial example and suppose that the discount rate is 8%:

	A	B	C	D
1	**RANKING PROJECTS WITH NPV AND IRR**			
2	Discount rate	8%		
3				
4	**Year**	**Project A**	**Project B**	
5	0	-500	-500	
6	1	100	250	
7	2	100	250	
8	3	150	200	
9	4	200	100	
10	5	400	50	
11				
12	NPV	216.64	212.11	<-- =C5+NPV(B2,C6:C10)
13	IRR	19.77%	27.38%	<-- =IRR(C5:C10)

In this case we know there is disagreement between the NPV (which would lead us to choose Project A) and the IRR (by which we choose Project B). Which is correct?

The answer to this question is that we should—for the case where the discount rate is 8%—choose using the NPV (that is, choose Project A). This is just one example of the general principal discussed in Section 3 that *using the NPV is always preferred*, since the NPV is the additional *wealth* that you get, whereas IRR is the compound rate of return. The economic assumption is that consumers maximize their wealth, not their rate of return.

Where is this chapter going?

Until this point in the chapter, we've discussed general principles of project choice using the NPV and IRR criteria. The following sections discuss some specifics:

- Ignoring sunk costs and using marginal cash flows (Section 3.6)
- Incorporating taxes and tax shields into capital budgeting calculations (Section 3.7)
- Incorporating the cost of foregone opportunities (Section 3.9)
- Incorporating salvage values and terminal values (Section 3.11)

3.6. Capital budgeting principle: Ignore sunk costs and consider only marginal cash flows

This is an important principle of capital budgeting and project evaluation: Ignore the cash flows you can't control and look only at the *marginal cash flows*—the outcomes of financial decisions you can still make. In the jargon of finance: Ignore *sunk costs*, costs that have already been incurred and are thus not affected by future capital budgeting decisions.

Here's an example: You recently bought a plot of land and built a house on it. Your intention was to sell the house immediately, but it turns out you did a horrible job. The house and land cost you \$100,000, but in its current state the house can't be sold. A friendly local contractor has offered to make the necessary repairs, but these will cost \$20,000; your real estate broker estimates that even with these repairs you'll never sell the house for more than \$90,000. What should you do?

- "My father always said 'Don't throw good money after bad.'" If this is your approach, you won't do anything. This attitude is typified in column B below, which shows that—if you make the repairs you will lose 25% on your money.
- "My mother was a finance prof, and she said "Don't cry over spilt milk. Look only at the marginal cash flows" These turn out to be pretty good. In column C below you see that making the repairs will give you a 350% return on your $20,000.

	A	B	C	D
1	**IGNORE SUNK COSTS**			
2	House cost	100,000		
3	Fix up cost	20,000		
4				
5	**Year**	**Cash flow wrong!**	**Cash flow right!**	
6	0	-120,000	-20,000	
7	1	90,000	90,000	
8	IRR	-25%	350%	<-- =IRR(C6:C7)

Of course your father was wrong and your mother right (this often happens): Even though you made some disastrous mistakes (you never should have built the house in the first place), you should—at this point—ignore the sunk cost of $100,000 and make the necessary repairs.

3.7. Capital budgeting principle: Don't forget the effects of taxes—Sally and Dave's condo investment

In this section we discuss the capital budgeting problem faced by Sally and Dave, two business-school grads who are considering buying a condominium apartment and renting it out for the income.

We use Sally and Dave and their condo to emphasize the place of taxes in the capital budgeting process. No one needs to be told that taxes are very important.[2] In the capital budgeting process, the cash flows that are to be discounted are *after-tax cash flows*. We postpone a fuller discussion of this topic to Chapters 6 and 7, where we define the concept of *free cash flow*. For the moment we concentrate on a few obvious principles, which we illustrate with the example of Sally and Dave's condo investment.

Sally and Dave—fresh out of business school with a little cash to spare—are considering buying a nifty condo as a rental property. The condo will cost \$100,000, and (in this example at least) they're planning to buy it with all cash. Here are some additional facts:

- Sally and Dave figure they can rent out the condo for \$24,000 per year. They'll have to pay property taxes of \$1,500 annually and they're figuring on additional miscellaneous expenses of \$1,000 per year.
- All the income from the condo has to be reported on their annual tax return. Currently Sally & Dave have a tax rate of 30%, and they think this rate will continue for the foreseeable future.
- Their accountant has explained to them that they can depreciate the full cost of the condo over 10 years—each year they can charge \$10,000 depreciation ($= \frac{\textit{condo cost}}{10\textit{-year depreciable life}}$) against the income from the condo.[3] This means that they can expect to pay \$3,450 in income taxes per year if they buy the condo and rent it out and have net income from the condo of \$8,050:

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[2] Will Rogers: "The difference between death and taxes is death doesn't get worse every time Congress meets."

[3] You may want to read the sidebar on depreciation before going on.

	A	B	C
1	**SALLY & DAVE'S CONDO**		
2	**Cost of condo**	100,000	
3	**Sally & Dave's tax rate**	30%	
4			
5	**Annual reportable income calculation**		
6	Rent	24,000	
7	Expenses		
8	Property taxes	-1,500	
9	Miscellaneous expenses	-1,000	
10	Depreciation	-10,000	
11	Reportable income	11,500	<-- =SUM(B6:B10)
12	Taxes (rate = 30%)	-3,450	<-- =-B3*B11
13	Net income	8,050	<-- =B11+B12

SIDEBAR: What is depreciation?

In computing the taxes they owe, Sally and Dave get to subtract expenses from their income. Taxes are computed on the basis of the *income before taxes* (=income – expenses – depreciation – interest). When Sally and Dave get the rent from their condo, this is *income*—money earned from their asset. When Sally and Dave pay to fix the faucet in their condo, this is an *expense*—a cost of doing business.

The cost of the condo is neither income nor an expense. It's a *capital investment*—money paid for an asset that will be used over many years. Tax rules specify that each year part of the capital investments can be taken off the income ("expensed," in the accounting jargon). This reduces the taxes paid by the owners of the asset and takes account of the fact that the asset has a limited life.

There are many depreciation methods in use. The simplest method is *straight-line depreciation*. In this method the asset's annual depreciation is a percentage of its initial cost. In the case of Sally and Dave, for example, we've specified that the asset is depreciated over 10 years. this results in annual depreciation charges of

$$straight\text{-}line\ depreciation = \frac{initial\ asset\ cost}{depreciable\ life\ span} = \frac{\$100,000}{10} = \$10,000\ annually$$

In some cases depreciation is taken on the asset cost minus its salvage value: If you think that the asset will be worth \$20,000 at the end of its life (this is the salvage value), then the annual straight-line depreciation might be \$8,000:

$$\begin{aligned} \begin{matrix} straight\text{-}line\ depreciation \\ with\ salvage\ value \end{matrix} &= \frac{initial\ asset\ cost - salvage\ value}{depreciable\ life\ span} \\ &= \frac{\$100,000 - \$20,000}{10} = \$8,000\ annually \end{aligned}$$

Accelerated depreciation

Although historically depreciation charges are related to the life span of the asset, in many cases this connection has been lost. Under United States tax rules, for example, an asset classified as having a 5-year depreciable life (trucks, cars, and some computer equipment is in this category) will be depreciated over 6 years (yes *six*) at 20%, 32%, 19.2%, 11.52%, 11.52%, 5.76% in each of the years 1, 2, … , 6. Notice that this method *accelerates* the depreciation charges—more than one-sixth of the depreciation is taken annually in years 1-3 and less in later years. Since—as we show in the text—depreciation ultimately saves taxes, this is in the interest of the asset's owner, who now gets to take more of the depreciation in the early years of the asset's life.

Two ways to calculate the cash flow

In the previous spreadsheet you saw that Sally and Dave's net income was $8,050. In this section you'll see that the *cash flow produced by the condo* is much more that this amount. It all has to do with depreciation: Because the depreciation is an expense for tax purposes but not a cash expense, the *cash flow* from the condo rental is different. So even though the net income from the condo is $8,050, the annual cash flow is $18,050—you have to add back the depreciation to the net income to get the cash flow generated by the property.

	A	B	C
16	**Cash flow, method 1 Add back depreciation**		
17	Net income	8,050	<-- =B14
18	Add back depreciation	10,000	<-- =-B11
19	Cash flow	18,050	<-- =B18+B17

In the above calculation, we've added the depreciation back to the net income to get the cash flow.

An asset's <u>cash flow</u> (the amount of cash produced by an asset during a particular period) is computed by taking the asset's net income (also called profit after taxes or sometimes just "income") and adding back non-cash expenses like depreciation.[4]

Tax shields

There's another way of calculating the cash flow which involves a discussion of *tax shields*. A tax shield is a tax saving that results from being able to report an expense for tax purposes. In general a tax shield just reduces the cash cost of an expense—in the above

[4] In Chapter 6 we introduce the concept of *free cash flow*, which is an extension of the cash flow concept discussed here.

example, since Sally & Dave's property taxes of $1,500 are an expense for tax purposes, the after-tax cost of the property taxes is:

$$(1-30\%)*\$1,500=\$1,500-\underbrace{30\%*1,500}_{\substack{\uparrow \\ \text{This \$450 is the} \\ \text{tax shield}}}=\$1,050.$$

The tax shield of $450 ($=30\%*\$1,500$) has reduced the cost of the property taxes.

Depreciation is a special case of a *non-cash expense* which generates a tax shield. A little thought will show you that the $10,000 depreciation on the condo generates $3,000 of cash. Because depreciation reduces Sally & Dave's reported income, each dollar of depreciation saves them $0.30 (30 cents) of taxes, without actually costing them anything in out-of-pocket expenses (the $0.30 comes from the fact that Sally & Dave's tax rate is 30%). Thus $10,000 of depreciation is worth $3,000 of cash. This $3,000 *depreciation tax shield* is a cash flow for Sally and Dave.

In the spreadsheet below we calculate the cash flow in two stages:

- We first calculate Sally and Dave's net income ignoring depreciation (cell B29). If depreciation were not an expense for tax purposes, Sally and Dave's net income would be $15,050.
- We then add to the this figure the depreciation tax shield of $3,000. The result (cell B32) gives the cash flow for the condo.

	A	B	C	D	E
21	**Cashflow, method 2 Compute after-tax income without depreciation, then add depreciation tax shield**				
22	Rent	24,000			
23	Expenses				
24	Property taxes	-1,500		This is what the net income would have been if depreciation were not an expense for tax purposes.	
25	Miscellaneous expenses	-1,000			
26	Depreciation	0			
27	Reportable income	21,500	<-- =SUM(B22:B26)		
28	Taxes (rate = 30%)	-6,450	<-- =-B3*B27		
29	Net income without depreciaiton	15,050	<-- =B27+B28		
30					
31	Depreciation tax shield	3,000	<-- =B3*10000	The effect of depreciation is to add a $3,000 tax shield.	
32	Cash flow	18,050	<-- =B31+B29		
33					

Is Sally and Dave's condo investment profitable?—a preliminary calculation

At this point Sally and Dave can make a preliminary calculation of the net present value and internal rate of return on their condo investment. Assuming a discount rate of 12% and assuming that they only hold the condo for 10 years, the NPV of the condo investment is $1,987 and its IRR is 12.48%:

	A	B	C
1	**SALLY & DAVE'S CONDO--preliminary valuation**		
2	Discount rate	12%	
3			
4	**Year**	**Cash flow**	
5	0	-100,000	
6	1	18,050	
7	2	18,050	
8	3	18,050	
9	4	18,050	
10	5	18,050	
11	6	18,050	
12	7	18,050	
13	8	18,050	
14	9	18,050	
15	10	18,050	
16			
17	Net present value, NPV	1,987	<-- =B5+NPV(B2,B6:B15)
18	Internal rate of return, IRR	12.48%	<-- =IRR(B5:B15)

Is Sally and Dave's condo investment profitable?—incorporating terminal value into the calculations

A little thought about the previous spreadsheet reveals that we've left out an important factor: The value of the condo at the end of the 10-year horizon. In finance an asset's value at the end of the investment horizon is called the asset's *salvage value* or *terminal value*. In the above spreadsheet we've assumed that the terminal value of the condo is zero, but this assumption implausible.

To make a better calculation about their investment, Sally and Dave will have to make an assumption about the condo's terminal value. Suppose they assume that at the end of the 10 years they'll be able to sell the condo for $80,000. The taxable gain relating to the sale of the condo is the difference between the condo's sale price and its book value at the time of sale—the initial price minus the sum of all the depreciation since Sally and Dave bought it. Since Sally and Dave have been depreciating the condo by $10,000 per year over a 10-year period, its book value at the end of 10 years will be zero.

In cell E10 below you can see that the sale of the condo for $80,000 will generate a cash flow of $56,000:

	A	B	C	D	E	F
1	**SALLY & DAVE'S CONDO: PROFITABILITY AND TERMINAL VALUE**					
2	**Cost of condo**	100,000				
3	**Sally & Dave's tax rate**	30%				
4						
5	**Annual reportable income calculation**			**Terminal value**		
6	Rent	24,000		Estimated resale value, year 10	80,000	
7	Expenses			Book value	0	
8	Property taxes	-1,500		Taxable gain	80,000	<-- =E6-E7
9	Miscellaneous expenses	-1,000		Taxes	24,000	<-- =B3*E8
10	Depreciation	-10,000		Net after tax--cashflow from terminal value	56,000	<-- =E8-E9
11	Reportable income	11,500	<-- =SUM(B6:B10)			
12	Taxes (rate = 30%)	-3,450	<-- =-B3*B11			
13	Net income	8,050	<-- =B11+B12			
14						
15	**Cash flow, method 1 Add back depreciation**					
16	Net income	8,050	<-- =B13			
17	Add back depreciation	10,000	<-- =-B10			
18	Cash flow	18,050	<-- =B17+B16			

To compute the rate of return of Sally and Dave's condo investment, we put all the numbers together:

	A	B	C	D
20	Discount rate	12%		
21				
22	**Year**	**Cashflow**		
23	0	-100,000		
24	1	18,050	<-- =B18, Annual cashflow from rental	
25	2	18,050		
26	3	18,050		
27	4	18,050		
28	5	18,050		
29	6	18,050		
30	7	18,050		
31	8	18,050		
32	9	18,050		
33	10	74,050	<-- =B32+E10	
34				
35	NPV of condo investment	20,017	<-- =B23+NPV(B20,B24:B33)	
36	IRR of investment	15.98%	<-- =IRR(B23:B33)	

Assuming that the 12% discount rate is the correct rate, the condo investment is worthwhile: It's NPV is positive and its IRR exceeds the discount rate.[5]

[5] When we say that a discount rate is "correct," we usually mean that it is appropriate to the riskiness of the cash flows being discounted. In Chapter 5 we have our first discussion in this book on how to determine a correct

Book value versus terminal value

The *book value* of an asset is its initial purchase price minus the accumulated depreciation. The *terminal value* of an asset is its assumed market value at the time you "stop writing down the asset's cash flows." This sounds like a weird definition of terminal value, but often when we do present value calculations for a long-lived asset (like Sally and Dave's condo, or like the company valuations we discuss in Chapters 7-9), we write down only a limited number of cash flows.

Sally and Dave are reluctant to make predictions about condo rents and expenses beyond a ten-year horizon. Past this point, they're worried about the accuracy of their guesses. So they write down ten years of cash flows; the terminal value is their best guess of the condo's value at the end of year 10. Their thinking is "let's examine the profitability of the condo if we hold on to it for 10 years and sell it."

This is what we mean when we say that "the terminal value is what the asset is worth when we stop writing down the cash flows."

Taxes: If Sally and Dave are right in their terminal value assumption, they will have to take account of taxes. The tax rules for selling an asset specify that the tax bill is computed on the *gain over the book value*. So, in the example of Sally and Dave:

$$\begin{aligned}&\textit{Terminal value} - \textit{taxes on gain over book} = \\ &\textit{Terminal value} - \textit{tax rate} * (\textit{Terminal value} - \textit{book value}) \\ &= 80,000 - 30\% * (80,000 - 0) = 56,000\end{aligned}$$

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discount rate. For the moment let's assume that the discount rate is appropriate to the riskiness of the condo's cash flows.

Doing some sensitivity analysis

If we really want to be fancy, we can do a sensitivity table (using Excel's **Data Table**, see Chapter 30). The table below shows the IRR of the investment as a function of the annual rent and the terminal value:

	A	B	C	D	E	F	G	H
38	**Data table--Condo IRR as function of annual rent and terminal value**							
39			**Rent**					
40		15.98%	**18,000**	**20,000**	**22,000**	**24,000**	**26,000**	**28,000**
41	**Terminal value -->**	**50,000**	9.72%	11.45%	13.15%	14.82%	16.47%	18.10%
42		**60,000**	10.26%	11.93%	13.59%	15.22%	16.84%	18.44%
43		**70,000**	10.77%	12.40%	14.01%	15.61%	17.19%	18.76%
44	=B36	**80,000**	11.25%	12.84%	14.42%	15.98%	17.54%	19.08%
45		**90,000**	11.71%	13.27%	14.81%	16.34%	17.87%	19.38%
46		**100,000**	12.15%	13.67%	15.19%	16.69%	18.19%	19.68%
47		**110,000**	12.58%	14.06%	15.55%	17.02%	18.50%	19.96%
48		**120,000**	12.98%	14.44%	15.90%	17.35%	18.80%	20.24%
49		**130,000**	13.37%	14.80%	16.23%	17.66%	19.09%	20.51%
50		**140,000**	13.75%	15.15%	16.56%	17.96%	19.37%	20.78%
51		**150,000**	14.11%	15.49%	16.87%	18.26%	19.65%	21.03%
52		**160,000**	14.46%	15.82%	17.18%	18.55%	19.91%	21.28%
53								
54		**Note**: The data table above computes the IRR of the condo investment for combinations of rent (from $18,000 to						
55		$26,000 per year) and terminal value (from $50,000 to $160,000).						
56		Data tables are very useful though not trivial to compute. See Chapter 30 for more information.						

The calculations aren't that surprising: For a given rent, the IRR is higher when the terminal value is higher, and for a given terminal value, the IRR is higher given a higher rent.

Building the data table[6]

Here's how the data table was set up:

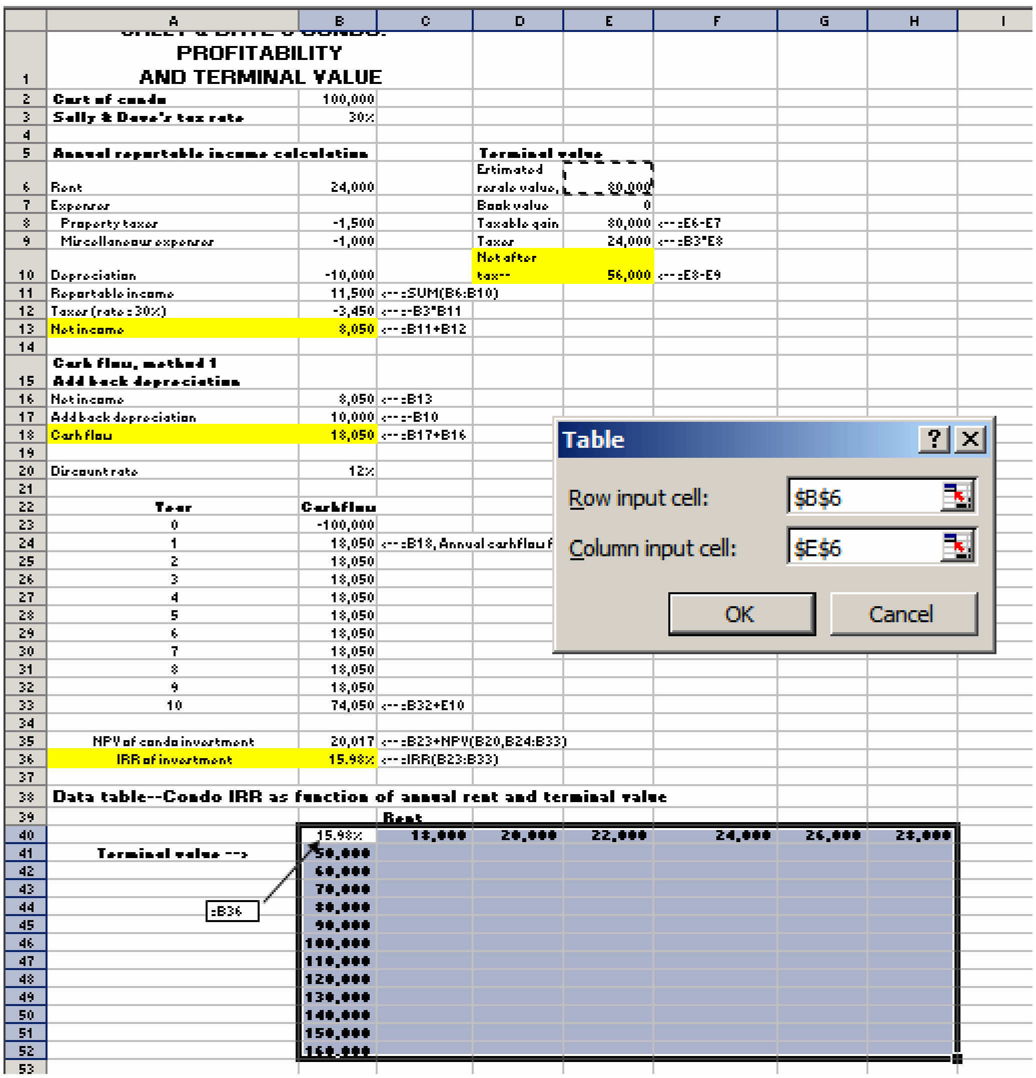
- We build a table with terminal values in the left-hand column and rent in the top row.
- In the top left-hand corner of the table (cell B40), we refer to the IRR calculation in the spreadsheet example (this calculation occurs in cell B36).

At this point the table looks like this:

[6] This subsection doesn't replace Chapter 30, but it may help you recall what we said there.

	A	B	C	D	E	F	G	H
38	Data table--Condo IRR as function of annual rent and terminal value							
39			Rent					
40		15.98%	18,000	20,000	22,000	24,000	26,000	28,000
41	Terminal value -->	50,000						
42		60,000						
43		70,000						
44	=B36	80,000						
45		90,000						
46		100,000						
47		110,000						
48		120,000						
49		130,000						
50		140,000						
51		150,000						
52		160,000						

Using the mouse we now mark the whole table. We use the **Data|Table** command and fill in the cell references from the original example:



The dialog box tells Excel to repeat the calculation in cell B36, varying the rent number in cell B6 and varying the terminal value number in cell E6. Pressing **OK** does the rest.

Mini case

A mini case for this chapter looks at Sally and Dave's condo once more—this time under the assumption that they take out a mortgage to buy the condo. Highly recommended!

3.8. Capital budgeting and salvage values

In the Sally-Dave condo example we've focused on the effect of non-cash expenses on cash flows: Accountants and the tax authorities compute earnings by subtracting certain kinds of expenses from sales, even though these expenses are *non-cash expenses*. In order to compute the cash flow, we add back these non-cash expenses to accounting earnings. We showed that these non-cash expenses create *tax shields*—they create cash by saving taxes.

In this section's example we consider a capital budgeting example in which a firm sells its asset before it is fully depreciated. We show that the asset's book value at the date of the terminal value creates a tax shield and we look at the effect of this tax shield on the capital budgeting decision.

Here's the example. Your firm is considering buying a new machine. Here are the facts:

- The machine costs $800.
- Over the next 8 years (the life of the machine) the machine will generate annual sales of $1,000.

- The annual cost of the goods sold (COGS) is $400 per year and other costs; selling, general, and administrative expenses (GS&A) are $300 per year.
- Depreciation on the machine is straight-line over 8 years (that is: $100 per year).
- At the end of 8 years, the machine's salvage value (or terminal value) zero.
- The firm's tax rate is 40%.
- The firm's discount rate for projects of this kind is 15%.

Should the firm buy the machine? Here's the analysis in Excel:

	A	B	C	D	E	F	G
1	**BUYING A MACHINE--NPV ANALYSIS**						
2	Cost of the machine	800					
3	Annual anticipated sales	1,000					
4	Annual COGS	400					
5	Annual SG&A	300			**NPV Analysis**		
6	Annual depreciation	100			Year	Cash flow	
7					0	-800	<-- =-B2
8	Tax rate	40%			1	220	<-- =$B$23
9	Discount rate	15%			2	220	
10					3	220	
11	**Annual profit and loss (P&L)**				4	220	
12	Sales	1,000			5	220	
13	Minus COGS	-400			6	220	
14	Minus SG&A	-300			7	220	
15	Minus depreciation	-100			8	220	
16	Profit before taxes	200	<-- =SUM(B12:B15)				
17	Subtract taxes	-80	<-- =-B8*B16		NPV	187	<-- =F7+NPV(B9,F8:F15)
18	Profit after taxes	120	<-- =B16+B17				
19							
20	**Calculating the annual cash flow**						
21	Profit after taxes	120					
22	Add back depreciation	100					
23	Cash flow	220					

Notice that we first calculate the profit and loss (P&L) statement for the machine (cells B12:B18) and then turn this P&L into a cash flow calculation (cells B21:B23). The annual cash flow is $220. Cells F7:F15 show the table of cash flows, and cell F17 gives the NPV of the project. The NPV is positive, and we would therefore buy the machine.

Salvage value—a variation on the theme

Suppose the firm can sell the machine for $300 at the end of year 8. To compute the cash flow produced by this salvage value, we must make the distinction between *book value* and *market value*:

Book value	An accounting concept: The book value of the machine is its initial cost minus the accumulated depreciation (the sum of the depreciation taken on the machine since its purchase). In our example, the book value of the machine in year 0 is $800, in year 1 it is $700, ..., and at the end of year 8 it is zero.
Market value	The market value is the price at which the machine can be sold. In our example the market value of the machine at the end of year 8 is $300.
Taxable gain	The taxable gain on the machine at the time of sale is the difference between the market value and the book value. In our case the taxable gain is positive ($300), but it can also be negative (see an example at the end of this chapter).

Here's the NPV calculation including the salvage value:

	A	B	C	D	E	F	G
1	**BUYING A MACHINE--NPV ANALYSIS with salvage value**						
2	Cost of the machine	800					
3	Annual anticipated sales	1,000					
4	Annual COGS	400					
5	Annual SG&A	300			**NPV Analysis**		
6	Annual depreciation	100			Year	Cash flow	
7					0	-800	<-- =-B2
8	Tax rate	40%			1	220	<-- =$B$23
9	Discount rate	15%			2	220	
10					3	220	
11	**Annual profit and loss (P&L)**				4	220	
12	Sales	1,000			5	220	
13	Minus COGS	-400			6	220	
14	Minus SG&A	-300			7	220	
15	Minus depreciation	-100			8	400	<-- =$B$23+B30
16	Profit before taxes	200	<-- =SUM(B12:B15)				
17	Subtract taxes	-80	<-- =-B8*B16		NPV	246	<-- =F7+NPV(B9,F8:F15)
18	Profit after taxes	120	<-- =B16+B17				
19							
20	**Calculating the annual cash flow**						
21	Profit after taxes	120					
22	Add back depreciation	100					
23	Cash flow	220					
24							
25	**Calculating the cash flow from salvage value**						
26	Machine market value, year 8	300					
27	Book value, year 8	0					
28	Taxable gain	300	<-- =B26-B27				
29	Taxes paid on gain	120	<-- =B8*B28				
30	Cash flow from salvage value	180	<-- =B26-B29				

Note the calculation of the cash flow from the salvage value (cell B30) and the change in the year 8 cash flow (cell F15).

One more example

Suppose we change the example slightly:

- The annual sales, SG&A, COGS, and depreciation are still as specified in the original example. The machine will still be depreciated on a straight-line basis over 8 years.
- However, we think we will sell the machine at the *end of year 7* at an estimated salvage value of $400. At the end of year 7 the book value of the machine is $100.

Here's how our calculations look now:

	A	B	C	D	E	F	G
1	**BUYING A MACHINE--NPV ANALYSIS with salvage value. Machine sold in year 7**						
2	Cost of the machine	800					
3	Annual anticipated sales	1,000					
4	Annual COGS	400					
5	Annual SG&A	300			**NPV Analysis**		
6	Annual depreciation	100			Year	Cash flow	
7					0	-800	<-- =-B2
8	Tax rate	40%			1	220	<-- =$B$23
9	Discount rate	15%			2	220	
10					3	220	
11	**Annual profit and loss (P&L)**				4	220	
12	Sales	1,000			5	220	
13	Minus COGS	-400			6	220	
14	Minus SG&A	-300			7	500	<-- =$B$23+B30
15	Minus depreciation	-100					
16	Profit before taxes	200	<-- =SUM(B12:B15)		NPV	221	<-- =F7+NPV(B9,F8:F15)
17	Subtract taxes	-80	<-- =-B8*B16				
18	Profit after taxes	120	<-- =B16+B17				
19							
20	**Calculating the annual cash flow**						
21	Profit after taxes	120					
22	Add back depreciation	100					
23	Cash flow	220					
24							
25	**Calculating the cash flow from salvage value**						
26	Machine market value, year 7	400					
27	Book value, year 7	100					
28	Taxable gain	300	<-- =B26-B27				
29	Taxes paid on gain	120	<-- =B8*B28				
30	Cash flow from salvage value	280	<-- =B26-B29				

Note the subtle changes from the previous example:

- The *cash flow from salvage value* is

$$Salvage\ value - tax * \underbrace{\left(Salvage\ value - Book\ value\right)}_{\substack{\uparrow \\ \text{Taxable gain at time} \\ \text{of machine sale}}}$$

In our example this is $280 (cell B30).

- Another way to write the cash flow from the salvage value is:

$$\underbrace{Salvage\ value * \left(1 - tax\right)}_{\substack{\uparrow \\ \text{After-tax proceeds from machine} \\ \text{sale } \textit{if} \text{ the whole salvage value is} \\ \text{taxed}}} + \underbrace{tax * book\ value}_{\substack{\uparrow \\ \text{Tax shield on book} \\ \text{value at time of machine} \\ \text{sale}}}$$

Using this example, you can see the role taxes play even if we sell the machine at a loss. Suppose, for example, that the machine is sold in year 7 for $50, which is less than the book value:

	A	B	C	D	E	F	G
1	**BUYING A MACHINE--NPV ANALYSIS with salvage value. Machine sold in year 7**						
2	Cost of the machine	800					
3	Annual anticipated sales	1,000					
4	Annual COGS	400					
5	Annual SG&A	300			**NPV Analysis**		
6	Annual depreciation	100			Year	Cash flow	
7					0	-800	<-- =-B2
8	Tax rate	40%			1	220	<-- =$B$23
9	Discount rate	15%			2	220	
10					3	220	
11	**Annual profit and loss (P&L)**				4	220	
12	Sales	1,000			5	220	
13	Minus COGS	-400			6	220	
14	Minus SG&A	-300			7	290	<-- =$B$23+B30
15	Minus depreciation	-100					
16	Profit before taxes	200	<-- =SUM(B12:B15)		NPV	142	<-- =F7+NPV(B9,F8:F15)
17	Subtract taxes	-80	<-- =-B8*B16				
18	Profit after taxes	120	<-- =B16+B17				
19							
20	**Calculating the annual cash flow**						
21	Profit after taxes	120					
22	Add back depreciation	100					
23	Cash flow	220					
24							
25	**Calculating the cash flow from salvage value**						
26	Machine market value, year 7	50					
27	Book value, year 7	100					
28	Taxable gain	-50	<-- =B26-B27				
29	Taxes paid on gain	-20	<-- =B8*B28				
30	Cash flow from salvage value	70	<-- =B26-B29				

In this case, the negative taxable gain (cell B28, the jargon often heard is "loss over book") produces a tax shield—the negative taxes of -$20 in cell B29. This tax shield is added to the market value to produce a salvage value cash flow of $70 (cell B30). Thus even selling an asset at a loss can produce a positive cash flow.

3.9. Capital budgeting principle: Don't forget the cost of foregone opportunities

This is another important principle of capital budgeting. An example: You've been offered the project below, which involves buying a widget-making machine for $300 to make a new product. The cash flows in years 1-5 have been calculated by your financial analysts:

	A	B	C
1	**DON'T FORGET THE COST OF FOREGONE OPPORTUNITIES**		
2	Discount rate	12%	
3			
4	**Year**	**Cashflow**	
5	0	-300	
6	1	185	
7	2	249	
8	3	155	
9	4	135	
10	5	420	
11			
12	NPV	498.12	<-- =NPV(B2,B6:B10)+B5
13	IRR	62.67%	<-- =IRR(B5:B10)

Looks like a fine project! But now someone remembers that the widget process makes use of some already existing but underused equipment. Should the value of this equipment be somehow taken into account?

The answer to this question has to do with whether the equipment has an alternative use. For example, suppose that, if you don't buy the widget machine, you can sell the equipment for $200. Then the true year 0 cost for the project is $500, and the project has a lower NPV:

	A	B	C
16	Discount rate	12%	
17			
18	**Year**	**Cashflow**	
19	0	-500	The $300 direct cost + $200 <-- value of the existing machines
20	1	185	
21	2	249	
22	3	155	
23	4	135	
24	5	420	
25			
26	NPV	298.12	
27	IRR	31.97%	

While the logic here is clear, the implementation can be murky: What if the machine is to occupy space in a building that is currently unused? Should the cost of this space be taken into account? It all depends on whether there are alternative uses, now or in the future.[7]

3.10. In-house copying or outsourcing? A mini-case illustrating foregone opportunity costs

Your company is trying to decide whether to outsource its photocopying or continue to do it in-house. The current photocopier won't do anymore—it either has to be sold or thoroughly fixed up. Here are some details about the two alternatives:

- The company's tax rate is 40%.
- Doing the copying in-house requires an investment of $17,000 to fix up the existing photocopy machine. Your accountant estimates that this $17,000 can be immediately booked as an expense, so that its after-tax cost is $(1-40\%)*17,000=10,200$. Given this

[7] There's a fine Harvard case on this topic: "The Super Project," Harvard Business School case 9-112-034.

investment the copier will be good for another five years. Annual copying costs are estimated to be $25,000 on a before-tax basis; after-tax this is $(1-40\%)*25,000=15,000$.

- The photocopy machine is on your books for $15,000, but its market value is in fact much less—it could only be sold today for $5,000. This means that the sale of the copier will generate a loss for tax purposes of $10,000; at your tax rate of 40%, this loss gives a tax shield of $4,000. Thus the sale of the copier will generate a cash flow of $9,000.
- If you decide to keep doing the photocopying in-house, the remaining book value of the copier will be depreciated over 5 years at $3,000 per year. Since your tax rate is 40%, this will produce a tax shield of 40%*$3,000 = $1,200 per year.
- Outsourcing the copying will be $33,000 per year—$8,000 more expensive than doing it in-house on the rehabilitated copier. Of course this $33,000 is an expense for tax purposes, so that the net savings from doing the copying in-house is

$$(1-tax\,rate)*outsourcing\ \cos ts=(1-40\%)*\$33,000=\$19,800.$$

- The relevant discount rate is 12%.

We will show you two ways to analyze this decision. The first method values each of the alternatives separately. The second method looks only at the differential cash flows. We recommend the first method—it's simpler and leads to fewer mistakes. The second method produces a somewhat "cleaner" set of cash flows that take explicit account of foregone opportunity costs.

Method 1: Write down the cash flows of each alternative

This is often the simplest way to do things; if you do it correctly, this method takes care of all the foregone opportunity costs without your thinking about them. Below we write down the cash flows for each alternative:

	In house	Outsourcing
Year 0	$-(1-tax\,rate)*machine\ rehab\ cost$ $=-(1-40\%)*17{,}000=-\$10{,}200$	$Sale\ price\ of\ machine$ $+tax\ rate*loss\ over\ book\ value$ $=\$5{,}000+40\%*(\$15{,}000-5{,}000)$ $=\$9{,}000$
Years 1-5 **annual cash flow**	$-(1\text{-}tax\,rate)*in\text{-}house\ costs$ $+tax\ rate*depreciation$ $=-(1-40\%)*\$25{,}000$ $+40\%*\$3{,}000=-\$13{,}800$	$-(1-tax\ rate)*outsourcing\ costs$ $=-(1-40\%)*\$33{,}000=$ $=-\$19{,}800$

Putting these data in a spreadsheet and discounting at the discount rate of 12% shows that it is cheaper to do the in-house copying. The NPV of the in-house cash flows is -$59,946, whereas the NPV of the outsourcing cash flows is -$62,375. Note that both NPVs are negative; but the in-house alternative is less negative (meaning: more positive) than the outsourcing alternative; therefore the in-house is preferred:

	A	B	C
1	**SELL THE PHOTOCOPIER OR FIX IT UP?**		
2	Annual cost savings (before tax) after fixing up the machine	8,000	
3	Book value of machine	15,000	
4	Market value of machine	5,000	
5	Rehab cost of machine	17,000	
6	Tax rate	40%	
7	Annual depreciation if machine is retained	3,000	
8	Annual copying costs		
9	In-house	25,000	
10	Outsourcing	33,000	
11	Discount rate	12%	
12			
13	**Alternative 1: Fix up machine and do copying in-house**		
14	Year	Cash flow	
15	0	-10,200	<-- =-B5*(1-B6)
16	1	-13,800	<-- =-$B$9*(1-$B$6)+$B$6*$B$7
17	2	-13,800	
18	3	-13,800	
19	4	-13,800	
20	5	-13,800	
21	NPV of fixing up machine and in-house copying	-59,946	<-- =B15+NPV(B11,B16:B20)
22			
23	**Alternative 2: Sell machine and outsource copying**		
24	Year	Cash flow	
25	0	9,000	<-- =B4+B6*(B3-B4)
26	1	-19,800	<-- =-(1-$B$6)*$B$10
27	2	-19,800	
28	3	-19,800	
29	4	-19,800	
30	5	-19,800	
31	NPV of selling machine and outsourcing	-62,375	<-- =B25+NPV(B11,B26:B30)

Method 2: Discounting the differential cash flows

In this method we subtract the cash flows of Alternative 2 from those of Alternative 1:

	A	B	C
34	**Subtract Alternative 2 CFs from Alternative 1 CFs**		
35	Year	Cash flow	
36	0	-19,200	<-- =B15-B25
37	1	6,000	<-- =B16-B26
38	2	6,000	
39	3	6,000	
40	4	6,000	
41	5	6,000	
42	NPV(Alternative 1 - Alternative 2)	2,429	<-- =B36+NPV(B11,B37:B41)

The NPV of the differential cash flows is positive. This means that Alternative 1 (in-house) is better than Alternative 2 (outsourcing):

$$NPV\left(In\text{-}house - Outsourcing\right) = NPV\left(In\text{-}house\right) - NPV(Outsourcing) > 0$$

This means that

$$NPV\left(In\text{-}house\right) > NPV(Outsourcing)$$

If you look carefully at the differential cash flows, you'll see that they take into account the cost of the foregone opportunities:

	Differential cash flow	**Explanation**
Year 0	-\$19,200	This is the after-tax cost of rehabilitating the old copier *plus* the foregone opportunity cost of selling the copier. In other words: This is the cost in year 0 of deciding to do the copying in-house.
Years 1-5	\$6,000	This is the after-tax saving of doing the copying in-house: If you do it in house, you save \$8,000 pre-tax (= \$4,800 after tax) and you get to take depreciation on the existing copier (= tax shield of \$1,200). Relative to in-house copying, the outsourcing alternative has a foregone opportunity cost of the loss of the depreciation tax shield.

If you examine the convoluted prose in the table above ("the outsourcing alternative has a foregone opportunity cost of the loss of the depreciation tax shield") you'll agree that it may just be simpler to list each alternative's cash flows separately.

3.11. Accelerated depreciation

As you know by now, the *salvage value* for an asset is its value at the end of its life; another term sometimes used is *terminal value*. Here's a capital budgeting example that illustrates the importance of accelerated depreciation in computing the cash:

- Your company is considering buying a machine for $10,000.
- If bought, the machine will produce annual cost savings of $3,000 for the next 5 years; these cash flows will be taxed at the company's tax rate of 40%.
- The machine will be depreciated over the 5 year period using the accelerated depreciation percentages allowable in the United States.[8] At the end of the 6th year, the machine will sold; your estimate of its salvage value at this point is $4,000, even though for accounting purposes its book value is $500.

You have to decide what the NPV of the project is, using a discount rate of 12%. Here are the relevant calculations:

[8] These accelerated depreciation percentages—termed ACRS (for "accelerated cost recovery system") are discussed briefly in Section 3.7.

	A	B	C	D	E	F	G	H
1	**CAPITAL BUDGETING WITH ACCELERATED DEPRECIATION**							
2	Machine cost	10,000						
3	Annual materials savings, before tax	3,000						
4	Salvage value, end of year 5	12,000						
5	Tax rate	40%						
6	Discount rate	12%						
7								
8	**Depreciation schedule (ACRS)**							
9	**Year**	**ACRS depreciation percentage**	**Actual depreciation**	**Depreciation tax shield**				
10	1	20.00%	2,000	800	<-- =$B$5*C10			
11	2	32.00%	3,200	1,280	<-- =$B$5*C11			
12	3	19.20%	1,920	768	<-- =$B$5*C12			
13	4	11.52%	1,152	461	<-- =$B$5*C13			
14	5	11.52%	1,152	461				
15	6	5.76%	576	230				
16								
17	**Terminal value**							
18	Year 5 sale price, estimated	12,000	<-- =B4					
19	Year 5 book value	576	<-- =B2-SUM(C10:C14)					
20	Taxable gain	11,424	<-- =B18-B19					
21	Taxes	4,570	<-- =B5*B20					
22	Net	7,430	<-- =B18-B21					
23								
24								
25	**Year**	**0**	**1**	**2**	**3**	**4**	**5**	
26	Purchase price	-10,000						
27	After-tax cost savings		1,800	1,800	1,800	1,800	1,800	<-- =$B$3*(1-$B$5)
28	Depreciation tax shield		800	1,280	768	461	461	<-- =D14
29	Terminal value						7,430	<-- =B22
30	Total cashflow	-10,000	2,600	3,080	2,568	2,261	9,691	<-- =SUM(G26:G29)
31								
32	Net present value	3,540.46	<-- =NPV(B6,C30:G30)+B30					
33	IRR	22.84%	<-- =IRR(B30:G30)					

The book value at the end of year 5 is the initial cost of the machine ($10,000) *minus* the sum of all the depreciation taken on the machine through year 5 ($9,424).

3.12. Conclusion

In this chapter we've discussed the basics of capital budgeting using NPV and IRR. Capital budgeting decisions can be crudely separated into "yes-no" decisions ("should we undertake a given project?") and into "ranking" decisions ("which of the following list of projects do we prefer?"). We've concentrated on two important areas of capital budgeting:

- The difference between NPV and IRR in making the capital budgeting decision. In many cases these two criteria give the same answer to the capital budgeting question. However, there are cases—especially when we rank projects—where NPV and IRR give different answers. Where they differ, NPV is the preferable criterion to use because the NPV is the additional wealth derived from a project.

- Every capital budgeting decision ultimately involves a set of anticipated cash flows, so when you do capital budgeting, it's important to get these cash flows right. We've illustrated the importance of sunk costs, taxes, foregone opportunities, and salvage values in determining the cash flows.

Exercises

1. You are considering a project whose cash flows are given below:

	A	B
3	Discount rate	25%
4		
5	**Year**	**Cashflow**
6	0	-1,000
7	1	100
8	2	200
9	3	300
10	4	400
11	5	500
12	6	600

a. Calculate the present values of the future cash flows of the project.

b. Calculate the project's net present value.

c. Calculate the internal rate of return.

d. Should you undertake the project?

2. Your firm is considering two projects with the following cash flows:

	A	B	C
5	Year	Project A	Project B
6	0	-500	-500
7	1	167	200
8	2	180	250
9	3	160	170
10	4	100	25
11	5	100	30

a. If the appropriate discount rate is 12%, rank the two projects.

b. Which project is preferred if you rank by IRR?

c. Calculate the crossover rate - the discount rate r in which the NPVs of both projects are equal.

d. Should you use NPV or IRR to choose between the two projects? Give a brief discussion.

3. Your uncle is a proud owner of an up-market clothing store. Because business is down he is considering replacing the languishing tie department with a new sportswear department. In order to examine the profitability of such move he hired a financial advisor to estimate the cash flows of the new department. After six months of hard work the financial advisor came up with the following calculation:

Investment (at t=0)

Rearranging the shop	40,000
Loss of business during renovation	15,000
Payment for financial advisor	12,000
Total	67,000

Profits (from t=1 to infinity)

Annual earnings from the sport department	75,000
Loss of earnings from the tie department	-20,000
Loss of earnings from other departments*	-15,000
Additional worker for the sport department	-18,000
Municipal taxes	-15,000
Total	7,000

* Some of your uncle's stuck up clients will not buy in a shop that sells sports wear

The discount rate is 12%, and there are no additional taxes. Thus, the financial advisor calculated the NPV as follows:

$$-67{,}000+\frac{7{,}000}{0.12}=-8{,}667$$

Your surprised uncle asked you (a promising finance student) to go over the calculation. What are the correct NPV and IRR of the project?

4. You are the owner of a factory that supplies chairs and tables to schools in Denver. You sell each chair for $1.76 and each table for $4.40 based on the following calculation:

	Chair department	**Table department**
No. of units	100,000	20,000
Cost of material	80,000	35,000
Cost of Labor	40,000	20,000
Fixed cost	40,000	25,000
Total cost	160,000	80,000
Cost per unit	1.60	4.00
Plus 10% profit	1.76	4.40

You have received an offer from a school in Colorado Springs to supply an additional 10,000 chairs and 2,000 tables for the price of $1.5 and $3.5 respectively. Your financial advisor advises you not to take up the offer because the price does not even cover the cost of production. Is the financial advisor correct?

5. A factory is considering the purchase of a new machine for one of its units. The machine costs $100,000. The machine will be depreciated on a straight line basis over its 10-year life to a salvage value of zero. The machine is expected to save the company $50,000 annually, but in order to operate it the factory will have to transfer an employee (with a salary of $40,000 a year) from one of its other units. A new employee (with a salary of $20,000 a year) will be required to

replace the transferred employee. What is the NPV of the purchase of the new machine if the relevant discount rate is 8% and corporate tax rate is 35%?

6. You are considering the following investment:

Year	EBDT (Earnings before depreciation and taxes)
0	-10,500
1	3,000
2	3,000
3	3,000
4	2,500
5	2,500
6	2,500
7	2,500

The discount rate is 11% and the corporate tax rate is 34%.

a. Calculate the project NPV using straight-line depreciation.

b. What will be the company's gain if it uses the MACRS depreciation schedule?

7. A company is considering buying a new machine for one of its factories. The cost of the machine is $60,000 and its expected life span is five years. The machine will save the cost of a worker estimated at $22,500 annually. The book value of the machine at the end of year 5 is $10,000 but the company estimates that the market value will be only $5,000. Calculate the NPV of the machine if the discount rate is 12% and the tax rate is 30%. Assume straight-line depreciation over the five-year life of the machine.

8. The ABD Company is considering buying a new machine for one of its factories. The machine cost is \$100,000 and its expected life span is 8 years. The machine is expected to reduce the production cost by \$15,000 annually. The terminal value of the machine is \$20,000 but the company believes that it would only manage to sell it for \$10,000. If the appropriate discount rate is 15% and the corporate tax is 40%:

   a. Calculate the project NPV.

   b. Calculate the project IRR.

9. You are the owner of a factory located in a hot tropical climate. The monthly production of the factory is \$100,000 except during June-September when it falls to \$80,000 due to the heat in the factory. In January 2003 you get an offer to install an air-conditioning system in your factory. The cost of the air-conditioning system is \$150,000 and its expected life span is 10 years. If you install the air-conditioning system, the production in the summer months will equal the production in the winter months. However the cost of operating the system is \$9,000 per month (only in the four months that you operate the system). You will also need to pay a maintenance fee of \$5,000 annually in October. What is the NPV of the air-conditioning system if the interest rate 12% and corporate tax rate is 35% (the depreciation costs are recognized in December of each year)?

10. The "Cold and Sweet" (C&S) company manufactures ice-cream bars. The company is considering the purchase of a new machine that will top the bar with high quality chocolate. The cost of the machine is \$900,000.

Depreciation and terminal value: The machine will be depreciated over 10 years to zero salvage value. However, the company intends to use the machine for only 5 years. Management thinks that the sale price of the machine at the end of 5 years will be $100,000.

The machine can produce up to one million ice cream bars annually. The marketing director of C&S believes that if the company will spend $30,000 on advertising in the first year and another $10,000 in each of the following years the company will be able to sell 400,000 bars for $1.30 each. The cost of producing of each bar is $0.50; and other costs related to the new products are $40,000 annually. C&S's cost of capital is 14% and the corporate tax rate is 30%.

a. What is the NPV of the project if the marketing director's projections are correct?

b. What is the minimum price that the company should charge for each bar if the project is to be profitable? Assume that the price of the bar does not affect sales.

c. The C&S Marketing Vice President suggested canceling the advertising campaign. In his opinion, the company sales will not be reduced significantly due to the cancellation. What is the minimum quantity that the company needs to sell in order to be profitable if the Vice President's suggestion is accepted.

d. Extra: Use a 2-dimensional data table to determine the sensitivity of the profitability to the price and quantity.

11. The "Less Is More" company manufactures swimsuits. The company is considering expanding to the bath robes market. The proposed investment plan includes:

- Purchase of a new machine: The cost of the machine is \$150,000 and its expected life span is 5 years. The terminal value of the machine is 0, but the chief economist of the company estimates that it can be sold for \$10,000.
- Advertising campaign: The head of the marketing department estimates that the campaign will cost \$80,000 annually.
- Fixed cost of the new department will be \$40,000 annually.
- Variable costs are estimated at \$30 per bathrobe but due to the expected rise in labor costs they are expected to rise at 5% per year.
- Each of the bathrobes will be sold at a price of \$45 at the first year. The company estimates that it can raise the price of the bathrobes by 10% in each of the following years.

The "Less Is More" discount rate is 10% and the corporate tax rate is 36%.

a) What is the break-even point of the bathrobe department?

b) Plot a graph in which the NPV is the dependent variable of the annual production.

12. The "Car Clean" company operates a car wash business. The company bought a machine 2 years ago at the price of \$60,000. The life span of the machine is 6 years and the machine has no disposal value, the current market value of the machine is \$20,000. The company is considering buying a new machine. The cost of the new machine is \$100,000 and its life span is 4 years. The new machine has a disposal value of \$20,000. The new machine is faster then the old one; thus the company believes the revenue will increase from \$1 million annually to \$1.03 million. In addition the new machine is expected to save the company \$10,000 in water and electricity costs.

The discount rate of "Car Clean" is 15% and the corporate tax rate is 40%. What is the NPV of replacing the old machine?

13. A company is considering whether to buy a regular or color photocopier for the office. The cost of the regular machine is $10,000, its life span is 5 years and the company has to pay another $1,500 annually in maintenance costs. The color photocopier's price is $30,000, its life span is also 5 years and the annual maintenance costs are $4,500. The color photocopier is expected to increase the revenue of the office by $8,500 annually. Assume that the company is profitable and pays 40% corporate tax, the relevant interest rate is 11%. Which photocopy machine should the firm buy?

14. The Coka company is a soft drink company. Until today the company bought empty cans from an outside supplier that charges Coka $0.20 per can. In addition the transportation cost is $1,000 per truck that transports 10,000 cans. The Coka company is considering whether to start manufacturing cans in its plant. The cost of a can machine is $1,000,000 and its life span is 12 years. The terminal value of the machine is $160,000. Maintenance and repair costs will be $150,000 for every 3 year period. The additional space for the new operation will cost the company $100,000 annually. The cost of producing a can in the factory is $0.17.

The cost of capital of Coka is 11% and the corporate tax rate is 40%.

a) What is the minimum number of cans that the company has to sell annually in order to justify self-production of cans?

b) Advanced: Use data tables in order to show the NPV and IRR of the project as a function of the number of cans.

15. The ZZZ Company is considering investing in a new machine for one of its factories. The company has two alternatives to choose from:

	Machine A	Machine B
Cost	\$4,000,000	\$10,000,000
Annual fixed cost per machine	\$300,000	\$210,000
Variable cost per unit	\$1.20	\$0.80
Annual production	400,000	550,000

The life span of each machine is 5 years. ZZZ sells each unit for a price of \$6. The company has a cost of capital of 12% and its tax rate is 35%.

a. If the company manufactures 1,000,000 units per year which machine should it buy?

b. Plot a graph showing the profitability of investment in each machine type depending on the annual production.

16. The Easy Sight company manufactures sunglasses. The company has two machines, each of which produces 1,000 sunglasses per month. The book value of each of the old machines is \$10,000 and their expected life span is 5 years. The machines are being depreciated on a straight-line basis to zero salvage value. The company assumes it will be able to sell a machine today (January 2004) for the price of \$6,000. The price of a new machine is \$20,000 and its expected life span is 5 years. The new machine will save the company \$0.85 for every pair of sunglasses produced.

Demand for sunglasses is seasonal. During the five months of the summer (May-September) demand is 2,000 sunglasses per month while during the winter months it falls down to 1,000 per month.

Assume that due to insurance and storage costs it is uneconomical to store sunglasses at the factory. How many new machines should "Easy Sight" buy if the discount rate is 10% and the corporate tax rate is 40%?

17. Poseidon is considering opening a shipping line from Athens to Rhodes. In order to open the shipping line Poseidon will have to purchase two ships that cost 1,000 gold coins each. The life span of each ship is 10 years, and Poseidon estimates that he will earn 300 gold coins in the first year and that the earnings will increase by 5% per year. The annual costs of the shipping line are estimated at 60 gold coins annually, Poseidon's interest rate is 8% and Zeus's tax rate is 50%.

a) Will the shipping line be profitable?

b) Due to Poseidon's good connections on Olympus he can get a tax reduction. What is the maximum tax rate at which the project will be profitable?

18. At the board meeting on Olympus, Hera tried to convince Zeus to keep the 50% tax rate intact due to the budget deficit. According to Hera's calculations, the shipping line will be more profitable if Poseidon will buy only one ship and sell tickets only to first class passengers. Hera estimated that Poseidon's annual costs will be 40 gold coins.

a) What are the minimum annual average earnings required for the shipping line to be profitable assuming that earnings are constant throughout the ten years?

b) Zeus, who is an old fashioned god, believes that "blood is thicker then money." He agreed to give Poseidon a tax reduction if he will only buy one ship. Use data tables in order to show the profitability of the project as dependent on the annual earnings and the tax rate.

19. Kane Running Shoes is considering the manufacturing of a special shoe for race walking which will indicate if an athlete is running (i.e. both legs are not touching the ground). The chief economist of the company presented the following calculation for the Smart Walking Shoes (SWS):

- R&D $200,000 annually in each of the next 4 years.

The Manufacturing project:

- Expected life span: 10 years
- Investment in machinery: $250,000 (at t=4) expected life span of the machine 10 years
- Expected annual sales: 5,000 pairs of shoes at the expected price of $150 per pair
- Fixed cost $300,000 annually
- Variable cost: $50 per pair of shoes

Kane's discount rate is 12%, the corporate rate is 40%, and R&D expenses are tax deductible against other profits of the company. Assume that at the end of project (that is. after 14 years) the new technology will have been superseded by other technologies and therefore have no value.

a) What is the NPV of the project?

b) The International Olympic Committee (IOC) decided to give Kane a loan without interest for 6 years in order to encourage the company to take on the project. The loan will have to be paid back in 6 equal annual payments. What is the minimum loan that the IOC should give in order that the project will be profitable?

20. (Continuation of previous problem) After long negotiations the IOC decided to lend Kane $600,000 at t=0. The project went ahead. After the research and development stage was completed (at t=4) but before the investment was made, the IOC decided to cancel race walking as an Olympic event. As a result Kane is expecting a large drop in sales of the SWS shoes. What is the minimum number of shoes Nike has to sell annually for the project to be profitable in each of the following two cases:

a) If in the event of cancellation the original loan term continues?

b) If in the event of cancellation the company has to return the outstanding debt to the IOC immediately?

21. The Aphrodite company is a manufacture of perfume. The company is about to launch a new line of products. The marketing department has to decide whether to use an aggressive or regular campaign.

Aggressive campaign

Initial cost - (production of commercial advertisement using a top model): $400,000

First month profit: $20,000

Monthly growth in profit (month 2-12): 10%

After 12 months the company is going to launch a new line of products and it is expected that the monthly profits from the current line would be $20,000 forever.

Regular Campaign

Initial cost (using a less famous model) $150,000

First month profit: $10,000

Monthly growth in profits (month 2-12): 6%

Monthly profit (month 13-∞): $20,000

a) The cost of capital is 7%. Calculate the NPV of each campaign and decide which campaign should the company undertake.

b) The manager of the company believes that due to the recession expected next year, the profit figures for the aggressive campaign (both first month profit, and 2-12 profit growth) are too optimistic. Use data table in order to show the differential NPV as a function of first month payment and growth rate of the aggressive campaign.

22. The Long-Life company has a 10 year monopoly for selling a new vaccine that is capable of curing all known cancers. The demand for the new drug is given by the following equation:

$$P = 10{,}000 - 0.03X\ ,$$

where P is the price per vaccine and X is the quantity. In order to mass-produce the new drug the company needs to purchase new machines. Each machine costs $70,000,000 and is capable of producing 500,000 vaccines per year. The expected life span of each machine is 5 years; over this time it will be depreciated on a straight-line basis to zero salvage value. The R&D cost for the new drug is $1,500,000,000, the variable costs are $1,000 per vaccine, fixed costs are $120,000,000 annually. If the discount rate is 12% and the tax rate is 30%, how many vaccines will the company produce annually? (Use either Excel's **Goal Seek** or its **Solver**—see Chapter 32.)

23. (Continuation of problem 22) The independent senator from Alaska Michele Carey has suggested that the government will pay Long-Life $2,000,000 in exchange for the company

guaranteeing that it will produce under the zero profit policy. (i.e. produce as long as NPV>=0).

How many vaccines will the company produce annually?

CHAPTER 6: DERIVING THE WEIGHTED AVERAGE COST OF CAPITAL (WACC)*

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Chapter contents

Overview

In Chapter 5 we discussed general principles of deriving discount rates. The basic principle is that the discount rate for a stream of cash flows should be appropriate to the riskiness of the cash flows. Although the measurement of risk is still vague (we will be specific about

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measuring risks only in Chapters 10 – 15), we saw in Chapter 5 that we often have a good intuitive feel for what constitutes a similar risk investment, and that this intuition allows us to determine a discount rate. For example, an investment whose cash flows are almost certain should be discounted at the bank lending rate. A real estate investment, on the other hand, should be discounted at the average rate of return we're likely to get from other, similar and risky, real estate investments.

In this chapter we discuss the **weighted average cost of capital** (WACC). The WACC is the average rate of return the firm has to pay its shareholders and its lenders. The WACC is often the appropriate risk-adjusted discount rate for a company's cash. Here are two examples:

- White Water Rafting Corporation is considering buying a new type of raft. The raft is more expensive than the existing rafts operated by the company because it is self-sealing—holes in the raft are automatically and permanently fixed by a new technology. During the rafting season, White Water's existing rafts spend a considerable amount of down time having their punctures fixed, and the company anticipates that the new self-sealing rafts will improve its profitability by increasing efficiency and decreasing costs. By being the first rafting company on the river to have the self-sealing rafts, White Water hopes to attract business away from other rafting companies—customers naturally hate to have their trips interrupted by "flat rafts" (the rafting equivalent of a "flat tire"), and when they hear of White Water's new rafts, they will prefer White Water over its competitors.

  The White Water financial analyst has derived the set of anticipated cash flows for the new raft. To complete the NPV analysis, the company needs to decide on an appropriate discount rate. Here's where the WACC comes in: Since the riskiness of the cash flows from the new rafts is similar to the riskiness of White Water Rafting's existing cash flows, the WACC is an appropriate discount rate.

- Gorgeous Fountain Water Company (GF) sells bottled water from the Gorgeous Fountain natural spring. The company is considering buying Dazzling Cascade Water Company. Dazzling Cascade (DC) operates in a neighboring area to that dominated by GF, and its operations, sales, and anticipated cash flows have been thoroughly analyzed by the Gorgeous Fountain financial analysis staff.

  In order to value DC, GF has to decide on an appropriate discount rate for the anticipated DC cash flows. Here's where the weighted average cost of capital comes in. GF's WACC is the average rate of return demanded by its investors; assuming that the riskiness of DC's cash flows is similar to that of GF, the WACC is an appropriate discount rate for the GF cash flows. Discounting the DC cash flows at GF's WACC allows Gorgeous Fountain to establish a bid price for Dazzling Cascade.

Some important terminology before we start

When we talk about "firms" in this book we generally mean corporations, companies that have shareholders and debtholders.[1] A typical firm is *incorporated*, which means that it is a legal entity which is separate from its shareholders and debtholders. The income of a corporation is taxed at the corporate income tax rate.

The shareholders own stock in the firm. When the firm is profitable, management may decide to pay dividends to the shareholders, but these dividend payments are not guaranteed. Shareholders can also sell their shares and in doing so may make a profit (called a "capital gain")

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[1] Equivalent terminology for shareholders: stockholders, equity owners; for debtholders: lenders, bondholders.

or a loss. As you can see, the cash flows of a shareholder in a firm are uncertain. The shareholders in the firm have *limited liability*; they are not responsible for repaying the debtholders if the firm cannot do so out of its cash flows.

The *cost of equity*, denoted r_E, is the discount rate applied by shareholders to their expected future cash flows from the firm. It goes without saying that this cost of equity depends—like the cost of equity of our real-estate investment in Chapter 5—both on the riskiness of the firm's free cash flows. The higher the riskiness of the shareholder's expected future cash flows, the higher the cost of equity r_E.

The firm's *debtholders* are its lenders. Debtholders are promised a fixed return (interest) on their lending to the firm. The debtholders may be banks, who have lent money to the firm, or they may be individuals or pension funds who have bought the firm's bonds. The interest payments to the firm's debtholders are expenses for tax purposes. The interest payments on the firm's debt and the firm's tax rate determine the *after-tax cost of debt* for the firm, which we denote $r_D(1\text{-}T)$.

Finance concepts discussed

- Cost of equity, r_E and the Gordon dividend model
- Cost of debt, r_D

Excel functions used

- This chapter uses no interesting Excel functions!

6.1. What does the firm's WACC mean?

The **weighted average cost of capital** (WACC) is the *average return that the company has to pay to its equity and debt investors*. Another way of putting this is that the WACC is the *average return shareholders and debtholders expect to receive from the company.*[2] The definition of the WACC is:

$$WACC = r_E * \underbrace{\frac{E}{E+D}}_{\substack{\uparrow \\ \text{the percentage} \\ \text{of equity used to} \\ \text{finance the firm}}} + r_D\left(1-T_C\right) * \underbrace{\frac{D}{E+D}}_{\substack{\uparrow \\ \text{the percentage} \\ \text{of debt used to} \\ \text{finance the firm}}}$$

where

r_E = *the firm's cost of equity--the return required by the firm's shareholders*

r_D = *the firm's cost of debt--the return required by the firm's debtholders*

E = *market value of the firm's equity*

D = *market value of the firm's debt*

T_C = *the firm's tax rate*

Here's a simple example to show what we mean: United Transport Inc. has 3 million shares outstanding; the current market price per share is \$10. The company has also borrowed \$10 million from its banks at a rate of 8%; this is the company's cost of debt r_D. United Transport has a tax rate of T_C = 40%.[3] The company thinks its shareholders want an annual

---

[2] In finance the *expected return*, the *required return*, the *cost of capital* (be it *cost of equity* or *cost of debt*), the *required rate of return* are all synonyms. They all represent the market-adjusted rate that investors get (or demand) on various investments or securities.

[3] We use the symbol T_C to indicate the *corporate* tax rate.

return on their investment of 20%; this 20% return is the company's cost of equity r_E.[4] To compute United Transport's WACC we use the formula:

$$
\begin{aligned}
&r_E = 20\% \\
&r_D = 8\% \\
&E = 3{,}000{,}000 \textit{ shares each worth } \$10 = \$30{,}000{,}000 \\
&D = \$10{,}000{,}000 \\
&T_C = 40\% \\
&WACC = r_E * \frac{E}{E+D} + r_D\left(1-T_C\right) * \frac{D}{E+D} \\
&\qquad = 20\% * \frac{30}{30+10} + 8\% * \left(1-40\%\right) * \frac{10}{30+10} = 16.2\%
\end{aligned}
$$

In a spreadsheet:

	A	B	C
1	**UNITED TRANSPORT--WACC**		
2	Number of shares	3,000,000	
3	Market price per share	10	
4			
5	E, market value of equity	30,000,000	<-- =B3*B2
6	D, market value of debt	10,000,000	
7			
8	r_E, cost of equity	20%	
9	r_D, cost of debt	8%	
10	T_C, firm's tax rate	40%	
11			
12	WACC, weighted average cost of capital: WACC=r_E*E/(E+D)+r_D*(1-T_C)*D/(E+D)	16.20%	<-- =B8*B5/(B5+B6)+B9*(1-B10)*B6/(B5+B6)

The United Transport WACC computation shows you that the WACC depends on five critical variables.

- r_E , the cost of equity. r_E is the return required by the firm's shareholders. Of the five parameters in the WACC calculation, r_E is the most difficult to calculate. A model for calculating r_E is given in Section 6.2.

[4] How did United Transport come to the conclusion that its shareholders want a 20% return? This is *the* question in the computation of the WACC, and we will spend a lot of this chapter discussing the answer. So be patient!

- E, the market value of a firm's equity. We will usually take E to equal the number of shares of the firm times the market price per share.
- r_D, the cost of debt. r_D is the cost of borrowing for the firm. In most cases we will take r_D to be the firm's marginal interest rate—the interest rate at which the firm could borrow additional funds from its banks or by selling bonds. A detailed example of the calculation of r_D for an actual firm is given in Section 6.4.
- D, the market value of the firm's debt. In most cases we will take D to be the total value of the firm's financial obligations. An actual example of a calculation for D is given below in Section 6.4.
- T_C, the firm's tax rate. Most often we calculate T_C by computing the average tax rate of the firm; see Section 6.4 for an example.

6.2. The Gordon dividend model: discounting anticipated dividends to derive the firm's cost of equity r_E

In this section we present a formula for computing the firm's cost of equity r_E. We will call the formula the *Gordon dividend model*, in honor of Myron Gordon who first set out the model in 1959.[5] This section has two subsections:

- In the first subsection we derive a model for calculating the value of a firm's shares based on their future anticipated dividends.

[5] The model is sometimes simply called the Gordon model; others call it the dividend discount model.

- In the second subsection, we use the share valuation model of the first section to derive the cost of equity r_E.

Valuing the firm's shares as the present value of the future anticipated dividends

We start by computing the fair market value of a stock that pays a growing dividend stream. Here is an example which presents most of the logic of our model: It is March 2, 2000, and you are thinking of purchasing a share of XYZ Corporation. Here are some facts about the company and its stock:

- XYZ is a steady payer of dividends; in the past it has paid dividends annually, and these dividends have tended to grow at an annual rate of 7%.
- The company just paid a dividend of \$10 per share. This dividend was paid on March 1, the company's traditional dividend payment date.

You want value XYZ shares by discounting the stream of future anticipated dividends. In predicting the future dividends of XYZ Corp., you assume that the dividends will grow at a rate of *7%* per year. Then the future anticipated dividends per share are:

$$
\begin{aligned}
Dividend\ \ today &= Div_0 = \$10.00 \\
Dividend\ \ next\ \ year &= Div_1 = Div_0\left(1+g\right) = \$10*\left(1+7\%\right) = \$10.70 \\
& Div_2 = Div_1\left(1+g\right) = Div_0\left(1+g\right)^2 = \$10*\left(1+7\%\right)^2 = \$11.45 \\
& Div_3 = Div_2\left(1+g\right) = Div_0\left(1+g\right)^3 = \$10*\left(1+7\%\right)^3 = \$12.25 \\
& \quad \ldots \\
& Div_t = Div_0\left(1+g\right)^t
\end{aligned}
$$

The three dots ... indicate that you think that the dividend stream is *very long* (when we write down the actual model, we will assume that the dividend stream goes on forever).

Suppose you think that the appropriate discount rate for the dividend stream is XYZ's cost of equity r_E = 15%. Using r_E to discount the future anticipated dividends, you get the fair value of the XYZ Corp's stock today (we will denote this by P_0):

$$\begin{aligned}
&\textit{valuing XYZ Corp. stock}: \\
&\textit{fair share value today}, P_0 = \frac{Div_1}{(1+r_E)} + \frac{Div_2}{(1+r_E)^2} + \frac{Div_3}{(1+r_E)^3} + \ldots \\
&\qquad = \frac{Div_0(1+g)}{(1+r_E)} + \frac{Div_0(1+g)^2}{(1+r_E)^2} + \frac{Div_0(1+g)^3}{(1+r_E)^3} + \ldots \\
&\qquad = \frac{Div_0(1+g)}{r_E - g}
\end{aligned}$$

The last line of the above formula uses a formula for the present value of a constant-growth annuity developed in Chapter 1 (p000): The present value of the cash flows $Div_0(1+g)$, $Div_0(1+g)^2$, $Div_0(1+g)^3$, ... at the discount rate r_E is :

$$P_0 = \sum_{t=1}^{\infty} \frac{Div_0(1+g)^t}{(1+r_E)^t} = \frac{Div_0(1+g)}{r_E - g}, \ \textit{when}\ |g| < |r_E|.^6$$

Applying the valuation model to XYZ stock gives:

$$\begin{aligned}
\textit{fair share value today}, P_0 &= \frac{10(1.07)}{(1.15)} + \frac{10(1.07)^2}{(1.15)^2} + \frac{10(1.07)^3}{(1.15)^3} + \ldots \\
&= \frac{\overbrace{10(1.07)}^{\text{This is } Div_0(1+g)}}{\underbrace{0.15-0.07}_{\text{This is } r_E - g}} = 133.75
\end{aligned}$$

Here's a spreadsheet implementation of the Gordon dividend model:

---

[6].The condition $|g| < |r_E|$ means that the absolute value of g less than the absolute value of r_E. If a firm's dividends have positive growth, then this is the same as assuming that $0 < g < r_E$.

	A	B	C
1	**VALUING XYZ CORP. SHARES**		
2	Current dividend, D_0	10	
3	Dividend growth rate, g	7%	
4	Cost of equity, r_E	15%	
5	Share value	133.75	<-- =B2*(1+B3)/(B4-B3)
6			
7	The formula used in cell B5 is usually called the Gordon dividend model, in honor of M. J. Gordon,who first stated its application to share valuation (1959).		

Using the Gordon dividend model to calculate the cost of equity r_E

In the previous subsection we derived the value of a share P_0 based on the current dividend per share Div_0, the anticipated growth rate of dividends g, and the cost of equity r_E. In this section we turn this formula around: We derive the cost of equity r_E based on the current value of a share P_0, the current dividend per share Div_0, the anticipated growth rate of dividends g.

According to the Gordon dividend model of the previous subsection, the stock price is given by $P_0 = \frac{Div_0(1+g)}{r_E - g}$. Turning this formula around to solve for the cost of equity r_E gives:

$$r_E = \frac{Div_0(1+g)}{P_0} + g \quad .$$

This is the *Gordon dividend model cost of equity formula*. In the Gordon dividend model the cost of equity r_E –the discount rate to be applied to equity cash flows—is the sum of two terms:

- $\frac{Div_0(1+g)}{P_0}$. This is the *anticipated dividend yield* of the stock. Suppose you buy the stock today, paying P_0. Then you anticipate getting a next-period dividend of $Div_0(1+g)$, where g is the anticipated growth rate of dividends. The term $\frac{Div_0(1+g)}{P_0}$ is the anticipated next period dividend return.
- g. This the *growth rate of all future dividends paid on the stock.*

Applying the Gordon dividend model cost of equity formula—a simple example

Consider a firm for which the current share price is $P_0 = \$25.00$ and which has just paid a per-share dividend of $Div_0 = \$3.00$. Shareholders of the firm believe that dividends will grow at a rate $g = 8\%$ per year. In this case the Gordon model cost of equity is $r_E = 20.96\%$:

	A	B	C
1	**USING THE GORDON MODEL TO COMPUTE THE COST OF EQUITY r_E**		
2	Current dividend, Div_0	3.00	
3	Current share price, P_0	25.00	
4	Anticipated dividend growth rate, g	8%	
5	Gordon model cost of equity, r_E	20.96%	<-- =B2*(1+B4)/B3+B4

6.3. Applying the Gordon cost of equity formula—Courier Corporation

Courier Corporation (stock symbol CRRC) is a book manufacturer that has experienced rapid growth of sales and profits. Courier's financial year ends September 30. We use the Gordon dividend model to calculate Courier's cost of equity at the end of September 2000.

Here's a spreadsheet which gives the relevant data and the calculations:

	A	B	C
1	**COURIER CORPORATION (CRRC)** **Calculation of Cost of Equity using Gordon Model**		
2	**Year ended 30 Sept**	**Dividend per share**	
3	1995	0.27	
4	1996	0.32	
5	1997	0.32	
6	1998	0.35	
7	1999	0.40	
8	2000	0.48	
9			
10	*g*, growth rate of dividends	12.47%	<-- =(B8/B3)^(1/5)-1
11	Div_0, current dividend	0.48	<-- =B8
12	$Div_0*(1+g)$, dividend anticipated in 2001	0.54	<-- =B11*(1+B10)
13	P_0, stock price, 30 Sept. 2000	21.66	
14			
15	r_E, Gordon dividend model cost of equity	14.97%	<-- =B12/B13+B10

In order to use the Gordon model to calculate the cost of equity in cell B15, we need the following assumptions:

- **The price of the share, P_0, is known**. In this case the P_0 is the stock price on the date of the calculation (30 September 2000). On this date $P_0 = \$21.66$.
- **The current dividend per share, Div_0, is known**. By "current dividend" we mean the last dividend paid by the firm, which in this case is the year 2000 Courier dividend Div_0 = $0.48 per share.
- **The average growth rate of the dividends, g, can be derived**. We derive this below from the dividend series in cells B3:B8. Our assumption is that

$$
\begin{aligned}
&Div_{1995} = 0.27 \\
&Div_{1996} = Div_{1995}(1+g) \\
&Div_{1997} = Div_{1995}(1+g)^2 \\
&\ldots \\
&Div_{2000} = Div_{1995}(1+g)^5 = 0.48
\end{aligned}
$$

This means that $g = \sqrt[5]{\frac{Div_{2000}}{Div_{1995}}} - 1 = \sqrt[5]{\frac{0.48}{0.27}} - 1 = 12.47\%$.

Given these assumptions, the cost of equity, r_E, for Courier is given by:

$$r_E = \frac{Div_0(1+g)}{P_0} + g = \frac{0.48*(1+12.47\%)}{21.66} + 12.47\% = 14.97\%$$

This is the calculation performed in cell B15.

Alternative calculations of the growth rate

The implementation of the Gordon model illustrated above uses the geometric growth rate $g = \sqrt[5]{\frac{D_{2000}}{D_{1995}}} - 1$ to calculate the g in the Gordon model. Here are two alternative ways to compute the growth rate g:

- *Alternative 1: Use a different time period.* In the example above, we've assumed that the future expected growth rate of dividends is predicted by the dividends between 1995 and 2000. However, we could—after some thought—decide that the dividends are better predicted by the period 1994 – 2000.[7] In this case the dividend growth rate is

$$g = \sqrt[6]{\frac{Div_{2000}}{Div_{1994}}} - 1 = \sqrt[6]{\frac{0.48}{0.13}} - 1 = 23.80\%$$

[7] If you look at the dividend series, you'll see that there was a very large increase in the Courier dividend between 1994 and 1995. In choosing 1995 as the base year, we've decided that the one-time increase in dividends of 100% between 1994 and 1995 isn't likely to be repeated. If—as in the current example—we use 1994 as the base year instead of 1995, we indicate that over a longer period the large dividend increase between 1994 and 1995 is likely to recur. As you can see, this assumption means that the cost of equity increases considerably.

This changes the cost of equity considerably—since anticipated dividend growth g is higher in this alternative than before, the cost of equity $r_E = \frac{Div_0(1+g)}{P_0} + g$ will be higher as shown below:

	A	B	C
1	**COURIER CORPORATION (CRRC)** **Alternative 1: Using a different base year**		
2	**Year ended 30 Sept**	**Dividend per share**	
3	1994	0.13	
4	1995	0.27	
5	1996	0.32	
6	1997	0.32	
7	1998	0.35	
8	1999	0.40	
9	2000	0.48	
10			
11	*g* , growth rate of dividends	23.80%	<-- =(B9/B3)^(1/6)-1
12	Div_0, current dividend	0.48	<-- =B9
13	$Div_0*(1+g)$, dividend anticipated in 2001	0.59	<-- =B12*(1+B11)
14	P_0, stock price, 30 Sept. 2000	21.66	
15			
16	r_E , Gordon dividend model cost of equity	26.54%	<-- =B13/B14+B11

- Alternative 2: Ignore historical dividends altogether. You might decide that the past history of Courier dividends is not indicative of its future dividend payouts. In this case, you might want to use a different number altogether for the anticipated dividend growth rate g.[8] In the example below you've decided that the growth rate for Courier's future dividends is 15%. This gives a cost of equity of 17.55%.

[8] In Chapters 8 - 9 we'll show how to build a model for the firm. You might want to use this model to project the dividend growth rate.

	A	B	C	D
1	**COURIER CORPORATION (CRRC)** **Alternative 2: Making up a future growth rate of dividends**			
2	*g*, growth rate of dividends	15.00%		
3	Div_0, current dividend	0.48		
4	Div_0*(1+g), dividend anticipated in 2001	0.55	<-- =B3*(1+B2)	
5	P_0, stock price, 30 Sept. 2000	21.66		
6				
7	r_E, Gordon dividend model cost of equity	17.55%	<-- =B4/B5+B2	

A final alternative to computing the cost of equity r_E: Using the total equity payout instead of per-share data

In some of the years 1995 – 2000, Courier purchased stock from its shareholders in open-market repurchase transactions. Look at the data below:

	A	B	C	D	E	F
1	**COURIER CORPORATION (CRRC)** **Computing the Total Equity Payout**					
2	**Year ended 30 Sept**	**Dividend per share**	**Total dividends**	**Share repurchases**	**Total equity payout: Dividends + repurchases**	
3	1995	0.27	793,000	0	793,000	
4	1996	0.32	970,000	0	970,000	
5	1997	0.32	969,000	882,000	1,851,000	
6	1998	0.35	1,205,000	0	1,205,000	
7	1999	0.40	1,354,000	455,000	1,809,000	
8	2000	0.48	1,572,000	114,000	1,686,000	
9						
10	Growth rate	12.47%	14.67%		16.28%	<-- =(E8/E3)^(1/5)-1
11						
12	Stock price, 30 Sept. 2000	21.66				
13	Number of shares, 30 Sept. 2000	2,938,000				
14	Market value of equity, 30sep00	63,637,080	<-- =B13*B12			
15						
16	2000 total dividend	1,686,000	<-- =E8			
17	Anticipated dividend growth rate	16.28%	<-- =E10			
18						
19	Gordon model cost of equity, r_E	19.36%	<-- =B16*(1+B17)/B14+B17			

The data in column C are for the total dividend paid out by Courier (*total dividend = dividend per share*number of shares*); in column D we see the amount of cash paid out to shareholders for the repurchase of their shares. Column D gives the *total equity payout*: the sum of the dividends + repurchases. Using the figures from column D and using the total market value of the equity on 30 September 2000, the Gordon model cost of equity for Courier is 19.36%.[9]

$$\begin{aligned}
&\textit{Gordon dividend model for cost of equity}\\
&\textit{using all payouts to equity holders:}\\
&r_E = \frac{\begin{bmatrix} \textit{Total current} \\ \textit{equity payout} = \\ \textit{total dividends} + \\ \textit{repurchases of stock} \end{bmatrix} \left(1 + \underbrace{g}_{\substack{\uparrow \\ \text{The anticipated} \\ \text{growth rate} \\ \text{of total equity} \\ \text{payouts}}} \right)}{\textit{Total equity value today}} + g \\
&\quad = \frac{1,686,000 * (1 + 16.28\%)}{63,637,080} + 16.28\% = 19.36\%
\end{aligned}$$

Although there is some controversy attached to the use of total equity payouts to compute the cost of equity r_E, we think it is the correct method. In the examples for Courier Corporation which follow, we will assume that the r_E for Courier is 19.36%.

---

[9] Of course our previous comments on alternative ways of computing the dividend growth rate g still apply—we could, for example, choose a different base year for the growth calculation.

Why do firms repurchase stock?

In recent years share buybacks have exceeded dividends as a form of distribution to shareholders. Firms repurchase stock instead of paying extra dividends for several reasons:

- Repurchases are used to "soak up" extra cash and keep dividend growth predictable. Most dividend-paying firms think their shareholders want to see a steady pattern of dividend growth. So if they have extra cash, they'll use it to buy back shares instead of increasing the dividend paid to shareholders.
- Repurchases help reduce shareholder taxes on cash paid out to shareholders. When a dividend is paid, all the shareholders receiving the dividend pay taxes on it at their ordinary income tax rate. Stock repurchases are voluntary (you don't have to sell your stock back to the company ...). If you let your stock be repurchased, the gain in most cases is taxed at your capital gains tax rate (lower than the ordinary income tax rate).
- Stock repurchases benefit both the shareholder who is bought out and the shareholder who does not let his shares be repurchased. Why? When some of the shares of the firm are repurchased, those shareholders who "stay in" the firm will get a larger share of its income and dividend payments in the future. So all parties gain.

6.4. Calculating the WACC for Courier

So far we've calculated Courier's cost of equity as $r_E = 19.36\%$. This is the return demanded by the company's shareholders. Now we want to calculate Courier's weighted average cost of capital $WACC = r_E \frac{E}{E+D} + r_D\left(1-T_C\right)\frac{D}{E+D}$. Before we can do this, however, we need to compute the values of the following variables:

- *E*: the market value of Courier's equity. As you can see from the previous spreadsheet, on September 30, 2000, Courier had 2,938,000 shares worth $21.66 per share. This gives *E = 2,938,000*21.66=$63,637,080*.
- *D*: The value of Courier's debt. On 30 September 2000, Courier had debt of $31,693,000 . This information comes from the company's annual report (see Figure 6.1). Courier's **debt** includes both the *current portion of long-term debt* and the *long-term debt* itself.[10]

[10] The calculation of the WACC actually calls for the *market value* of the firm's debt. However, this is a number which is very difficult to calculate; instead it is standard practice to use the book value of the debt as illustrated.

COURIER CORPORATION

CONSOLIDATED BALANCE SHEETS

	September 30, 2000	September 25, 1999
LIABILITIES AND STOCKHOLDERS' EQUITY		
Current liabilities:		
Current maturities of long-term debt (Note D)	$ 366,000	$ 338,000
Accounts payable	18,023,000	11,644,000
Accrued payroll	6,708,000	5,173,000
Accrued taxes	5,303,000	5,162,000
Other current liabilities	7,606,000	5,034,000
Total current liabilities	38,006,000	27,351,000
Long-term debt (Note D)	31,327,000	1,193,000
Deferred income taxes (Note C)	2,428,000	2,693,000
Other liabilities	2,709,000	2,716,000
Total liabilities	74,470,000	33,953,000

Figure 6.1: Courier's liabilities from its balance sheet. The debt items are marked.

- r_D, the cost of Courier's borrowing. In theory r_D ought to be the marginal cost of debt—the borrowing rate of the company for additional debt. However, this rate is usually difficult to derive. A plausible alternative is use information about the current borrowing rate of the company. In Figure 6.2 you can see what the company reports about its long term debt. You can see that the debt has been borrowed at various interest rates. We use the borrowing rate of 7.13% (the rate applicable to most of the debt) as the company's cost of debt r_D .

COURIER CORPORATION

NOTES TO CONSOLIDATED FINANCIAL STATEMENTS

D. LONG-TERM DEBT

Long-term debt consisted of the following:

	2000	1999
Obligation under revolving bank credit facility at 7.13% as of September 30, 2000	$30,500,000	$ –
Obligation under industrial development bond arrangement at 3%, payable in monthly installments through May 2011	901,000	972,000
9.5% secured promissory note, payable in monthly installments through October 2001	292,000	559,000
	31,693,000	1,531,000
Less: Current maturities	366,000	338,000
Total	$31,327,000	$1,193,000

Figure 6.2: Courier's borrowing terms, as set out in the notes to its financial statements. As you can see, most of the company's borrowing is at the rate of 7.13%. We use this rate as the firm's cost of debt r_D .

- T_C, Courier's tax rate. We can calculate Courier's tax rate from its provision for income taxes. Courier's provision for income taxes in 2000 was $\frac{5,249,000}{15,886,000} = 33.04\%$. We use this as an estimate for the firm's tax rate T_C.

COURIER CORPORATION

CONSOLIDATED STATEMENTS OF INCOME

For the Years Ended	September 30, 2000	September 25, 1999	September 26, 1998
Net sales	$188,320,000	$163,991,000	$151,591,000
Cost of sales	140,226,000	123,184,000	113,923,000
Gross profit	48,094,000	40,807,000	37,668,000
Selling and administrative expenses	32,002,000	27,726,000	26,653,000
Interest expense	325,000	524,000	1,303,000
Other income (expense) (Note J)	119,000	–	2,043,000
Income before taxes	15,886,000	12,557,000	11,755,000
Provision for income taxes (Note C)	5,249,000	4,181,000	4,030,000
Net income	$ 10,637,000	$ 8,376,000	$ 7,725,000

Figure 6.3: Courier's income statements show the taxes paid. By dividing the company's year 2000 taxes by its income before taxes, we arrive at a tax rate of

$$T_C = 33.04\% = \frac{5,249,000}{15,886,000}.$$

So what's Courier's WACC?

Here's our calculation for Courier's WACC:

	A	B	C
1	**COURIER CORPORATION (CRRC)** **Calculating the WACC, Sept. 2000**		
2	Cost of equity, r_E	19.36%	<-- Computed from total equity payouts
3	Cost of debt, r_D	7.13%	<-- From Courier Corp. financial statements
4			
5	Sept. 2000 equity value, E	63,637,080	<-- Number of shares times current share price
6	Sept. 2000 debt value, D	31,693,000	<-- From Courier Corp. financial statements
7	Total: Equity + Debt, E+D	95,330,080	<-- =SUM(B5:B6)
8			
9	Percentage of equity, E/(E+D)	67%	<-- =B5/B7
10	Percentage of debt, D/(E+D)	33%	<-- =B6/B7
11			
12	Tax rate, T_C	33.04%	<-- From Courier Corp. financial statements
13			
14	WACC	14.51%	<-- =B2*B9+B3*(1-B12)*B10

In the next section we'll use the WACC of 14.51% for Courier to calculate the value of its equity.

6.5. Two uses of the WACC

The weighted average cost of capital (WACC) is the weighted average rate of return required by a company's shareholders and debtholder. We presume that this rate of return reflects the average risk of shareholder and debtholder future cash flows. This is plausible, since we have derived the cost of equity r_E from anticipated future payouts to shareholders, and we have derived the cost of debt r_D from the rate demanded on the firm's debts by its lenders. Thus the WACC represents a weighted average of the riskiness of shareholder and debtholder cash flows.

When the riskiness of a stream of cash flows is similar to the riskiness of the cash flows received by shareholders and debtholders, the WACC is the appropriate risk-adjusted discount rate. There are two important cases where this is often true:

- In capital budgeting situations. When a company is considering investing in a project whose risk is comparable to the riskiness of the company as a whole, the WACC is an appropriate discount rate for the project's cash flows.
- To value the company as a whole. Below we define the concept of *free cash flow* (FCF). The value of the Courier is the discounted value of its future anticipated FCFs, where the WACC is the discount rate.

In this section we illustrate both these uses of the WACC for Courier.

Using the WACC as a discount rate for projects

The WACC of Courier Corporation is 14.51%—this is the weighted average return demanded by the firm's shareholders and bondholders. Recall that Courier is in the book printing business. Suppose the company is thinking of investing in a project whose riskiness is like the riskiness of its current business. This could be something as simple as another printing press to print more books or a warehouse to house them, but it could also be something much more complicated—like the acquisition of another printing company.

In all of these cases, the WACC is the natural starting point as a discount rate. What we mean by "starting point" is that—in discounting the cash flows of the project—Courier should assume that initial discount rate is 14.51% and then "tweak" the discount rate a bit to adjust for perceived risks.

Let's say that the company is considering buying a machine that will allow them to print more books. The cash flows, NPV, and IRR of the machine are given below. If the riskiness of the machine's cash flows is similar to the riskiness of Courier's overall cash flows, then the WACC is a reasonable discount rate. The analysis below shows that company should not undertake the investment—the investment's NPV is negative (-$16,460) and its IRR (7.80%) is less than the WACC of 14.51%:

	A	B	C
1	**COURIER CORPORATION (CRRC)** **Using the WACC as a discount rate**		
2	WACC	14.51%	
3			
4	**Year**	**Cash flow**	
5	0	-100,000	
6	1	15,000	
7	2	22,000	
8	3	33,000	
9	4	44,000	
10	5	12,000	
11			
12	NPV	-16,460	<-- =NPV(B2,B6:B10)+B5
13	IRR	7.80%	<-- =IRR(B5:B10)

Of course there's always room for "tweaking," since some of the assumptions we made may not be as accurate as we thought. Suppose, for example, that the machine's cash flows are perceived to be much less risky than the overall cash flows of Courier. As an extreme case we might consider the case where the machine cash flows are only as risky as Courier's debt. Since the company's after-tax cost of debt is 7.13%*(1-33.04%) = 4.77%, this would then be an appropriate discount rate for the project and the company should accept it (since the IRR of 7.80% is higher than 4.77%).

Valuing Courier Corporation using its WACC and predicted free cash flows (FCFs)

In the previous subsection we used the weighted average cost of capital (WACC) to value a typical project of the firm. The second major use of the WACC is to value companies. A complete explanation of this use of the WACC will have to wait until Chapter 9, where we explain the concept of free cash flow (FCF) in detail. For our purposes in this chapter, the **free cash flow** (FCF) is the amount of cash generated by the company's business activities, by its operations as opposed to its financing activities. The FCF is "free" in the sense that it can be used to provide cash to the firm’s shareholders and debtholders in the form of dividends and share repurchases (payments to shareholders) and interest payments (to debtholders).

To accurately define the FCF, you need some knowledge of accounting. If the following table gives you problems, you should read the accounting refresher in Chapter 7.

Here’s the definition of the FCF:

Defining the Free Cash Flow (FCF)	
Profit after taxes	This is the basic measure of the profitability of the business, but it is an accounting measure that includes financing flows (such as interest), as well as non-cash expenses such as depreciation. Profit after taxes does not account for either changes in the firm's working capital or purchases of new fixed assets, both of which can be important cash drains on the firm. The FCF definition takes changes in working capital and purchases of new fixed assets into account separately.
+ Depreciation	This non-cash expense is added back to the profit after tax.
The sum of the next two items is the *change in net working capital*, often denoted by ΔNWC	
- Increase in current assets related to the firm's operations.	When the firm's sales increase, more investment is needed in inventories, accounts receivable, etc. This increase in current assets is not an expense for tax purposes (and is therefore ignored in the profit after taxes), but it is a cash drain on the company. For purposes of calculating the FCF, the increase in current assets does not include changes in cash and marketable securities.
+ Increase in current liabilities related to the firm's operations	An increase in the sales often causes an increase in financing related to sales (such as accounts payable or taxes payable). This increase in current liabilities—when related to sales—provides cash to the firm. The FCF includes all current liability items related to operations; it does not include financial items such as short-term borrowing, the current portion of long-term debt, and dividends payable.
- Capital expenditures (CAPEX)	An increase in fixed assets (the long-term productive assets of the company) is a use of cash, which reduces the firm's free cash flow.
+ after-tax interest payments (net)	FCF measures the cash produced by the business activity of the firm. The FCF should not include any items related to the firm's financing. In particular we need to neutralize the effect of interest payments which appear in the firm's profit after taxes. We do this by: • Adding back the after-tax cost of interest on debt (*after-tax* since interest payments are tax-deductible), • Subtracting out the after-tax interest payments on cash and marketable securities.
FCF = sum of the above	

In 2000 Courier Corporation had a free cash flow (FCF) of $6,381,240:

	A	B	C
1	**COURIER CORPORATION** **Calculation of FCF for 2000**		
2	Profit after taxes	10,637,000	
3	Add back depreciation	8,062,000	
4	Changes in working capital		
5	Subtract increases in current assets	-1,141,000	
6	Add increases in current liabilities	4,962,000	
7	Subtract out capital expenditures	-16,347,000	
8	Add back after-tax interest	208,240	
9	Free cash flow (FCF)	6,381,240	<-- =SUM(B2:B8)

Using FCFs and WACC to value Courier

In finance theory, the market value of a company's debt and equity is the value of its free cash flows discounted at its weighted average cost of capital:

$$Value\ of\ \ Debt + Equity = PV\ \ value\ of\ \ future\ FCFs,\ discounted\ at\ WACC$$

Suppose that you've performed a careful analysis of Courier Corporation and that you think the future growth of Courier's FCF is 8% per year. Since Courier's WACC is 14.51%, we can value the company as the present value of its future FCFs in the following way:

$$\begin{aligned}
&Courier\ value = \\
&Value\ of\ \ Equity + Debt = PV\left(FCFs,\ discounted\ at\ WACC\right) \\
&\qquad = \sum_{t=1}^{\infty} \frac{FCF_t}{\left(1+WACC\right)^t} = \sum_{t=1}^{\infty} \frac{FCF_{2000} * \left(1+FCF\ \ growth\ rate\right)^t}{\left(1+WACC\right)^t} \\
&\qquad = \sum_{t=1}^{\infty} \frac{6,381,240 * \left(1+8\%\right)^t}{\left(1+14.51\%\right)^t} = \frac{6,381,240 * \left(1+8\%\right)}{14.51\% - 8\%} = 105,849,458
\end{aligned}$$

Notice that this valuation—like the Gordon dividend model of Section 6.2—makes use of the constant-growth annuity formula developed in Chapter 1 (p000):

$$\sum_{t=1}^{\infty} \frac{FCF_{2000} * \left(1+FCF\ \ growth\ rate\right)^t}{\left(1+WACC\right)^t} = \frac{FCF_{2000} * \left(1+FCF\ \ growth\ rate\right)}{WACC - FCF\ \ growth\ rate}$$

Since the debt of Courier is worth \$31,693,000, we value the equity at \$104,849,458-\$31,693,000=\$74,156,458. Since there are 2,938,000 shares, each share is worth $\frac{74,156,458}{2,938,000} = 25.24$. This compares favorably with the current share price of Courier, \$21.66, so this makes Courier (in the parlance of stock market analysts) a "buy" recommendation.

Here's all of this in a spreadsheet:

	A	B	C
1	**VALUING COURIER**		
2	Year 2000 FCF	6,381,240	
3	Anticipated FCF growth	8%	
4	WACC	14.51%	
5			
6	Firm value	105,849,458	<-- =B2*(1+B3)/(B4-B3)
7	Debt value	31,693,000	
8	Equity value	74,156,458	<-- =B6-B7
9			
10	Number of shares	2,938,000	
11	Per-share value	25.24	<-- =B8/B10

One further note—mid-year discounting

We introduced this topic in Chapter 4 and subsequently dropped it like a hot potato. The idea was that because most cash flows occur throughout the year—the appropriate discounting process should discount them as if they occur in mid-year. In terms of the computation just done for Courier, instead of calculating:

$$\begin{aligned} Debt + Equity &= \frac{FCF_{2000}(1+FCF\ growth)}{(1+WACC)} + \frac{FCF_{2000}(1+FCF\ growth)^2}{(1+WACC)^2} + \dots \\ &= \frac{FCF_{2000}(1+FCF\ growth)}{WACC - FCF\ growth} \end{aligned},$$

we should be calculating the

$$Debt + Equity = \frac{FCF_{2000}\left(1+FCF\ \ growth\right)}{\left(1+WACC\right)^{0.5}} + \frac{FCF_{2000}\left(1+FCF\ \ growth\right)^{2}}{\left(1+WACC\right)^{1.5}} + \dots$$
$$= \left[\frac{FCF_{2000}\left(1+FCF\ \ growth\right)}{WACC - FCF\ \ growth}\right] * \left(1+WACC\right)^{0.5}$$

As explained in Chapter 4 (p000), mid-year discounting raises our valuation of cash flows, since the earlier a cash flow occurs, the more it is worth. If we implement mid-year discounting for Courier, then our valuation of Courier's shares increases from $25.25 to $27.77:

	A	B	C
1	**VALUING COURIER** **Using mid-year discounting**		
2	Year 2000 FCF	6,381,240	
3	Anticipated FCF growth	8%	
4	WACC	14.51%	
5			
6	Firm value	113,269,251	<-- =(1+B4)^0.5*B2*(1+B3)/(B4-B3)
7	Debt value	31,693,000	
8	Equity value	81,576,251	<-- =B6-B7
9			
10	Number of shares	2,938,000	
11	Per-share value	27.77	<-- =B8/B10

Summing up

In this chapter we have calculated the firm's weighted-average cost of capital (WACC). The WACC is the risk-adjusted discount rate for the firm's free cash flows. It is often used to value projects whose riskiness is similar to the riskiness of the firm's existing activities, and it is also used to derive the value of the firm. Both of these uses have been illustrated in this chapter.

The WACC is defined as:

$$WACC = r_E \frac{E}{E+D} + r_D\left(1-T_C\right)\frac{D}{E+D}$$

In the table below we summarize how we derived each of the elements of this formula:

Cost of equity r_E	We've used the Gordon model to determine the cost of equity: $r_E = \frac{Div_0(1+g)}{P_0} + g$, where Div_0 = total dividends + stock repurchases of the current year; g = anticipated growth rate of dividends+repurchases; P_0 = total equity value on current date
Cost of debt r_D	In principle, this should be the firm's marginal borrowing rate, but this is often difficult to determine. For Courier, we used a number representative of the firm's cost of borrowing. An alternative is to use the firm's average borrowing cost over the previous year: $r_D = \frac{Interest\ \ paid\ \ in\ current\ \ year}{Average\ \ debt,\ this\ year\ \ and\ \ last}$
Market value of equity E	Current number of shares * current market price per share
Market value of debt D	The market value of a firm's debt is difficult to calculate. We almost always substitute the *book value* of the firm's debt for this number. In the Courier example we showed how to determine this book value from the firm's balance sheets.
Firm's tax rate T_C	T_C ought to be the firm's *marginal tax rate*. In practice we usually use either: a) The firm's average tax rate, measured by: $average\ tax\ rate = \frac{Taxes\ \ from\ \ Profit\ \ and\ Loss\ \ Statement}{Profit\ \ before\ \ taxes} = 33.04\%$ b) The firm's *statutory tax rates*. Courier's statutory Federal tax rate is 34%. State taxes are another 2.98% of its income. Another estimate of its tax rates might thus be 36.98% .

A final warning

Cost of capital calculations are critical for valuations and controversial. They involve a mixture of theory and judgment. Almost every number in the WACC calculation above can be determined in several ways. In many cases professionals do extensive sensitivity analysis on the WACC and the FCF growth to establish a *price range*—the range of valuations which appears to be reasonable, given the variation in plausible assumptions.

The most important modification you might want to make to the WACC calculation above involves the cost of equity r_E. An important competing model to the Gordon model is the capital asset pricing model (CAPM). In Chapter 14 we will show you how to use this model to calculate the cost of equity.

Exercises

1. Compute the weighted average cost of capital (WACC) for a company having:

Market value of debt	\$200,000
Market value of equity	\$300,000
Cost of debt, r_E	7.5%
Cost of debt, r_D	13%
Tax rate, T_C	40%

2. Calculate the cost of equity r_E for a company having:

Market value of debt	\$2,500,000
Market value of equity	\$1,000,000
Cost of debt, r_D	5%
Tax rate, T_C	25%
Weighted average cost of capital, WACC	10%

3. Aboudy Corporation's stock price is currently \$22.00 per share. The company has just paid a dividend of \$0.55 per share, and shareholders anticipate that this dividend will grow in the future at a rate of 6% per year. Use the Gordon model to calculate the company's cost of equity r_E.

4. Gradcom's anticipated next-year dividend is $1.20. Analysts anticipate that this dividend will grow at a 4% annual rate.

4.a. If the stock's current share price is $30, what is its cost of equity r_E according to the Gordon Model?

4.b. Show in an Excel graph the cost of equity as a function of the dividend growth rate (let the growth rate be 0%, 2%, 4%, …, 20%).

5. Consider the following data regarding Cinema Company.

	A	B	C	D	E	F
1	**Cinema Company**					
2	Year	Dividend per share	Total dividends	Number of share repurchases	Payments from share repurchases	Total
3	1995	0.25	???	0	0	???
4	1996	0.25	???	115,000	140,000	???
5	1997	0.3	???	0	0	???
6	1998	0.31	???	200,000	260,000	???
7	1999	0.35	???	120,000	180,000	???
8	2000	0.37	???	0	0	???
9	2001	0.39	???	0	0	???
10	2002	0.42	???	120,000	220,000	???
11						
12	Stock price, end of 2002	1.83				
13	Number of shares, January 1995	4,300,000				

5.a. Complete the ??? in the spreadsheet above (assume that the dividend payment was before the share repurchase).

5.b. Find the cost of equity r_E of Cinema using the Gordon dividend model for the total equity payout.

5.c. What would be Cinema's cost of equity if we consider only the dividend payments without the share repurchases?

6. It is 1 January 2005, and you are interested in finding the cost of equity r_E of your company. After a quick search you have found the following data:

- The company currently has 1,600,000 shares outstanding. The current share price is $3.
- The company's earnings for 2004 were $2,000,000. The company policy has just paid out $300,000 in dividends, and it intends to continue this 15% dividend payout from earnings in the future.
- During 2004 the company spent $600,000 on share repurchases. It is the company's intention to increase the amount spent on share repurchases at the same growth rate as the amount spent on dividends.
- Projected earnings growth is 2% per year.

Using the Gordon model for the total equity payout, what is the company's cost of equity r_E?

****

1. You wish to estimate the share's price of 'softy', your favorite underwear company. You know that tomorrow the company will pay its annual dividend in the amount of 1.5$ per share, a growth in the company's cash dividends of 4% comparing to last year. As an experienced investor, you demand a yield of 12% on your investment in the company. What should be the company's share price?

2. Your boss asked you to find the WACC of 'Welcome to Paradise' company. After a quick research you have come up with the following data:
   • The company has 1,600,000 shares, currently sold for 2$ per share.
   • The company's debt is 2,500,000$. The amount of interest paid last year by the company was $300,000.
   • The corporate tax rate is 40%.
   • The cost of capital requested by the investors is 13%.
   What is the company's WACC?

3. You are interested in calculating the cost of capital of 'The lions' Company, based on the average WACC of its industry, which is 11%. You know that the company stock price is $11, and it has 5,500,000 shares. The company cost of debt is 9%, its debt is $4,000,000 and the company's tax rate is 40%. What is the company cost of capital?

4. You are interested in calculating the WACC of ABC Company. Its stock price is $8, and it has a debt to equity ratio of 1. ABC's cost of debt is 9%, its cost of capital is 12% and the company's tax rate is 40%. What is the company's WACC?

5. Assume the following data concerning 'ZZZ' Company.

   • The company has 2,000,000 shares, currently sold for 2.5$ per share.

   • The company's debt is 3,000,000 from the company market value. The interest rate paid last year by the company was $250,000.

   • The company paid total dividend of $600,000 last year, and its expected dividend growth is 3%. In addition, the company repurchases 150,000 of its shares.

   • The corporate tax rate is 30%.

   What is the company's WACC?

6. You have come up with the following data concerning 'Zion' Company.

   • The company has 2,500,000 shares.

   • The company's debt is 90% from the company market value. The interest rate paid last year by the company was $500,000.

   • The company paid total dividend of $800,000 last year, which is 25% of its pre-tax profit, and its expected growth next year is in $50,000 more.

   • The company paid tax in the amount of 950,000.

   • The cost of capital requested by the investors is 13%.

   What is the company's WACC?

7. You are considering a new project to your firm. This project requires investment of $500,000 and generates cash flow of $70,000 for the next 10 years. To your judgment,

the project has no risk. You know that your company's WACC is 14%, and that the risk free rate is 6%. Should you take this project?

8. 'The sauce', a well-known pizza factory, have asked you to evaluate the factory free cash flow (FCF) activity. You have estimated that the FCF of the factory is $4,500,000, its WACC is 12.5% and its estimated growth is 5% each year. If you know that 'The sauce' debt is $19,000,000 and it has 6,500,000 outstanding shares, what should be the share's price? Repeat the question by using mid-year discounting as well.

1. XYZ Corp. has just paid a dividend of $5 per share. You think this dividend will grow at 8% per year. If you think the correct discount rate for the dividend stream of XYZ is 25%, how much should you be willing to pay for the stock?

2. You just bought a share of ABC Corp. for $28. The company has just paid a dividend of $2 per share, and you anticipate that this dividend will grow at a rate of 12% per year. What is your implied cost of equity for ABC?

3. You are considering purchasing a stock of ABC Corp, which has just paid a $3 annual dividend per share. The company does not repurchase any of its shares. You anticipate that the company's dividends will grow at a rate of 20% per year for the next 5 years. After this time, you think that the growth of the annual dividends will slow to 5% per year. If your cost of equity for ABC is 10%, what price should you be prepared to pay for the stock?

4. Suppose that a firm is financed with 70% equity and 30% debt. The interest rate on debt is 8%, and the expected return on the common stocks is 17%. The firm's tax rate is 40%. What is the firm's weighted average cost of capital?

5. You are given the following information for Twin Inc.

Long-Term Debt Outstanding:	$300,000
Current Yield to Maturity (r_D):	8%
Number of Shares of Common Stock:	10,000
Price per Share:	$50
Book Value per Share:	$25
Expected Rate of Return on Stock (r_E):	15%

a) Calculate Twin Inc.'s weighted-average cost of capital (assuming that the firm pays no taxes).

b) How would r_E and the weighted-average cost of capital change if Twin Inc.'s stock price falls to $25 due to declining profits? Assume that business risk is unchanged.

CHAPTER 7: AN ACCOUNTING PRIMER*

this version: October 31, 2003

Chapter contents

Overview

Accounting is an important component of financial analysis. Although you've probably had an accounting course before taking the principles of finance course for which this book is

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intended, you may have forgotten some basic accounting principles.[1] In this chapter we remind you of these accounting principles, using a series of simple examples.

One of the things you will notice is that—with the exception of this sentence—we never refer to *debits* and *credits*. This is an accounting chapter for finance students: We write all of our accounts and information directly onto the balance sheet.

This chapter will help you understand three basic accounting statements: The balance sheet, the profit and loss statement and the consolidated statement of cash flows. In addition we will explain in detail the concept of free cash flow, which we introduced in Chapter 6 and which plays a central role in the valuation of companies.

Accounting concepts discussed

- Balance sheet, profit and loss statement, consolidated statement of cash flows
- Assets, liabilities
- Equity, debt
- Fixed assets, depreciation
- Accounts receivable, accounts payable
- Accrual accounting

Finance concepts discussed

- Free cash flow (FCF)
- Discounting FCFs and residual value to value the firm

[1] Without disparaging our accounting colleagues, the introductory accounting courses are often badly taught.

Excel functions used

This chapter uses only the most basic Excel functions.

7.1. Three basic accounting statements

This chapter aims to explain the three basic accounting statements that you need to understand if you're to do financial accounting. In this section we summarize these statements; if you do not understand this summary, skip this section and read on—more explanations will follow (if you *do* understand everything, perhaps this section is superfluous for you).

The balance sheet

The balance sheet is a *double-columned* statement.

- The left-hand column ("Assets") gives details of what the company owns. It includes items such as: cash on hand, inventories, and equipment. It also includes items such as the billings the company has sent out for which it has not received payment (accounts receivable).
- The right-hand column ("Liabilities and Equity") gives details of how the company's assets are financed—who put up the money to finance the assets. This column includes items like: borrowing from banks and other debtholders, money raised from shareholders (equity), bills not yet paid (accounts payable).

BASIC BALANCE SHEET—What assets does the firm own and how are these assets financed?	
Assets: What the company owns	**Liabilities and equity**: Who put up the money for the assets
Short-term assets—assets with a short life (generally less than one year, see footnote, p000) Cash Marketable securities Inventories Accounts receivable: Customer billings that are not yet paid **Fixed assets**—assets with a longer life Land Plant, property and equipment net of depreciation	**Short-term liabilities**—financing which the company will repay in the short term (generally within one year) Short-term borrowing from banks Accounts payable: Bills that the company must pay Taxes payable: Taxes that the company knows it will have to pay in the short term **Long-term liabilities**—debt financing that has to be repaid over a period of more than one year **Equity**—money provided by shareholders Stock: Money paid by shareholders to the company for shares in the company Accrued retained earnings: Firm profits not paid out to shareholders
TOTAL ASSETS	**TOTAL LIABILITIES AND EQUITY**
NOTE: Total assets and total liabilities-equity are *always equal*	

The profit and loss statement (P&L)

The profit and loss statement (also called an "income statement") is a statement of how much the firm has earned over a given period. This period is most often a year, but it can also be a quarter or a month. The accountant's aim in the P&L is to provide a statement of earnings that accurately reflects the underlying economics of the firm's operations. Many readers of P&L statements view them as providing a statement of the amount of cash left in the company's till at the end of the period. These two aims are not equivalent, and the divergence between them provides much material for the conflict between accountants and finance professionals. We've already noted this conflict in Chapter 4, where we back the depreciation to the profits when we derived the cash flow. In section 7.3 of this chapter, we show the accounting profession's solution to this problem—the consolidated statement of cash flows. In section 7.4 we discuss the free cash flow (FCF), which is the finance profession's answer.

But these issues are for later! In the meantime here's a typical profit and loss statement:

PROFIT AND LOSS—how much money did the firm make?
Sales Subtract **costs of goods sold (COGS)—**the direct cost of producing the sales Subtract **selling, general, and administrative expenses (SG&A)—**the overheads involved in producing the sales Subtract **depreciation—**the cost of using the firm's fixed assets Subtract **interest expenses—**the cost of the firm's borrowings Add **interest income** and **other income from** cash and marketable securities
Profits before taxes (PBT) Subtract **taxes**
Profits after taxes (PAT) Subtract **Dividends paid to shareholders**
Retained earnings—firm profits not paid to shareholders. These are added to the "accrued retained earnings" item on the Liabilities and Equity side of the balance sheet

A basic misunderstanding—the corner grocery store versus IBM or: Accrual versus Cash Accounting

"My grandparents operated a corner grocery store. Gramps gave no credit and paid all his bills in cash. At the end of every day he walked to the bank and deposited whatever he had in the cash register drawer ("the till"). This was his profit for the day. I don't understand all this b.s. about the profit and loss not reflecting the cash realities of a business. Why can't everyone be like Gramps?"

If you have a simple business, you can still be like Gramps. You can do your accounting using the **cash accounting method**—whatever remains in the till (or in your bank account) is your profit over a given period. But most business use the **accrual accounting method** discussed in this chapter. In this method, certain non-cash items count as either income or as expenses. The system applies economic logic to make the income/expense determination. Here are some examples which even Gramps would agree with:

- On January 15, 1953, Gramps took in $1000 and paid out bills of $600. But when writing down the profit at the end of the day, he saw that Mrs. Smith, one of his best and most reliable clients, had promised to pay him her $25 grocery bill tomorrow. By the logic of cash accounting, Gramps had made $400 for the day, but by any other logic, his actual profits for the day were $425.
- On the next day, the milk man came by before the store opened and left $50 of dairy products on the stoop. Gramps will pay him a day later. By economic logic, the unpaid bill of $50 should be attributed to today (and lower today's profits), even if only paid the next day.

These examples can be multiplied, until even Gramps would agree that **accrual accounting**, though more complicated than cash accounting, is a more logical way to determine his profits.

The consolidated statement of cash flows

The firm's consolidated statement of cash flows explains *where the money came from and where it went*. Because the profit and loss statement does not necessarily reflect the realities of how cash flows into and out of the firm as a result of its activies, the cash flow statement is necessary to close the loop.

CONSOLIDATED STATEMENT OF CASH FLOWS Where did the cash come from and where did it go?
Operating cash flows—cash implications of the firm's activities Profits after taxes Add back depreciation (this is an expense on the P&L, but doesn't cost cash) Subtract increases in inventories, accounts receivable, etc. (these don't appear on the P&L, but they require cash) Add increases in accounts payable, taxes payable, etc. (these appear on the P&L as expenses but they weren't actually paid, and so they supply cash)
Investment cash flows—cash implications of the firm's investment activities Subtract out purchases of equipment Subtract purchases of subsidiaries, other companies, etc. Add back sales of equipment, subsidiaries, etc. Add or subtract sales or purchases of financial investments (such as securities) by the firm
Financing cash flows—cash implications of the firm's financing activities Add new debt financing Subtract repayment of debts Add sales of new shares (and subtract repurchases of shares by the firm) Subtract dividends paid to shareholders
Adding all these items together should give the *change in the firm's cash balances* over the accounting period

In the following sections, we show how to construct these three statements for Anytown Travel Services (ATS), a new company which is just being started in Anytown, U.S.A.

7.2. Starting a firm

Brother and Sister live in Anytown, U.S.A.. They've just graduated from college, and have decided to start a taxi service. There's no taxi company in Anytown, and they think it will be a great success. They've got $25,000 in cash with which to start the business.

It's 2 January 2003. Brother and sister:

- Go to a lawyer and incorporate themselves as Anytown Travel Services (ATS).
- They open a bank account and deposit $25,000.

At this point their balance sheet looks like this:

	A	B	C	D	E
2	1-Jan-03				
3	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**				
4	**Assets**			**Liabilities and equity**	
5	Cash	25,000		Equity	25,000
6	Total assets	25,000		Total liabilities and equity	25,000

Now they need a taxi, so they go to a local used car dealer and pick out a nice car. Along with the taxi sign on top of the car, some minor repairs, and a tank of gas, the car costs $18,000. They pay for the car in cash. Here's their new balance sheet:

	A	B	C	D	E
10	1-Jan-03				
11	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**				
12	**Assets**			**Liabilities and equity**	
13	Cash	7,000		Equity	25,000
14	Taxi	18,000			
15	Total assets	25,000		Total liabilities and equity	25,000

31 January 2003

During January 2003, their first month of operation, brother and sister have:

- Collected $12,000 in taxi fares
- Paid $4,000 in gasoline and other costs

- Paid themselves $1,000 each as a salary.[2]

This means that the balance sheet on 31 January 2003 looks like:

	A	B	C	D	E
19	31-Jan-03				
20	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**				
21	**Assets**			**Liabilities and equity**	
22	Cash			Equity	25,000
23	Cash at beginning of month	7,000			
24	Fares	12,000			
25	Gas, etc.	-4,000			
26	Salaries	-2,000			
27	Cash at end of month	13,000			
28					
29	Taxi	18,000			
30	Total assets	31,000		Total liabilities and equity	25,000

Unfortunately, this balance sheet doesn't *balance*—the total assets don't equal the total liabilities and equity. This is probably the worst transgression you can make in an accounting framework! In this case the solution to this problem is easy—ATS has actually made $6,000 during the month ($12,000 in receipts minus $6,000 in costs). This profit is added to the initial equity of the firm as *retained earnings* (meaning: profits not paid out):

	A	B	C	D	E	F	G	H	I
33	31-Jan-03								
34	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**								
35	**Assets**			**Liabilities and equity**			**Profit and loss for the period**		
36	Cash						Sales	12,000	<-- =B38
37	Cash at beginning of month	7,000					Owner salaries	-2,000	<-- =B40
38	Fares	12,000					Fuel	-4,000	<-- -
39	Gas, etc.	-4,000		**Equity**			Profit	6,000	<-- =SUM(H36:H38)
40	Salaries	-2,000		Initial stock	25,000		Dividends	0	
41	Cash at end of month	13,000		Accumulated retained earnings	6,000		Retained earnings	6,000	<-- =H39+H40
42									
43	Taxi	18,000							
44	**Total assets**	**31,000**		**Total liabilities and equity**	**31,000**				

The retained earnings from the profit and loss—that part of the profits that ATS doesn't pay out as dividends—ends up on the balance sheet in "accumulated retained earnings." This means that we've now got 2 equity accounts—"Initial stock" is the amount of money brother-sister initially

[2] Do Anytown residents give tips? Yes, but Brother and Sister decided that whoever's driving can keep the tips she/he gets.

put into the firm, and "accumulated retained earnings" is that part of profits they've decided not to pay out.

A Note on Terminology

Almost every accounting item in our example has another (equally valid) name. For example:

- Profit is also called Income and a Profit and Loss statement is often called an Income Statement.
- Initial Stock is also called "Stock issued at par" or sometimes "Stock issued at par and additional premium" or "Paid-in capital" (in this book we often use just "Stock")

We'll try to be consistent in this book, but we can't always keep our promise!

28 February 2003

Receipts, costs and salaries during the month were the same as those in January. On the last day of February, however, ATS underwent a dramatic expansion: Brother and Sister bought another taxi. Like the first, this one cost $18,000. They paid for this taxi by using $10,000 of their cash and borrowing $8,000 from the bank. The loan has to be paid off over the next 10 months ($800 per month) and ATS has to pay 1% interest per month on the outstanding loan balance. Because the loan has to be paid back in the short-term, it is classified as a *current liability*—an obligation of the firm that has to be paid back within a year.

Here's what the balance sheet looks like:

	A	B	C	D	E	F	G	H
47	28-Feb-03							
48	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**							
49	**Assets**			**Liabilities and equity**			**Profit and loss for the period**	
50	Cash			**Current liabilities**			Sales	12,000
51	Cash at beginning of month	13,000		Taxi loan from bank	8,000		Owner salaries	-2,000
52	Fares	12,000					Fuel	-4,000
53	Gas	-4,000					Profit	6,000
54	Salaries	-2,000		**Equity**			Dividends	0
55	Used to buy new taxi	-10,000		Initial stock	25,000		Retained earnings	6,000
56	Cash at end of month	9,000		Accumulated retained earnings				
57				January 2003	6,000			
58	Taxis	36,000		February 2003	6,000			
59	**Total assets**	**45,000**		**Total liabilities and equity**	**45,000**			

Notice that on the right-hand side of the balance sheet we've started to distinguish between "liabilities" and "equity." The former represents financing from outside sources, while "equity" is financing by the owners of the company.

31 March 2003

ATS had another very successful month:

- They hired an extra driver for each of the two taxis (Brother and Sister still drive a couple of hours per day, but the drivers bear most of the brunt). The drivers are getting $1,500 each. Each taxi brought in $20,000 for the month. However, $8,000 of this derives from a contract signed to provide transportation for the local flour mill and its executives/guests. The flour mill hasn't paid this money yet; they pay their bills on the 15th of the month following. As you will see below, this unpaid bill generates an *account receivable* on ATS's balance sheet—a bill to customers which is outstanding and is anticipated to be paid within a year. An account receivable is an asset of the firm (some firms sell their accounts receivable—if you can sell it, it must be an asset).
- One of the drivers was promised a signing bonus of $800 which would be paid only if she proved herself for at least 6 weeks. If she's still working for ATS by 20 May 2003, this driver will be paid the $800 bonus. The bonus (as yet unpaid) is listed as a *Current*

liability; this is accounting terminology for an unpaid bill which will be paid within a year.

- Brother and Sister bought a computer for the office (still located in the spare room of their house). The computer cost $2,000.
- Other expenses: Gas $6,000; Brother-Sister raised their salaries to $2,000 each, for a total of $4,000.
- They paid off $800 on the car loan. The bank charges them 1% per month, so that they've also paid $80 interest.

	A	B	C	D	E	F	G	H
62	31-Mar-03							
63	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**							
64	**Assets**			**Liabilities and equity**			**Profit and loss for the period**	
65	**Current assets**			**Current liabilities**			Sales	40,000
66	Cash			Unpaid signing bonus	800		Driver salaries	-3,000
67	Cash at beginning of month	9,000		Taxi loan from bank	7,200		Owner salaries	-4,000
68	Fares paid	32,000					Signing bonus	-800
69	Driver salaries	-3,000					Fuel	-6,000
70	Owner salaries	-4,000					Interest	-80
71	Fuel	-6,000					Profit	26,120
72	Interest	-80						
73	Repayment of principal	-800						
74	Computer	-2,000						
75	Cash at end of month	25,120						
76				**Equity**				
77	Accounts receivable	8,000		Initial stock	25,000			
78				Retained earnings				
79	**Fixed assets**			January 2003	6,000			
80	Computer	2,000		February 2003	6,000			
81	Taxis	36,000		March 2003	26,120			
82								
83	**Total assets**	**71,120**		**Total liabilities and equity**	**71,120**			

Here are some notes on the balance sheet:

- Look at how the computer is listed (cells B74 and B80). On the one hand, it was paid for out of cash and therefore there's a -$2,000 item in the cash lines (B74). On the other hand, the computer is *not* an expense—instead it's a capital investment (like the two taxis) and is listed under fixed assets (B80). Accountants (and the tax authorities) define a capital asset as an asset that: i) has a life longer than a year and ii) produces income over its lifetime. Capital assets are not immediately listed on the profit and loss as expenses; instead, their cost is written off over their lifetime as depreciation.[3]

- Notice that the $880 paid to the bank is *fully deducted* from the cash in the balance sheet (cells B72 and B73) but that only $80 of this sum (the interest payment) is an expense on the profit and loss statement (H68). Repayment of loan principal is not an expense (you're only paying back what you got—it's not a cost, although it is a negative cash flow, see below). The $800 repayment of loan also materializes on the right-hand side of the balance sheet as a reduction in the auto loan under current liabilities (cell E67).
- We've started to distinguish between *current assets* and *fixed assets* on the left-hand side of the balance sheet. "Current assets" are short-lived assets, which can or will be liquidated at short notice, usually a year or less. The cash account on the balance sheet is obviously a current asset. AST's accounts receivable are also current assets because they relate to the unpaid bills of the flour mill which will be paid off within the month.
- Note that the receivable of $8,000 is part of the sales of the firm on the profit and loss statement (cell H65). That is—not all the firm's sales are in cash. Any *reasonably anticipated receipt* should be put in the profit and loss statement as a sale. Similarly—see next bullet—any reasonably anticipated expense should be recorded in the profit and loss statement as a cost. On the other hand the repayment of this $8,000 receivable in the coming month will not affect the profit and loss statement.

[3] As we write this chapter, a financial scandal in the United States involves WorldCom, which misclassified some expenses of doing business as the purchase of long-term assets. (This would be similar to ATS listing the salaries of their taxi drivers as the purchase of a fixed asset.) By doing this WorldCom *over-reported* its income.

- In a similar way, the $800 unpaid signing bonus is listed as a *current liability* (E66) and reduces the month's profits (H68), even though it has not yet actually been paid. In the next statement the company will pay this $800 bonus, and at that point it will not affect the profit and loss statement.

April - June

The following events occurred during this three-month period:

- The two taxis produced sales of $25,000/month each, $150,000 for the whole period. $30,000 ($10,000 per month) of this figure is for the transportation contract with the local flour mill. This company always pays one month in arrears, so that at the end of June, they still have a $10,000 bill outstanding.[4]
- The drivers were paid $1,500 each per month = $9,000.
- Brother and Sister salaries stayed the same as in March ($2,000 each per month) = $12,000.
- Gasoline for the two taxis cost $7,000 per month per taxi = $42,000. The gas station has agreed to extend credit to ATS—the company can now pay its gasoline bills on the 10th of the month following. At the end of June, $14,000 in gasoline bills remained to be paid (this will be recorded as an account payable on the liabilities side of the balance sheet).
- In each of the three months, ATS paid $800 off on the taxi loan, so that the total loan outstanding at the end of June is $4,800 (this is the initial $8,000 loan, minus repayments

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[4] During the April-June period, the flour mill i) paid off the $8,000 bill left from March, ii) was billed $10,000 per month, and iii) paid off April's $10,000 bill in May and paid off May's $10,000 bill in June. At the end of the period, the company still owed ATS $10,000 for services rendered in June.

of $800/month for March – June). The interest payments (1% per month on outstanding balances) during the April – June period were:

- $72 for April (1% of $7200)
- $64 for May (1% of $6400)
- $56 for June (1% of $5600)

- ATS was billed by its insurance company for a policy that has been in force since January. The cost is $12,000. They paid this whole amount in May 2003. At the end of June, $6,000 of this amount relates to insurance for the period January – June; this will be included in the profit and loss statement as an expense. The other $6,000 relates to insurance for the rest of the year. As you will see below, the *unused portion* of the insurance ($6,000) is recorded as a *prepaid expense* on the asset side of the balance sheet.
- On 30 June 2003 Brother and Sister bought a small building to house the ATS offices. The building (an old gas station) includes a garage, some office space, and fuel pumps for the taxis. The cost of the building was $80,000. They financed the purchase with $20,000 in cash and a $60,000, 10-year mortgage from bank. The mortgage conditions are:
    - Monthly principle repayment $\dfrac{\$60,000}{\underbrace{10*12}_{\text{10 years*12 months/year}}} = \$500$.
    - Monthly interest payment: ½% on outstanding mortgage balance.
- They paid the signing bonus to the driver ($800).

Here are the new balance sheet and profit and loss statement:

	A	B	C	D	E	F	G	H	I
86	30-Jun-03								
87	**ANYTOWN TRAVEL SERVICES (ATS), Inc.**								
88	**Assets**			**Liabilities and equity**			**Profit and loss for the period**		
89	**Current assets**			**Current liabilities**			Sales	150,000	<-- =B92+B104
90	Cash			Taxi loan from bank	4,800		Driver salaries	-9,000	<-- =B94
91	Cash at beginning of April	25,120		Unpaid signing bonus	0		Owner salaries	-12,000	<-- =B95
92	Fares paid	140,000		Accounts payable (gasoline)	14,000		Fuel	-42,000	<-- =B96-E92
93	Payment of outstanding receivable	8,000		Total current liabilities	18,800		Interest	-192	<-- =B97
94	Driver salaries	-9,000					Insurance	-6,000	<-- =B99+B105
95	Owner salaries	-12,000		**Long-term liabilities**			Profit	80,808	<-- =SUM(H89:H94)
96	Fuel	-28,000		Mortgage to buy building	60,000		Dividends	0	
97	Interest	-192					Retained earnings	80,808	
98	Repayment of principal	-2,400							
99	Insurance	-12,000							
100	Building, cash payment	-20,000							
101	Signing bonus	-800							
102	Cash at end of June	88,728							
103									
104	Accounts receivable	10,000		**Equity**					
105	Prepaid expenses (insurance)	6,000		Initial stock	25,000				
106	Total current assets	104,728		Retained earnings					
107				January 2003	6,000				
108	**Fixed assets**			February 2003	6,000				
109	Computer	2,000		March 2003	26,120				
110	Taxis	36,000		April-June 2003	80,808				
111	Building	80,000		Total equity	143,928				
112	Total fixed assets	118,000							
113									
114	**Total assets**	222,728		**Total liabilities and equity**	222,728				

In columns G and H we've given a profit and loss statement for the period which ignores depreciation and taxes (we'll get to these in a moment). An outlay is recorded as an expense only if it is required in the current period to produce income. Here are some examples:

- The $20,000 paid in cash for the building is balanced by an asset of $80,000 (the building itself) and the $60,000 mortgage which is recorded as a liability. Thus the $20,000 is not an expense, even though it's a cash outlay. When the building starts to be depreciated, this depreciation will be recorded as an expense (it represents the cost of the building in producing the current period's income).
- The $800 for the signing bonus is a cash outlay balanced by a reduction in a corresponding current liability. The signing bonus was an expense in the April accounting period.
- The $12,000 spent for insurance is partially offset by a $6,000 prepaid expense. Thus only $6,000 of the insurance outlay is an expense on the profit and loss statement.

Preparing a profit and loss statement for January - June

Their accountant insists that Brother and Sister prepare a profit and loss statement for the first half-year of the company's operations. The accountant explains that:

- It is important they know how the new company has performed thus far
- Brother and Sister have to pay an estimated tax payment to the IRS on July 15 based on their profit for the first half year.

The main difference between the profit and loss statements shown thus far and the statement that the accountant prepares is *depreciation*. There are several different interpretations of depreciation:

- Depreciation is a cost allowed by the tax authorities for the use of a fixed asset. Since we have not thus far included the costs of fixed assets (the taxis, the computer, and the building) in our profit and loss statements, depreciation is a way to spread out these costs over the useful life of the assets.
- Depreciation represents the *economic cost* of using a fixed asset over the life of the asset.
  - The accountant depreciates the computer over a 2-year useful life. The monthly depreciation of the computer is therefore $\$2,000/24 = \$83.33$. Since they've owned the computer for three months, the total depreciation on the computer for the period is 3*83.33=$250.
  - The taxis are depreciated over a 3-year useful life; this works out to $\frac{\$18,000}{3*12} = \$500$ per month. They bought the first taxi on 1 January, so that it has 6 months of depreciation (= $3,000). The second taxi is 4 months old (it was

purchased the last day of February), so that its depreciation is $2,000. Thus total depreciation on the taxis is $5,000.

- Finally, the building, bought on 30 June 2003, is going to be depreciated over 10 years. However, since it has just been put on the balance sheet, there is no depreciation to be taken for this building yet.

The corporate tax rates applicable to ATS are 5% state tax and 36% Federal tax. For purposes of computing the Federal tax, the state tax is an expense (see cells H129 and H130 below). Here's the way the balance sheet and the profit and loss look after taking into account depreciation and the tax rates:

	A	B	C	D	E	F	G	H	I
117	30-Jun-03								
118	**ANYTOWN TRAVEL SERVICES (ATS), Inc., Jan-June 2003 financial statements including depreciation and taxes**								
119	**Assets**			**Liabilities and equity**			**Profit and loss for the period**		
120	**Current assets**			**Current liabilities**			Sales	214,000	<-- =H36+H50+H65+H89
121	Cash			Taxi loan from bank	4,800		Driver salaries	-12,000	<-- =H66+H90
122	Cash at beginning of period	25,000		Unpaid signing bonus	0		Owner salaries	-20,000	<-- =H37+H51+H67+H91
123	Cash sales	204,000		Accounts payable (gasoline)	14,000		Signing bonus	-800	<-- =H68
124	Driver salaries	-12,000		Taxes payable	44,562		Fuel	-56,000	<-- =H38+H52+H69+H92
125	Owner salaries	-20,000		Total current liabilities	63,362		Interest	-272	<-- =H70+H93
126	Signing bonus	-800					Insurance	-6,000	<-- =H94
127	Fuel	-42,000		**Long-term liabilities**			Depreciation	-5,250	<-- =B142+B144+B146
128	Interest	-272		Mortgage to buy building	60,000		Profit	113,678	<-- =SUM(H120:H127)
129	Repayment of car loan principal	-3,200					State income tax (5%)	-5,684	<-- =-5%*H128
130	Insurance	-12,000					Federal income tax (36%)	-38,878	<-- =-36%*(H128+H129)
131	Building, cash payment	-20,000					Profit after taxes	69,116	<-- =H128+H129+H130
132	Cash paid for taxis	-28,000					Dividends	0	
133	Cash paid for computer	-2,000					Retained earnings	69,116	<-- =H131-H132
134	Cash at end of period	88,728							
135									
136	Accounts receivable	10,000							
137	Prepaid expenses (insurance)	6,000							
138	Current assets	104,728							
139									
140	**Fixed assets**			**Equity**					
141	Computer	2,000		Initial stock	25,000				
142	Minus accumulated depreciation	-250		Accumulated retained earnings	69,116				
143	Taxis	36,000		Total equity	94,116				
144	Minus accumulated depreciation	-5,000							
145	Building	80,000							
146	Minus accumulated depreciation	0							
147	Net fixed assets	112,750							
148									
149	**Total assets**	217,478		**Total liabilities and equity**	217,478				

Note that on 1 July 2003, ATS hasn't actually paid the taxes (they're due only on 15 July). To take care of this, the accountant creates a category called *taxes payable*; this is a current liability for taxes i) due within a short period of time, ii) that have already been accounted for in the profit and loss statement, and iii) that have not yet been paid. The taxes payable account (cell E124) of $44,562 is the sum of the state and Federal taxes owed ($5,684 + $38,878).

7.3. The consolidated statement of cash flows

This is the third accounting statement we're mastering in this chapter. The purpose of the consolidated statement of cash flows is to explain the growth over the period of the cash balances on the balance sheet. The statement of cash flows accomplishes this by classifying all the firm's cash inflows and outflows into three categories: cash flow from operating activities, cash flow from investing activities, and cash flow from financing activities.

Cash flow from operating activities

Cash flow from operating activities includes the profit after taxes for the period minus increases in operating current assets plus increases in operating current liabilities:

	A	B
153	**Consolidated statement of cash flows**	
154	**Cash flow from operating activities**	
155	Profit after taxes	69,116
156	Add back depreciation	5,250
157	Subtract increase in current assets	-16,000
158	Add increases in current liabilities	58,562
159	**Cash provided from operating activities**	116,928

- In the period January – June, ATS had profits of $69,116. It recorded depreciation of $5,250 on its taxis and computer; this depreciation is not a cash expense, and it is added back in the cash flow statement.
- In addition, the company's current assets excluding cash grew by $16,000. This is the sum of the end-June account receivable from the flour mill plus the pre-paid insurance

expenses. This $16,000 is a cost of business not recorded in the profit and loss; the cash flow statement subtracts this amount.

- At the end of June, the company had $44,562 of unpaid taxes and $14,000 of unpaid gasoline bills. This *increase in current liabilities* is an expense which was recorded on the P&L but which has (as yet—ultimately these bills will, of course, be paid) no cash implications. Therefore we add it back.

Cash flow from investing activities

Cash flow from investing activities includes acquisition of fixed assets (land, property, machines) and investments made by the company in marketable securities.

	A	B
161	**Cash flow from investing activities**	
162	Payments for fixed assets	
163	Taxis	-36,000
164	Computer	-2,000
165	Building	-80,000
166	Purchases of marketable securities	0
167	Proceeds from sales of marketable securities	0
168	**Cash used in investing activities**	-118,000

?????Meni: why isn't depreciation here?

Cash flow from financing activities

This item includes money raised by the company from sale of stock, from taking loans, and so on.

	A	B
170	**Cash flow from financing activities**	
171	Proceeds from new debt	68,000
172	Debt repayments	-3,200
173	Cash dividends paid	0
174	New stock sold	0
175	Stock repurchased	0
176	**Cash used in financing activities**	64,800

?????Meni: explain more. Why isn't interest here? Why only debt repayments?

Here's the whole statement:

	A	B	C	D
153	**Consolidated statement of cash flows**			
154	**Cash flow from operating activities**			
155	Profit after taxes	69,116		<-- =H131
156	Add back depreciation	5,250		<-- =-H127
157	Subtract increase in current assets	-16,000		<-- =-SUM(B136:B137)
158	Add increases in current liabilities	58,562		<-- =E122+E123+E124
159	**Cash provided from operating activities**	116,928		
160				
161	**Cash flow from investing activities**			
162	Payments for fixed assets			
163	Taxis	-36,000		
164	Computer	-2,000		
165	Building	-80,000		
166	Purchases of marketable securities	0		
167	Proceeds from sales of marketable securities	0		
168	**Cash used in investing activities**	-118,000		
169				
170	**Cash flow from financing activities**			
171	Proceeds from new debt	68,000		
172	Debt repayments	-3,200		
173	Cash dividends paid	0		
174	New stock sold	0		
175	Stock repurchased	0		
176	**Cash used in financing activities**	64,800		
177				
178	**Net change in cash over period**	63,728	<-- =B159+B168+B176	
179	Initial cash balances	25,000		
180	**Ending cash balance**	88,728		

During the period January – June, ATS had a net cash inflow of $63,728 (cell B178). Added to the initial cash balance of the company over this period, $25,000, the ending cash balances should be $88,728. And indeed they are—this is the cash balance listed on the company's end-June balance sheet.

7.4. Computing the free cash flow (FCF)

The consolidated statement of cash flows gives the amount of cash generated by ATS during its first half year of existence. For finance purposes (recall, dear reader, that this is a finance and not an accounting book!), it is useful to know *how much cash was generated by the firm's operations*. Our measure of this is the *free cash flow* (FCF). The consolidated statement of cash flows does not give this information, since it mixes operational and financial cash flows.

The FCF is defined as:

DEFINITION OF THE FREE CASH FLOW (FCF)	
	Explanation
Profit after taxes	
Add back depreciation	Depreciation is a non-cash expense and is therefore added back.
Subtract increase in current assets used for operations	For purposes of the FCF, this item does not include cash or marketable securities
Add increase in current liabilities from operations	Accounting current liabilities include items like short-term debt and current portion of long-term debt. These financial items are not included in the FCF.
Subtract increase in fixed assets at cost	This represents the amount spent on new assets over the period. In the jargon of Wall Street it is often called "capital expenditures" (CAPEX).
Add back after-tax interest expenses	The FCF is an *operating concept*: It relates to cash generated by the firm's operations. Interest expenses are a financial (non-operating) item and should therefore be added back. On the other hand, the profit after taxes includes only *after-tax interest*; this is the amount added back in the FCF calculation. Note that in our case the firm's *effective tax rate* is $\underbrace{36\%}_{\text{Federal tax rate}}+\underbrace{(1-36\%)}_{\text{Federal tax deductability of state tax}}*\underbrace{5\%}_{\text{State tax rate}}=39.2\%$
FCF	The free cash flow is the amount of cash generated by the firm's operations or business activities. Another way of looking at this is that the FCF is the amount of cash generated by the firm if everything were financed with equity.

To compute the FCF it is handy to put the profit and loss and the balance sheet in two side-by-side columns, one for the start of the period (1 January 2003) and one for the end of the period (30 June 2003).

	A	B	C	D
1	**ANYTOWN TRAVEL SERVICES (ATS), INC. Calculating the Free Cash Flow (FCF)**			
2	**Profit and loss for period ending**	**1-Jan-03**	**30-Jun-03**	
3	Sales		214,000	
4	Cost of sales		-94,800	
5	Interest		-272	
6	Depreciation		-5,250	
7	Profit		113,678	
8	State income tax (5%)		-5,684	
9	Federal income tax (36%)		-38,878	
10	Profit after taxes		69,116	
11	Dividends		0	
12	**Retained earnings**		69,116	
13				
14	**Balance sheet--Assets**	**1-Jan-03**	**30-Jun-03**	
15	Current assets			
16	Cash	25,000	88,728	
17	Accounts receivable	0	10,000	
18	Prepaid expenses (insurance)	0	6,000	
19	Total current assets	25,000	104,728	
20				
21	Fixed assets			
22	At cost	0	118,000	
23	Accumulated depreciation	0	-5,250	
24	Net fixed assets	0	112,750	
25	**Total assets**	**25,000**	**217,478**	
26				
27	**Balance sheet--Liabilities and equity**	**1-Jan-03**	**30-Jun-03**	
28	Current liabilities			
29	Short-term loan from bank (taxi loan)		4,800	
30	Taxes payable	0	44,562	
31	Accounts payable (gasoline)	0	14,000	
32	Total current liabilities	0	63,362	
33				
34	Long-term liabilities			
35	Mortgage to buy building	0	60,000	
36				
37	Equity			
38	Stock	25,000	25,000	
39	Retained earnings	0	69,116	
40	Total equity	25,000	94,116	
41	**Total liabilities and equity**	**25,000**	**217,478**	
42				
43	**Free cash flow (FCF)**			
44	Profit after taxes		69,116	
45	Add back depreciation		5,250	
46				
47	Change in net working capital			
48	Subtract increase in operating current assets		-16,000	<-- =-SUM(C17:C18)-SUM(B17:B18). Changes in cash and marketable securities are not included
49	Add increase in operating current liabilities		58,562	<-- =C32-C29-B32. The taxi loan is financing and not operational
50	Change in net working capital		42,562	<-- =C49+C48
51				
52	Change in fixed assets at cost		-118,000	<-- =-(C22-B22)
53				
54	Add back after-tax net interest paid		165	<-- =-(1-39.2%)*C5
55	**Free cash flow**		**-906**	<-- =C44+C45+C50+C52+C54

So what does the FCF of -$906 mean? If ATS had paid all its operating expenses in cash and collected all its operating payments in cash during the first 6 months of its existence, it would be "in the hole" $906.

7.5. Stop and think!

So . . . Did Brother-Sister do well this half year? Or should we be worried by their negative free cash flow?

- The negative FCF tells us that the business did not produce enough cash to pay all of its expenses and capital costs. But, of course, the "changes in fixed asset at cost" item of $118,000 (cell C52) includes several long-term investments (two taxis, a computer, and a building) which are expected to produce revenues over a long period of time.
- The accounting profits of $69,611 deal with these long-term investments by including only their *depreciation* as a cost of operating the business. Conceptually, this depreciation attributes the cost of having a long-term asset to the current period's income.

7.6. The next half year

The months July – December 2003 were a period of stabilization for ATS. During this period the company bought no more new assets. Here's what happened:

- Sales continued at $50,000 per month, for a total of $300,000.

- The flour mill continued to rack up $10,000 of fares per month. It paid its bills faithfully at the end of each month, so that at end-December 2003 it had only December's $10,000 outstanding.
- The two drivers continued to make $1,500 per month each. This made their total salaries for the period $18,000.
- The two owners continued to take $2,000 per month salary, for a total of $24,000 for these six months.
- Gasoline bills continued to be $7,000 per taxi per month. Total fuel bills for the period were 7*$14,000 = $98,000. ATS was paying its gasoline bill at the end of the following month, so that at year end it had $14,000 of gasoline bills outstanding. The remainder ($84,000) had been paid in full.
- The "prepaid expense (insurance)" became an actual expense:
  - The balance sheet line called "prepaid expense" became 0 instead of $6,000
  - $6,000 of insurance expenses were charged against profits as an expense
- The company continued to pay off its taxi loan. During the period July – December, it paid off the loan in full and paid $168 in interest on the loan (1% per month on the outstanding balance).

	A	B	C	D
38	**Taxi loan**	**Principal outstanding, beg. Month**	**Principal paid, end month**	**Interest**
39	31-Jul-03	4,800	800	48.00
40	31-Aug-03	4,000	800	40.00
41	30-Sep-03	3,200	800	32.00
42	31-Oct-03	2,400	800	24.00
43	30-Nov-03	1,600	800	16.00
44	31-Dec-03	800	800	8.00
45	1-Jan-04	0		
46	**Total**		4,800	168

- The company started to pay off its mortgage. The mortgage payments were $500 per month, with ½% interest on the outstanding balance:

	A	B	C	D
48	**Mortgage**	**Principal outstanding, beg. Month**	**Principal paid, end month**	**Interest**
49	31-Jul-03	60,000	500	300.00
50	31-Aug-03	59,500	500	297.50
51	30-Sep-03	59,000	500	295.00
52	31-Oct-03	58,500	500	292.50
53	30-Nov-03	58,000	500	290.00
54	31-Dec-03	57,500	500	287.50
55	1-Jan-04	57,000		
56	**Total**		3,000	1,762.50

- At the end of the year, when the owners saw that the business was profitable, they declared a $30,000 dividend. In order not to stress the business, they decided to pay out $15,000 of this dividend immediately and the remainder at the end of March 2004. This created:
    - A charge of $15,000 against the cash accounts of the business
    - An item called "dividends payable" in the current liabilities of the firm

Putting all these numbers together, here are the company's financial statements for the second half year:

	A	B	C	D	E	F	G	H
1	**ANYTOWN TRAVEL SERVICES (ATS), INC., July - December 2003 financial statements including depreciation and taxes**							
2								
3	**Assets**			**Liabilities and equity**			**Profit and loss**	
4	**Current assets**			**Current liabilities**				
5	Cash			Taxi loan from bank	0		Sales	300,000
6	Cash at beginning of period	88,728		Accounts payable (gasoline)	14,000		Driver salaries	-18,000
7	Cash sales	290,000		Taxes payable			Owner salaries	-24,000
8	Driver salaries	-18,000		For Jan - June			Fuel	-98,000
9	Owner salaries	-24,000		For July - Dec	55,103		Interest	-1,931
10	Receivable paid	10,000		Dividend payable	15,000		Insurance	-6,000
11	Fuel	-84,000		Total current liabilities	84,103		Depreciation	
12	Interest						Computer	-500
13	Auto loan	-168		**Long-term liabilities**			Building	-4,000
14	Mortgage	-1,763		Mortgage to buy building	57,000		Taxis	-7,000
15	Repayment of principal							
16	Auto loan	-4,800					Profit	140,570
17	Mortgage	-3,000					State income tax (5%)	-7,028
18	Taxes paid, 15 Jul 2003	-44,562					Federal income tax (36%	-48,075
19	Payment of outstanding gas bill	-14,000					Profit after taxes	85,466
20	Dividend paid	-15,000					Dividends	-30,000
21	Cash at end of period	179,436					Retained earnings	55,466
22								
23	Accounts receivable	10,000						
24	Prepaid expenses (insurance)	0						
25	Current assets	189,436						
26								
27	**Fixed assets**			**Equity**				
28	Computer	2,000		Initial stock	25,000			
29	Minus accumulated depreciation	-750		Accumulated retained earnings				
30	Taxis	36,000		Jan - June	69,116			
31	Minus accumulated depreciation	-12,000		July - December	55,466			
32	Building	80,000		Total equity	149,582			
33	Minus accumulated depreciation	-4,000						
34	Net fixed assets	101,250						
35								
36	Total assets	290,686		Total liabilities and equity	290,686			

7.7. Computing the free cash flow for the second half year

In rows 44-58 below we calculate the free cash flow for the company at the end of December 2003:

	A	B	C	D	E
1	**ANYTOWN TRAVEL SERVICES (ATS), INC. Free Cash Flow for 2003**				
2	**Profit and loss for period ending**	**1-Jan-03**	**30-Jun-03**	**31-Dec-03**	
3	Sales		214,000	300,000	
4	Cost of sales		-94,800	-146,000	
5	Interest		-272	-1,931	
6	Depreciation		-5,250	-11,500	
7	Profit		113,678	140,570	
8	State income tax (5%)		-5,684	-7,028	
9	Federal income tax (36%)		-38,878	-48,075	
10	Profit after taxes		69,116	85,466	
11	Dividends		0	-30,000	
12	**Retained earnings**		**69,116**	**59,114**	
13					
14	**Balance sheet--Assets**	**1-Jan-03**	**30-Jun-03**	**31-Dec-03**	
15	Current assets				
16	Cash	25,000	88,728	179,436	
17	Accounts receivable	0	10,000	10,000	
18	Prepaid expenses (insurance)	0	6,000	0	
19	Total current assets	25,000	104,728	189,436	
20					
21	Fixed assets				
22	At cost	0	118,000	118,000	
23	Accumulated depreciation	0	-5,250	-16,750	
24	Net fixed assets	0	112,750	101,250	
25	**Total assets**	**25,000**	**217,478**	**290,686**	
26					
27	**Balance sheet--Liabilities and equity**	**1-Jan-03**	**30-Jun-03**	**31-Dec-03**	
28	Current liabilities				
29	Short-term loan from bank (taxi loan)		4,800	0	
30	Taxes payable	0	44,562	55,103	
31	Accounts payable (gasoline)	0	14,000	14,000	
32	Dividend payable			15,000	
33	Total current liabilities	0	63,362	84,103	
34					
35	Long-term liabilities				
36	Mortgage to buy building	0	60,000	57,000	
37					
38	Equity				
39	Stock	25,000	25,000	25,000	
40	Retained earnings	0	69,116	124,582	
41	Total equity	25,000	94,116	149,582	
42	**Total liabilities and equity**	**25,000**	**217,478**	**290,686**	
43					
44	**Free cash flow (FCF)**				
45	Profit after taxes		69,116	59,114	
46	Add back depreciation		5,250	11,500	<-- =-D6
47					
48	Change in net working capital				
49	Subtract increase in operating current assets		-16,000	6,000	<-- =-(SUM(D17:D18)-SUM(C17:C18))
50	Add increase in operating current liabilities		58,562	25,541	<-- =(D33-D29)-(C33-C29)
51	Change in net working capital		42,562	31,541	<-- =D50+D49
52					
53	Change in fixed assets at cost		-118,000	0	
54					
55	Add back after-tax net interest paid		165	1,174	
56	**Free cash flow**		**-906**	**103,329**	<-- =D45+D46+D51+D53+D55
57					
58	**Year total FCF**		**102,423**	**<-- =C56+D56**	

During the year, the company had a total FCF of $102,423. Not bad for a new company!

Which depreciation do you add back in the cash flow?

When calculating the free cash flow (FCF) for ATS for the period July-December 2003, the depreciation we add back in the free cash flow (cell D46) computation is the depreciation which appears in the profit and loss statement for the period (cell D6), and not the accumulated depreciation which appears on the balance sheet (cell D23) for the same period. The reason for this is that D6 refers to the depreciation applied to the profits of the period, whereas D23 refers to the accumulated depreciation for all of the fixed assets which appear on the balance sheet.

7.8. Using the FCF in a valuation exercise

Suppose we wanted to use the FCFs to value the Brother Sister Travel Services. In financial theory, the value of a business is the present value of its free cash flows, discounted at an appropriate risk-adjusted discount rate.

At the end of 2003, Brother and Sister perform such a valuation, in a quick-and-dirty manner. They make the following assumptions:

- The FCF of $103,329 which occurred in the second half of 2003 will recur in each half year. This makes the annual anticipated FCF $206,658.
- *Except* that every 3 years they will have to buy new taxis for $36,000 and some additional equipment for another $4,000 (they're a bit unspecific about this, but the thought is that the computer will need replacing every few years and perhaps there'll be some other expenses).

- The business will last another 6 years, until 31 December 2010. At this date it will quietly expire.[5] Brother-Sister assume that at the end of 2010 they'll be able to sell the building for $150,000. At that point the building will have been on ATS's books for 6 ½ years. Since its monthly depreciation is $666.67, its accumulated depreciation will be $52,000 (= 78 months *$666.67). Because the building was purchased for $80,000, its book value at the end of 2010 will be $28,000 (book value = initial cost minus accumulated depreciation). The net, after-tax cash flow from selling the building is given below:

	A	B	C
1	**ANYTOWN TRAVEL SERVICES (ATS), INC. Building residual value at end-2010**		
2	Book value of building		
3	Initial cost	80,000	
4	Accumulated depreciation, end 2010	52,000	<-- =78*666.67
5	Book value	28,000	<-- =B3-B4
6			
7	Market value	150,000	
8	Taxable gain	122,000	<-- =B7-B5
9	State tax on marketable gain (5%)	6,100	<-- =5%*B8
10	Gain for Federal tax purposes	115,900	<-- =B8-B9
11	Federal tax (36%)	41,724	<-- =36%*B10
12	Total taxes	47,824	<-- =B11+B9
13	Net after-tax cash flow from sale of building	102,176	<-- =B7-B12

[5] A lot of the value of the business derives from the fact that Anytown didn't have a taxi service. Brother-sister figure that in another few years, more taxi operators will enter the business and make the business less profitable. They've already started lobbying the Anytown city council to introduce strict taxi licensing regulations (with existing operators grandfathered in ...).

Valuing the business

To value the business, ATS need to compute the cash flows and discount them by an appropriate cost of capital. They assume that the appropriate discount rate is 30%. Here's their calculation:

	A	B	C	D	E
1	**ANYTOWN TRAVEL SERVICES (ATS), INC.** **Quick and dirty valuation**				
2	**Date**	**FCF**	**Additional capex**	**Residual value**	**Total**
3	31-Dec-04	206,658			206,658
4	31-Dec-05	206,658	-40,000		166,658
5	31-Dec-06	206,658			206,658
6	31-Dec-07	206,658			206,658
7	31-Dec-08	206,658	-40,000		166,658
8	31-Dec-09	206,658			206,658
9	31-Dec-10	206,658		102,176	308,834
10					
11	Discount rate	30%			
12					
13	Enterprise value = PV of future free cash flows	560,922	<-- =NPV(B11,E3:E9)		
14	Add back cash balances on 31 December 2003	179,436			
15	Asset value of firm, 31 December 2003	740,358			
16	Debt on 31 December 2003	57,000	<-- Outstanding mortgage		
17	Value of equity	683,358	<-- =B15-B16		

Here are the details of the valuation:

- Assuming a discount rate of 30% for the cash flows, the enterprise value on 31 December 2003 at $560,922. We use the term *enterprise value* to denote the present value of the firm's future free cash flows and terminal value.[6]

- In addition to this enterprise value, the firm has $179,436 cash on hand on 31 December 2003. Adding this cash to the enterprise value gives the asset value of the firm on this date as $740,358.

[6] We have an extended discussion of the enterprise value in the next chapter.

- Since there is a mortgage outstanding on this date of $57,000, brother-sister's equity stake in the business is worth $683,358 = $740,358 - $57,000.

  Not bad for a year's work!

Technical comment: mid-year discounting

In Chapter 4, section000, we discussed the concept of mid-year discounting to account for the fact that cash flows occur throughout the year and not at year's end. Mid-year discounting assumes that the annual cash flows occur in mid-year, and not at year's end.

Had Brother and Sister valued their firm by using mid-year discounting, they would have concluded that the equity value of the Anytown Travel Services was $761,985:

	A	B	C	D	E
1	**ANYTOWN TRAVEL SERVICES (ATS), INC. Valuation assuming mid-year discounting**				
2	**Date**	**FCF**	**Additional capex**	**Residual value**	**Total**
3	31-Dec-04	206,658			206,658
4	31-Dec-05	206,658	-40,000		166,658
5	31-Dec-06	206,658			206,658
6	31-Dec-07	206,658			206,658
7	31-Dec-08	206,658	-40,000		166,658
8	31-Dec-09	206,658			206,658
9	31-Dec-10	206,658		102,176	308,834
10					
11	Discount rate	30%			
12					
13	Enterprise value = PV of future free cash flows	639,549	<-- =NPV(B11,E3:E9)*(1+B11)^0.5		
14	Add back cash balances on 31 December 2003	179,436			
15	Asset value of firm, 31 December 2003	818,985			
16	Debt on 31 December 2003	57,000	<-- Outstanding mortgage		
17	Value of equity	761,985	<-- =B15-B16		

Conclusion

In this chapter we've reviewed the basic methodology of financial accounting. We've reviewed the construction of the balance sheet, the profit and loss statement, and the statement of cash flows of a business. In addition we showed how to construct a free cash flow statement and how to use this statement to value a business.

CHAPTER 8: FINANCIAL PLANNING MODELS*

this version: 24 July 2003

Chapter contents

Overview

This chapter explains how to build spreadsheet models that allow you to predict the future performance of a firm. These models are called *financial planning models* or *pro forma models*. In accounting jargon a "pro forma" statement is something that looks like an accounting statement but that is *forward looking*. Financial planning models look like accounting statements; however, whereas accounting statements report what *happened* to the firm in the

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past, a financial planning model *predicts* what the firm's accounting statements will look like in the future.

Financial planning models have a variety of uses:

- Projecting future financing needs of the firm: Building a financial planning model helps you predict whether the firm will need financing in the future. It also helps you tie the firm's financing needs to its future performance. For example—does an increase in the growth rate of sales create cash or use cash? The answer is not always clear: More sales produce more profits (and hence produce more cash). However, an increase in the growth rate of sales may also require more capital investment (machines, land, etc.) and may require greater net working capital. A financial planning model can help us sort out these two opposing trends.
- Business plans: When you make a business plan (which you then take to investors to get financing or to a bank to explain why you need a loan and can pay it back), you'll often need to build a pro forma model of your firm. The model you build illustrates your assumptions about the financial and business environment in which your firm will operate in the future. Valuation: They can be used to predict the future free cash flows, dividends, and profits of a firm. Chapter 9 shows how to use the pro forma prediction of future cash flows to value a firm.

In this chapter we develop a simple model that illustrates the methodology of financial planning models. In the next chapter we will use a financial planning model to value the firm.

Finance and accounting concepts used in this chapter

- This chapter assumes familiarity with the accounting concepts reviewed in Chapter 7.
- Net present value, present value
- Cash flows and free cash flows (FCF)

Excel concepts and functions used in this chapter

- Excel formulas and model building
- Relative versus absolute copying
- Circular references
- Data tables

8.1. Initial accounting statements for a financial planning model

Financial planning models are predictions of what a firm's *future* financial statements will look like. To build such a model we start with the *present*—the firm's current financial statements. To illustrate the process by which financial planning models are constructed, in the next section we will project five years of financial statements for Whimsical Toenails, a company which runs a chain of toenail-painting parlors. Whimsical's management and bankers want to project the firm's future performance, and we will help them by constructing a financial planning model.

Our starting point is Whimsical Toenail's current income statement and balance sheet:

WHIMSICAL TOENAILS INITIAL INCOME STATEMENT	
Sales	1,000
Cost of goods sold	-500
Depreciation	-100
Interest payments on debt	-32
Interest earned on cash	6
Profit before tax	375
Taxes	-150
Profit after tax	225
Dividends	-90
Retained earnings	135

WHIMSICAL TOENAILS INITIAL BALANCE SHEET			
Assets		**Liabilities and equity**	
Cash	80	Current liabilities	80
Current assets	150	Debt	320
Fixed assets			
Fixed assets at cost	1,070	Equity	
Accumulated depreciation	-300	Stock (paid-in capital)	450
Net fixed assets	770	Accumulated retained earnings	150
Total assets	**1,000**	**Total liabilities and equity**	**1,000**

Accounting terminology versus financial planning model terminology

Before proceeding further, some comments on our use of terminology. While most of the terminology in this chapter follows the standard accounting nomenclature, some changes are necessary to accommodate the structure of financial planning models. For example, while accountants use “current assets” to denote both *operating* and *financial* short-term assets, financial planning models use “current assets” to mean only operating short-term assets (to emphasize this point, the terminology “operating current assets” is sometimes used). Similarly, in the accounting framework “current liabilities” includes both operational items (like accounts payable—bills which are as yet unpaid by the firm) and financial items (like short-term debt and current portion of long term debt). Financial planning models use “current liabilities” to denote operational items. To emphasize this point, we sometimes use the terminology "operating current liabilities."

Current assets—what's included in the financial planning model and what's not?

In financial planning models the "current assets" category contains only items that are related to the operations of the firm. Here are several typical items that would be included in the financial planning model definition of current assets.

- Accounts receivable: These are payments due from customers and are generated by the operations of the firm. Since accounts receivable are generated by the firm's sales, they are included in the operating current assets of the financial planning model.
- Inventories: Inventories include both raw materials to be used for production and unsold finished products. Inventories are part of the operating current assets of the financial planning model.
- Prepaid expenses: Prepaid expenses are costs which the firm pays before it actually receives the associated services. An example might be rent paid by the firm for future periods: If the firm pays this rent in advance (for example, not month-by-month, but 6 months in advance), then this prepayment of the rent is recorded by the accountant as a prepaid expense and is recorded as a current asset. For our financial planning model, we usually assume that these are part of *operating* current assets.

Which accounting current assets are not included in the financial planning model definition of current assets? There are two important examples:

- Cash: The "cash" item on the balance sheet refers to money kept in the firm's bank accounts. Sometimes the accounting line item is called "cash and equivalents," with the second term denoting assets like certificates of deposit and money market accounts which can be easily converted into cash. "Cash" is an operating current asset to the extent that it is needed by the firm for its daily operations. In most cases, however, the cash accounts on the balance sheets simply refer to non-operating assets which are kept in liquid form by the firm.
- Marketable securities: This item on the balance sheet refers to other financial assets—such as stocks and bonds—bought by the firm. Marketable securities are not needed for the firm's operations, and are thus not an operating current asset.

The distinction between cash as an operating asset and cash as a store of value is usually obvious once you understand the business of the firm. A taxi driver needs to keep some cash on hand in order to make change for his customers, and a supermarket needs to keep some cash in the till for the same reason; in these cases at least some of the cash is an operating current asset (although even for a taxi or supermarket, most of the cash is likely to be a financial, non-operating current asset). On the other hand, in March 2003, Microsoft reported having $4.3 billion in cash and another $41.9 billion in marketable securities. It is unlikely that almost any of this $46.2 billion is needed for daily operations. It is not an operating current asset, but rather a financial current asset.

Current liabilities

For purposes of the financial planning model, "current liabilities" contains only items that are related to the operations of the firm. Here are two typical items that would be included in the current liabilities of our financial planning model:

- Accounts payable: These are unpaid bills the firm owes to its suppliers. Since this item is related to the operations of the firm, we include it in the financial planning model definition of current liabilities.
- Taxes payable: When a firm's payment of taxes does not coincide with the accounting period, the taxes owed are entered into the balance sheet as a current liability. For example, for the accounting year ending 31 December 2005, XYZ Corp. owes the $2,000 in taxes, but it won't pay this tax bill until 15 January 2006. The financial statements of XYZ Corp. for 2005 will report taxes of $2,000 in the profit and loss statement; the firm's balance sheet will report taxes payable of $2,000 in the current liabilities. Taxes payable relate to the firm's operations and are included in the financial planning model definition of current liability.

Accounting current liability items that are not included in the financial planning model definition of current liabilities are typically financial items. Here are two examples:

- Short-term debt: These are borrowings by the firm that are due within one year. A bank overdraft (a credit line on a business's checking account) is a good example of a short-term debt. Accountants include this item in current liabilities, but financial planning models include them as *debt*.
- Current portion of long-term debt: This is the portion of the firm's debt that is due for payment within the current financial year. Accountants include this item in

current liabilities; financial planning models include the current portion of long-term debt in the *debt* category.

8.2. Building a financial planning model

Now that we have our terminology straight, we can build our financial planning model for Whimsical Toenails. A typical financial planning model has three major components:

- The model parameters. Also called the *value drivers*, a financial planning model's parameters include the major assumptions of the model. For example, we might assume that the *sales growth* parameter is 10% per year. Or we might assume that the *current assets to sales* parameter is 15%—meaning that an increase of $1,000 in sales requires an additional $150 of current assets. Typically, financial-statement models are *sales-driven*; this term means that many of the most important financial statement value drivers are assumed to be functions of the firm's sales.
- The financial policy assumptions. We will make assumptions about how the firm finances itself in the future. What is the mix between debt and new equity issued? Does excess cash produced by the firm go towards repaying debt or does it end up in the firm's cash balances? These assumptions are important determinants of the firm's future financial statements.
- The pro forma financial statements. Once we decide on the financial model's parameters, we will build the pro forma financial statements for the firm we are modeling—the income statement, balance sheets, and free cash flows.

When we've used the model's parameters and the financing assumptions to project the future financial statements of the firm, we can then use the model. By varying the model's assumptions, we can use the financial planning model to build different scenarios of how the firm will perform in the future. In Chapter 9 we will use the financial planning model to project the future free cash flows of the firm in order to value the firm. We might also want to use the model to evaluate the ability of the firm to repay its debts (there's an end-of-chapter exercise which illustrates this use).

The model's parameters—the value drivers

The *sales growth* parameter is usually the most important parameter of the financial planning model. In our example Whimsical Toenails current (year 0) level of sales is 1,000. Over the five-year horizon of the financial planning model, the firm expects its sales to grow at a rate of 10 percent per year.

Other model parameters are derived from the following financial statement relations.[1]

- Current assets: We assume that Whimsical's end-year current assets on the balance sheet will be 15 percent of the annual firm sales.
- Current liabilities: We assume that Whimsical's end-year current liabilities on the balance sheet will be 8 percent of the annual firm sales
- Net fixed assets: End-year net fixed assets are assumed to be 77 percent of annual sales.
- Depreciation: The annual depreciation charge is 10 percent of the average value of the fixed assets on the books during the year.

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[1] In practice the model's parameters are often derived from an analysis of the company's historic financial statements.

- Cost of goods sold: Assumed to be 50 percent of sales.
- Interest rate on debt: 10 percent.
- Interest earned on cash: Whimsical Toenails earns 8 percent on the average balances of cash.
- Tax rate: 40 percent of the firm's profit before taxes.
- Dividends paid: We assume that Whimsical Toenails pays out 40 percent of its profits after taxes as dividends to shareholders.

The model's financial policy assumptions

The second component of a financial planning model is the model's financial policy assumptions. In this initial financial planning model we make the following assumptions:

- Debt: Whimsical currently has debt of 320 on its balance sheet. The company's agreement with the bank specifies that it will repay 80 of this debt in each of the next four years. Once the debt is fully repaid, the company intends to stay debt-free.
- Stock: Company management does not intend to either issue new stock nor repurchases stock over the five-year model horizon. The stock item in the firm's balance sheets thus remains at its year-0 level of 450.
- Cash. In our model this item is the *plug*: The cash item is defined so that the left-hand side of the balance sheet always equals the right-hand side of the balance sheet:

$$Cash = Total\ liabilities\ and\ equity - Current\ assets - Net\ \ fixed\ assets$$

The "plug" is the balance sheet item which guarantees the equality of the future projected total assets and the future projected total liabilities and equity. Every financial planning model has a plug, and the plug is almost always either cash (as in this case) or debt or stock.

To see how the plug fits into our model, consider the projected future balance sheets:

WHIMSICAL TOENAILS balance sheet model assumptions	
Assets	**Liabilities and equity**
Cash [PLUG]	Current liabilities [8% of sales]
Current assets [15% of sales]	Debt [repaid by 80/year until zero]
Fixed assets Fixed assets at cost - Accumulated depreciation [10% of average assets] Net fixed assets [77% of sales]	Equity Stock (paid in capital) [constant] Accumulated retained earnings [previous year's accumulated retained + this year's retained from income statement]
Total assets	**Total liabilities and equity**

The "plug" assumption has two meanings:

1. The *mechanical* meaning of the plug: The plug guarantees the equality of total assets and total liabilities and equity. For the Whimsical Toenails financial model, cash is the plug. By defining cash to equal the total liabilities and equity minus current assets and minus net fixed assets, we guarantee that future projected assets and liabilities will always be equal. This is important, since the two sides of the balance sheet must always be equal.
2. The *financial* meaning of the plug: Whimsical Toenails sells no additional stock and is locked into a debt repayment schedule. By defining the plug to be cash, we are also making a statement about how the firm finances itself. For the case of Whimsical Toenails, this means that all incremental financing (if needed) for the firm will come from the cash; it also means that if the firm has additional cash, it will go into this account.

Projecting the year 1 balance sheet and income statement

Given our assumptions we can now develop the pro forma model and project the financial statements for year 1:

	A	B	C	D
1	**WHIMSICAL TOENAILS SETTING UP THE FINANCIAL STATEMENT MODEL for year 1**			
2	Sales growth	10%		
3	Current assets/Sales	15%		
4	Current liabilities/Sales	8%		
5	Net fixed assets/Sales	77%		
6	Costs of goods sold/Sales	50%		
7	Depreciation rate	10%		
8	Interest rate on debt	10.00%		
9	Interest earned on cash balances	8.00%		
10	Tax rate	40%		
11	Dividend payout ratio	40%		
12				
13	**Year**	**0**	**1**	
14	**Income statement**			
15	Sales	1,000	1,100	<-- =B15*(1+$B$2)
16	Costs of goods sold	(500)	(550)	<-- =-C15*$B$6
17	Depreciation	(100)	(117)	<-- =-$B$7*(C30+B30)/2
18	Interest payments on debt	(32)	(28)	<-- =-$B$8*(B36+C36)/2
19	Interest earned on cash and marketable securities	6	6	<-- =$B$9*(B27+C27)/2
20	Profit before tax	374	411	<-- =SUM(C15:C19)
21	Taxes	(150)	(164)	<-- =-C20*$B$10
22	Profit after tax	225	247	<-- =C21+C20
23	Dividends	(90)	(99)	<-- =-$B$11*C22
24	Retained earnings	135	148	<-- =C23+C22
25				
26	**Balance sheet**			
27	Cash	80	64	<-- =C39-C28-C32
28	Current assets	150	165	<-- =C15*$B$3
29	Fixed assets			
30	At cost	1,070	1,264	<-- =C32-C31
31	Depreciation	(300)	(417)	<-- =B31-$B$7*(C30+B30)/2
32	Net fixed assets	770	847	<-- =C15*$B$5
33	**Total assets**	1,000	1,076	<-- =C32+C28+C27
34				
35	Current liabilities	80	88	<-- =C15*$B$4
36	Debt	320	240	<-- =B36-80
37	Stock	450	450	<-- =B37
38	Accumulated retained earnings	150	298	<-- =B38+C24
39	**Total liabilities and equity**	1,000	1,076	<-- =SUM(C35:C38)

Excel note: Relative versus absolute copying

The dollar signs within a formula indicate that when the formulas are copied the cell references to the model parameters should not change. The technical jargon for this in Excel is *absolute copying* as opposed to the *relative copying* when variables are indicated without dollar signs. The distinction between absolute and relative copying is critical for financial planning models—if you fail to put the dollar signs correctly in the model, it will not copy correctly when you project years 2 and beyond.

The use of relative versus absolute copying is explained in Chapter 27.

Income statement equations

Here are the relations for our financial planning model, with model parameters in bold face. These relations will end up as the formulas in the cells of our Excel model.

- Sales = Initial sales * (1+ **Sales growth**)^{year} .

  Alternatively: Sales(t) = Sales(t-1)*(1+**Sales growth**)

- Costs of goods sold = Sales * **Costs of goods sold/Sales**

  We assume that Whimsical's only expenses related to sales are costs of goods sold. Most companies also book an expense item called selling, general, and administrative expenses (SG&A). Exercise 2 at the end of this chapter illustrates how you would introduce SG&A into the model.

- Interest payments on debt = **Interest rate on debt** * Average debt over the year. We use this formula to estimate Whimsical's interest payments on the debt. For example: If the company's debt at the end of year 0 is 320 and its debt at the end of year 1 is 240, the financial planning model estimates its year-1 interest payments as:

$$10\% * \frac{320+240}{2} = 10\% * \underbrace{280}_{\substack{\uparrow \\ \text{Whimsical's} \\ \text{average debt} \\ \text{in year 1}}} = 28 .$$

- Interest earned on cash = **Interest rate on cash** * Average cash over the year.
- Depreciation = **Depreciation rate** * Average fixed assets at cost over the year. This assumes that all new fixed assets are purchased during the year. We also assume that there is no disposal of fixed assets. Looking at the financial model may help you understand the calculation of the depreciation: Whimsical's year-0 fixed assets at cost are $1,070 and its projected year-1 fixed assets at cost are $1,264. Since the company's depreciation rate is 10%, its year-1 depreciation in the income statement is:

$$10\% * \frac{1,070+1,264}{2} = 10\% * \underbrace{1,167}_{\substack{\uparrow \\ \text{Average fixed} \\ \text{assets at cost} \\ \text{in year 1}}} = 117$$

- Profit before taxes = Sales - Costs of goods sold - Interest payments on debt + Interest earned on cash & marketable securities – Depreciation.
- Taxes = **Tax rate** * Profit before taxes
- Profit after taxes = Profit before taxes - Taxes
- Dividends = **Dividend payout ratio** * Profit after taxes.

  Whimsical Toenails has a policy of paying out a fixed percentage of its profits as dividends. In the exercises for this chapter we explore some alternative dividend policies.
- Retained earnings = Profit after taxes - Dividends

Balance sheet equations

- Cash = Total liabilities – Current assets – net fixed assets.

  As explained above, this definition means that cash is the balance sheet plug.

- Current assets = **Current assets/sales** * Sales

- Net fixed assets = **Net fixed assets/sales** * Sales .

- Accumulated depreciation = Previous year's accumulated depreciation + **Depreciation rate** * average fixed assets at cost over the year.

- Fixed assets at cost = Net fixed assets + accumulated depreciation.

  Note that this model does not distinguish between plant, property, and equipment (PP&E) and other fixed assets such as land.

- Current liabilities = **Current liabilities/Sales** * Sales .[2]

- Debt is assumed to be unchanged. This means that during the model's 5 year horizon, we assume that the firm neither increases its borrowing nor pays back any of its initial debt principal. An alternative model which assumes that debt is the balance sheet plug; this model is the subject of one of our end-of-chapter exercises.

- Stock is assumed to be unchanged. The company is assumed to issue no new stock.

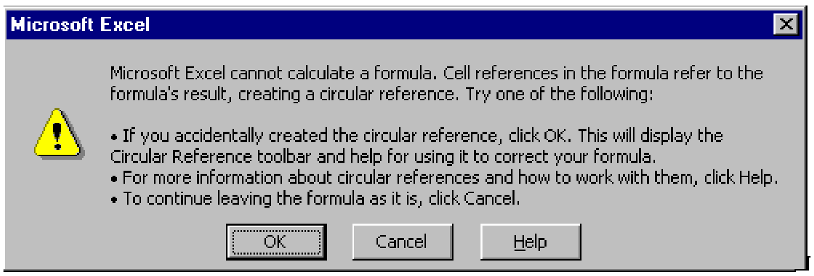
[2] Some modellers prefer to model current liabilities as a percentage of the firm's costs of goods sold (COGS). The thinking here is that—because current liabilities include the firm's accounts payable (which in turn include the firm's unpaid bills for inventories and the like)—current liabilities are largely dependent on the level of the firm's costs of goods sold. While it is easy to incorporate this assumption in our model (see example ??? at the end of the chapter), it doesn't make much difference: If COGS are a percentage of sales and current liabilities are a percentage of sales, then the current liabilities are also a percentage of the COGS.

- Accumulated retained earnings = Previous year's accumulated retained earnings + current year's additions to retained earnings

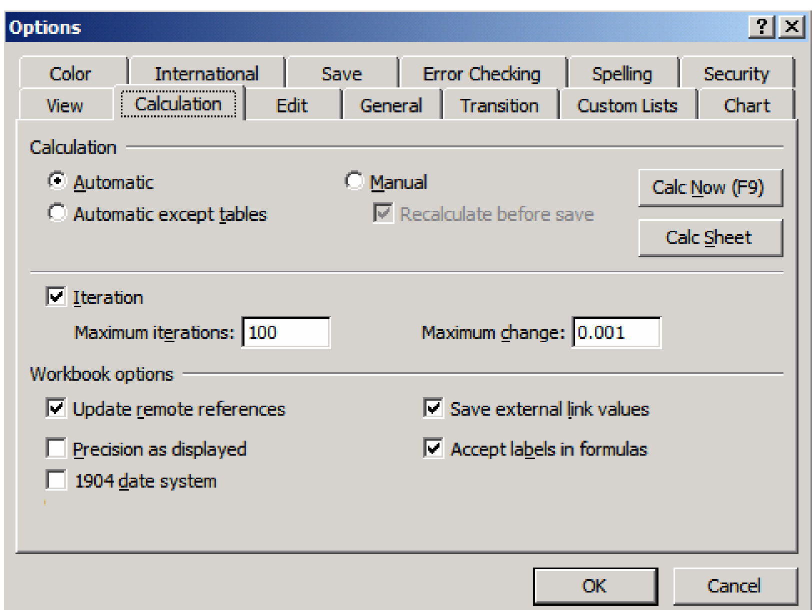
Excel note—solving circular references

Financial statement models in Excel always involve cells that are mutually dependent. In our model, for example, the interest earned on cash depends on the profits of the firm, but the profits depend on the interest earned on cash. Another example of mutual dependence in our model involves the fixed asset accounts: Fixed assets at cost are the sum of net fixed assets plus accumulated depreciation, but accumulated depreciation is a function of the fixed assets at cost.

As a result of these inevitable mutual dependencies, the solution of the model depends on the ability of Excel to solve circular references. To make sure your spreadsheet recalculates, you have to go to the **Tools|Options|Calculation** box and click **Iteration**. If you open a spreadsheet that involves iteration, and if this box is not clicked, you will see the following Excel error message:



Depending on where you are in Excel when you open the file with the circular references, you may get a slightly different version of the above message. Whatever message you see, get out of it by pressing **Cancel** and go to **Tools|Options|Calculation|Iteration**. In this dialog box click the box labeled **Iteration**:



8.3. Extending the model to years 2 and beyond

Now that you have the model set up, you can extend it by copying the columns:

	A	B	C	D	E	F	G
1	**WHIMSICAL TOENAILS--FINANCIAL MODEL**						
2	Sales growth	10%					
3	Current assets/Sales	15%					
4	Current liabilities/Sales	8%					
5	Net fixed assets/Sales	77%					
6	Costs of goods sold/Sales	50%					
7	Depreciation rate	10%					
8	Interest rate on debt	10.00%					
9	Interest earned on cash balances	8.00%					
10	Tax rate	40%					
11	Dividend payout ratio	40%					
12							
13	**Year**	**0**	**1**	**2**	**3**	**4**	**5**
14	**Income statement**						
15	Sales	1,000	1,100	1,210	1,331	1,464	1,611
16	Costs of goods sold	(500)	(550)	(605)	(666)	(732)	(805)
17	Depreciation	(100)	(117)	(137)	(161)	(189)	(220)
18	Interest payments on debt	(32)	(28)	(20)	(12)	(4)	-
19	Interest earned on cash and marketable securities	6	6	5	4	4	8
20	Profit before tax	374	411	452	496	544	594
21	Taxes	(150)	(164)	(181)	(199)	(217)	(237)
22	Profit after tax	225	247	271	298	326	356
23	Dividends	(90)	(99)	(109)	(119)	(130)	(142)
24	Retained earnings	135	148	163	179	196	214
25							
26	**Balance sheet**						
27	Cash	80	64	54	51	55	146
28	Current assets	150	165	182	200	220	242
29	Fixed assets						
30	At cost	1,070	1,264	1,486	1,740	2,031	2,364
31	Depreciation	(300)	(417)	(554)	(715)	(904)	(1,124)
32	Net fixed assets	770	847	932	1,025	1,127	1,240
33	**Total assets**	1,000	1,076	1,168	1,276	1,402	1,628
34							
35	Current liabilities	80	88	97	106	117	129
36	Debt	320	240	160	80	0	0
37	Stock	450	450	450	450	450	450
38	Accumulated retained earnings	150	298	461	640	835	1,049
39	**Total liabilities and equity**	1,000	1,076	1,168	1,276	1,402	1,628

The most common Excel mistake to make in the transition between the two-columned financial model and this one is the failure to mark the model parameters with dollar signs. If you commit this error, you will get zeros in places where there should be numbers.[3]

[3] If this paragraph is mysterious to you, change the model by putting in the following mistake: In cell C28, write the formula =C15*B3 (instead of the correct formula =C15*$B$3). Then copy cell C28 to D28:G28. Now you'llunderstand the importance of *dollarizing* the correct cell references!

Understanding the model—doing some sensitivity analysis

The financial model we've built shows that our firm's profits after tax will grow from $225 in year 0 to $352 in year 5. The balances of cash grow from $80 to $459, the firm's total assets grow to $1,941, and so on … .

We can use the model to do some *sensitivity analysis*. For example, what would happen to profits if the growth rate of sales were to be 8 per cent instead of 10 per cent and if the cost of goods sold were to be 55 per cent of sales instead of the 50 per cent currently in the model? Given our Excel model, we simply have to make the relevant changes in the parameters in cells B2 and B6. Our intuition is that these performance changes will make the firm's financial results worse, and this is indeed confirmed in the model, as shown below:

	A	B	C	D	E	F	G
1	**WHIMSICAL TOENAILS MODEL WITH SOME CHANGES**						
2	Sales growth	8%	<-- Changed from 10%				
3	Current assets/Sales	15%					
4	Current liabilities/Sales	8%					
5	Net fixed assets/Sales	77%					
6	Costs of goods sold/Sales	55%	<-- Changed from 50%				
7	Depreciation rate	10%					
8	Interest rate on debt	10.00%					
9	Interest earned on cash balances	8.00%					
10	Tax rate	40%					
11	Dividend payout ratio	40%					
12							
13	**Year**	**0**	**1**	**2**	**3**	**4**	**5**
14	**Income statement**						
15	Sales	1,000	1,080	1,166	1,260	1,360	1,469
16	Costs of goods sold	(550)	(594)	(642)	(693)	(748)	(808)
17	Depreciation	(100)	(116)	(135)	(156)	(181)	(208)
18	Interest payments on debt	(32)	(28)	(20)	(12)	(4)	-
19	Interest earned on cash and marketable securities	6	6	4	3	2	4
20	Profit before tax	324	348	374	401	429	457
21	Taxes	(130)	(139)	(150)	(160)	(172)	(183)
22	Profit after tax	195	209	224	241	258	274
23	Dividends	(78)	(83)	(90)	(96)	(103)	(110)
24	Retained earnings	117	125	135	144	155	165
25							
26	**Balance sheet**						
27	Cash	80	58	40	26	16	89
28	Current assets	150	162	175	189	204	220
29	Fixed assets						
30	At cost	1,070	1,247	1,449	1,677	1,935	2,227
31	Depreciation	(300)	(416)	(551)	(707)	(888)	(1,096)
32	Net fixed assets	770	832	898	970	1,048	1,131
33	**Total assets**	1,000	1,052	1,113	1,185	1,268	1,441
34							
35	Current liabilities	80	86	93	101	109	118
36	Debt	320	240	160	80	0	0
37	Stock	450	450	450	450	450	450
38	Accumulated retained earnings	150	275	410	554	709	873
39	**Total liabilities and equity**	1,000	1,052	1,113	1,185	1,268	1,441

If you compare the model above to our previous version of the model, you'll see that the firm's sales growth has slowed (from 10 percent to 8 percent) and that its sales have become more expensive (cost of goods sold is 55 percent of sales instead of 50 percent). The result is that profits after taxes (row 22) are lower than before. Cash balances (row 27) are also lower than in the previous version of the model.

8.4. Free cash flow (FCF): measuring the cash produced by the firm's operations

In this section we use our model to measure the firm's projected *free cash flow* (FCF). We discussed the concept of free cash flow in Chapter 7 (page000). A good way to think of FCF is that it is the amount of cash the firm would produce if it had no debt whatsoever. This is equivalent to the amount of cash produced by the firm if the shareholders have to finance *all* of the operations of the firm. For short, we'll say that the FCF is a measure of the *cash produced by the firm's operations*.

The FCF is the measure on which we base our valuation of the firm. We gave an example of this in Chapter 7 (p000, where we used the future predicted FCFs of Anytown Travel Services to value the company). In Chapter 9 we return to this topic and show how a financial planning model's predictions of FCFs can be used to value a company.

In this section we merely use the financial planning model to project the firm's future free cash flows. Before we do so, however, let's recap the definition and the terminology we use.

The definition of the free cash flow is:

Defining the Free Cash Flow (FCF)	
Profit after taxes	This is the basic measure of the profitability of the business, but it is an accounting measure that includes financing flows (such as interest), as well as non-cash expenses such as depreciation. Profit after taxes does not account for changes in the firm's working capital or purchases of new fixed assets, both of which can be important cash drains on the firm.
+ Depreciation	This non-cash expense is added back to the profit after tax.
- Increase in current assets	When the firm's sales increase, more investment is needed in inventories, accounts receivable, etc. This increase in current assets is not an expense for tax purposes (and is therefore ignored in the profit after taxes), but it is a cash drain on the company. Note that our use of the term "current assets" is slightly different from the standard accounting usage—see the discussion following this table.
+ Increase in current liabilities	An increase in the sales often causes an increase in financing related to sales (such as accounts payable or taxes payable). This increase in current liabilities—when related to sales—provides cash to the firm. Since it is directly related to sales, we include this cash in the free cash flow calculations. Note that our use of the term "current liabilities" is slightly different from the standard accounting usage—see the discussion following this table.
- Increase in fixed assets at cost (also called "capital expenditures"—CAPEX)	An increase in fixed assets (the long-term productive assets of the company) is a use of cash, which reduces the firm's free cash flow.
+ after-tax interest payments (net)	FCF is an attempt to measure the cash produced by the business activity of the firm. To neutralize the effect of interest payments on the firm's profits, we: • Add back the after-tax cost of interest on debt (*after-tax* since interest payments are tax-deductible), • Subtract out the after-tax interest payments on cash.
FCF = sum of the above	The free cash flow measures the cash produced by the firm's operations.

Here is the FCF calculation for Whimsical Toenails.

	A	B	C	D	E	F	G
42	**Free cash flow calculation**						
43	**Year**	**0**	**1**	**2**	**3**	**4**	**5**
44	Profit after tax		247	271	298	326	356
45	Add back depreciation		117	137	161	189	220
46	Subtract increase in current assets		(15)	(17)	(18)	(20)	(22)
47	Add back increase in current liabilities		8	9	10	11	12
48	Subtract increase in fixed assets at cost		(194)	(222)	(254)	(291)	(333)
49	Add back after-tax interest on debt		17	12	7	2	0
50	Subtract after-tax interest on cash		(3)	(3)	(3)	(3)	(5)
51	**Free cash flow**		176	188	201	214	228

The FCFs in row 51 are substantially lower than the firm's profits after taxes in row 44. The major reasons for this are the large capital expenditure (row 48) which outweigh the cash effect of the depreciation (row 45).

The FCF calculations are sensitive to the model assumptions. Suppose that Whimsical Toenail's sales growth is 8% (instead of 10%) and that its cost of goods sold is 55% of sales (instead of 50%). You might suspect that these negative changes in the model assumptions will make Whimsical's future projected FCFs substantially lower, and you're right:

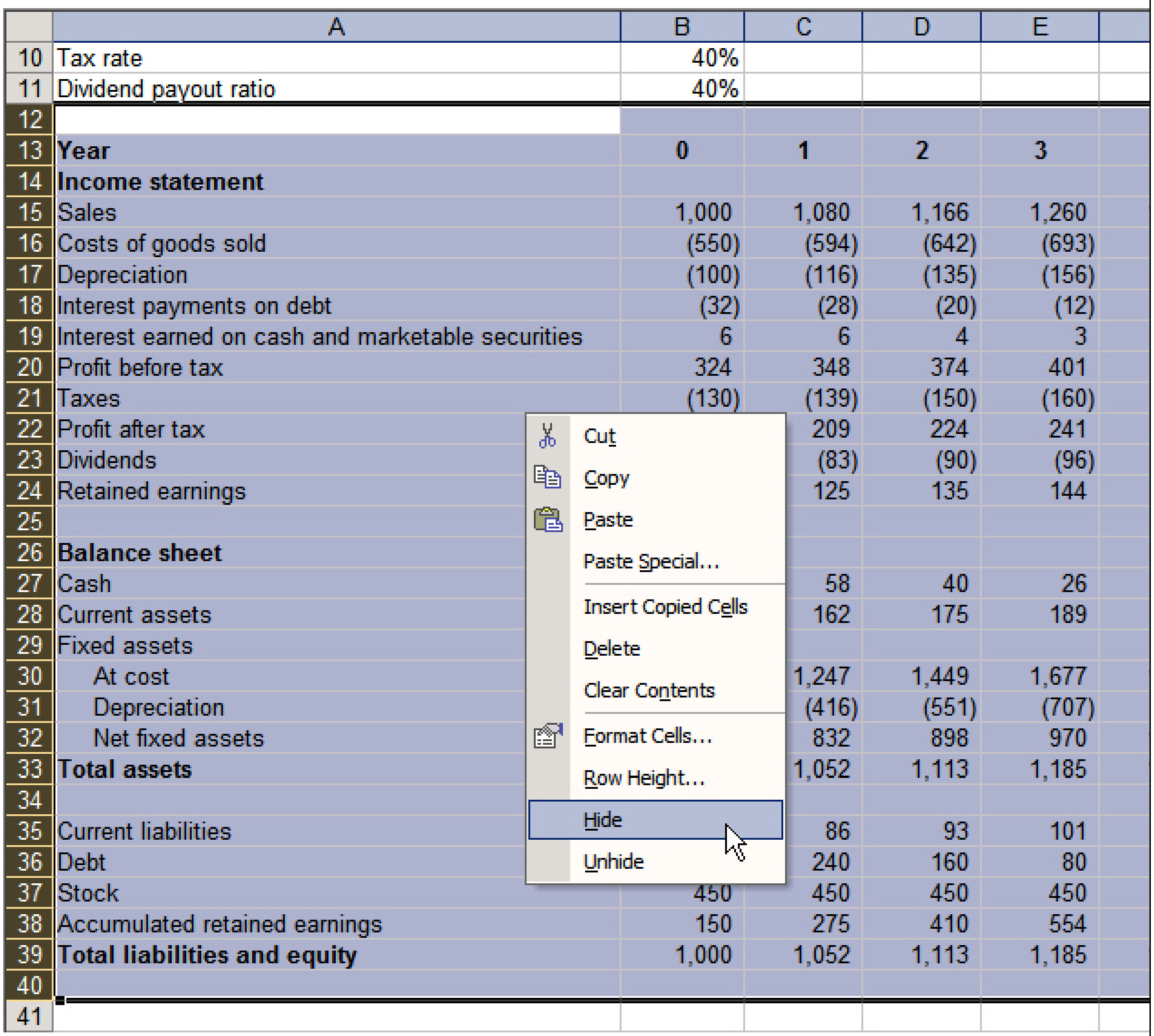
	A	B	C	D	E	F	G
1	**WHIMSICAL TOENAILS MODEL WITH SOME CHANGES**						
2	Sales growth	8%	<-- Changed from 10%				
3	Current assets/Sales	15%					
4	Current liabilities/Sales	8%					
5	Net fixed assets/Sales	77%					
6	Costs of goods sold/Sales	55%	<-- Changed from 50%				
7	Depreciation rate	10%					
8	Interest rate on debt	10.00%					
9	Interest earned on cash balances	8.00%					
10	Tax rate	40%					
11	Dividend payout ratio	40%					
41							
42	**Free cash flow calculation**						
43	**Year**	**0**	**1**	**2**	**3**	**4**	**5**
44	Profit after tax		209	224	241	258	274
45	Add back depreciation		116	135	156	181	208
46	Subtract increase in current assets		(12)	(13)	(14)	(15)	(16)
47	Add back increase in current liabilities		6	7	7	8	9
48	Subtract increase in fixed assets at cost		(177)	(201)	(228)	(258)	(292)
49	Add back after-tax interest on debt		17	12	7	2	0
50	Subtract after-tax interest on cash		(3)	(2)	(2)	(1)	(3)
51	**Free cash flow**		155	161	168	174	180

Excel Note: Hiding rows

In the above example we've hidden rows 12-40. To do this in Excel:

- Mark the rows you want to hide.
- Right-click on the mouse and click **Hide**

Here's what the screen looks like:



	A	B	C	D	E
10	Tax rate	40%			
11	Dividend payout ratio	40%			
12					
13	**Year**	**0**	**1**	**2**	**3**
14	**Income statement**				
15	Sales	1,000	1,080	1,166	1,260
16	Costs of goods sold	(550)	(594)	(642)	(693)
17	Depreciation	(100)	(116)	(135)	(156)
18	Interest payments on debt	(32)	(28)	(20)	(12)
19	Interest earned on cash and marketable securities	6	6	4	3
20	Profit before tax	324	348	374	401
21	Taxes	(130)	(139)	(150)	(160)
22	Profit after tax		209	224	241
23	Dividends		(83)	(90)	(96)
24	Retained earnings		125	135	144
25					
26	**Balance sheet**				
27	Cash		58	40	26
28	Current assets		162	175	189
29	Fixed assets				
30	At cost		1,247	1,449	1,677
31	Depreciation		(416)	(551)	(707)
32	Net fixed assets		832	898	970
33	**Total assets**		1,052	1,113	1,185
34					
35	Current liabilities		86	93	101
36	Debt		240	160	80
37	Stock	450	450	450	450
38	Accumulated retained earnings	150	275	410	554
39	**Total liabilities and equity**	1,000	1,052	1,113	1,185
40					
41					

Marking the rows and clicking **Unhide** reverses the action.

8.5. Reconciling the cash balances—the consolidated statement of cash flows

The free cash flow calculation is different from the "consolidated statement of cash flows" that is a part of every accounting statement (see Chapter 7, page000). The FCF calculation shows you how much cash is produced by the firm's operations. On the other hand, the purpose of the accounting statement of cash flows is to explain the increase in the cash accounts in the balance sheet as a function of the cash flows from the firm's operating, investing, and financing activities. Here's the consolidated statement of cash flows for our model:

	A	B	C	D	E	F	G	H
1	**WHIMSICAL TOENAILS--reconciliation of cash balances** **Note that the profit and loss statement and FCF statement have been hidden**							
2	Sales growth	10%						
3	Current assets/Sales	15%						
4	Current liabilities/Sales	8%						
5	Net fixed assets/Sales	77%						
6	Costs of goods sold/Sales	50%						
7	Depreciation rate	10%						
8	Interest rate on debt	10.00%						
9	Interest earned on cash balances	8.00%						
10	Tax rate	40%						
11	Dividend payout ratio	40%						
12								
13	**Year**	**0**	**1**	**2**	**3**	**4**	**5**	
25								
26	**Balance sheet**							
27	Cash	80	64	54	51	55	146	
28	Current assets	150	165	182	200	220	242	
29	Fixed assets							
30	At cost	1,070	1,264	1,486	1,740	2,031	2,364	
31	Depreciation	(300)	(417)	(554)	(715)	(904)	(1,124)	
32	Net fixed assets	770	847	932	1,025	1,127	1,240	
33	**Total assets**	1,000	1,076	1,168	1,276	1,402	1,628	
34								
35	Current liabilities	80	88	97	106	117	129	
36	Debt	320	240	160	80	0	0	
37	Stock	450	450	450	450	450	450	
38	Accumulated retained earnings	150	298	461	640	835	1,049	
39	**Total liabilities and equity**	1,000	1,076	1,168	1,276	1,402	1,628	
40								
41								
54	**CONSOLIDATED STATEMENT OF CASH FLOWS--RECONCILING THE CASH BALANCES**							
55	**Cash flow from operating activities**							
56	Profit after tax		247	271	298	326	356	
57	Add back depreciation		117	137	161	189	220	
58	Adjust for changes in net working capital:							
59	Subtract increase in current assets		(15)	(17)	(18)	(20)	(22)	
60	Add back increase in current liabilities		8	9	10	11	12	
61	Net cash from operating activities		356	401	451	506	566	
62								
63	**Cash flow from investing activities**							
64	Aquisitions of fixed assets--capital expenditures		(194)	(222)	(254)	(291)	(333)	
65	Purchases of investment securities		0	0	0	0	0	
66	Proceeds from sales of investment securities		0	0	0	0	0	
67	Net cash used in investing activities		(194)	(222)	(254)	(291)	(333)	
68								
69	**Cash flow from financing activities**							
70	Net proceeds from borrowing activities		-80	-80	-80	-80	0	
71	Net proceeds from stock issues, repurchases		0	0	0	0	0	
72	Dividends paid		(99)	(109)	(119)	(130)	(142)	
73	Net cash from financing activities		(179)	(189)	(199)	(210)	(142)	
74		=C73+C67+C61						
75	Net increase in cash and cash equivalents		-16	-10	-3	4	91	
76	Cash balances at end of previous year	=C27	80	64	54	51	55	
77	Cash balances at end of current year		64	54	51	55	146	
78	=C75+C76							
79	This number should be							
80	equal to cell C27 .							
81								

Row 77 checks that the ending balances in the cash accounts derived through the consolidated statement of cash flows match those derived in row 27 of the balance sheets (which use cash as a plug). The fact that row 77 is the same as row 27 shows that our model correctly accounts for all the accounting relations. To see this, look at cells C75, C76, and C77:

- C76 shows that at the end of year 0 the firm's cash balances were 80.
- C75 shows that *everything the firm did during the year*—sales, costs of sales, interest paid, new financing through debt and equity, …. *everything*—produced a net decrease in cash of 16.
- C77 is the sum of the previous two cells: If the firm started off the year with 80 in cash and if its total activities produced -16 in cash, then the ending cash balances should be 64. And so they are! Our model accounts for all the firm's activities.

What's more useful—the consolidated statement of cash flows or the free cash flow?

What's more useful—the cash increment in row 75 of the consolidated statement of cash flows or the free cash flow we derived in section 8.4? Although they both have their purposes, there's no doubt that for finance purposes, the FCF is a more useful and more widely-used number. The FCF measures the cash produced by the firm's business activities. It is the relevant finance measure for the effectiveness of the firm at doing what it was founded to do—make something and sell it. The cash increment in row 75 is also important, however: First of all, it allows us to check that we've done our calculations correctly by giving a check and balance on the cash line in the balance sheet. Second, it shows us why the cash line in the balance changed.

Conclusion

In this chapter we've used Excel to construct financial planning models. These models, also called pro forma models or financial planning models, have a variety of uses in finance. Financial planning models are at the heart of most business plans, the financial projections which firms use to persuade banks to loan them money and to persuade investors to buy their shares. Financial planning models are used to value firms (see next chapter) and to build scenarios showing how the firm will perform under various operating and financial assumptions.

Building a financial planning model is a powerful intellectual exercise: It forces you to combine accounting statements, a firm's operational parameters, and the firm's financing into one integrated model of the firm.

This chapter has concentrated on the "nuts and bolts" of building a financial planning model. In Chapter 9 we show you how to use these models to value the firm.

Exercises

Note: The CD-ROM which comes with *Principles of Finance with Excel* contains a spreadsheet entitled **Chapter08 template.xls**. although you may have to make some changes in the template, this can be used as the basis for answering many of the exercises below.

1. Here's a basic exercise that will help you understand what's going on in the modeling of financial statements. Replicate the model in section 8.3. That is, enter the correct formulas for the cells and see that you get the same results as the book. (This turns out to be more of an exercise in accounting than in finance. If you're like many financial modelers, you'll see that there are some aspects of accounting that you've forgotten!)

2. a. The model of section 8.3 includes costs of goods sold but not selling, general and administrative (SG&A) expenses. Suppose that the firm has $200 of these expenses each year, irrespective of the level of sales. Change the model to accommodate this new assumption. Show the resulting profit and loss statements, balance sheets, the free cash flows, and the valuation.

b. Do a data table in which you show the sensitivity of the equity value to the level of SG&A. Let SG&A vary from $0 per year to $500 per year.

3. Suppose that in the model of Section 8.3 the fixed assets *at cost* for years 1 – 5 are 100% of sales (in the current model, it is *net* fixed assets which are a function of sales). Change the model accordingly. Show the resulting Profit and Loss Statements, Balance Sheets, and Free Cash Flows for years 1 – 5. (Assume that in year 0, the fixed assets accounts are as shown in section 2. Note that since year 0 is given—it is the current situation of the firm, whereas years

1 - 5 are the predictions for the future—there is no need for the year 0 ratios to conform to the predicted ratios for years 1 – 5.)

4. Back to the model of section 8.3 as given in the book. Suppose that the fixed assets at cost follow the following step function:

$$Fixed\ \ Assets\ at\ \ Cost = \begin{cases} 100\% * Sales & if\ \ Sales \le 1200 \\ 1200 + 90\% * (Sales - 1200) & 1200 < Sales \le 1400 \\ 1{,}380 + 80\% * (Sales - 1400) & Sales > 1400 \end{cases}$$

Incorporate this function into the model.

5. a. Consider the model in section 8.3. Make two changes in the model: i) Let debt be the plug and keep cash constant at its year-0 level. ii) Suppose that the firm has 1000 shares and that it decides to pay, in year 1, a dividend per share of 15 cents. In addition, suppose that it wants this dividend per share to grow in subsequent years by 12% per year. Incorporate these changes into the pro forma model.

b. Do a sensitivity analysis in which you show the effect on the debt/equity ratio of the annual growth rate of dividends. Vary this rate from 0% to 18%, in steps of 2%.

Additional exercises:

- **Debt capacity**
- **CL as percent of COGS**
- **small case**

CHAPTER 9: DISCOUNTED CASH FLOW (DCF) VALUATION WITH FINANCIAL PLANNING MODELS*

this version: July 27, 2003

Chapter contents

Overview

In Chapter 8 we learned how to use accounting concepts to build a financial planning model of a company. In this chapter we use financial planning models to value a company. This is something that almost every finance specialist has to do occasionally. The valuation technique we employ—called *discounted cash flow* (DCF) *valuation*—is the valuation technique

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it.

universally favored by the finance profession. DCF valuations are often based on the financial planning models discussed in Chapter 8. When these models are used to do a DCF valuation, they are also used to do much of the sensitivity analysis which helps determine if the valuation is reasonable.

Valuation is not intrinsically difficulty, but because there are several competing definitions of what constitutes the "value of a firm," people often get confused. To shed some light on this issue, Section 9.1 discusses the different concepts of firm value. As you will see in Section 9.1, finance specialists often identify the value of the firm with the present value of its future cash flows. We will use the financial planning models of Chapter 8 to determine these cash flows.

After discussing the concept of firm value in Section 9.1, we summarize the steps involved in a DCF valuation in Section 9.2. We then go on to show you how to value a company by building a full-blown DCF valuation model (Section 9.3).

Finance concepts used

- Present value
- Free cash flow
- Gordon model (Chapter 6)
- Terminal value
- Mid-year valuation

Excel functions used

- NPV

- Data tables

9.1. What does "value of the firm" mean?

The terms "value of a company" or "value of a firm" are often used interchangeably by finance professionals. Even finance professionals, however, can use a confusing variety of meanings for these terms. Here are a few of the meanings which are often intended:

- In finance the definition most often used for "firm value" is the following: *The value of a firm is the market value of the firm's equity plus the market value of the firm's financial debt.* This section illustrates two methods of computing the firm value.
    - The simplest method is to value the firm's equity (its shares) using the firm's share price in the market, and to add to this the value of the firm's debt.
    - A second method, the DCF method, is based on discounted cash flows. In a DCF valuation firm value equals the present value of the firm's futures FCFs *plus* the value of its currently available liquid assets.
- Often when individuals discuss the *firm value*, they really mean the *value of its shares*. It is better to use the term *equity value* for the value of a company's shares and to use the term *firm value* (or *company value*) to denote the market value of the firm's equity plus its debt. In our calculations we also show you how to compute the value of a firm's shares.
- Sometimes the term *firm value* is used to denote the *accounting value* of the firm. Also known as the *book value*, this value is based on the firm's balance sheets. Because accounting statements are based on historical values, people in finance

generally prefer not to use this definition. At the end of this section we illustrate why we do not like this valuation method.

Motherboard Shoes: What's it worth?

To illustrate the different concepts of firm value, we'll tell the story of Motherboard Shoes. Motherboard is listed on the Chicago Stock Exchange, but the majority of the stock is owned by the Motherboard family, which founded the company and still runs it. The current date is 1 January 2005, and the Motherboards have received an offer for their shares from Century Shoe International. They would like to know if the offer is a fair one.

Their investment advisor, John Mba has advised them that there are two plausible ways to value the company (John just finished business school and liked it so much that he changed his last name to reflect his new status). Each of the two methods has advantages and disadvantages.

The share price valuation: Valuing a Motherboard by using current share price

The simplest way to value Motherboard is look at the value of its share. Motherboard Shoes has one million shares, which were trading on 31 December 2004 at $50 per share. Thus the market value of the firm's equity is $50 million. In addition the company's balance sheet shows that it has short-term debt of $2.5 million and long-term debt of $7.5 million; John Mba uses these balance sheet values (also called book values) of the debt as an approximation to the debt's market value.[1]

Using the current share price, the *firm value* of Motherboard Shoes is $60 million:

	A	B	C
1	**MOTHERBOARD SHOES**		
2	Number of shares	1,000,000	
3	Current share price	50.00	
4	Market value of equity	50,000,000	<-- =B2*B3
5			
6	Short-term debt	2,500,000	
7	Long-term debt	7,500,000	
8			
9	**Firm value**	**60,000,000**	**<-- =B4+B6+B7**

The discounted cash flow (DCF) valuation: Valuing Motherboard by discounting its future free cash flows

The advantage of the share-price valuation method illustrated above is that it is very simple: The firm value equals the market value of the firm's shares plus the book value of its debt. Valuing the company at its current price of $50 per share is perfectly acceptable for someone considering buying a few shares of the company, but it makes less sense if Motherboard Shoes is selling a controlling block of shares. In this case the purchaser would probably expect to pay more for the following reasons:

- If the purchaser tried to buy a big block of shares of Motherboard shares on the open market, he would probably have to offer more than the current market price per share. As he bought more and more shares, the price would go up; in addition, the

[1] This is common practice. Most company debt is not traded on financial markets, and therefore there is no easily-available market value for the debt. As a first approximation, most finance professionals use the book value of a firm's debt as a proxy for the debt's market value.

announcement that someone was trying to take over Motherboard Shoes would—in many cases—force the share price up.

- There are benefits to controlling a company that are not priced in the market price per share. The market price of a share reflects the value of a company's future dividends to a *passive* shareholder who has no control over the company. In general the value of a controlling block of shares is larger than the market value, since the controlling shareholder can actually decide what the company will do. He can also derive considerable *private benefits* from running the company.[2]

To deal with these problems, John Mba proposes to use the *discounted cash flow* (DCF) valuation method to value the shares. DCF valuations are a standard finance methodology, which defines the value of the firm as the present value of the firm's future free cash flows (FCF), discounted at the weighted average cost of capital (WACC), plus the firm's initial cash and marketable securities. Section 9.4 discusses the theory behind this method of valuation, but for the moment we skip all the theory and simply present the formula:

$$DCF\ \ firm\ value = \begin{matrix} Market\ value \\ of\ \ firm's\ debt \end{matrix} + \begin{matrix} Market\ value \\ of\ \ firm's\ equity \end{matrix}$$

$$= \underbrace{PV\begin{pmatrix} all\ \ future\ FCFs \\ discounted\ \ at\ WACC \end{pmatrix}}_{\substack{\uparrow \\ \text{Often called the "enterprise value"} \\ \text{of the firm}}} + \begin{matrix} Current\ cash\ and \\ marketable\ securities \end{matrix}$$

[2] Economists use the term *private benefits* to discuss all kinds of financial and non-financial benefits associated with firm ownership. The big car with a driver that the company gives its president is a private benefit of ownership, and so is the feeling of ownership—a psychological benefit, perhaps, but nonetheless valuable.

(Notice that the present value of the firm's future FCFs is often called the firm's *enterprise value*.) After some work to estimate the future free cash flows, John comes up with the following valuation:

	A	B	C	D	E	F
1	**MOTHERBOARD SHOES--DCF VALUATION**					
2	**Year**	**2005**	**2006**	**2007**	**2008**	**2009**
3	Estimated free cash flow	9,210,135	10,052,522	10,966,397	11,956,842	13,029,110
4	Terminal value					91,203,773
5	Total	9,210,135	10,052,522	10,966,397	11,956,842	104,232,883
6						
7	Weighted average cost of capital, WACC	20%				
8	Enterprise value, PV of future FCFs + terminal value	68,657,407	<-- =NPV(B7,B5:F5)			
9	Add current cash & marketable securities	500,000				
10	Firm value	69,157,407	<-- =B9+B8			
11						
12	Subtract out debt					
13	Short-term debt	2,500,000				
14	Long-term debt	7,500,000				
15	Estimated value of equity	59,157,407	<-- =B10-B13-B14			
16						
17	Number of shares	1,000,000				
18	Estimated value per share	59.16	<-- =B15/B17			

There are a few things to explain about this valuation:

- John has projected 5 years of future FCFs and has also projected a *terminal value* at the end of the 5 years. He explains that the finance methodology requires him to estimate the present value of all the future free cash flows: $PV\begin{pmatrix} all\ \ future\ FCFs \\ discounted\ \ at\ WACC \end{pmatrix}$. However, he thinks this is too much guesswork. Instead of estimating all future FCFs, he's estimated 5 years of FCFs and then estimated the *terminal value*, the value of Motherboard at the end of year 5:

$$Enterprise\ value = PV\begin{pmatrix} all\ \ future\ FCFs \\ discounted\ \ at\ WACC \end{pmatrix}$$
$$= \frac{FCF_{2005}}{(1+WACC)} + \frac{FCF_{2006}}{(1+WACC)^2} + \ldots + \frac{FCF_{2009}}{(1+WACC)^5} + \frac{Terminal\ Value}{(1+WACC)^5}$$

If the weighted average cost of capital is 20%, the enterprise value, the present value of the FCFs and the terminal value, is $68,657,407.[3]

- Adding current balances of cash and marketable securities to the present value of the FCFs and subtracting out the value of the firm's debts gives an *equity valuation* of $59,157,407 (cell B15). Since there are one million shares outstanding, this values each share at $59.16 (cell B18).

The firm's book value—a definition we'd rather not use

There's another valuation method which John explains to the Motherboard family—the *accounting definition* of firm value uses the balance sheet to arrive at the value of the firm. For the case of Motherboard Shoes, the balance sheets at the end of 2004 look like:

	A	B	C	D	E	F
1	**MOTHERBOARD SHOES, BALANCE SHEETS, END 2004**					
2	**Assets**			**Liabilities and equity**		
3	Cash and marketable securities	500,000		Accounts payable	3,750,000	
4	Accounts receivable	2,500,000		Short-term debt	2,500,000	
5	Inventories	3,750,000				
6				Long-term debt	7,500,000	
7	Fixed assets at cost	12,000,000				
8	Accumulated depreciation	-1,000,000		Common stock	1,000,000	
9	Net fixed assets	11,000,000		Accumulated retained earnings	3,000,000	
10	Total assets	17,750,000	<-- =B3+B4+B5+B9	Total liabilities and equity	17,750,000	<-- =SUM(E3:E9)

By the accounting definition of firm value, the firm is worth

$$\begin{aligned}
&\textit{Firm value, accounting definition} = \textit{Debt} + \textit{Equity}\\
&= \underbrace{2,500,000 + 7,500,000}_{\text{Debt}} + \underbrace{\underbrace{1,000,000}_{\text{Stock}} + \underbrace{3,000,000}_{\text{Accumulated retained earnings}}}_{\text{Book value of equity}}\\
&= 14,000,000
\end{aligned}$$

[3] We don't explain how to compute the WACC. See Chapters 6 and 15.

The accounting definition of firm value relies on *book values*, the value of the firm's debt and equity as listed in the firm's balance sheet. Recall from Chapter 7 that the accounting definition, which is based on historical values, is a *backward looking* definition. The finance definition of firm value is a *forward looking* definition (it discounts the future anticipated values of the cash flows). John Mba thinks that the accounting definition gives an inappropriate valuation, and he's right.[4] In the case of Motherboard Shoes, the forward-looking DCF valuation of the firm is $68,657,407 whereas the backward-looking accounting definition is $14,000,000.

9.2. Using the DCF valuation—summary

The DCF valuation of a firm is based on discounting the firm's future expected free cash flows (FCFs), using the weighted average cost of capital (WACC) as the discount rate. In this section we summarize the steps for implementing this valuation, and in the next section we illustrate a DCF valuation using a financial planning model we learned in Chapter 8.

Step 1: Estimate the weighted average cost of capital

The WACC is the discount rate for the future FCFs. We discussed the WACC in Chapter 6 and gave an example of how to estimate it.[5] In this chapter we will not go into the details of estimating the WACC; calculating the WACC entails many assumptions and in many cases the calculation itself becomes a topic of controversy among the parties involved in the valuation. For this example, we assume that John Mba's estimate of a 20% WACC is correct. In Section

[4] Not to disparage accounting (very important) or accountants (most of whom would readily agree).

[5] Later in the book, Chapter 15 gives another approach to estimating the WACC.

9.3 we will perform some sensitivity analysis (using an Excel Data Table) to show how changes in the WACC affect the valuation.

Step 2: Project a reasonable number of FCFs

A financial planning model's predictions of future FCFs are based on the assumption that the parameters of the model will not change by too much. Most financial analysts define "reasonable" to mean number of periods over which this basic assumption is not too silly.[6] Everyone recognizes that a firm's environment is dynamic and that the model parameters will change over time, a fact that is usually addressed by doing sensitivity analysis (see section 9.4). John has assumed that he can reasonably project the next 5 years of cash flows.

Step 3: Project the long-term FCF growth rate and the terminal value

Valuation using the DCF method in principle requires us to project an *infinite number* of future FCFs, but in a standard financial planning model we project only a limited number of FCFs. A solution to this problem is to define the firm's terminal value as the firm value at the end of year 5. The definition John uses is contained in Figure 1.

---

[6] The author defines "not too silly" as something he can explain to his mother with a straight face (and that she won't laugh at him).

Schematic: DCF Valuation of the Firm

$$DCF\ \ firm\ value = PV\begin{pmatrix} all\ \ future\ \ FCFs \\ discounted\ \ at\ \ WACC \end{pmatrix} + \begin{matrix} Current\ \ cash\ \ and \\ marketable\ \ securities \end{matrix}$$

$$= \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + ... + \begin{matrix} Current\ \ cash\ \ and \\ marketable\ \ securities \end{matrix}$$

$$= \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \frac{FCF_4}{(1+WACC)^4} + \frac{FCF_5}{(1+WACC)^5} \leftarrow Line\ 1:$$ John Mba estimates these FCFs with a financial planning model

$$+ \frac{FCF_6}{(1+WACC)^6} + \frac{FCF_7}{(1+WACC)^7} + \ . \ . \ .$$

$\leftarrow Line\ \ 2:$ John Mba uses the terminal value in place of these numbers:

$$\frac{1}{(1+WACC)^5} \underbrace{\frac{FCF_5 * (1+long\text{-}term\ \ FCF\ \ growth)}{WACC - long\text{-}term\ \ FCF\ \ growth}}_{\substack{\uparrow \\ \text{This is the "terminal value"}}}$$

$$+ \begin{matrix} Current\ \ cash\ \ and \\ marketable\ \ securities \end{matrix}$$

$\leftarrow Line\ \ 3:$ The last term in the valuation

Figure 1: A DCF valuation

As you can see, there are 3 parts to this valuation equation:

- Line 1 is the present value of the first 5 years of free cash flows. John has projected these cash flows one-by-one, using a financial planning model (details to come in Section 9.3).
- Instead of projecting the present value of each of the cash flows in years 6, 7, 8, …, infinity, John has chosen to summarize them in the *present value of the terminal value*. In Line 2 this is given as $\frac{1}{(1+WACC)^5} \underbrace{\frac{FCF_5 * (1+long\text{-}term\ FCF\ growth)}{WACC - long\text{-}term\ FCF\ growth}}_{\substack{\uparrow \\ \text{This is the "terminal value"}}}$. Terminal value is what we project the firm to be worth at the end of projection horizon. In Section 9.4 we explain how this expression for the terminal value is derived. For now we assume that John's prediction of Motherboard's terminal value is correct.
- Line 3 gives the value of the cash and marketable securities.

The terminal value formula requires us to estimate the long-term FCF growth rate. In the financial planning model for the Motherboard Shoes FCFs, this long-term growth rate is different from the sales growth rate projected for the company's next five years. As you will see in Section 9.3, John projects a relatively high growth rate of sales of 10% for Motherboard over the 5 year horizon of the planning model. John's criterion for choosing the long-term FCF growth rate of the company is that a company's cash flows cannot grow forever at a rate greater than the economy in which it operates. He estimates that the long-term growth of the U.S. economy is 5 percent, and that this rate is also the long-term rate for Motherboard Shoes.

Using his model, John Mba estimates that Motherboard's year-5 FCF is $13,029,110. Using the WACC of 20 percent and the long-term FCF growth rate of 5 percent, the company's terminal value is $91,203,773:

$$Terminal\ value = \frac{FCF_5 * (1 + long\text{-}term\ FCF\ growth)}{WACC - long\text{-}term\ FCF\ growth} = \frac{\$13{,}029{,}110 * (1+5\%)}{20\% - 5\%} = \$91{,}203{,}773$$

Step 4: Determine the value of the firm

At this point all the elements of the firm valuation formula are in place:

- WACC: the discount rate for the FCFs and the terminal value
- Five years of FCFs projected from the financial planning model
- The terminal value of the firm
- The firm's initial (year 0) balances of cash and marketable securities

We can now value the firm:

	A	B	C	D	E	F
1	**MOTHERBOARD SHOES--DCF VALUATION**					
2	**Year**	**2005**	**2006**	**2007**	**2008**	**2009**
3	Estimated free cash flow	9,210,135	10,052,522	10,966,397	11,956,842	13,029,110
4	Terminal value					91,203,773
5	Total	9,210,135	10,052,522	10,966,397	11,956,842	104,232,883
6						
7	Weighted average cost of capital, WACC	20%				
8	Enterprise value, PV of future FCFs + terminal value	68,657,407	<-- =NPV(B7,B5:F5)			
9	Add current cash & marketable securities	500,000				
10	Firm value	69,157,407	<-- =B9+B8			
11						
12	Subtract out debt					
13	Short-term debt	2,500,000				
14	Long-term debt	7,500,000				
15	Estimated value of equity	59,157,407	<-- =B10-B13-B14			
16						
17	Number of shares	1,000,000				
18	Estimated value per share	59.16	<-- =B15/B17			

The value of the firm is $69,157,407 (cell B9). In cells B15 and B18 we've added two more steps:

Step 5: Value the firm's equity by subtracting the value of the firm's debt today from the firm value

The firm value is the value of the firm's debt + equity. We are often interested in valuing only the firm's equity—our estimate of the market value of the firm's shares.

$$\textit{Firm value} = \textit{Debt} + \textit{Equity} = \$69{,}157{,}407$$

This means that

$$\textit{Equity} = \textit{Firm value} - \textit{Debt} = \$69{,}157{,}407 - \$10{,}000{,}000 = \$59{,}157{,}407$$

Stock market analysts often use the estimate of a firm's equity value to arrive at a per-share valuation of the firm. They then compare this estimated per-share value to the current market price to come up with a buy or sell recommendation for the stock. Since Motherboard Shoes has 1,000,000 shares outstanding, the estimated market value per share is $\frac{\$59{,}157{,}407}{1{,}000{,}000} = \$59.16$.

This share valuation is higher than the current market value per share of $50. If the DCF valuation analysis were being used to make recommendations about the stock, we would expect the analyst would make a "buy" recommendation for the Motherboard Shoes. In this case the analysis is used by John Mba to recommend that Motherboard be taken over for more than its current price per share.

Step 6: Adding mid-year valuation

In Chapter 4 (page000) we discussed *mid-year valuation* of cash flows. The idea was that when cash flows occur over the course of the year and not at the end of the year, we should take the standard present value formula and multiply it by $(1+WACC)^{0.5}$. For Motherboard Shoes, mid-year valuation makes sense, since the company's sales occur throughout the year and not just at year-end. In the spreadsheet below you can see how mid-year valuation affects the value of the firm and projected share valuation: Cell B8 shows that the present value of future cash flows and terminal value firm value increases from $69 million to $75 million. In cell B18 you can see that the projected share value increases to $65.71.

	A	B	C	D	E	F
1	**MOTHERBOARD SHOES--DCF VALUATION using mid-year discounting (see cell B8)**					
2	**Year**	**2005**	**2006**	**2007**	**2008**	**2009**
3	Estimated free cash flow	9,210,135	10,052,522	10,966,397	11,956,842	13,029,110
4	Terminal value					91,203,773
5	Total	9,210,135	10,052,522	10,966,397	11,956,842	104,232,883
6						
7	Weighted average cost of capital, WACC	20%				
8	PV of future FCFs + terminal value	75,210,421	<-- =NPV(B7,B5:F5)*(1+B7)^0.5			
9	Add current cash & marketable securities	500,000				
10	Firm value	75,710,421	<-- =B9+B8			
11						
12	Subtract out debt					
13	Short-term debt	2,500,000				
14	Long-term debt	7,500,000				
15	Estimated value of equity	65,710,421	<-- =B10-B13-B14			
16						
17	Number of shares	1,000,000				
18	Estimated value per share	65.71	<-- =B15/B17			

Step 7: Don't trust anything! Do a sensitivity analysis

Valuations are based on a formidable number of assumptions! When we do sensitivity analysis, we evaluate the effect of changing values of the main variables on the value of the firm. Our "weapon of choice" for sensitivity analysis is the **Data Table** feature of Excel (see Chapter 30). We leave our demonstrations of sensitivity analysis for the next section.

9.3. Projecting the FCFs and doing the DCF valuation with a financial planning model

So far we've shown how John Mba performs his valuation, but we haven't shown the financial planning model which produces the free cash flows. The model looks a lot like those discussed in Chapter 8. Here it is, with the mid-year valuation discussed in the previous section:

	A	B	C	D	E	F	G	H
1	**MOTHERBOARD SHOES, FINANCIAL MODEL using mid-year valuation**							
2	Sales growth	10%						
3	Current assets/Sales	25%		Additional model assumptions:				
4	Current liabilities/Sales	15%		1. Net fixed assets are assumed constant				
5	Net fixed assets	Constant		2. Debt principal is repaid by $1 million/year				
6	Costs of goods sold/Sales	40%		3. Cash is the plug				
7	Depreciation rate	15%		4. Mid-year discounting				
8	Interest rate on debt	9.00%						
9	Interest earned on cash balances	4.00%						
10	Tax rate	35%						
11	Dividend payout ratio	40%						
12								
13	**Year**	**0**	**1**	**2**	**3**	**4**	**5**	
14	**Income statement**							
15	Sales	25,000,000	27,500,000	30,250,000	33,275,000	36,602,500	40,262,750	
16	Costs of goods sold		(11,000,000)	(12,100,000)	(13,310,000)	(14,641,000)	(16,105,100)	
17	Depreciation		(1,945,946)	(2,261,505)	(2,628,235)	(3,054,436)	(3,549,749)	
18	Interest payments on debt		(855,000)	(765,000)	(675,000)	(585,000)	(495,000)	
19	Interest earned on cash and marketable securities		102,653	279,954	482,274	711,756	970,697	
20	Profit before tax		13,801,707	15,403,449	17,144,039	19,033,821	21,083,597	
21	Taxes		(4,830,598)	(5,391,207)	(6,000,414)	(6,661,837)	(7,379,259)	
22	Profit after tax		8,971,110	10,012,242	11,143,625	12,371,983	13,704,338	
23	Dividends		(3,588,444)	(4,004,897)	(4,457,450)	(4,948,793)	(5,481,735)	
24	Retained earnings		5,382,666	6,007,345	6,686,175	7,423,190	8,222,603	
25								
26	**Balance sheet**							
27	Cash	500,000	4,632,666	9,365,011	14,748,686	20,839,126	27,695,704	
28	Current assets	6,250,000	6,875,000	7,562,500	8,318,750	9,150,625	10,065,688	
29	Fixed assets							
30	At cost	12,000,000	13,945,946	16,207,451	18,835,686	21,890,121	25,439,871	
31	Depreciation	(1,000,000)	(2,945,946)	(5,207,451)	(7,835,686)	(10,890,121)	(14,439,871)	
32	Net fixed assets	11,000,000	11,000,000	11,000,000	11,000,000	11,000,000	11,000,000	
33	**Total assets**	17,750,000	22,507,666	27,927,511	34,067,436	40,989,751	48,761,391	
34								
35	Current liabilities	3,750,000	4,125,000	4,537,500	4,991,250	5,490,375	6,039,413	
36	Debt	10,000,000	9,000,000	8,000,000	7,000,000	6,000,000	5,000,000	
37	Stock (1,000,000 shares, par value $1 each)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	
38	Accumulated retained earnings	3,000,000	8,382,666	14,390,011	21,076,186	28,499,376	36,721,979	
39	**Total liabilities and equity**	17,750,000	22,507,666	27,927,511	34,067,436	40,989,751	48,761,391	
40								
41								
42	**Year**	**0**	**1**	**2**	**3**	**4**	**5**	
43	**Free cash flow calculation**							
44	Profit after tax		8,971,110	10,012,242	11,143,625	12,371,983	13,704,338	
45	Add back depreciation		1,945,946	2,261,505	2,628,235	3,054,436	3,549,749	
46	Subtract increase in current assets		(625,000)	(687,500)	(756,250)	(831,875)	(915,063)	
47	Add back increase in current liabilities		375,000	412,500	453,750	499,125	549,038	
48	Subtract increase in fixed assets at cost		(1,945,946)	(2,261,505)	(2,628,235)	(3,054,436)	(3,549,749)	
49	Add back after-tax interest on debt		555,750	497,250	438,750	380,250	321,750	
50	Subtract after-tax interest on cash		(66,725)	(181,970)	(313,478)	(462,642)	(630,953)	
51	**Free cash flow**		9,210,135	10,052,522	10,966,397	11,956,842	13,029,110	
52								
53								
54	**Valuing the firm--using mid-year discounting**							
55	Weighted average cost of capital	20%						
56	Long-term FCF growth rate	5%						
57								
58	**Year**	**0**	**1**	**2**	**3**	**4**	**5**	
59	FCF		9,210,135	10,052,522	10,966,397	11,956,842	13,029,110	
60	Terminal value						91,203,773	<-- =G59*(1+B56)/(B55-B56)
61	Total		9,210,135	10,052,522	10,966,397	11,956,842	104,232,883	
62								
63	NPV of row 80	75,210,421	<-- =NPV(B76,C81:G81)*(1+B76)^0.5					
64	Add in initial (year 0) cash and mkt. securities	500,000						
65	Enterprise value	75,710,421						
66	Subtract out value of firm's debt today	-10,000,000						
67	Equity value	65,710,421						
68	Value per share	65.71	<-- =B67/1000000					

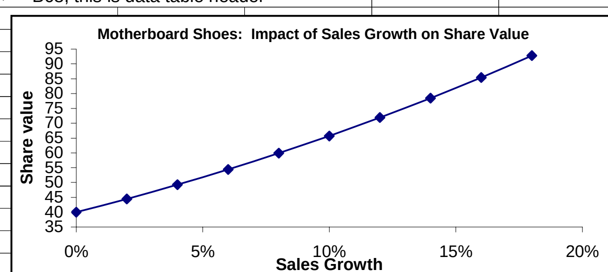
Several features of the model used by John Mba to value Motherboard Shoes are:

- Net fixed assets are assumed constant. John assumes that—as long as depreciation is invested back into fixed assets—Motherboard Shoes will need no more fixed assets. Another way of thinking about this assumption is that the major expenses incurred for fixed assets are equal to the depreciation expenses. As you can see in row 30 of the model this does not mean that fixed assets at cost are constant. It does mean, however, that in the FCF calculations lines 45 (depreciation) and 48 (capital expenses) cancel out.

- John assumes that in each of the next five years, Motherboard Shoes will repay $1 million of its $10 million debt.
- Cash is the plug.

Given the full-blown financial planning model, there are obviously many sensitivity analyses we can perform. Below we show two data tables. The first table analyzes the effect of the sales growth assumption (cell B2 of the model) on the share valuation. John Mba has estimated sales growth of 10% annually for the next 5 years. As you can see, the effect of this assumption is quite dramatic. The greater the sales growth (cell B2), the greater the valuation of Motherboard's shares:

	A	B	C	D	E	F	G
71	**Sensitivity analysis: Effect of sales growth (cell B2) on value per share**						
72							
73	**Sales growth**	**Share value**					
74		65.71	<-- =B68, this is data table header				
75	0%	39.96					
76	2%	44.48					
77	4%	49.29					
78	6%	54.43					
79	8%	59.89					
80	10%	65.71					
81	12%	71.89					
82	14%	78.46					
83	16%	85.43					
84	18%	92.82					
85							
86							
87							



Excel Note: Data tables

This may be the appropriate place to review **Data Tables**, which are covered in Chapter000. What may be confusing in the previous data table is the "65.71" in cell B74. This is a reference to the share price calculation in the initial model The data table asks Excel to redo this calculation for the sales growths in cells A75:A84.

A second sensitivity analysis performed by John Mba is the effect of the weighted average cost of capital and the long-term growth rate (cells B55 and B56) on the per-share valuation. Notice that these two parameters affect the valuation in two ways:

- The terminal value calculation in cell G60 is $\dfrac{FCF_5 * (1 + long\text{-}term\ FCF\ \ growth)}{WACC - long\text{-}term\ FCF\ \ growth}$. This computation is affected by both the long-term growth and the WACC parameters.
- The present value calculation in cell B63 is affected by the WACC.

To examine the effect of these two parameters, John builds a two-dimensional data table:

	A	B	C	D	E	F	G
90							
91	=IF(B55>B56,B68,"nmf")						
92			**WACC**				
93		65.71	**10%**	**12%**	**14%**	**16%**	**18%**
94		**0%**	118.54	97.01	81.68	70.22	61.33
95	**Long-term growth rate**	**2%**	141.87	111.62	91.49	77.14	66.40
96		**4%**	180.76	133.52	105.21	86.36	72.91
97		**6%**	258.54	170.04	125.80	99.28	81.60
98		**8%**	491.87	243.06	160.12	118.65	93.77
99		**10%**	nmf	462.14	228.76	150.95	112.02
100		**12%**	nmf	nmf	434.68	215.53	142.44
101		**14%**	nmf	nmf	nmf	409.29	203.27
102		**16%**	nmf	nmf	nmf	nmf	385.77
103							
104		**Note**: Data tables are discussed in Chapter 30.					

The results produced by this sensitivity analysis are not surprising:

- *Going across rows* shows that as the WACC increases, the value per share decreases. Since a larger WACC means that the present value of a future cash flow is less, this is to be expected.

- *Going down columns* shows that the larger the long-term growth rate expected from Motherboard Shoes, the more the shares are worth. Again, this is not a surprise, since larger long-term growth rates mean higher FCFs after the year-5 model horizon. As noted in the box below, our terminal value model only works when the long-term growth rate is less than the WACC. When this assumption is not true (meaning: the long-term growth $\geq$ WACC), we've had Excel write "nmf" ("no meaningful figure"). The technique for doing this is explained below.

Excel/Finance Note

Notice our use of the **If** function in cell B93 of the data table above. The terminal value formula is:

$$Terminal\ value = \frac{FCF_5 * (1 + long\text{-}term\ FCF\ growth\ rate)}{WACC - long\text{-}term\ FCF\ growth\ rate}$$

As noted in Chapter 6, this formula is only valid when $WACC > long\text{-}term\ FCF\ growth$. Since some of the combinations of growth and WACC in the data table violate this condition, we've used the **If** function to isolate them. As used in cell B93, this function says:

$$\text{If(B55>B56,}\ \underbrace{\text{B68}}_{\substack{\uparrow \\ \text{If WACC>long-term} \\ \text{FCF growth,} \\ \text{put in the valuation} \\ \text{as performed in cell} \\ \text{B78}}}\ ,\ \underbrace{\text{"nmf"}}_{\substack{\uparrow \\ \text{If WACC} \leq \text{long-term} \\ \text{FCF growth,} \\ \text{write "no meaninful} \\ \text{figure"}}}\ )$$

9.4. Advanced section: What's the theory behind the model?

In this section we explain some theoretical points about the valuation model illustrated in the previous section. Not all of this is easy, and you may (understandably) want to skip this section.[1]

Why is the firm's value related to the PV of the future FCFs?

Our basic valuation formula is:

$$Firm\ value = Debt + Equity$$

$$= \begin{matrix} Initial\ cash \\ and\ marketable \\ securities \end{matrix} + \frac{FCF_1}{(1+WACC)^1} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \dots$$

The *enterprise value* of the firm is defined to be the value of the firm's operations. In financial theory, the enterprise value is the present value of the firm's future anticipated cash flows. In this section we explain these concepts.

The valuation process

One way of viewing valuation is through the use of the accounting paradigm, but using market values. We rewrite the balance sheet by moving the current liabilities from the liabilities/equity side to the asset side of the balance sheet:

[1] Why would an author put a section like this in this book? Our experience is that ultimately almost all finance professionals are called upon to do valuations. At some point in every valuation, someone is going to question your techniques and theory. That's the time to come back to this section.

USING THE BALANCE SHEET AS AN ENTERPRISE VALUATION MODEL

ORIGINAL BALANCE SHEET			
Assets		**Liabilities**	
Cash and marketable securities		Operating current liabilities	
Operating current assets		Debt	
Net fixed assets		Equity	
Goodwill			
Total assets		**Total liabilities and equity**	
THE ENTERPRISE VALUATION "BALANCE SHEET"			
Assets		**Liabilities**	
Cash and marketable securities			
Operating current assets		Debt	
- Operating current liabilities			Enterprise value:
=Net working capital			PV of FCFs
Net fixed assets		Equity	discounted at WACC
Goodwill			
Market value		**Market value**	

To value a company, we set:

$$\begin{aligned} Market\ \ value &= Initial\ \ cash\ \ balances + \sum_t \frac{FCF_t}{(1+WACC)^t} \\ &= Initial\ \ cash\ \ balances + \ Enterprise\ \ value \end{aligned}$$

If we are valuing the equity of the firm, we subtract the value of the debt:

$$\begin{aligned} Equity\ \ value &= Market\, value - Debt \\ &= Initial\ \ cash\ \ balances + \sum_t \frac{FCF_t}{(1+WACC)^t} - Debt \\ &= \sum_t \frac{FCF_t}{(1+WACC)^t} - (Debt - Initial\ \ cash) \end{aligned}$$

Note that this means that we can write the enterprise balance sheet in a slightly different form:

THE ENTERPRISE VALUATION "BALANCE SHEET"
A slight variation: Cash and marketable securities netted out from debt

Assets		**Liabilities**	
Operating current assets		Debt - cash & marketable securities	
- Operating current liabilities			
=Net working capital			
Net fixed assets		Equity	
Goodwill			
Enterprise value		**Enterprise value**	

Enterprise value:
PV of FCFs discounted at WACC

We can use the FCF projections and a cost of capital to determine the enterprise value of the firm. Suppose we have determined that the firm's weighted average cost of capital (WACC) is 20%.[2] Then the *enterprise value* of the firm is the discounted value of the firm's projected FCFs plus its terminal value:

$$Enterprise\ value = \frac{FCF_1}{(1+WACC)^1} + \frac{FCF_2}{(1+WACC)^2} + \ldots + \frac{FCF_5}{(1+WACC)^5} + \frac{Year\text{-}5\ Terminal\ Value}{(1+WACC)^5}$$

In this formula, the *Year-5 Terminal Value* is a proxy for the present value of all FCFs from year 6 onwards.[3]

Terminal value

In determining the terminal value we use a version of the Gordon model described in Chapter 6. We have assumed that—after the year 5 projection horizon—the cash flows will grow at a rate of growth equal to the sales growth of 10%. This gives the terminal value as:

---

[2] In Chapter 6 we introduced the topic of the WACC and showed you how to calculate this using the Gordon dividend model. In Chapter 14 we show an alternative calculation of the WACC which uses the security market line. In this chapter we simply assume a value for the WACC.

[3] We don't actually project these cash flows. We determine the terminal value based on year-5 FCF.

$$Terminal\ Value\ at\ end\ of\ \ year\ 5 = \sum_{t=1}^{\infty} \frac{FCF_{t+5}}{(1+WACC)^t} = \sum_{t=1}^{\infty} \frac{FCF_5 * (1+growth)^t}{(1+WACC)^t}$$
$$= \frac{FCF_5 * (1+growth)}{WACC - growth}$$

The last equality is derived in a manner similar to the dividend valuation of shares (the Gordon model) discussed in Chapter 6.

Summary

The valuation of a business ("how much is it worth") is one of the most important activities of a financial analyst. In this chapter we have described how to do a discounted cash flow (DCF) valuation using the financial planning models of Chapter 8. To do a DCF valuation you have to understand almost all facets of the business:

- How the business works—this affects the financial parameters used in the financial planning model. The composition of the firm's current assets and current liabilities (meaning: its net working capital needed to do its business) and the amount of fixed assets (buildings and equipment and land) needed to do this business—all of these factors affect a firm's valuation.
- How to compute the cost of capital. The weighted average cost of capital (WACC) is the discount rate used to value the future FCFs of the firm. In this chapter we have not discussed its computation (Chapters 6 and 15 give different aspects of the WACC computation).
- How to use Excel to do the relevant computations.

Exercises

**The present value calculation above assumes that cash flows occur at the end of the year. However, they actually occur throughout the year. To account for this fact, John assumes that each of the projected free cash flows occurs in *mid-year*. As discussed in Chapter000, this means that the present value calculation??????????????? [leave as exercise?]

CHAPTER 10: WHAT IS RISK?*

slight bug fix: July 27, 2003

Chapter contents

Overview

Risk is the magic word in finance. Whenever finance people can't explain something, we try to look confident and say "it must be the risk." If we want to appear intelligent when hearing

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance with Excel* by Simon Benninga (http://finance.wharton.upenn.edu/~benninga/pfe.html). Check with the author (benninga@wharton.upenn.edu) before distributing this draft (though you will probably get permission), and check the website to make sure the material is updated.

a financial presentation, we look skeptical and say "Have you considered the risks?" Usually that's enough to score a point or two.[1]

Our intuition usually relates financial risks to *unpredictability*. A financial asset like a savings account is thought to be not risky because its future value is known, whereas a financial asset like a stock is risky because we do not know what it will be worth in the future. Financial assets of different types have different gradations of risk: Our intuitions tell us that a savings account is less risky than a share in a company, and a share in a high-tech start-up is more risky than a share in a well-established blue-chip company.

The intuition which ties unpredictability and risk together is valid, but can have some surprising aspects. For example, in section 10.??? we show that a Treasury bill, a bond issued by the United States government, can sometime be risky. The T-bill becomes risky if you need to sell it before it matures. We illustrate this risk with an example. We also look at the risk of holding a share, and show that it can be quantified statistically. This is an important insight for Chapters 11-15, where we use a statistical description of stock price risk to talk about choosing portfolios of stocks.

Although we have tried to make the chapter unstatistical and non-mathematical, inevitably the measurement of risk involves calculations.[2]

---

[1] I give my students the following hint about taking finance exams: Suppose you have to answer a question to which you absolutely don't know the answer ("What is the zeta function of the annual returns?" "How do you explain the difference between XYZ Corp's annual returns over time?"). If you know nothing about the question, make up a meaningless sentence which includes the word "risk" ("The zeta function of the annual returns relates to the riskiness of the returns." "XYZ's annual returns vary because of the changing risk of the company.") You're bound to get a point or two.

Finance concepts discussed

- Ex-post and ex-ante returns
- Holding-period returns
- Treasury bond returns
- Return statistics—mean, variance, and standard deviation

Excel functions used

- Month
- Sqrt
- Average
- Varp
- Frequency

10.1. The risk characteristics of financial assets—some introductory blather

In the course of your life you'll be exposed to many financial assets. You've already been exposed to them, even if you didn't know that they were "financial assets": When you were small, your parents might have opened a savings account for you at the local bank, or your grandparents bought you a few shares of stock. Now that you're a student, you're stuck with

[2] Students reading this book will generally have had a statistics course. This chapter assumes some familiarity with basic statistical concepts and the next chapter reviews these concepts in the context of financial assets. In this sense

student loans, and each month you're trying to decide whether to pay off your credit card balances or let them ride for another month and pay interest on them. Once you finish school, you'll be taking a car loan, buying a house and taking a mortgage, buying stocks and bonds, … .

All financial assets have different characteristics of *horizon*, *safety*, and *liquidity*. As you will see, all three of these terms are in some basic sense indicative of the asset's riskiness. In this section we briefly review these concepts.

Horizon

Some assets are *short-term* and others *long-term*. Money deposited in a checking account is a good example of a *very short-term* financial asset; the money can be withdrawn at any time. On the other hand, many savings accounts require you to deposit the money for a given period of time. Look at Figure 1, which shows the rates offered on certificates of deposit (CD) by Metropolitan Bank of Chicago. A CD is a time-deposit at a bank—a deposit which cannot be withdrawn for a certain period of time.[3] Not surprisingly, longer-term CDs offer higher interest rates.

You're not always "locked in" to a financial asset with a long horizon. Many long-horizon assets can be sold in the open market. Suppose, for example, that you buy a 10-year government bond. You can "cash out" of the bond at almost any time by selling the bond in the open market, but selling the bond before its 10-year maturity exposes you to the riskiness of an unknown market price. This subject is explored in detail in Section 10.2 below.

Chapters 10 and 11 are twins.

[3] Most banks will allow you to withdraw your money from a CD before the horizon date, but only if you pay a penalty.

Some assets have a long and indeterminate horizon. A share of stock in a company is a good example. Holding a share of McDonald's stock, for example, entitles you to whatever dividends the company pays its shareholders for as long as you hold the stock and the company exists. You can, of course, sell the stock in the stock market, but this exposes you to the risks of the stock price fluctuations. In Section 10.3 below we discuss how to analyze the riskiness of stock holding; this is a topic to which we return in much greater detail in Chapters 11 – 15.

Safety

Financial assets differ in the certainty with which you get back your money. The Metropolitan Bank CDs in Figure 1 are guaranteed by the Federal Deposit Insurance Corporation, an agency of the United States government, up to a limit of $100,000. Up to this limit, the purchaser of a Metropolitan Bank CD will get his money back (including interest), even if the Metropolitan Bank fails to meet its obligations.

CDs issued by the Millenium Bank and Trust (MB&T) of St. Vincents (a small Carribean country) pay much higher interest rates (see Figure 2) but are not guaranteed by the U.S. government. The return on the MB&T CDs is less certain and consequently the interest rates offered by the bank are higher.

The issuer of a CD announces the interest rate to be paid on the CD and will, presumably, keep this promise if possible. The same holds for a bond issues by a company or a government. On the other hand, the issuer of a stock does not give any undertaking about either the stock's dividends or the market price of the stock. In this sense the safety of a stock is much less than the safety of a CD or a bond.

In general, the less safe an asset, the greater the return investors will demand and expect from the asset. Thus, for example, if Metropolitan Bank's CDs pay interest of between 1% - 3%, intelligent holders of McDonalds stock (less safe and more uncertain than a CD) should expect a return greater than 1% - 3%.

This business of "expected return" is complicated:

- If you buy a Metropolitan Bank 5-year CD, you are promised an annual return of 3%. You will get this annual return with absolute certainty (well, *almost absolutely*: There's always the remote possibility of a catastrophe which prevents both Metropolitan and the U.S. government from honoring their obligations …). For the Metropolitan Bank 5-year CD, the *expected return* and the *actual return received* (in economists' jargon, these are called the *ex-ante* and the *ex-post* returns) are the same.
- If you buy a share of McDonald's stock, you will *expect* to get more than 3% annual return. However, in this case this *expectation* is merely an *anticipated average future return*. In other words: You would be disappointed but not surprised if the actual annual return on the stock after 5 years was less than 3%.

Liquidity

The *ease* with which an asset can be bought or sold is the asset's *liquidity*. In general, the more liquid an asset, the easier it is to "get rid off," and the less its risk.

Listed stocks of major American companies have very high liquidity. For the period 1990-1999, the average daily number of McDonalds shares traded (meaning: shares bought and sold) on the New York Stock Exchange was 1.5 million shares. This is the average; the highest number of shares traded daily was almost 12 million and the lowest number of shares was

63,000.. If you want to buy or sell a single share of stock (or even several thousand shares), you'll have no trouble doing so: McDonalds stock is very liquid.

Liquidity has another aspect, which financial economists call *price impact*. Suppose you decided to sell the 1,000 shares of McDonalds stock your father gave you. You'll have no trouble selling the stock, but will your sale affect the market price? For McDonalds stock the answer is "no."

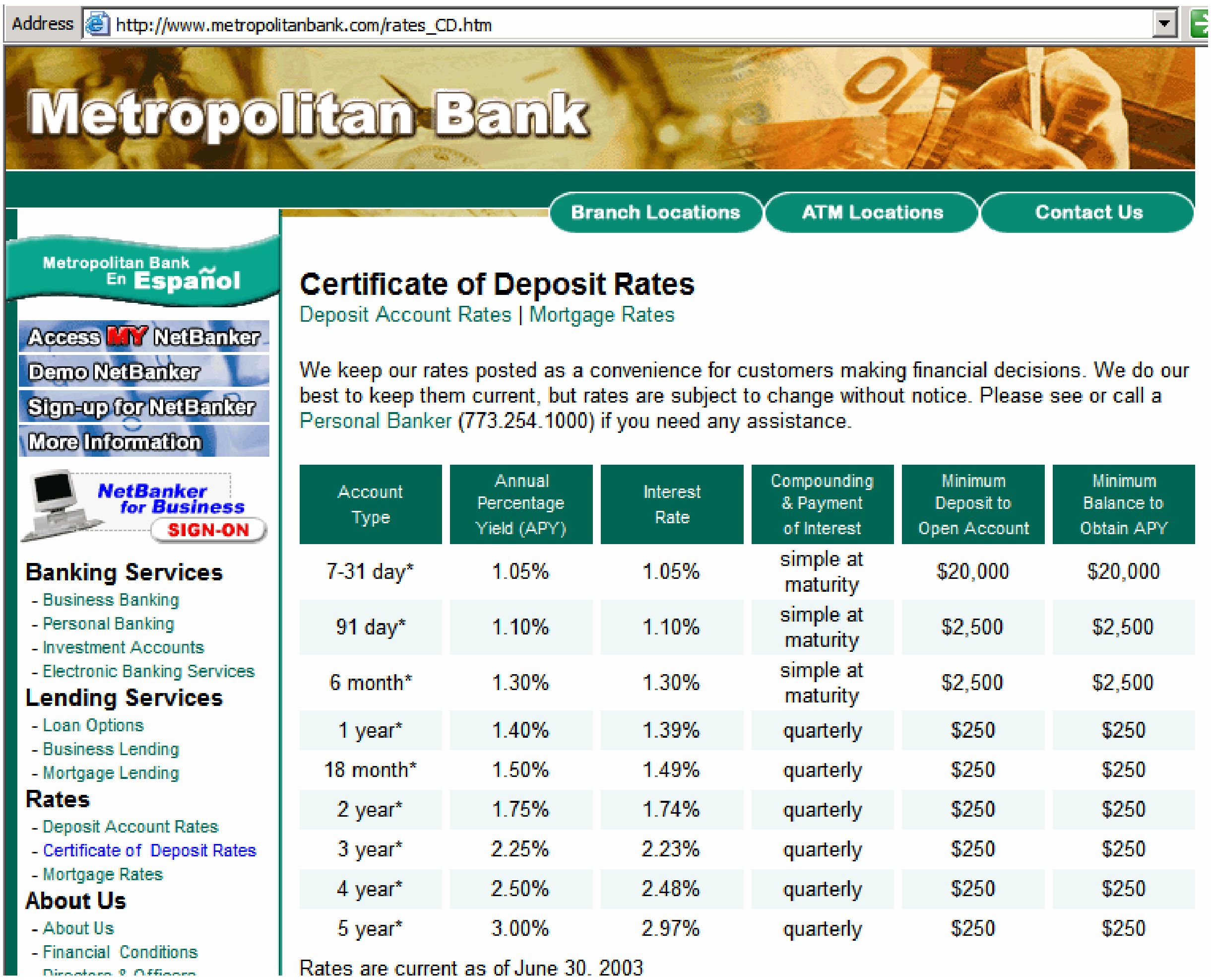
Not all stocks are equally liquid: Aladdin Knowledge Systems is a small company which trades on the Nasdaq stock exchange. On an average day around 60,000 shares of Aladdin are bought and sold, but this number has been as low as 100 shares per day. You would have relatively little trouble buying or selling several thousand shares of Aladdin stock, but your action might well affect the market price of the stock. Aladdin is not nearly as liquid as McDonalds and consequently has greater liquidity risk.

What now?

Horizon, safety, and liquidity all determine the *risk* of a financial asset. In the succeeding sections we'll give some concrete examples. We start by looking at the risks inherent in holding a U.S. Treasury bill. A T-Bill is completely *safe*, in the sense that the U.S. Treasury will keep its obligation to pay back the money borrowed. It's also very *liquid*—billions of dollars of T-bills are bought and sold every day in financial market. However, we'll show that the *horizon* of a T-bill means that it is *somewhat risky*—if you try to sell it before it matures, the market price is unpredictable.

From the T-bill we move on to an analysis of the risks inherent in McDonalds stock. McDonalds stock is *not safe* in the sense that the company makes no promises about either dividends or the future market price of the stock. We'll analyze the returns on McDonald stock over the decade 1990-2000 and we'll try to make some statistical sense out of these returns.

Metropolitan Bank (Chicago) Certificates of Deposit


Address http://www.metropolitanbank.com/rates_CD.htm

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We keep our rates posted as a convenience for customers making financial decisions. We do our best to keep them current, but rates are subject to change without notice. Please see or call a Personal Banker (773.254.1000) if you need any assistance.

Account Type	Annual Percentage Yield (APY)	Interest Rate	Compounding & Payment of Interest	Minimum Deposit to Open Account	Minimum Balance to Obtain APY
7-31 day*	1.05%	1.05%	simple at maturity	$20,000	$20,000
91 day*	1.10%	1.10%	simple at maturity	$2,500	$2,500
6 month*	1.30%	1.30%	simple at maturity	$2,500	$2,500
1 year*	1.40%	1.39%	quarterly	$250	$250
18 month*	1.50%	1.49%	quarterly	$250	$250
2 year*	1.75%	1.74%	quarterly	$250	$250
3 year*	2.25%	2.23%	quarterly	$250	$250
4 year*	2.50%	2.48%	quarterly	$250	$250
5 year*	3.00%	2.97%	quarterly	$250	$250

Rates are current as of June 30, 2003

Metropolitan Bank offers a variety of certificates of deposit (CD). CDs differ by interest rates and by the amount of time the money is locked up. CDs with longer lock-up times offer higher interest rates. APY is Metropolitan Bank's terminology for the effective annual interest rate (EAIR) discussed in Chapter 2. For example, the 5-year CD pays 2.97% quarterly. This makes the EAIR 3.00%:

$$EAIR = \left(1 + \frac{2.97\%}{4}\right)^4 - 1 = 3.00\%$$

Figure 1

Millenium Bank and Trust (St. Vincents) Certificates of Deposit

Premium Certificate	Minimum Deposit	Interest Rate
3 Year CD	$5,000	6.00%
	$25,000	6.25%
	$100,000	7.00%
4 Year CD	$5,000	6.25%
	$25,000	6.50%
	$100,000	7.25%
5 Year CD	$5,000	6.75%
	$25,000	7.00%
	$100,000	7.75%
7 Year CD	$5,000	7.00%
	$25,000	7.25%
	$100,000	8.00%

Figure 2

IS IT RISK OR UNCERTAINTY?

Frank H. Knight (1885-1972) wrote a dissertation in 1921 called *Risk, Uncertainty and Profit*. Knight used *risk* to mean *randomness with knowable probabilities* and *uncertainty* to mean *randomness which is unmeasurable*. In finance the distinction between these two concepts is often blurred and the words "risk" and "uncertainty" are used interchangeably.

10.2. A safe security can be risky because it has a long horizon

Finance people use the words "risk-free" to describe an asset whose value in the future is known with certainty; "riskless" is a synonym. One classic textbook example of a risk-free asset is a bank savings account. If you deposit $100 in your bank savings account, which currently earns 10%, then you *know* that one year from now there will be $110 in the account. It's risk-free.

A United States Treasury bill is another example of a riskless asset. Treasury bills are short-term bonds issued by the government of the United States.[4] Unlike bank CDs, Treasury bills do not have an explicit interest rate. Instead they are sold at a discount—a bill with a face value of $1,000 which matures one year from now might be sold today for $950. In this case the purchaser of the bill who holds the bill to maturity would be paid $1,000 by the U.S. Treasure and would thus earn a rate of return of $\frac{1,000}{950}-1=5.2632\%$. Since Treasury bills are issue by the U.S. government, at least one kind of risk—default risk—is absent from these instruments:

[4] There are many different kinds of bonds. For a more complete discussion, see Chapter 000.

Since the government owns the printing machines which produce dollar bills, they can always run off a few dollars to make good on their promises.

The purpose of this section is to point out that even Treasury bills—and other financial instruments which are free of default risk—may have elements of *price risk*.

Suppose that on 1 January 2001 you buy a one-year $1,000 U.S. Treasury bill, intending to hold the bill until its maturity on 1 January 2002. As we said, a Treasury bill doesn't pay any interest; instead, it is bought at a discount—that is, for less than its face value. In the case at hand, suppose you buy the bill for $953.04; since it matures in one year after the purchase, you anticipated getting interest of 4.93%:

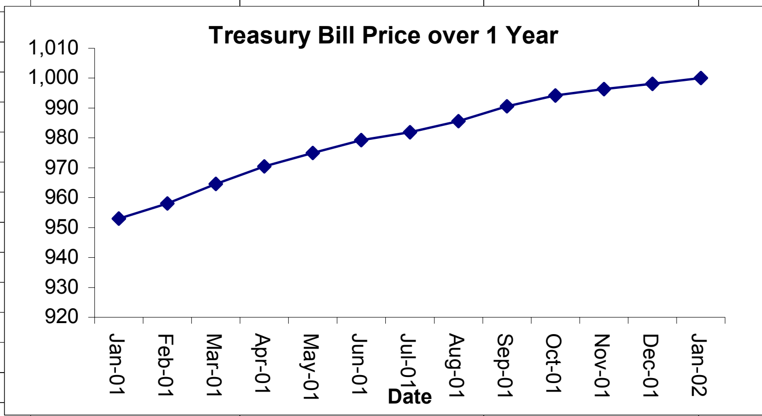
	A	B	C	D	E
1	**INTEREST ON THE TREASURY BILL**				
2					
3	Purchase price	953.04			
4	Payoff on maturity	1,000.00	<-- This is the Treasury bill's face value		
5					
6	Interest	4.93%	<-- =B4/B3-1		

Now before we start doing fancier calculations, let's make one thing perfectly clear: *If you hold the Treasury bill from 1 January 2001 until its maturity 1 year later, you will* *<u>absolutely, definitely</u> earn 4.93% interest*. T-bills are obligations of the United States government and it has never defaulted on them.

In finance jargon the *ex-ante* return (sometimes called the *anticipated* or *expected* return) is the return you think you're going to get. The *ex-post* return (also called the *realized* return) is the actual return that you get when you sell the asset. For the Treasury bill illustrated here, the ex-ante return equals the ex-post return *if you hold the bill until maturity*. This is always true for riskless bonds.

Out of curiosity, you track the market price of the bill on the first of each month during the year. Here's what you find:

	A	B	C	D	E	F	G
1	**THE PRICE OF THE TREASURY BILL THROUGHOUT THE YEAR**						
2							
3	**Date**	**Bill price**					
4	1-Jan-01	953.04					
5	1-Feb-01	958.08					
6	1-Mar-01	964.59					
7	1-Apr-01	970.46					
8	1-May-01	974.95					
9	1-Jun-01	979.23					
10	1-Jul-01	981.92					
11	1-Aug-01	985.56					
12	1-Sep-01	990.62					
13	1-Oct-01	994.14					
14	1-Nov-01	996.36					
15	1-Dec-01	998.12					
16	1-Jan-02	1,000.00					
17							



We use this monthly price data to compute some returns.

What ex-post rate of return would you have earned if you'd sold the Treasury bill early?

Suppose you had sold the T-bill on 1 May 2001 for \$974.95. What would you have earned? A relatively simple calculation provides the answer. The monthly rate of return—the *ex-post* return—is defined by:

$$1+\underline{ex\text{-}post}\ monthly\ rate\ of\ return=\left(\frac{Price\ on\ 1May01}{Initial\ price\ on\ 1Jan01}\right)^{1/4}$$

$$=\left(\frac{974.95}{953.04}\right)^{1/4}=1.0057$$

The exponent of ¼ is there because of the 4 month interval between January and May. If we raise this to the 12th power, we will get 1+annual rate of return:

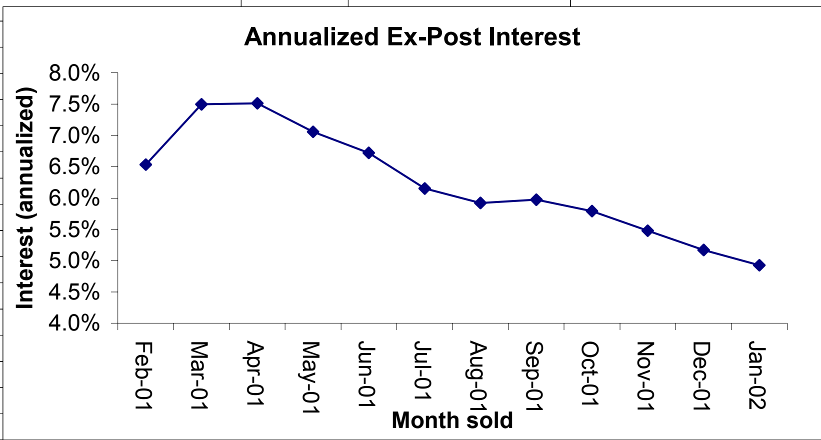
	A	B	C
1	**ANNUALIZED EX-POST RETURN, JAN-MAY**		
2			
3	Bought, 1 January 2001	953.04	
4	Sold, 1 May 2001	974.95	
5	Monthly return	0.57%	<-- =(B4/B3)^(1/4)-1
6	Annualized return	7.06%	<-- =(1+B5)^12-1

If, instead, you had sold the Treasury bill on April 1, one month earlier, you would have made 7.51% in annual terms:

	A	B	C
1	**ANNUALIZED EX-POST RETURN, JAN-APRIL**		
2			
3	Bought, 1 January 2001	953.04	
4	Sold, 1 April 2001	970.46	
5	Monthly return	0.61%	<-- =(B4/B3)^(1/3)-1
6	Annualized return	7.51%	<-- =(1+B5)^12-1

We can do this exercise for each of the months from February – December:

	A	B	C	D
1	**EX-POST INTEREST ON THE TREASURY BILL**			
2				
3	**Date**	**Bill price**	**Annualized return if sold at beginning of month**	
4	Jan-01	953.04		
5	Feb-01	958.08	6.53%	<-- =(B5/$B$4)^(12/(MONTH(A5)-MONTH($A$4)))-1
6	Mar-01	964.59	7.50%	<-- =(B6/$B$4)^(12/(MONTH(A6)-MONTH($A$4)))-1
7	Apr-01	970.46	7.51%	
8	May-01	974.95	7.06%	
9	Jun-01	979.23	6.72%	
10	Jul-01	981.92	6.15%	
11	Aug-01	985.56	5.92%	
12	Sep-01	990.62	5.97%	
13	Oct-01	994.14	5.79%	
14	Nov-01	996.36	5.48%	
15	Dec-01	998.12	5.17%	
16	Jan-02	1,000.00	4.93%	



As you can see, if the T-bill is sold before its maturity, there is a considerable amount of risk—defined here as the possible variation in the ex-post rate of return.

An Excel Note

The interest rate calculations above use the following formula:

$$r_{monthly} = 1 + \text{monthly interest rate} = \left(\frac{T\text{-}bill\ price, month\ t}{T\text{-}bill\ purchase\ price} \right)^{\frac{1}{number\ of\ months\ held}}$$

$$r_{annual} = 1 + \text{annual interest rate} = \left(1 + r_{monthly} \right)^{12}$$

In order to calculate the number of months the T-bill has been held, we use the Excel function **Month**. This function, when applied to a date, identifies the month by a number (January = 1, February = 2, ...):

	A	B	C	D	E
1	USING EXCEL'S MONTH FUNCTION				
2					
3	**Date**	**Month**			
4	3-Jan-03	1	<-- =MONTH(A4)		
5	16-Sep-06	9	<-- =MONTH(A5)		

What ex-ante rate of return would you have earned if you'd bought the T-bill during the year?

In the previous exercise we calculated the ex-post rate of return which you would have earned if you had bought the T-bill on 1 January 2001 and had sold it before the bill's maturity on 1 January 2002. There's a second "game" we can play with the T-bill prices. Suppose you had bought the bill at the beginning of May for 974.95 and suppose you intended to hold it until the end of the year. What's the annualized interest ex-ante return you could expect?

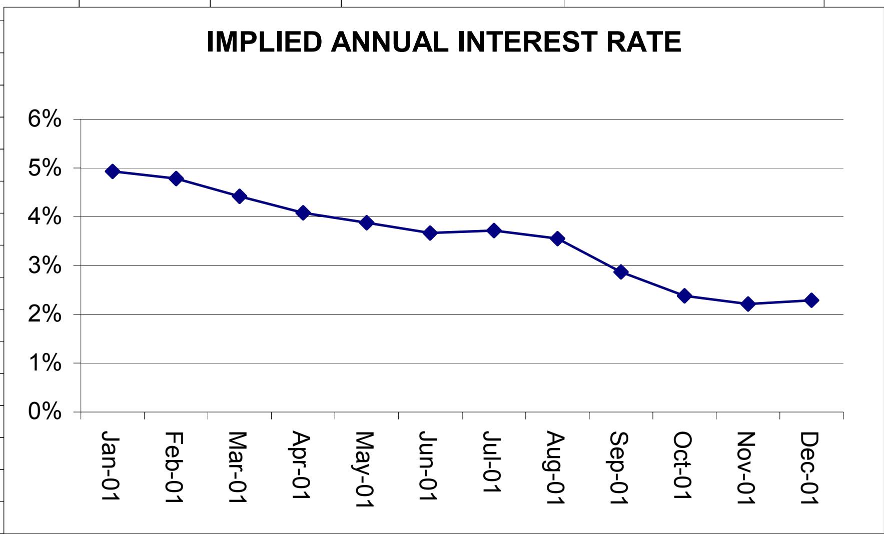
$$1 + \underline{ex\text{-}ante}\ monthly\ rate\ of\ return = \left(\frac{1{,}000}{Price\ on\ 01May01} \right)^{1/8}$$

$$= \left(\frac{1{,}000}{974.95} \right)^{1/8} = 1.0032$$

$$Annualized\ ex\text{-}ante\ return = (1.0032)^{12} - 1 = 3.88\%$$

If we do this for each of the months:

	A	B	C	D	E	F
21					=($B$36/B24)^(1/C24)-1	
22						
23	**Date**	**Bill price**	**Months till maturity**	**Implied monthly interest ex-ante rate**	**Ex-ante rate annualized**	
24	Jan-01	953.04	12	0.40%	4.93%	<-- =(1+D24)^12-1
25	Feb-01	958.08	11	0.39%	4.78%	
26	Mar-01	964.59	10	0.36%	4.42%	
27	Apr-01	970.46	9	0.33%	4.08%	
28	May-01	974.95	8	0.32%	3.88%	
29	Jun-01	979.23	7	0.30%	3.66%	
30	Jul-01	981.92	6	0.30%	3.72%	
31	Aug-01	985.56	5	0.29%	3.55%	
32	Sep-01	990.62	4	0.24%	2.87%	
33	Oct-01	994.14	3	0.20%	2.38%	
34	Nov-01	996.36	2	0.18%	2.21%	
35	Dec-01	998.12	1	0.19%	2.29%	
36	Jan-02	1,000.00	0			



What's the message?

This simple example, which illustrates the riskiness of a "riskless" U.S. Treasury bill, illustrates that financial risk depends on horizon: A financial asset can be riskless over one horizon and risky over another. In our example, buying the Treasury bill at any point during the year and holding it until maturity *guarantees* that the *ex-ante* return will equal the *ex-post* return. On the other hand, selling the bill before its maturity involves risk—in this case the realized return (the ex-post return) varies.

A final word: What caused the riskiness of the Treasury bills?

We've shown that holding a T-bill during 2001 could have been pretty risky—if you were thinking of selling the bill before maturity. The cause of all this uncertainty was the Federal Reserve Bank's Open Market Committee. This powerful committee sets short-term interest rates, which have a dramatic effect on the value of all bonds, but especially on short-term bonds like Treasury bills. In an effort to shore up the flagging U.S. economy, the Fed's Open Market Committee reduced interest rates *eleven times* during the course of 2001! These changes in interest rate caused the changes in the ex-post and ex-ante returns which we've documented in this section.

10.3. Risk in stock prices—McDonald's stock

A U.S. Treasury bill is a relatively simple security: The issuer is very well-known and has never defaulted, the *ex-ante* return can be derived from the price, and this return is guaranteed if you hold the bill until maturity. A stock has none of these properties, and is thus in every sense riskier. The problem is how to quantify this risk.

Here's an example—Figure 3 shows how the stock price of McDonald's varied over the decade of the 90s:

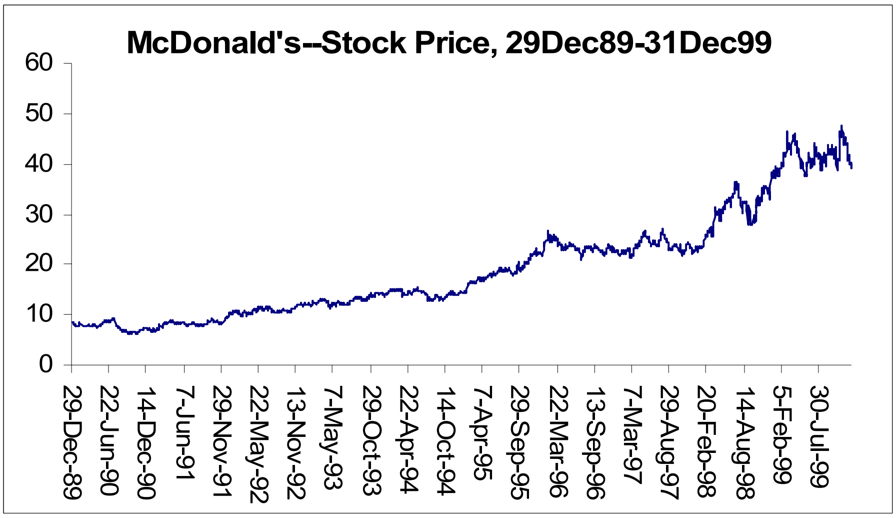


Figure 3: The stock price of McDonald's, 1990 - 1999

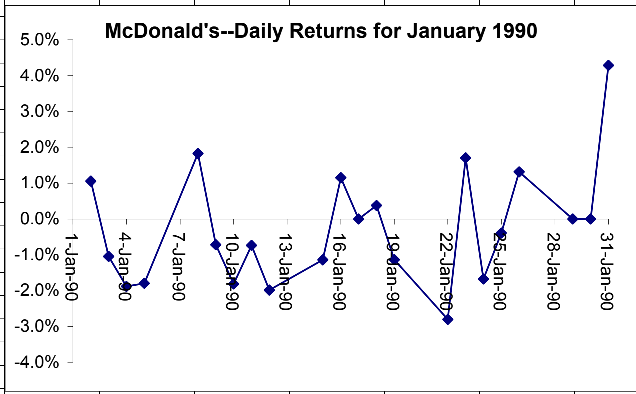
The fact that the stock's price goes up and down is an indication of the stock's riskiness.[5] If we calculate the daily returns, we see a different kind of risk. Below we calculate the *daily return* from holding McDonald's stock—this is what you would earn in percentages if you bought the stock at its closing price on day *t* and sold it at its closing price on day *t+1*:

$$Daily\ return,\ day\ t = \frac{P_{t+1}}{P_t} - 1$$

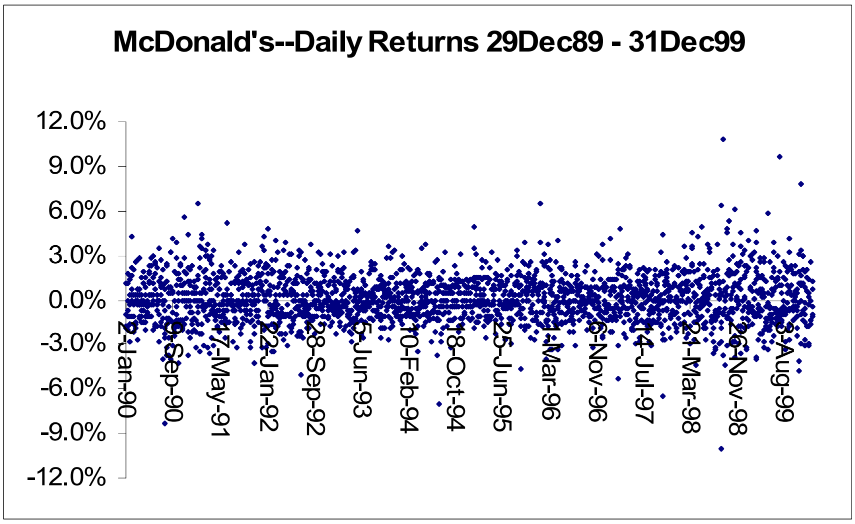
If you plot the daily returns for one month, you get a very spiky pattern:

[5] A technical note which you can ignore but which may make your finance prof happy: The prices of the McDonald's stock have been adjusted to include dividends.

	A	B	C	D	E	F	G	H	I	J	K
1	**McDONALD'S--DAILY STOCK PRICES, 29Dec89 - 31Dec99**										
2											
3	Date	Closing stock price									
4	Date	Close									
5	29-Dec-89	8.50									
6	2-Jan-90	8.59	1.059%	<-- =B6/B5-1							
7	3-Jan-90	8.50	-1.048%	<-- =B7/B6-1							
8	4-Jan-90	8.34	-1.882%								
9	5-Jan-90	8.19	-1.799%								
10	8-Jan-90	8.34	1.832%								
11	9-Jan-90	8.28	-0.719%								
12	10-Jan-90	8.13	-1.812%								
13	11-Jan-90	8.07	-0.738%								
14	12-Jan-90	7.91	-1.983%								
15	15-Jan-90	7.82	-1.138%								
16	16-Jan-90	7.91	1.151%								
17	17-Jan-90	7.91	0.000%								
18	18-Jan-90	7.94	0.379%								
19	19-Jan-90	7.85	-1.134%								
20	22-Jan-90	7.63	-2.803%								
21	23-Jan-90	7.76	1.704%								
22	24-Jan-90	7.63	-1.675%								
23	25-Jan-90	7.60	-0.393%								
24	26-Jan-90	7.70	1.316%								
25	29-Jan-90	7.70	0.000%								
26	30-Jan-90	7.70	0.000%								
27	31-Jan-90	8.03	4.286%								



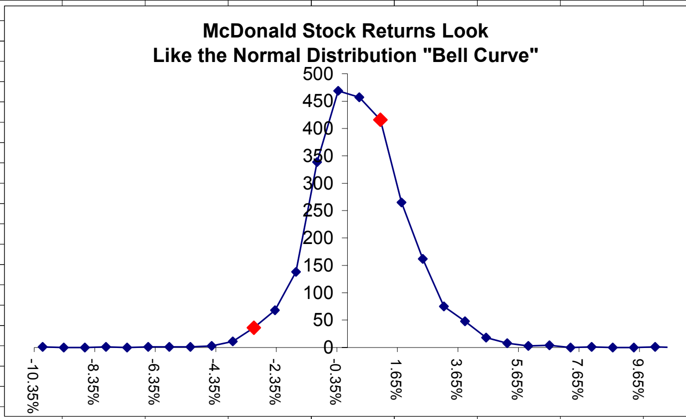
If you plot the daily returns for all 2,528 data points, you get a very "noisy" pattern (there are dots all over the place, though there seem to be slightly more dots above the x-axis than below it):



The distribution of McDonald's stock returns

The previous two graph show the return on McDonald's stock on each specific date. These graphs clearly show that the stock is risk—the returns vary from day to day—but they don't give much insight into the statistical nature of the riskiness of the stock. A different way to think about the riskiness of McDonald's stock is to look at the frequency distribution of the daily returns. Of the 2528 daily returns, how many were between 1.09% and 1.79%? The answer turns out to be 416, which is 16.462% of the total number of returns. As another example: A total of 36 of the returns (1.425% of the total), were between -3.09% and -2.40%. In the spreadsheet below we've used Excel's **Frequency** function to calculate the whole frequency distribution of the returns (see Box0.00 for more on how to use this function). The plots of the returns look very much like the normal distribution (the "bell curve") you've probably studied in a statistics course.

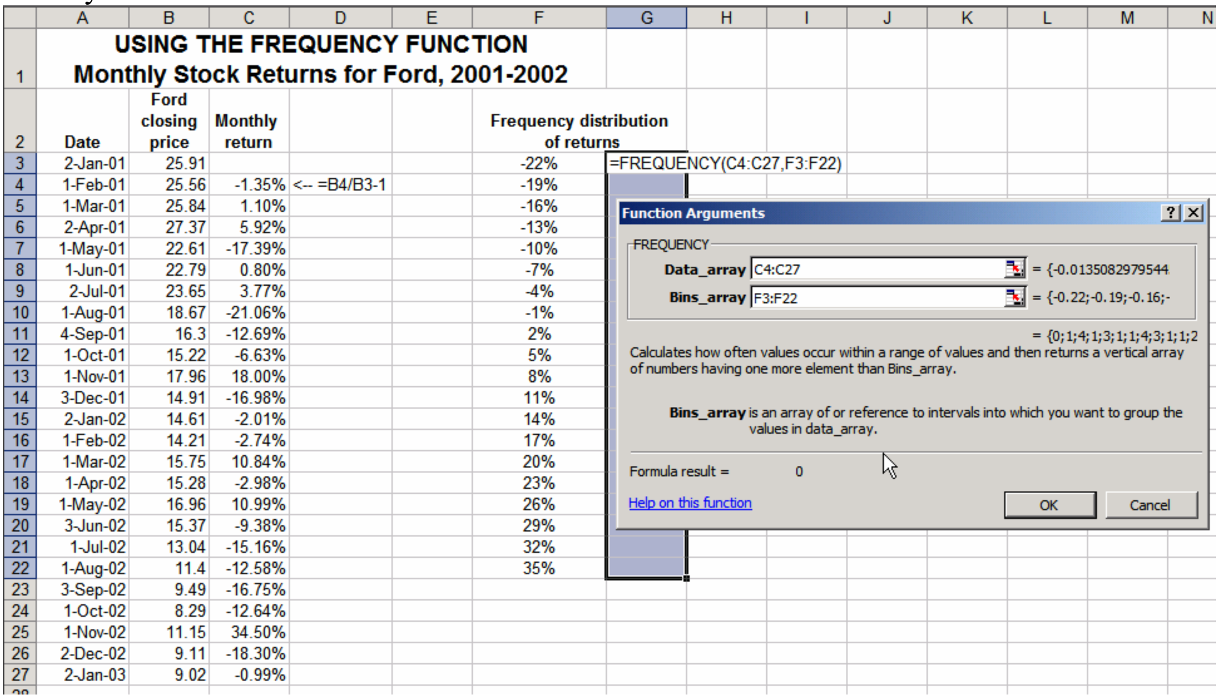
	F	G	H	I	J
6	**Computing the frequency distribution of MCD returns**				
7	Largest daily return	10.86%	<-- =MAX(C7:C2534)		
8	Smallest daily return	-10.07%	<-- =MIN(C7:C2534)		
9					
10	**Bin**	**How many?**	**Percentage**		
11	-10.07%	1	0.040%		
12	-9.37%	0	0.000%		
13	-8.68%	0	0.000%		
14	-7.98%	1	0.040%		
15	-7.28%	0	0.000%		
16	-6.58%	1	0.040%		
17	-5.88%	1	0.040%		
18	-5.19%	1	0.040%		
19	-4.49%	3	0.119%		
20	-3.79%	11	0.435%		
21	-3.09%	36	1.425%		
22	-2.40%	68	2.691%		
23	-1.70%	138	5.461%		
24	-1.00%	339	13.415%		
25	-0.30%	469	18.560%		
26	0.40%	457	18.085%		
27	1.09%	416	16.462%		
28	1.79%	265	10.487%		
29	2.49%	162	6.411%		
30	3.19%	75	2.968%		Note: The yellow highlighted data in the table is marked
31	3.88%	48	1.899%		in red in the graph.
32	4.58%	18	0.712%		
33	5.28%	8	0.317%		
34	5.98%	3	0.119%		
35	6.68%	4	0.158%		
36	7.37%	0	0.000%		
37	8.07%	1	0.040%		
38	8.77%	0	0.000%		
39	9.47%	0	0.000%		
40	10.16%	1	0.040%		
41	10.86%	0	0.000%		
42					
43	Note: We've used the Excel **Frequency** function (see separate page).				
44	To understand the numbers: there is 1 daily return between -10.07% and -9.37%;				
45	There are 3 daily returns between -4.49% and -3.79%; etc.				



[Separate Page]

An Excel Note: The Frequency Function

The frequency distribution which gave rise to the "bell curve" for McDonald's stock returns was calculated with an Excel function called **Frequency**. To use this function, consider the following example, which gives the monthly returns on Ford stock between January 2001 and January 2003:



	A	B	C	D	E	F
1	**USING THE FREQUENCY FUNCTION Monthly Stock Returns for Ford, 2001-2002**					
2	**Date**	**Ford closing price**	**Monthly return**			**Frequency distribution of returns**
3	2-Jan-01	25.91				-22%
4	1-Feb-01	25.56	-1.35%	<-- =B4/B3-1		-19%
5	1-Mar-01	25.84	1.10%			-16%
6	2-Apr-01	27.37	5.92%			-13%
7	1-May-01	22.61	-17.39%			-10%
8	1-Jun-01	22.79	0.80%			-7%
9	2-Jul-01	23.65	3.77%			-4%
10	1-Aug-01	18.67	-21.06%			-1%
11	4-Sep-01	16.3	-12.69%			2%
12	1-Oct-01	15.22	-6.63%			5%
13	1-Nov-01	17.96	18.00%			8%
14	3-Dec-01	14.91	-16.98%			11%
15	2-Jan-02	14.61	-2.01%			14%
16	1-Feb-02	14.21	-2.74%			17%
17	1-Mar-02	15.75	10.84%			20%
18	1-Apr-02	15.28	-2.98%			23%
19	1-May-02	16.96	10.99%			26%
20	3-Jun-02	15.37	-9.38%			29%
21	1-Jul-02	13.04	-15.16%			32%
22	1-Aug-02	11.4	-12.58%			35%
23	3-Sep-02	9.49	-16.75%			
24	1-Oct-02	8.29	-12.64%			
25	1-Nov-02	11.15	34.50%			
26	2-Dec-02	9.11	-18.30%			
27	2-Jan-03	9.02	-0.99%			

When you finish putting in range C4:C27 and F3:F22 as shown above, *don't click on **OK***! Instead, simultaneously press the keys [Ctrl]+[Shift]+[Enter]. This will put the frequency in the spreadsheet as shown below:

	F	G
2	**Frequency distribution of returns**	
3	-22%	0
4	-19%	1
5	-16%	4
6	-13%	1
7	-10%	3
8	-7%	1
9	-4%	1
10	-1%	4
11	2%	3
12	5%	1
13	8%	1
14	11%	2
15	14%	0
16	17%	0
17	20%	1
18	23%	0
19	26%	0
20	29%	0
21	32%	0
22	35%	1

This table says that in the period January 2001 – January 2003, there was 1 month when Ford stock had a return between -22% and -19%, 4 months when Ford stock had a return between -19% and -16%, and so on.

Computing the mean and standard deviation of the McDonald's returns

If we concentrate on the end-year prices of McDonald's, we can compute the average annual return of 18.86 percent; on average, a McDonald's shareholder got an annual return of 18.86 percent per year over the period 1990-1999. The standard deviation of the annual returns is 23.28 percent. The standard deviation is a statistical measure of the variation of the stock's returns—the greater the standard deviation, the greater the riskiness of the stock.

	A	B	C	D	E	F	G	H
1	**McDONALD'S--End-Year Stock Prices, 1989 - 1999**							
2	**Date**	**Closing stock price**	**Return**					
3	29-Dec-89	8.50				**Statistics**		
4	31-Dec-90	7.17	-15.647%	<-- =B4/B3-1		Largest annual return	60.88%	<-- =MAX(C4:C13)
5	31-Dec-91	9.36	30.544%	<-- =B5/B4-1		Smallest annual return	-15.65%	<-- =MIN(C4:C13)
6	31-Dec-92	12.01	28.312%					
7	31-Dec-93	14.04	16.903%			Average annual return	18.86%	<-- =AVERAGE(C4:C13)
8	30-Dec-94	14.41	2.635%			Variance of annual returns	0.0542	<-- =VARP(C4:C13)
9	29-Dec-95	22.23	54.268%			Standard deviation of annual returns	23.28%	<-- =STDEVP(C4:C13)
10	31-Dec-96	22.35	0.540%					
11	31-Dec-97	23.52	5.235%					
12	31-Dec-98	37.84	60.884%					
13	31-Dec-99	39.71	4.942%					
14								
15		**STATISTICAL REVIEW**						
16		**McDonald's return**	**Return minus average, squared**			This statistical review shows you how to compute the average, variance, and standard deviation using the formulas described in the text and (hopefully) learned in your statistics course.		
17	31-Dec-90	-15.65%	11.908%	<-- =(B17-$B$28)^2		See the "Excel and Statistical Review" box in the text for more details.		
18	31-Dec-91	30.54%	1.365%					
19	31-Dec-92	28.31%	0.893%					
20	31-Dec-93	16.90%	0.038%					
21	30-Dec-94	2.64%	2.633%					
22	29-Dec-95	54.27%	12.536%					
23	31-Dec-96	0.54%	3.357%					
24	31-Dec-97	5.23%	1.857%					
25	31-Dec-98	60.88%	17.659%					
26	31-Dec-99	4.94%	1.938%					
27								
28	Average	18.86%	<-- =SUM(B17:B26)/10					
29	Variance	0.0542	<-- =SUM(C17:C26)/10					
30	Standard deviation	23.28%	<-- =SQRT(B29)					

EXCEL AND STATISTICAL REVIEW

We delve deeper into statistics in Chapter 11. To remind you of the meanings of the terms:

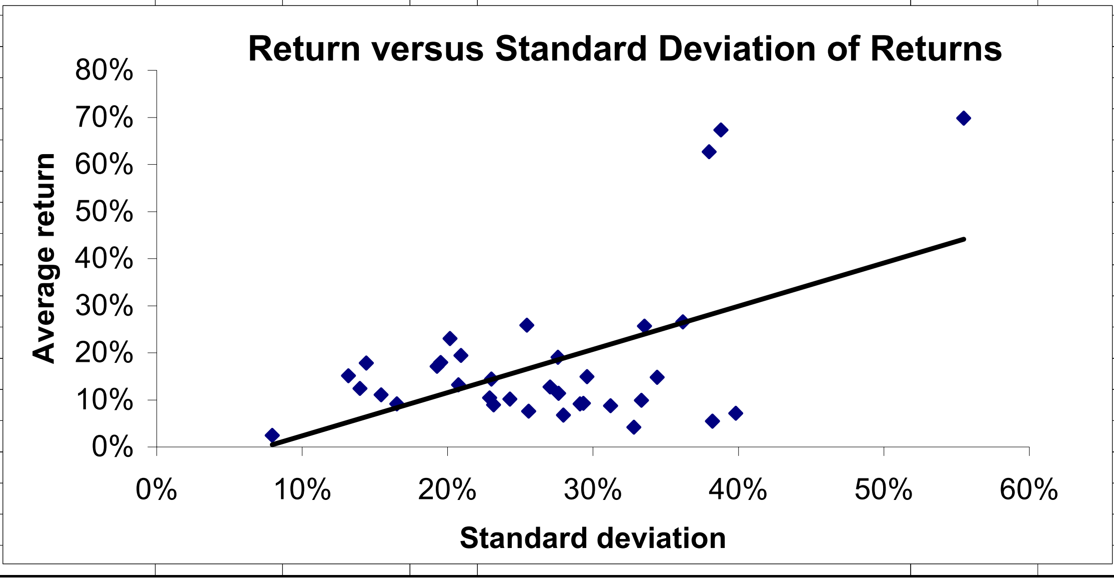
- The *average* (also called the *mean*) annual return for McDonald's is computed by summing the annual returns and dividing by 10, the number of returns. In cell G7 we've computed the average by using the Excel function **Average**; in cell B28 we compute the average by using **=Sum(B17:B26)/10**.
- The *variance* of the McDonald annual returns is computed in three steps: i) Subtract each return from the average. Then: ii) Square the result; these "squared deviations from the average" are shown in cells C17:C26. The third step in computing this variance is to: iii) Average the sum of the squared deviations. This is illustrated in cell B29. Cell G8 shows that the Excel function **Varp** gives the same result.
- Since the returns are in percent, the variance has units "percent squared." This is a bit difficult to understand. The *standard deviation* of the returns is the square root of the variance (this has units which are percent). Informally, you can think of the standard deviation as representing the average percentage variability of the individual returns. Cell B30 used the Excel function **Sqrt** to derive the standard deviation, and cell G9 uses the Excel function **Stdevp**.

How risky are other assets?

To give you some feel for how risky different assets are, here is a table of the annualized returns and standard deviations for various assets:

AVERAGE RETURN VERSUS STANDARD DEVIATION OF RETURNS
A Somewhat Arbitrary List of Assets, 1990-1999

	Average return	Standard deviation		Average return	Standard deviation
Abbott	17.12%	19.27%	Marriott	7.15%	39.81%
Airproducts	12.73%	27.05%	McGraw-Hill	17.96%	19.52%
Alcoa	19.03%	27.59%	Microsoft	62.72%	37.99%
AmericanAirlines	9.26%	29.34%	Nasdaq	23.00%	20.17%
ATT	6.76%	27.96%	Nicor	9.15%	16.50%
Boeing	7.57%	25.57%	Nordstrom	5.44%	38.21%
Cisco	67.31%	38.80%	Northrop	14.76%	34.40%
Coke	10.18%	24.29%	PPG	14.42%	23.02%
Dell	69.86%	55.48%	Procter & Gamble	19.41%	20.91%
Duke	11.07%	15.43%	Safeway	25.65%	33.53%
Exxon	12.42%	13.96%	Standard & Poor's 500 Index	15.09%	13.18%
Ford	9.12%	29.11%	Teva	26.57%	36.17%
GM	11.37%	27.63%	U.S. Steel	4.17%	32.80%
Hershey	13.17%	20.74%	UST	10.39%	22.89%
IBM	14.89%	29.57%	Vanguard Long-Term Treasury Fund	2.43%	7.94%
Kellogg	8.92%	23.16%	Walmart	25.84%	25.45%
Magellan Fund	17.82%	14.40%	WR Grace	8.73%	31.20%
Manpower	9.93%	33.32%			



A closer look at the four highlighted financial assets can give you some better intuitions on the relation between financial risk and return:

- Vanguard's Long-Term Treasury Fund is a mutual fund which invests in long-term U.S. Treasury bonds. As you saw in Section 10.2, the absence of default risk in these bonds does not mean that they are riskless: Their prices can vary considerably, and as a result the holder of the Vanguard Long-Term Treasury Fund may experience uncertainty in her returns. Nevertheless, our intuition tells us (correctly, as you'll see in the succeeding bullets) that this fund ought to be less risky than most stocks. During the decade of 1990-1999, the Vanguard Long-Term Treasury Fund gave an average annual return of 2.43% and this return had a standard deviation of 7.94%.

- The Standard & Poor's 500 Index is a broad-based index of the largest U.S. stocks and is often used as a measure of the performance of the U.S. stock markets. During the decade of 1990s, the S&P 500 Index had an average annual return of 15.09% and a standard deviation of 13.18%. As you might expect, there's a clear tradeoff between the average return of the S&P 500 and the Vanguard Long-Term Treasury Fund: The S&P 500 gives more return, but you pay for this return with greater variability (measured by the standard deviation):

$$\begin{aligned} &\textit{Treasury Fund average return} &&= 2.43\% < 15.09\% = S\&P\ 500\ \textit{average return} \\ &\textit{Treasury Fund standard deviation} &&= 7.94\% < 13.18\% = S\&P\ 500\ \textit{standard deviation} \end{aligned}$$

- The riskiest asset in our table is Dell stock. This stock performed spectacularly over the decade, giving an annual average return of 69.86%.[6] On the other hand Dell was also the riskiest stock in the table, with a standard deviation of return of 55.48%.

- After the fact (or, as economists like to say *ex-post*) some assets were clearly inferior to other assets. Manpower's stock turned out to have lower average return and higher risk (measured by standard deviation) than the S&P 500.

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[6] This number deserves some thought and admiration: If you'd invested \$100 in Dell stock in 1990, it would have grown to $\$100*(1+69.86\%)^{10} = \$19,955$ by the end of the decade!

Over this period there seems to be a positive relation between the standard deviation of the returns and the annualized return of the assets, although a closer look at the graph will show you that there is a large group of assets with different standard deviations and roughly the same average returns. The graph should be taken as somewhat indicative of a possible relation between risk (as measured here by the standard deviation of returns) and average returns. From this graph you might be tempted to conclude that when an asset's standard deviation of returns increase, the return expected from the asset increases. This is not far from the truth—but in Chapter 12 we define another measure of asset risk, called *beta*, which works better.[7]

10.4. Advanced topic: Using continuously-compounded returns to compute annualized return statistics

We discussed continuous compounding in Chapter 2. As explained there, the continuously-compounded return is calculated using the **Ln** function. For reasons that are beyond the purview of this book, the continuously-compounded return is the only consistent method of computing return statistics (by "consistent" we mean two things: there's a theory behind the numbers, and this theory gives the same results whether you're computing the annual statistics from daily, weekly, or monthly data). In the spreadsheet below, we've computed the continuously-compounded return statistics for McDonald's.

---

[7] Actually if you look really carefully at the trendline in the graph, you'll see it's mostly influenced by the three rightmost points (these are for Microsoft, Cisco, and Dell). Without these points, there would be no relation between the return and the standard deviation of the returns—we need a better measure of asset risk, which we develop in the following chapters.

	A	B	C	D	E	F	G	H
1	**McDONALD'S--DAILY STOCK PRICES, 29Dec89 - 31Dec99**							
2	**This spreadsheet uses the continously compounded return**							
3								
4	Date	Closing stock price						
5	Date	Close				**Statistics**		
6	29-Dec-89	8.50				**Daily returns**		
7	2-Jan-90	8.59	1.053%	<-- =LN(B7/B6)		Largest daily return	10.31%	<-- =MAX(C7:C2534)
8	3-Jan-90	8.50	-1.053%	<-- =LN(B8/B7)		Smallest daily return	-10.62%	<-- =MIN(C7:C2534)
9	4-Jan-90	8.34	-1.900%					
10	5-Jan-90	8.19	-1.815%			Average daily return	0.0610%	<-- =AVERAGE(C7:C2534)
11	8-Jan-90	8.34	1.815%			Variance of daily returns	0.0257%	<-- =VARP(C7:C2534)
12	9-Jan-90	8.28	-0.722%			Standard deviation of daily returns	1.6039%	<-- =SQRT(G11)
13	10-Jan-90	8.13	-1.828%					
14	11-Jan-90	8.07	-0.741%			**Annualized**		
15	12-Jan-90	7.91	-2.003%			Average annual return	15.43%	<-- =G10*253
16	15-Jan-90	7.82	-1.144%			Variance of annual returns	6.51%	<-- =G11*253
17	16-Jan-90	7.91	1.144%			Standard deviation of annual returns	25.51%	<-- =SQRT(G16)
18	17-Jan-90	7.91	0.000%					
19	18-Jan-90	7.94	0.379%					
20	19-Jan-90	7.85	-1.140%					
21	22-Jan-90	7.63	-2.843%					
22	23-Jan-90	7.76	1.689%					
23	24-Jan-90	7.63	-1.689%					
24	25-Jan-90	7.60	-0.394%					
25	26-Jan-90	7.70	1.307%					
26	29-Jan-90	7.70	0.000%					
27	30-Jan-90	7.70	0.000%					

The average daily continuously compounded return (cell G10) is 0.0610%. To *annualize* this return, we multiply by 253, the average number of business days per year.[8] The annualized average continuously compounded return is 15.43%. Similarly the annualized return variance is 6.51% (cell G16), and the annualized standard deviation of returns is 25.51% (cell G17).[9]

[8] Over the 10-year period 1990-1999, there were 2528 days on which McDonald's stock was transacted. This averages out to 253 days per year.

[9] Note that the continuously-compounded average return of 15.43% is less than the discretely-compounded return of 18.86% computed in the previous sub-section. One reason for this is that continuous compounding builds up faster than discretely-compounded interest. Another reason, as noted in Chapter 2, is that there are legitimate alternative ways to compute returns. To compare the returns of two assets, make sure that the basis of the computation is the same.

10.5. Risk and return depend on the your unit of account

As we've shown in the examples above, risk and return depend on the kind of security you're considering. Returns can also depend on *what currency you're calculating in*. Investors these days are putting their money in many stock markets around the world, and their returns are affected both by fluctuations in stock prices and in the rates of exchange.

In the example below we calculate the return—in Euros and in dollars—from holding the Amsterdam Stock Exchange index (symbol: AEX). In the table below we use the continuously-compounded returns. As shown in Section 10.4, using continuously-compounded returns makes it much easier to go from monthly data to annual data. For example:

- The average *monthly* Euro return on the AEX index is -0.14%. To compute the average *annual* return, we simply multiply this number by 12: 12*-0.14% = -1.67%.
- The *monthly* standard deviation of AEX Euro returns is 5.10%. To compute the average *annual* standard deviation, we multiply this number by $\sqrt{12}$: $\sqrt{12}*5.10\% = 17.68\%$.

	A	B	C	D	E	F	G	H
1	**AMSTERDAM STOCK EXCHANGE INDEX (AEX)** **in Euros and Dollars**							
2	**Date**	**Index price in Euros**	**Monthly return in Euros**		**Euro/$ exchange rate**	**Index price in $**	**Monthly return in $**	
3	January-99	532.09			1.161	617.76		
4	February-99	536.12	0.75%	<-- =LN(B4/B3)	1.121	600.99	-2.75%	<-- =LN(F4/F3)
5	March-99	536.93	0.15%	<-- =LN(B5/B4)	1.088	584.18	-2.84%	<-- =LN(F5/F4)
6	April-99	573.52	6.59%		1.07	613.67	4.92%	
7	May-99	554.06	-3.45%		1.063	588.97	-4.11%	
8	June-99	561.19	1.28%		1.038	582.52	-1.10%	
9	July-99	552.77	-1.51%		1.035	572.12	-1.80%	
10	August-99	572.42	3.49%		1.06	606.77	5.88%	
11	September-99	547.45	-4.46%		1.05	574.82	-5.41%	
12	October-99	571.82	4.36%		1.071	612.42	6.34%	
13	November-99	602.11	5.16%		1.034	622.58	1.65%	
14	December-99	671.41	10.89%		1.011	678.80	8.64%	
15	January-00	612.38	-9.20%		1.014	620.95	-8.91%	
16	February-00	664.28	8.14%		0.983	652.99	5.03%	
17	March-00	662.29	-0.30%		0.964	638.45	-2.25%	
18	April-00	661.38	-0.14%		0.947	626.33	-1.92%	
19	May-00	655.5	-0.89%		0.906	593.88	-5.32%	
20	June-00	672.14	2.51%		0.949	637.86	7.14%	
21	July-00	668.18	-0.59%		0.94	628.09	-1.54%	
22	August-00	689.52	3.14%		0.904	623.33	-0.76%	
23	September-00	661.52	-4.15%		0.872	576.85	-7.75%	
24	October-00	680.56	2.84%		0.855	581.88	0.87%	
25	November-00	649.92	-4.61%		0.856	556.33	-4.49%	
26	December-00	637.6	-1.91%		0.897	571.93	2.76%	
27	January-01	639.98	0.37%		0.938	600.30	4.84%	
28	February-01	597.33	-6.90%		0.922	550.74	-8.62%	
29	March-01	558.36	-6.75%		0.91	508.11	-8.06%	
30	April-01	593.09	6.03%		0.892	529.04	4.04%	
31	May-01	585.15	-1.35%		0.874	511.42	-3.39%	
32	June-01	573.5	-2.01%		0.853	489.20	-4.44%	
33	July-01	548.72	-4.42%		0.861	472.45	-3.48%	
34	August-01	523.63	-4.68%		0.9	471.27	-0.25%	
35	September-01	453.87	-14.30%		0.911	413.48	-13.08%	
36	October-01	460.33	1.41%		0.906	417.06	0.86%	
37	November-01	492.67	6.79%		0.888	437.49	4.78%	
38	December-01	506.78	2.82%		0.892	452.05	3.27%	
39								
40	**Return statistics**		**In Euros**				**In Dollars**	
41	Monthly average		-0.14%				-0.89%	<-- =AVERAGE(G4:G38)
42	Monthly standard deviation		5.10%				5.15%	<-- =STDEVP(G4:G38)
43								
44	Annual average		-1.67%				-10.71%	<-- =12*G41
45	Annual standard deviation		17.68%				17.83%	<-- =SQRT(12)*G42

A "Euro investor"—someone living in Euro-land and who thinks in Euros—would have lost 1.67 percent per year (cell C44) on her investment in the AEX over the two years surveyed. Over the same time period a "dollar investor"—say an American investing in the Amsterdam AEX—would have lost almost 10.71 percent per year (cell G44).

Why do the Euro returns and the dollar returns differ so radically? Take a look at what happened between 1 January 1999 and 1 February 1999. A Euro investor who bought the index on 1 January 1999 would have paid € 532.09; if she sold the index one month later, she would have gotten € 536.12. This is a Euro return of $Ln\left(\frac{€536.12}{€532.09}\right) = 0.75\%$.

On the other hand, a dollar investor who bought the Amsterdam index on 1 January 1999 would have paid $617.76 (at the point he purchased the index, $1 was worth € 1.161, so that the Euro price of the index becomes $€532.09 * 1.161 = \$617.76$). When this dollar investor sold the index after one month, it was at € 536.12 and the value of a dollar had fallen to $1 = € 1.121, so that the dollar price of the index was $€536.12 * 1.121 = \$600.99$. As a result the investor's dollar return was $Ln\left(\frac{\$600.99}{\$617.76}\right) = -2.75\%$.

The conclusion: Whether the Amsterdam Stock Exchange index was just a bad or a very bad investment depends very much on whether you were a Euro investor (in which case it was a bad investment) or a dollar investor (much worse).

The unit of account (dollar or Euro) matters.

Conclusion

In this chapter we have tried to give you some intuitions into the nature of financial risk by a series of examples. Risk—the variability of returns from an asset over time—depends on a number of factors. Broadly speaking the characteristics of an asset's risk are its *horizon*, its *safety*, and its *liquidity*. As we've shown, even safe assets like U.S. Treasury bills can be risky because their prices can change over the asset's horizon. With our example of McDonald's stock we've shown that some statistical sense can be made of the variability of the stock's return over time—by using Excel's **Frequency** function, we were able to show that McDonald's stock returns look very much like the familiar statistical "bell curve." Finally, with our example of the Amsterdam stock exchange index, we've shown that risks can differ depending on who's measuring them: The dollar investor in Amsterdam stocks did much worse than the Euro investor.

Risk is the most problematic concept in finance: The variability of financial asset returns is the main fact of financial life, but risk is not easy to define or measure. In the chapters which follow we will develop a model to *price risks*; by this we mean a model which will help us determine the risk-adjusted discount rate. The important innovation of this model (to come) is that risk depends on a *portfolio context*—it is not just the asset's returns by themselves that determine the asset's riskiness, but the asset's returns in the context of the portfolio of all the assets held by the investor. To some extent we have already hinted at this model in this chapter—by showing that there is a relation between the historic returns of assets and the standard deviation of these returns.

In the next chapters we will refine this intuition. We'll show you that it's not the standard deviation but rather the asset's *beta* (a measure of risk which we'll define) which helps us determine the risk-adjusted return. Beta is a widely used measure of risk, and as a finance student you should understand how to use it. But before you do this, you'll need a brush-up on your statistics skills. This is the task we set ourselves in the next chapter.

Exercises

1. It's 1 January 2001 and you're considering buying a $1,000 face-value U.S. Treasury bill which matures in 1 year. The interest rate is 7% annually.

   1.a. If you buy the T-Bill now, how much will you pay?

   1.b. If the interest rate remains 7% annually, how much will the bill be worth on 1 February 2001? 1 March? 1 April? ... 1 December?

On March 15, 2002, you purchased a 2-year Treasury bond with face value $10,000 and a 4% coupon (payable semi-annually). The price of the bond was $9,750; it promises a coupon of $200 on 15 September 2002, 15 March 2003, 15 September 2003, and 15 March 2004 (on this last date the bond will repay its face value).

a. Based on the following, compute the annualized IRR of the bond purchase:

	A	B	C
4	**Date**	**Cash flow**	
5	15-Mar-02	-9,750	
6	15-Sep-02	200	
7	15-Mar-03	200	
8	15-Sep-03	200	
9	15-Mar-04	10,200	
10			
11		2.67%	<-- =IRR(B5:B9)

b. On 16 September 2002 you sold the bond for $10,000. What was the ex-post annualized yield that you got? What was the ex-ante annualized yield of the buyer of the bond?

CHAPTER 11: STATISTICS FOR PORTFOLIOS*

this version: July 11, 2003

Chapter contents

Overview

In order to understand and work through Chapters 12 – 15, you will need to know some statistics. If you're like a lot of finance students, you've had a statistics course and forgotten much of what you learned there. This chapter is a refresher—it show you exactly what you need

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance with Excel* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it. All the material is copyright and the rights belong to the author.

in order to proceed with the succeeding chapters, using Excel to do all the calculations. (Excel is a great statistical toolbox—someday all business-school statistics courses will use it. In the meantime you're stuck with this chapter.)

Finance concepts

- How to calculate stock returns and adjust them for dividends and stock splits
- Return mean, variance, and standard deviation for an asset
- Return mean and variance for a portfolio of two assets
- Regressions

Excel functions and techniques

- **Average**
- **Var( )** and **Varp( )**
- **Stdev( )** and **Stdevp( )**
- **Covar( ) and Correl( )**
- **Trendlines** (Excel's term for regressions)
- **Slope( )**, **Intercept( )**, **Rsq( )**

11.1. Basic statistics for asset returns: mean, standard deviation, covariance, and correlation

In this section you will learn to calculate the return on a stock and its statistics: the mean (interchangeably referred to as the average or expected return), the variance, and the standard deviation. You will also learn the meaning of the covariance of the return between two stocks and of the correlation coefficient of the returns.

General Motors stock and its returns

The following spreadsheet shows data for General Motors stock during the decade of 1990. For each year, we've given the closing price of GM stock and the dividend the company paid during the year. We've also calculated the annual returns and their statistics; these calculations are explained after the table:

	A	B	C	D	E
1	**PRICE AND DIVIDEND DATA FOR GENERAL MOTORS (GM)**				
2	**Date**	**Closing Price**	**Dividend**	**Annual return**	
3	29-Dec-89	42.2500	-		
4	31-Dec-90	34.3750	3.00	-11.54%	<-- =(C4+B4)/B3-1
5	31-Dec-91	28.8750	1.60	-11.35%	<-- =(C5+B5)/B4-1
6	31-Dec-92	32.2500	1.40	16.54%	
7	31-Dec-93	54.8750	0.80	72.64%	
8	30-Dec-94	42.1250	0.80	-21.78%	
9	29-Dec-95	52.8750	1.10	28.13%	
10	31-Dec-96	55.7500	1.60	8.46%	
11	31-Dec-97	60.7500	5.59	19.00%	
12	31-Dec-98	71.5625	2.00	21.09%	
13	31-Dec-99	72.6875	14.15	21.34%	
14					
15	Average return			14.25%	<-- =AVERAGE(D4:D13)
16	Variance of return			0.0638	<-- =VARP(D4:D13)
17	Standard deviation of return			25.25%	<-- =STDEVP(D4:D13)

Suppose you had bought a share of GM at the end of December 1989 for $42.25 and sold it a year later, at the end of December 1990, for $34.375. During this year, GM paid a per-share dividend of $3.[1] Your return from holding GM throughout 1990 would have been:

$$r_{GM,1990} = \frac{P_{GM,1990} + Div_{GM,1990} - P_{GM,1989}}{P_{GM,1989}} = \frac{34.375 + 3.00 - 42.25}{42.25} = -11.54\%.$$

Several notes:

- We use $r_{GM,1990}$ to denote the return on GM stock in 1990 and we use $Div_{GM,1990}$ to denote GM's dividend in 1990.
- The numerator of $r_{GM,1990}$ is

  $$P_{GM,1990} + Div_{GM,1990} - P_{GM,1989} = 34.375 + 3.00 - 42.25 = -4.875$$

  This is the gain on holding GM during the year (in this case it's a negative gain: a loss of $4.875). The denominator is of $r_{GM,1990}$ is the initial investment from buying GM stock at the beginning of the year.
- In cell D4 of the spreadsheet we've written $r_{GM,1990}$, the return for 1990, in a slightly different form as **(C4+B4)/B3-1**. This is equivalent to:

  $$r_{GM,1990} = \frac{P_{GM,1990} + Div_{GM,1990}}{P_{GM,1989}} - 1$$

Cells D15, D16, and D17 give the return statistics for GM:

- **D15**: The average return over the decade is 14.25% per year. This number is also called the *mean return* and it's calculated with the Excel function **=Average(D4:D13)** . We

[1] Actually the company paid 4 quarterly dividends of $0.75, but we've added these together to get the annual dividend.

often use the past returns to predict future returns. When we make this use of the data, we also call the mean the *expected return*, meaning that we use the historic average of GM's stock returns as a prediction of what the stock will return in the future. We will sometimes use the notations $E\left(r_{GM}\right)$ or $\overline{r}_{GM}$ In this book the terms mean, average, and expected return will be used almost interchangeably. The formal definition is:

$$Mean\ GM\ \ return = E\left(r_{GM}\right) = \overline{r}_{GM} = \frac{r_{GM,1990} + r_{GM,1991} + \ldots + r_{GM,1999}}{10}$$

You might wonder at the number of expressions (mean, average, expected return) and the number of symbols $\left(E\left(r_{GM}\right), \overline{r}_{GM}\right)$ for the same idea. We've introduced them all both for convenience and because, in your further finance studies, you're likely to see them used synonymously.

- **D16**: The variance of the annual returns is 6.38%. Variance and standard deviation are statistical measures of the variability of the returns. The variance is calculated with the Excel function **=Varp(D4:D13)**. (See the "Excel note" box further on to see more information about this function and its cousin **=Var(D4:D13)**.) The variance is often denoted by the Greek symbol σ_{GM}^2 (pronounced "sigma squared of GM"); sometimes it's written as $Var\left(r_{GM}\right)$. The formal definition of the variance is:

$$Var\left(r_{GM}\right) = \sigma_{GM}^2 = \frac{\left(r_{GM,1990} - \overline{r}_{GM}\right)^2 + \left(r_{GM,1991} - \overline{r}_{GM}\right)^2 + \ldots + \left(r_{GM,1999} - \overline{r}_{GM}\right)^2}{10}$$

- **D17**: The standard deviation of the annual returns is the square root of the variance: $\sqrt{0.0638} = 25.25\%$. Excel has two functions, **Stdevp( )** and **Stdev( )**, to do this calculation directly. Since we usually use **Varp( )** for the variance, we will use **Stdevp( )**

for the standard deviation. It is common to use the Greek letter sigma for the standard deviation, writing σ_{GM} .

Statistical Note (skip until later, or perhaps forever, if you like)

Excel has two variance functions, **Varp** and **Var**. The former measures the "population variance," and the latter measures the "sample variance." Similarly Excel has two functions for the standard devation, **Stdevp** and **Stdev**. In this book we use only the functions **Varp** and **Stdevp**. This box is a reminder but not an explanation of the difference between the two concepts.

If you have return data $\{r_{stock,1}, r_{stock,2}, \ldots, r_{stock,N}\}$ for a *stock*, then the mean return is $\overline{r}_{stock} = \frac{1}{N}\sum_{t=1}^{N} r_{stock,t}$. The definitions of the two variance functions are:

$$VarP\left(\{r_{stock,1}, r_{stock,2}, \ldots, r_{stock,N}\}\right) = \frac{1}{N}\sum_{j=1}^{N}\left(r_{stock,j} - \overline{r}_i\right)^2$$

$$Var\left(\{r_{stock,1}, r_{stock,2}, \ldots, r_{stock,N}\}\right) = \frac{1}{N-1}\sum_{j=1}^{N}\left(r_{stock,j} - \overline{r}_i\right)^2$$

There's a long story about the difference between these two concepts which we'll leave for someone else (like your statistics instructor) to explain. Suffice it to say that in the examples covered in this book we'll use **VarP** and its standard deviation equivalent **StdevP**.

Finally, you might wonder why there are two expressions—the variance and the standard deviation—which measure the variability. The answer has to do with the units of these expressions. Each term in the variance is squared in order to make everything positive. But this means that the units of the variance are "percent squared," which is a bit difficult to understand. The standard deviation, the square root of the variance, reduces the squared percentages of the variance back to "percent." This way the mean and the standard deviation have the same units.

Microsoft stock and its returns

The GM example above illustrated the adjustment of the stock return data to include dividends. We now use Microsoft stock to show you how stock returns are affected by stock splits. A *stock split* occurs when stock holders get multiple shares of stock for each share they own. The most typical stock split is a “2-for-1” split, in which shareholders get 1 additional share for each share they already own (see Figure 1 for a Microsoft stock split announcement in 1996).

MICROSOFT STOCK SPLIT ANNOUNCEMENT

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Bill Gates Web Site

Essays on Technology

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Microsoft Announces Stock Split

Microsoft Corporation today announced that its Board of Directors approved a 2-for-1 stock split.

REDMOND, WA, November 12, 1996 -- Microsoft Corporation today announced that its Board of Directors approved a 2-for-1 stock split. Shareholders will receive one additional share for every share held on the record date of November 22, 1996.

"We're pleased that customers continue to find our products compelling and innovative, and have rewarded us with good earnings and a good stock price," said Mike Brown, Chief Financial Officer. "This is the sixth time the stock has split since the company went public on March 13, 1986. This split should make our stock more accessible to a broader base of investors."

As of October 31, 1996, Microsoft had approximately 600 million shares outstanding. Upon completion of the split, the number will increase to approximately 1.2 billion shares outstanding. The additional shares will be mailed or delivered on or about December 6, 1996 by the Company's transfer agent, Chase/Mellon Shareholder Services L.L.C.

Figure 1: On 12 November 1996 Microsoft announced a 2-for-1 stock split. Shareholders owning one share on 22 November 1996 would be mailed an additional share of stock. This increased the number of shares of the company from 600 million to 1.2 billion. The Microsoft statement hints at the company's reasoning for the split: With its stock trading at almost \$150 per share before the split, Microsoft used the split to reduce the price of the share in order to put it into a range which would make it "more accessible to a broader base of investors."

Microsoft (MSFT) paid no dividends in the 1990-1999 decade, but the stock split several times. Here are some data:

	A	B	C
1	**PRICE AND STOCK SPLIT DATA FOR MICROSOFT (MSFT)**		
2	**Date**	**Closing Price**	**Stock split during year?**
3	29-Dec-89	87.0000	
4	31-Dec-90	75.2500	2.0 for 1
5	31-Dec-91	111.2500	1.5 for 1
6	31-Dec-92	85.3750	1.5 for 1
7	31-Dec-93	80.6250	no
8	30-Dec-94	61.1250	2.0 for 1
9	29-Dec-95	87.7500	no
10	31-Dec-96	82.6250	2.0 for 1
11	31-Dec-97	129.2500	no
12	31-Dec-98	138.6875	2.0 for 1
13	31-Dec-99	116.7500	2.0 for 1

Here's what these stock splits mean for the Microsoft shareholders: Suppose you had bought one share of MSFT on 29 December 1989 for $87.00 and held it throughout 1990. During 1990, Microsoft *split* its stock, giving shareholders two shares for every one share they owned. At the end of 1990, each of these (split) shares was worth $75.25, so that your $87 investment had grown to $150.25! The return for the year is therefore:

$$r_{MSFT,1990} = \frac{\left(P_{MSFT,31Dec90}\right)*2}{P_{MSFT,29Dec89}} - 1 = \frac{150.50}{87} - 1 = 72.99\%$$

The "2" in the formula above is the stock split *adjustment factor* which shows that Microsoft one share owned at the beginning of 1990 became two shares by the end of the year. In the spreadsheet below we calculate the *cumulative adjustment factor*. This shows you how your end-1989 $87.00 investment in MSFT would have grown throughout the decade if you correctly account for the stock splits.

	A	B	C	D	E	F	G
16	**Date**	**Closing Price**	**Stock split during year?**	**Cumulative adjustment factor**	**Adjusted price**	**Annual return**	
17	29-Dec-89	87.0000		1	87.00		
18	31-Dec-90	75.2500	2.0 for 1	2	150.50	72.99%	<-- =E18/E17-1
19	31-Dec-91	111.2500	1.5 for 1	3	333.75	121.76%	<-- =E19/E18-1
20	31-Dec-92	85.3750	1.5 for 1	4.5	384.19	15.11%	
21	31-Dec-93	80.6250	no	4.5	362.81	-5.56%	
22	30-Dec-94	61.1250	2.0 for 1	9	550.13	51.63%	
23	29-Dec-95	87.7500	no	9	789.75	43.56%	
24	31-Dec-96	82.6250	2.0 for 1	18	1,487.25	88.32%	
25	31-Dec-97	129.2500	no	18	2,326.50	56.43%	
26	31-Dec-98	138.6875	2.0 for 1	36	4,992.75	114.60%	
27	31-Dec-99	116.7500	2.0 for 1	72	8,406.00	68.36%	
28							
29	Average return					62.72%	<-- =AVERAGE(F18:F27)
30	Variance of return					14.43%	<-- =VARP(F18:F27)
31	Standard deviation of return					37.99%	<-- =SQRT(F30)
32							
33							
34							

The cumulative adjustment factor is the product of all the splits: 72 = 2*1.5*1.5*2*2*2*2

Taking into account the stock splits, your $87.00 investment in MSFT would have grown to $8,406 by the end of the decade! During the 1990s, MSFT gave an average annual return of 62.72%; this return had a standard deviation of 37.99%.[2]

[2] Adding or subtracting the standard deviation from the average gives a plausible range for Microsoft stock returns. Roughly speaking, the 37.99% standard deviation indicates that with a 68% probability, the Microsoft stock returns are in the range between 24.73% and 100.71%. These two numbers are computed by 24.73% = 62.72% - 37.99% and 100.71% = 62.72% + 37.99%.

Stock Splits and the Cumulative Adjustment Factor

On 31 January 2002, you bought one share of XYZ stock for \$50. One year minus one day later, on 30 January 2003, your share of XYZ stock is trading at \$80. At the end of the day the stock *splits*: For every share you own, you now have *two* shares. In a logical world, this would mean that the price of the share should fall by 50%, so that on 31 January 2003, it XYZ trades at \$40.[3]

Now suppose you're trying to calculate your return on the stock. Is it $\frac{\$40}{\$50}-1=-20\%$ or is the return $\frac{2*\$40}{\$50}-1=60\%$? The latter, of course! *You adjusted the stock price by the adjustment factor*.

Suppose that in July 2003 XYZ splits 1.5 for 1 and that on 31 January 2004 the price per share is \$25. Then your return since you bought the stock is $\frac{2*1.5*\$25}{\$50}-1=\frac{3*\$25}{\$50}-1=50\%$.

The cumulative adjustment factor is the product of all the splits since you bought the stock.

11.2. Downloaded data from commercial sources is adjusted for dividends and splits

The author's two favorite data sources for information about stock prices, dividends and stock splits are Yahoo, which is free, and the Center for Research in Security Prices (CRSP) data

[3] The world is not all that logical, but this in fact usually happens—when a stock splits 2 for 1, its post-split price is usually close to half its pre-split price. If the stock splits on a 1.5 for 1 basis, the post-split price is close to two-thirds (2/3 = 1/1.5) its pre-split price.

base which originates from the University of Chicago (many universities subscribe to CRSP—ask your data manager).[4] When you download data from these sources, they automatically adjust the price data to account for dividends and splits. So you don't have to do all the adjustment calculations illustrated in the previous section.[5]

It is important to note, however, that the adjustments made by Yahoo and CRSP may look different from the ones we made above. For example, here's the adjusted Microsoft data from Yahoo:

	A	B	C	D
1	**DOWNLOADED ADJUSTED DATA FROM YAHOO FOR MICROSOFT**			
2	**Date**	**MSFT adjusted price**		
3	29-Dec-89	1.2083		
4	31-Dec-90	2.0903	73.00%	<-- =B4/B3-1
5	31-Dec-91	4.6354	121.76%	<-- =B5/B4-1
6	31-Dec-92	5.3359	15.11%	<-- =B6/B5-1
7	31-Dec-93	5.0391	-5.56%	<-- =B7/B6-1
8	30-Dec-94	7.6406	51.63%	<-- =B8/B7-1
9	29-Dec-95	10.9688	43.56%	<-- =B9/B8-1
10	31-Dec-96	20.6562	88.32%	<-- =B10/B9-1
11	31-Dec-97	32.3125	56.43%	<-- =B11/B10-1
12	31-Dec-98	69.3438	114.60%	<-- =B12/B11-1
13	31-Dec-99	116.7500	68.36%	<-- =B13/B12-1
14				
15	Average return		62.72%	<-- =AVERAGE(C4:C13)
16	Variance of return		14.43%	<-- =VARP(C4:C13)
17	Standard deviation of return		37.99%	<-- =STDEVP(C4:C13)

The annual return statistics are the same, but the method of price adjustment is different: Yahoo has adjusted the stock prices so that the stock price on the last date ($116.75) is the same

[4] For penniless students, Yahoo is especially useful. An appendix to this chapter shows you how to download financial data from Yahoo.

[5] If it's all in the downloaded data, why the heck did we do all the work in this section? The answer, of course, is that it helps to understand what the numbers are telling you.

as the market price on that date. All previous prices have been adjusted accordingly. For example: the 29 December 1989 Yahoo price for MSFT of $1.2083 is 1/72 times the actual market price on this date—this adjustment is made since the stock split 72 times in the period 1990 – 1999.

The bottom line on downloaded data

Don't worry too much about how the adjustment is done. Calculate your returns from the adjusted stock price data given by your data provider. They usually do the corrections right.

11.3 Covariance and correlation—two additional statistics

So far we've looked at statistics—mean, variance, standard deviation—that relate to the returns of an individual stock. In this section we examine two statistics—covariance and correlation—that relate the returns of two stocks to each other. We continue to use our data for GM and MSFT. In the following spreadsheet, we've put the returns for both stocks on one spreadsheet and calculated the covariance and correlation (cells B17:B19):

	A	B	C	D
1	**GM AND MSFT, ANNUAL RETURN DATA**			
2	**Date**	**GM return**	**MSFT return**	
3	31-Dec-90	-11.54%	72.99%	
4	31-Dec-91	-11.35%	121.76%	
5	31-Dec-92	16.54%	15.11%	
6	31-Dec-93	72.64%	-5.56%	
7	30-Dec-94	-21.78%	51.63%	
8	29-Dec-95	28.13%	43.56%	
9	31-Dec-96	8.46%	88.32%	
10	31-Dec-97	19.00%	56.43%	
11	31-Dec-98	21.09%	114.60%	
12	31-Dec-99	21.34%	68.36%	
13				
14	Average return	14.25%	62.72%	
15	Variance of return	6.38%	14.43%	
16	Standard deviation of return	25.25%	37.99%	
17	Covariance of returns	-0.0552		<-- =COVAR(B3:B12,C3:C12)
18	Correlation of returns	-0.5755		<-- =CORREL(B3:B12,C3:C12)
19		-0.5755		<-- =B17/(B16*C16)

The *covariance* between two series is a measure of how much the series (in our case, the returns on GM and MSFT) move up or down together. The formal definition is:

$$Cov\left(r_{GM}, r_{MSFT}\right) = \sigma_{GM,MSFT} = \frac{1}{10}\left\{\begin{array}{l}\left(r_{GM,1}-\overline{r}_{GM}\right)\left(r_{MSFT,1}-\overline{r}_{MSFT}\right)+\left(r_{GM,2}-\overline{r}_{GM}\right)\left(r_{MSFT,2}-\overline{r}_{MSFT}\right)+ \\ \ldots+\left(r_{GM,10}-\overline{r}_{GM}\right)\left(r_{MSFT,10}-\overline{r}_{MSFT}\right)\end{array}\right\}$$

The idea, as you can see from the formula, is to measure the deviations of each data point from its average and to multiply these deviations. As you can see from cell B18, Excel has a function **Covar( )** which—when applied directly to the returns in columns B and C, calculates the covariance.

Calculating the covariance the long way using the definition may give you some more insight into what the covariance measures and what Excel's **Covar** function does.

	A	B	C	D	E	F	G	H
1	CALCULATING THE COVARIANCE THE LONG TEDIOUS WAY							
2	Date	GM return	MSFT return		GM return minus average	MSFT return minus average	=C4-$C$15 Product	
3	31-Dec-90	-11.54%	72.99%	=B3-$B$14-->	-25.79%	10.27%	-0.0265	<-- =E3*F3
4	31-Dec-91	-11.35%	121.76%		-25.60%	59.04%	-0.1511	
5	31-Dec-92	16.54%	15.11%		2.28%	-47.61%	-0.0109	
6	31-Dec-93	72.64%	-5.56%		58.38%	-68.28%	-0.3987	
7	30-Dec-94	-21.78%	51.63%		-36.03%	-11.09%	0.0400	
8	29-Dec-95	28.13%	43.56%		13.88%	-19.16%	-0.0266	
9	31-Dec-96	8.46%	88.32%		-5.79%	25.60%	-0.0148	
10	31-Dec-97	19.00%	56.43%		4.74%	-6.29%	-0.0030	
11	31-Dec-98	21.09%	114.60%		6.84%	51.88%	0.0355	
12	31-Dec-99	21.34%	68.36%		7.09%	5.64%	0.0040	
13								
14	Average return	14.25%	62.72%	<-- =AVERAGE(C3:C12)		Covariance	-0.0552	<-- =AVERAGE(G3:G12)
15						Covariance	-0.0552	<-- =COVAR(B3:B12,C3:C12)
16						Correlation	-0.5755	<-- =CORREL(B3:B12,C3:C12)
17						Correlation	-0.5755	<-- =G14/(STDEVP(B3:B12)*STDEVP(C3:C12))

In cell E3, we've subtracted GM's 1990 return of -11.54% from its decade average return of 14.25% (cell B14); the resulting number indicates that in 1990 GM stock underperformed its average by -25.79%. During the same year, MSFT overperformed its average by 10.27%. The covariance takes the product of these two numbers ($-25.79\%*10.27\% = -0.0265$) and similar numbers for each of the other years and averages them (cell G14). As you can see, Excel's **Covar** function gives the same result (cell G15) and saves a lot of work. The covariance of -0.0552 for GM and MSFT tells us that, on average, when GM exceeded its mean, MSFT was below its mean, and vice versa.

Another common measure of how much two data series move up or down together is the *correlation coefficient*. The correlation coefficient is always between -1 and +1, which—as you'll see in the next subsection—makes it possible for us to be more precise about how the two sets of returns move together. Roughly speaking, two sets of returns which have a correlation coefficient of -1 vary *perfectly inversely*, by which we mean that when one return goes up (or down), we can perfectly predict how the other return goes down (or up). A correlation coefficient of +1 means that the returns vary in *perfect tandem*, by which we mean that when one return goes up (or down), we can perfectly predict how the other return goes up (or down). A correlation coefficient between -1 and +1 means that the two sets of returns vary together less than perfectly.

The correlation coefficient is defined as:

$$Correlation\left(r_{GM}, r_{MSFT}\right) = \rho_{GM,MSFT} = \frac{Cov\left(r_{GM}, r_{MSFT}\right)}{\sigma_{GM}\sigma_{MSFT}}.$$

Notice that the Greek letter ρ (pronounced “rho”) is often used as a symbol for the correlation coefficient. In the spreadsheet above, we calculate the correlation coefficient in two ways: In cell G19 we use the Excel function **Correl( )** to compute the correlation. Cell G17 applies the formula $\frac{Cov\left(r_{GM}, r_{MSFT}\right)}{\sigma_{GM}\sigma_{MSFT}}$ (and, of course, gets the same result).

Some facts about covariance and correlation

Here are some facts about covariance and correlation. We state them without much attempt at elaborate explanation or proof.

Fact 1. Covariance is affected by units, correlation isn’t. Here’s an example: In the spreadsheet below, we’ve presented the annual returns as whole numbers instead of percentages (writing GM’s 1990 return as -11.54 instead of -11.54%). The covariance (cell B18) is now -552.10, which is 10,000 times our previous calculation. But the correlation coefficient (B19) remains the same as before, -0.5755.

	A	B	C	D
1	**GM AND MSFT, ANNUAL RETURN DATA percentages presented as whole numbers**			
2	**Date**	**Annual return**	**Annual return**	
3	29-Dec-89			
4	31-Dec-90	-11.54	72.99	
5	31-Dec-91	-11.35	121.76	
6	31-Dec-92	16.54	15.11	
7	31-Dec-93	72.64	-5.56	
8	30-Dec-94	-21.78	51.63	
9	29-Dec-95	28.13	43.56	
10	31-Dec-96	8.46	88.32	
11	31-Dec-97	19.00	56.43	
12	31-Dec-98	21.09	114.60	
13	31-Dec-99	21.34	68.36	
14				
15	Average return	14.25	62.72	
16	Variance of return	637.80	1442.92	
17	Standard deviation of return	25.25	37.99	
18	Covariance of returns	-552.10		<-- =COVAR(B4:B13,C4:C13)
19	Correlation of returns	-0.5755		<-- =CORREL(B4:B13,C4:C13)
20		-0.5755		<-- =B18/(B17*C17)
21				
22	Correlation is symmetric	-0.5755		<-- =CORREL(C4:C13,B4:B13)

Statistical note: Why does covariance depend on the units of measurement whereas correlation doesn't?

Why is the covariance measured in whole numbers 10,000 times bigger than the covariance measured in percentages? Since we've represented percentages as whole numbers, we've essentially multiplied each percentage return by 100. This is how -11.54% becomes -11.54. Since the covariance multiplies the percentages for GM and MSFT together, this means that we've multiplied our previous calculations by 100*100 = 10,000.

The correlation coefficient divides the covariance by the product of the standard deviations, $Correlation\left(r_{GM}, r_{MSFT}\right) = \frac{Cov\left(r_{GM}, r_{MSFT}\right)}{\sigma_{GM}\sigma_{MSFT}}$. In our new calculation, the covariance is 10,000 times bigger, but each standard deviation is 100 times bigger, so that the denominator is also 10,000 times bigger. The result is that the correlation is the same, no matter if the data is measured in percentages or whole numbers.

Fact 2. The correlation between GM and MSFT is the same as the correlation between MSFT and GM. The same holds for the covariance: $Cov\left(r_{GM}, r_{MSFT}\right) = Cov\left(r_{MSFT}, r_{GM}\right)$ The technical jargon for this is that "correlation and covariance are symmetric." To see this in Excel, note that cells B19 (**=Correl(B4:B13,C4:C13)**) and B22 (**=Correl(C4:C13,B4:B13)**) are equal in the above spreadsheet.

Fact 3. The correlation will always be between +1 and –1. The higher the correlation coefficient is in absolute value, the more the two series move together. If the correlation is either -1 or +1, then the two series are *perfectly correlated*, which means that knowing one series allows you to predict completely the value of the second series. If the correlation coefficient is between -1 and +1, then the two series move in tandem less than perfectly.

Fact 4. If the correlation coefficient is either +1 or –1, this means that the two returns have a linear relation between them. Since this is not easy to understand, we illustrate with a numerical example: Adams Farm and Morgan Sausage are two shares listed on the Farmers Stock Exchange. For reasons that are difficult to determine, each Morgan Sausage's stock return is equal to 60% of that of Adams Farm plus 3%. We can thus write: $r_{Morgan\ Sausage,t} = 3\% + 0.6 * r_{Adams\ Farm,t}$. This means that the return on Morgan Sausage stock is *completely predictable* given the return on Adams Farm stock. Thus the correlation is either -1 or +1. Since, when Adams Farm's return moves up, so does the return of Morgan Sausage, the correlation is +1.[6]

The Excel spreadsheet which follows confirms that the correlation is +1.

---

[6] The Farmers Stock Exchange has two other stocks whose returns are related by the equation $r_{Chicken\ Feed,t} = 50\% - 0.8 * r_{Poulty\ Delight,t}$. In this case, the negative coefficient (-0.8) tells us that the correlation between the two sets of returns is -1. (See end-of-chapter exercise.)

	A	B	C	D
1	**CORRELATION +1** **Adams Farm and Morgan Sausage Stocks** $r_{\text{Morgan Sausage},t} = 3\% + 0.6*r_{\text{Adams Farm},t}$			
2	**Year**	**Adams Farm stock return**	**Morgan Sausage stock return**	
3	1990	30.73%	21.44%	<-- =3%+0.6*B3
4	1991	55.21%	36.13%	
5	1992	15.82%	12.49%	
6	1993	33.54%	23.12%	
7	1994	14.93%	11.96%	
8	1995	35.84%	24.50%	
9	1996	48.39%	32.03%	
10	1997	37.71%	25.63%	
11	1998	67.85%	43.71%	
12	1999	44.85%	29.91%	
13				
14	Correlation		1.00	<-- =CORREL(B3:B12,C3:C12)
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
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30				
31				
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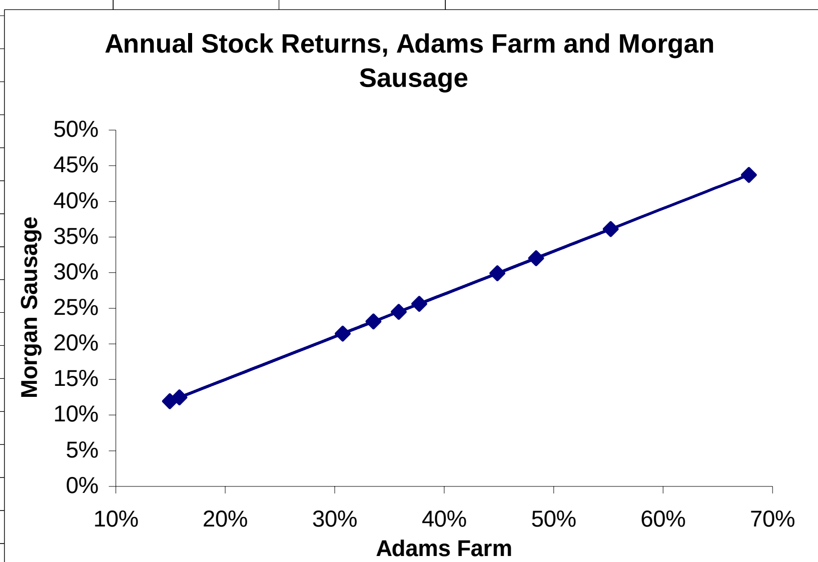
33				



Fact 4 can be written mathematically as follows: Suppose Stock 1 and Stock 2 are *perfectly correlated* (meaning that the correlation is either +1 or -1). Then:

$$r_{Stock1,t} = a + b * r_{Stock2,t} \left. \right\} \begin{array}{l} \leftarrow \text{b>0 if the correlation =+1} \\ \leftarrow \text{b<0 if the correlation =-1} \end{array}$$

11.4. Portfolio mean and variance for a two-asset portfolio

A *portfolio* is a set of stocks or other financial assets. Most people don't just own one stock, they own portfolios of stocks, and the risks they bear relate to the riskiness of their portfolio. In the next chapter we'll start our economic analysis of portfolios. In this section we'll show you how to compute the mean and variance of a portfolio composed of two stocks. Suppose that between 1990-99 you held a portfolio invested 50% in GM and 50% in MSFT. Column E of the spreadsheet below shows what the annual returns would have been on this portfolio. In cells E17:E21 we calculate the portfolio return statistics in the same way we calculated the return statistics for the individual assets GM and MSFT.

	A	B	C	D	E	F
1	**CALCULATING PORTFOLIO RETURNS AND THEIR STATISTICS**					
2	Proportion of GM	0.5				
3	Proportion of MSFT	0.5	<-- =1-B2			
4						
5	**Date**	**General Motors GM**	**Microsoft MSFT**		**Portfolio return**	
6	Dec-90	-11.54%	72.99%		30.73%	<-- =$B$2*B6+$B$3*C6
7	Dec-91	-11.35%	121.76%		55.21%	
8	Dec-92	16.54%	15.11%		15.82%	
9	Dec-93	72.64%	-5.56%		33.54%	
10	Dec-94	-21.78%	51.63%		14.93%	
11	Dec-95	28.13%	43.56%		35.84%	
12	Dec-96	8.46%	88.32%		48.39%	
13	Dec-97	19.00%	56.43%		37.71%	
14	Dec-98	21.09%	114.60%		67.85%	
15	Dec-99	21.34%	68.36%		44.85%	
16						
17	Mean	14.25%	62.72%		38.49%	<-- =AVERAGE(E6:E15)
18	Variance	6.38%	14.43%		2.44%	<-- =VARP(E6:E15)
19	St. dev.	25.25%	37.99%		15.62%	<-- =STDEVP(E6:E15)
20	Covariance		-0.0552			
21	Correlation		-0.5755			
22						
23	**Direct calculation of portfolio mean and variance**					
24	Portfolio mean	38.49%	<-- =B2*B17+B3*C17			
25	Portfolio variance	2.44%	<-- =B2^2*B18+B3^2*C18+2*B2*B3*C20			
26	Portfolio st. dev.	15.62%	<-- =SQRT(B25)			

Cells B24:B26 show that these portfolio statistics can be calculated directly from the statistics for the individual assets. To calculate the portfolio mean using these short-cuts, we first need some notation: Let x_{GM} stand for the proportion of GM stock in the portfolio and let x_{MSFT} denoted for the proportion of MSFT stock in the portfolio. In our example $x_{GM} = 0.5$ and $x_{MSFT} = 0.5$ and the portfolio mean return is given by:

$$\begin{aligned} \textit{Portfolio mean return} &= E\left(r_p\right) = x_{GM} E\left(r_{GM}\right) + x_{MSFT} E\left(r_{MSFT}\right) \\ &= x_{GM} E\left(r_{GM}\right) + \left(1 - x_{GM}\right) E\left(r_{MSFT}\right) \end{aligned}$$

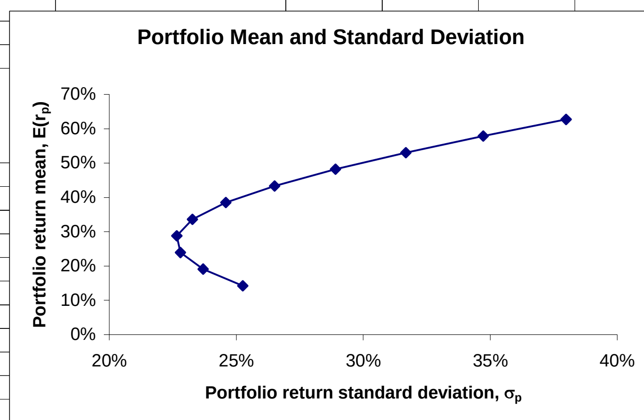
Notice the second line of the formula: If we only have two assets in the portfolio, then the proportion of the second asset is "one minus" the proportion of the first asset: $x_{MSFT} = 1 - x_{GM}$.

The formula for the portfolio variance is given by:

$$Portfolio\ variance = Var(r_p) = x_{GM}^2 Var(r_{GM}) + x_{MSFT}^2 Var(r_{MSFT}) + 2x_{GM}x_{MSFT}Cov(r_{GM}, r_{MSFT}) \ .$$

In the spreadsheet below we've built a table of the portfolio statistics using the formulas. In the table we vary the proportion of GM stock in the portfolio from 0% to 100% (which means, of course, that the proportion of MSFT stock goes from 100% to 0%).

	A	B	C	D	E
1	**CALCULATING PORTFOLIO RETURNS AND THEIR STATISTICS FROM THE FORMULAS**				
2		**General Motors GM**	**Microsoft MSFT**		
3	Mean	14.25%	62.72%		
4	Variance	6.38%	14.43%		
5	St. dev.	25.25%	37.99%		
6	Covariance		-5.52%		
7					
8	**Proportion of GM in portfolio**	**Portfolio Variance**	**Portfolio standard deviation**	**Portfolio mean**	
9	0%	14.43%	37.99%	62.72%	
10	10%	12.06%	34.72%	57.87%	
11	20%	10.03%	31.67%	53.03%	
12	30%	8.36%	28.91%	48.18%	
13	40%	7.03%	26.51%	43.33%	
14	50%	6.05%	24.59%	38.49%	
15	60%	5.42%	23.28%	33.64%	
16	70%	5.14%	22.66%	28.79%	
17	80%	5.20%	22.81%	23.95%	
18	90%	5.62%	23.70%	19.10%	
19	100%	6.38%	25.25%	14.25%	
20					
21					
22			=SQRT(B19)	=A19*$B$3+(1-A19)*$C$3	
23	=A19^2*$B$4+(1-A19)*$C$4+2*A19*(1-A19)*$C$6				
24					
25					

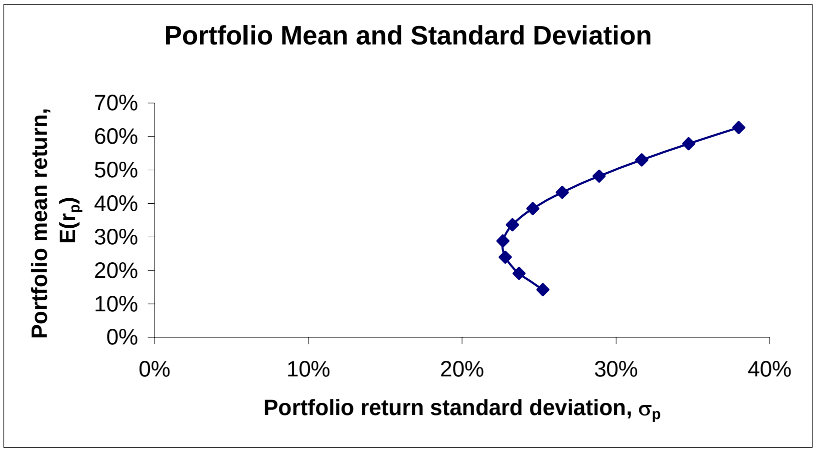


The graph is one which you will see again (lots!) in Chapters 12 and 13. It plots the portfolio standard deviation σ_p on the x-axis and the portfolio mean return $E(r_p)$ on the y-axis. The parabolic shape of the graph is the subject of much discussion in finance, but this is a purely technical chapter—the finance part of the discussion will have to wait until the following chapters.

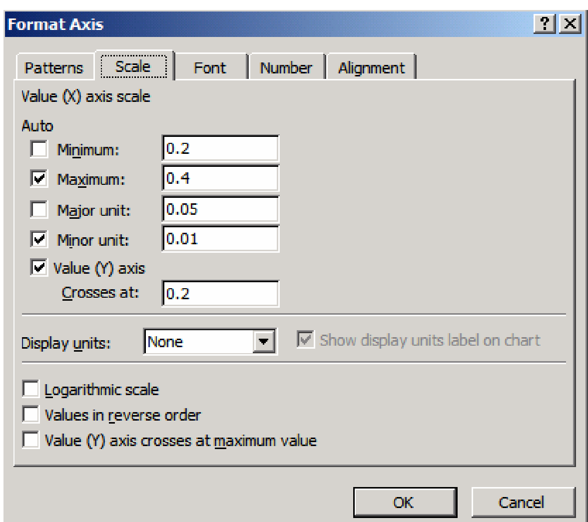
Two Excel Notes About the Graph

Note 1: The graph above is an Excel XY (Scatter) plot of the data in the range C9:D19. Notice that we've put the data in a somewhat "unnatural" order: We first compute the variance (cells B9:B19), then the standard deviation (C9:C19), and only then the expected return (D9:D19). All this is done to make it easier to use Excel's XY charts, which by default use the left-most data column as the data for the x-axis and data in columns to the right for y-axis data. (There are other work-arounds, but they're too cumbersome to explain right here).

When we originally made this graph, it looked like this:



Note 2: We "shortened" the x-axis by: i) Clicking on the axis, ii) Right-clicking the mouse and bringing up the menu for **Format axis**, and iii) Changing the settings to the following:



11.5. Using regressions

Linear regression (for short: regression) is a technique for fitting a line to a set of data. Regressions are used in finance to examine the relation between data series. In the chapters that follow we often need to use regressions; we introduce the basic concepts here. We do not discuss the statistical theory behind regressions, but instead we show you how to run a regression and how to use it.

We've divided the discussion into three sub-sections: First we discuss the mechanics of doing a regression in Excel, then we discuss the meaning of the regression, and finally we discuss alternative ways of doing the regression.

The mechanics of doing a regression in Excel

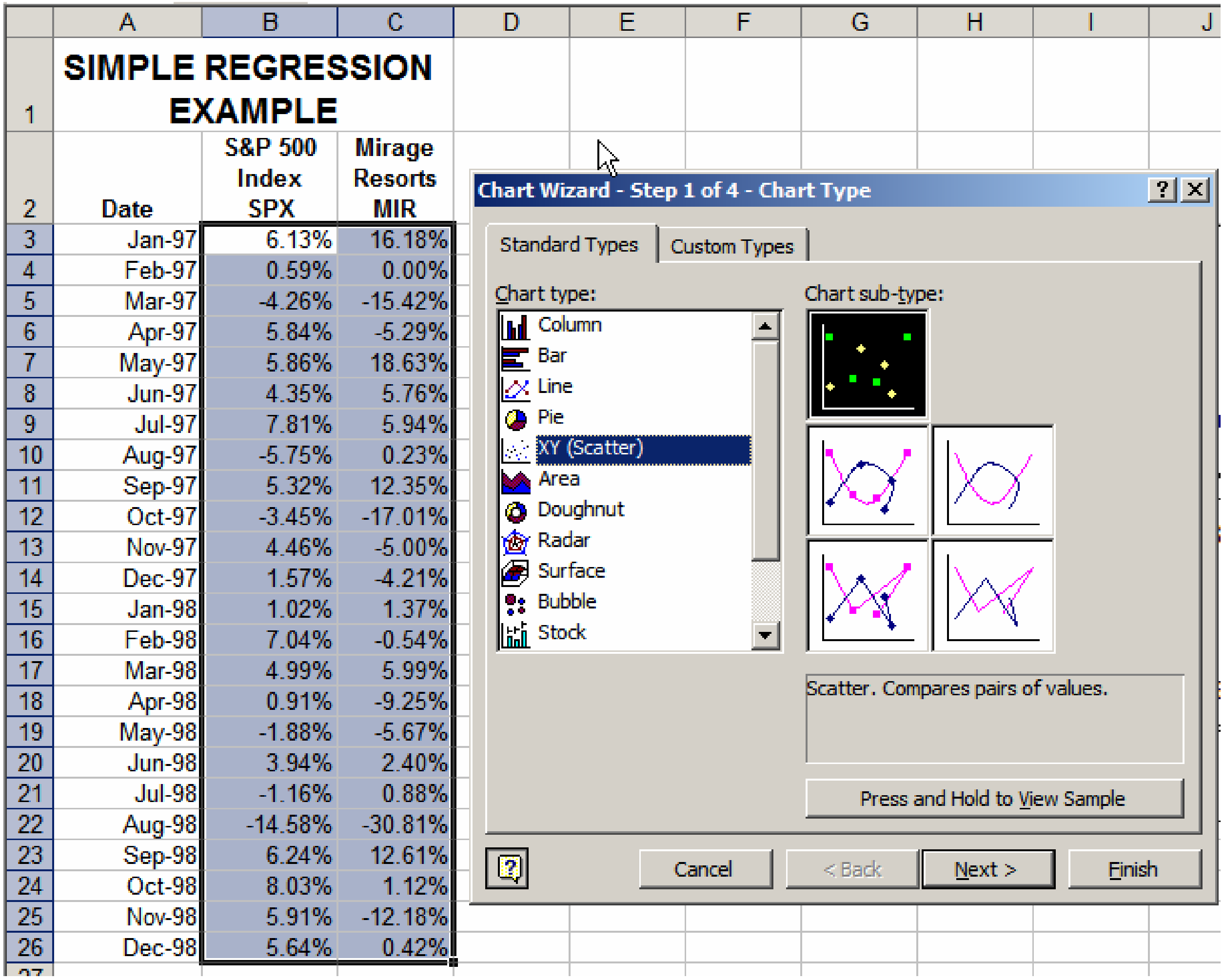
In this subsection we discuss a simple regression example and make little attempt to explain the economic meaning of the regression. Instead we focus on the mechanics of doing the regression in Excel and leave the economic interpretation for the next subsection.

The table below gives the monthly returns for the S&P 500 Index (stock symbol SPX) and for Mirage Resorts (stock symbol MIR) for 1997 and 1998. The S&P 500 Index includes the 500 largest stocks traded on U.S. stock exchanges, and its performance is roughly indicative of the performance of the U.S. stock market as a whole. We will use the regression analysis to see if we can understand the relation between the S&P's returns and MIR's returns—that is, if we can understand the effect of the U.S. stock market on the returns of MIR stock.

Here's the data we will examine:

	A	B	C
1	**SIMPLE REGRESSION EXAMPLE**		
2	**Date**	**S&P 500 Index SPX**	**Mirage Resorts MIR**
3	Jan-97	6.13%	16.18%
4	Feb-97	0.59%	0.00%
5	Mar-97	-4.26%	-15.42%
6	Apr-97	5.84%	-5.29%
7	May-97	5.86%	18.63%
8	Jun-97	4.35%	5.76%
9	Jul-97	7.81%	5.94%
10	Aug-97	-5.75%	0.23%
11	Sep-97	5.32%	12.35%
12	Oct-97	-3.45%	-17.01%
13	Nov-97	4.46%	-5.00%
14	Dec-97	1.57%	-4.21%
15	Jan-98	1.02%	1.37%
16	Feb-98	7.04%	-0.54%
17	Mar-98	4.99%	5.99%
18	Apr-98	0.91%	-9.25%
19	May-98	-1.88%	-5.67%
20	Jun-98	3.94%	2.40%
21	Jul-98	-1.16%	0.88%
22	Aug-98	-14.58%	-30.81%
23	Sep-98	6.24%	12.61%
24	Oct-98	8.03%	1.12%
25	Nov-98	5.91%	-12.18%
26	Dec-98	5.64%	0.42%

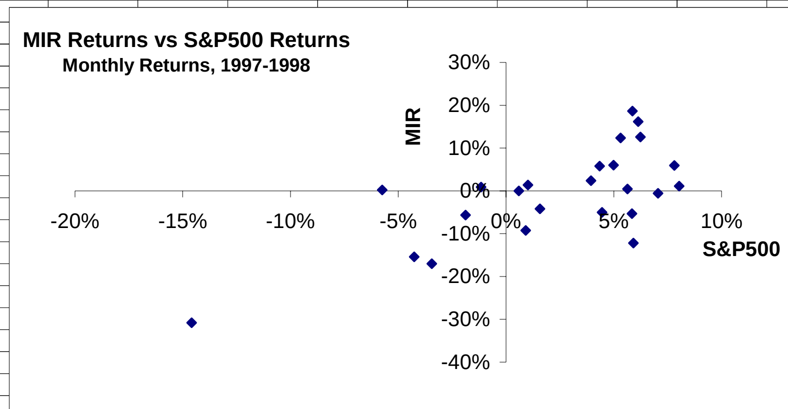
We now use Excel to produce an XY scatter plot of these returns. We use the command **Insert|Chart**, and then the Chart Wizard to produce the desired graph:



	A	B	C
1	SIMPLE REGRESSION EXAMPLE		
2	Date	S&P 500 Index SPX	Mirage Resorts MIR
3	Jan-97	6.13%	16.18%
4	Feb-97	0.59%	0.00%
5	Mar-97	-4.26%	-15.42%
6	Apr-97	5.84%	-5.29%
7	May-97	5.86%	18.63%
8	Jun-97	4.35%	5.76%
9	Jul-97	7.81%	5.94%
10	Aug-97	-5.75%	0.23%
11	Sep-97	5.32%	12.35%
12	Oct-97	-3.45%	-17.01%
13	Nov-97	4.46%	-5.00%
14	Dec-97	1.57%	-4.21%
15	Jan-98	1.02%	1.37%
16	Feb-98	7.04%	-0.54%
17	Mar-98	4.99%	5.99%
18	Apr-98	0.91%	-9.25%
19	May-98	-1.88%	-5.67%
20	Jun-98	3.94%	2.40%
21	Jul-98	-1.16%	0.88%
22	Aug-98	-14.58%	-30.81%
23	Sep-98	6.24%	12.61%
24	Oct-98	8.03%	1.12%
25	Nov-98	5.91%	-12.18%
26	Dec-98	5.64%	0.42%

Here's what the chart looks like. As described in Chapter 28 on graphs in Excel, we've gotten rid of the grey background which is the Excel default.

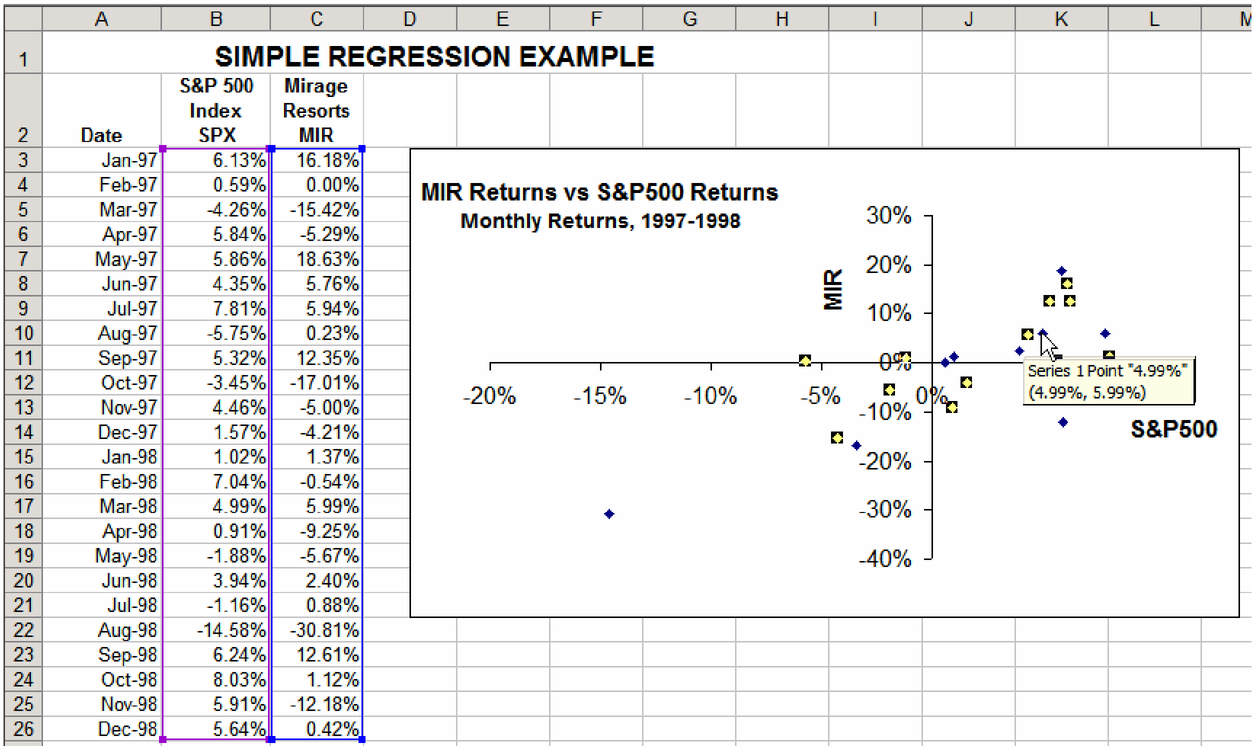
	A	B	C
1	**SIMPLE REGRESSION EXAMPLE**		
2	**Date**	**S&P 500 Index SPX**	**Mirage Resorts MIR**
3	Jan-97	6.13%	16.18%
4	Feb-97	0.59%	0.00%
5	Mar-97	-4.26%	-15.42%
6	Apr-97	5.84%	-5.29%
7	May-97	5.86%	18.63%
8	Jun-97	4.35%	5.76%
9	Jul-97	7.81%	5.94%
10	Aug-97	-5.75%	0.23%
11	Sep-97	5.32%	12.35%
12	Oct-97	-3.45%	-17.01%
13	Nov-97	4.46%	-5.00%
14	Dec-97	1.57%	-4.21%
15	Jan-98	1.02%	1.37%
16	Feb-98	7.04%	-0.54%
17	Mar-98	4.99%	5.99%
18	Apr-98	0.91%	-9.25%
19	May-98	-1.88%	-5.67%
20	Jun-98	3.94%	2.40%
21	Jul-98	-1.16%	0.88%
22	Aug-98	-14.58%	-30.81%
23	Sep-98	6.24%	12.61%
24	Oct-98	8.03%	1.12%
25	Nov-98	5.91%	-12.18%
26	Dec-98	5.64%	0.42%



We want to draw a line through the points above, and we want this line to be to be the "best" line in the sense that it is the closest line you could draw through the points.[7] There are several ways to do this in Excel (as usual ...). Here's what we do:

- Click on the points of the graph so that Excel marks all of them. (If you have a lot of data points, Excel may mark only some of the points; just ignore this and proceed to the next step.) After you do this, the graph looks like:

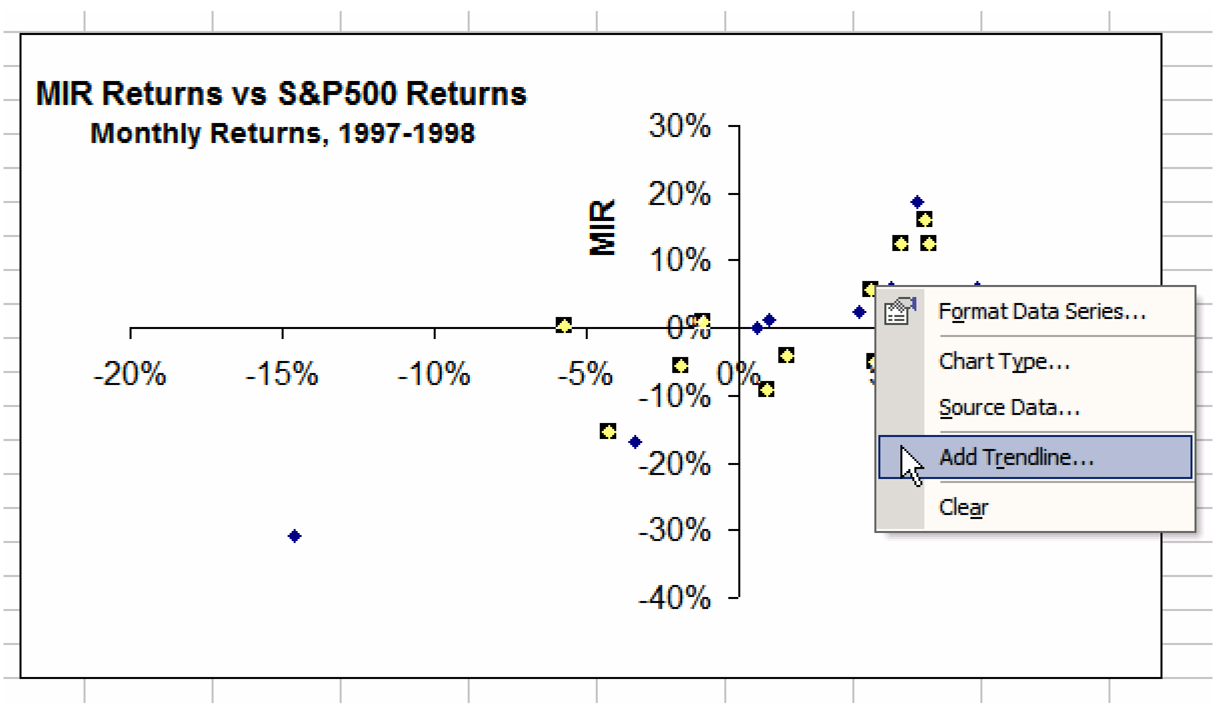
[7] There's a formal statistical definition of "best" and "closest," but we'll leave that to another course.



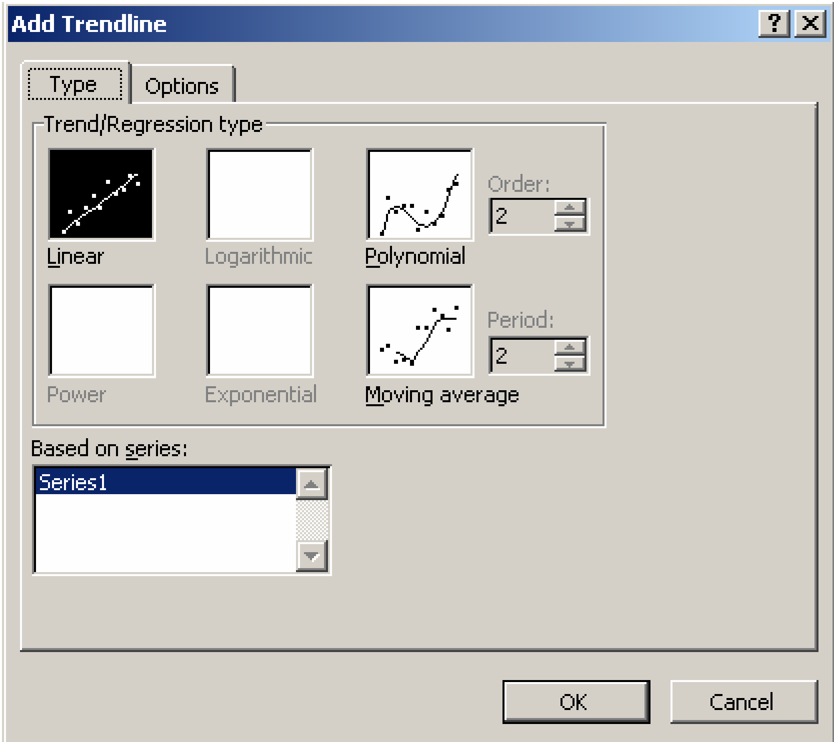
SIMPLE REGRESSION EXAMPLE

Date	S&P 500 Index SPX	Mirage Resorts MIR
Jan-97	6.13%	16.18%
Feb-97	0.59%	0.00%
Mar-97	-4.26%	-15.42%
Apr-97	5.84%	-5.29%
May-97	5.86%	18.63%
Jun-97	4.35%	5.76%
Jul-97	7.81%	5.94%
Aug-97	-5.75%	0.23%
Sep-97	5.32%	12.35%
Oct-97	-3.45%	-17.01%
Nov-97	4.46%	-5.00%
Dec-97	1.57%	-4.21%
Jan-98	1.02%	1.37%
Feb-98	7.04%	-0.54%
Mar-98	4.99%	5.99%
Apr-98	0.91%	-9.25%
May-98	-1.88%	-5.67%
Jun-98	3.94%	2.40%
Jul-98	-1.16%	0.88%
Aug-98	-14.58%	-30.81%
Sep-98	6.24%	12.61%
Oct-98	8.03%	1.12%
Nov-98	5.91%	-12.18%
Dec-98	5.64%	0.42%

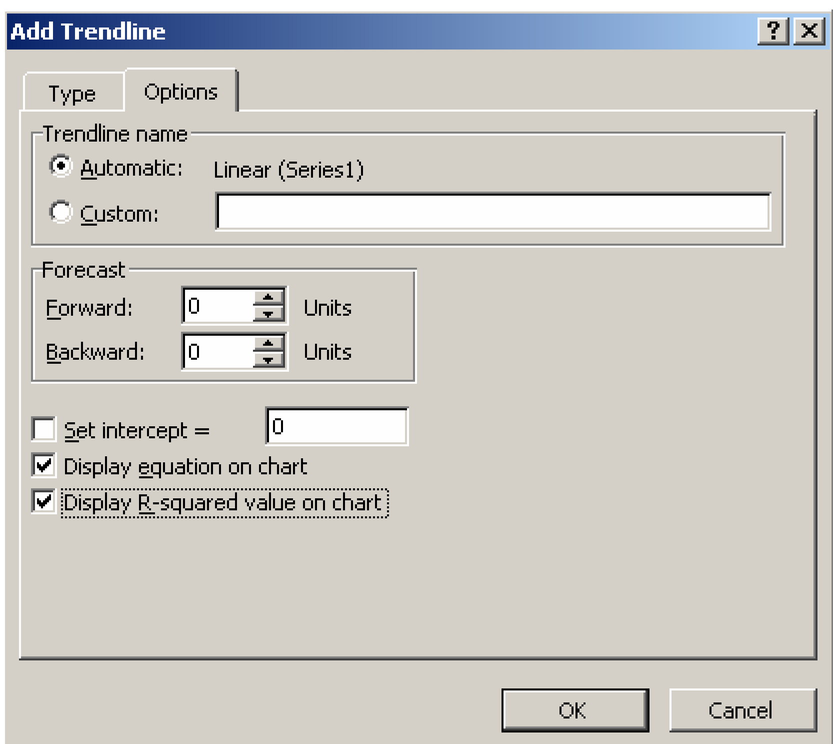
- With the points marked, right-click the mouse and choose **Add Trendline:**



- **Add Trendline** brings up the following box, in which we leave the choice **Linear** regression.

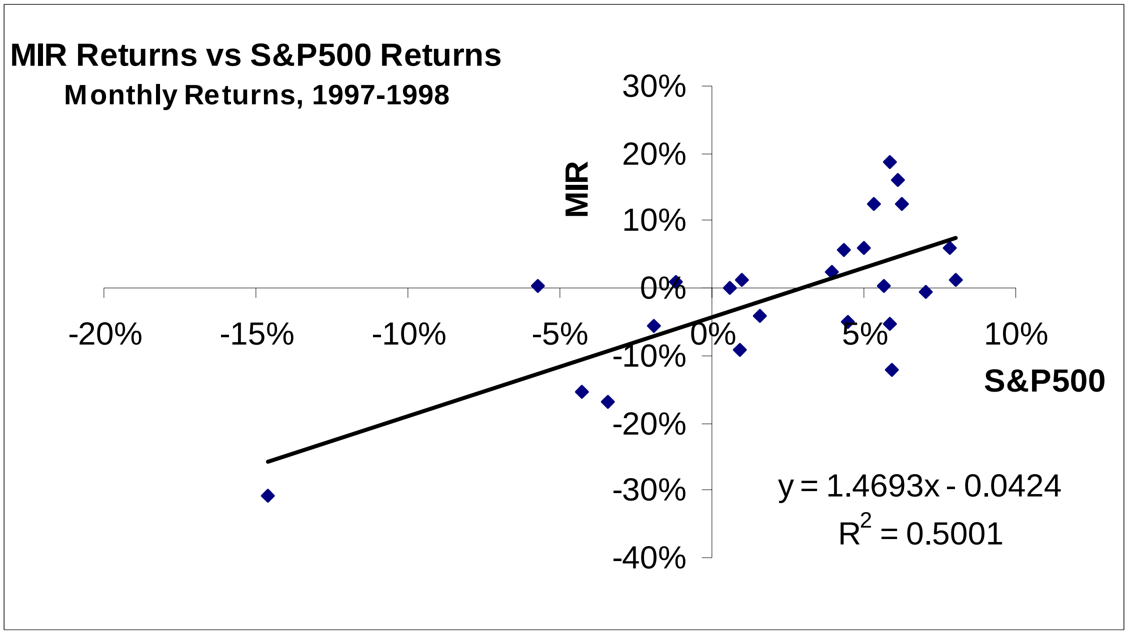


- Before clicking **OK**, we move to the **Options** tab and mark **Display equation on chart** and **Display R-squared value on chart**.



- Now you can click **OK**.

Excel displays the following chart:



The box with the regression results can be moved by a held-down left click of the mouse.

What does the regression mean?

The graph above shows the regression line as: $y = 1.4693x - 0.0424, R^2 = 0.5001$. Since we're trying to understand the effect of the S&P Index on MIR stock, we can attach the following meaning to the variables of the regression line:

- The "y" of the regression line stands for the monthly percentage return of MIR and the "x" stands for the monthly percentage return of the S&P 500 index.
- The *slope* of the regression line is 1.4693. This tells us that, on average, a 1% increase in the S&P monthly return caused a 1.4693% increase in the MIR monthly return. Of course this also goes the other direction: On average a 1% decrease in the S&P is related to a 1.4693% decrease in MIR's return.

- The fact that the slope of the regression is greater than 1 means that MIR is very sensitive to the S&P: Variations (increases or decreases) in the S&P return cause larger variations in the MIR return. We return to this topic in Chapter 12.
- The *intercept* of the regression line is -0.0424. The intercept tells us that in months when the S&P 500 doesn't "move," MIR's return tends to decrease by 4.24%.
- The R^2 (pronounced "r squared") of the regression line says that 50.01% of the variability in the MIR returns is explained by the variability of the S&P500 returns. This may seem sort of low but it's actually quite respectable: The R^2 of 50% says that half of MIR's return variability is explained by the variability of the S&P 500 index. The other 50% of the return variability is presumably explained by factors which are unique to MIR. You wouldn't expect much more: If for some strange reason the R^2 were 100%, this would mean that *all* of MIR's returns are explained by the S&P returns, which is clearly nonsense.

The regression line thus allows you to make some interesting predictions about the MIR return based on the S&P return. Suppose you're a financial analyst and you think that this month the S&P index will go up by 20%. Then based on the regression, you'd expect MIR to increase by $1.4693*20\% - 0.0424 = 25.146\%$. Knowing that the R^2 is approximately 50%, only about half of the variability in MIR stock returns is explained by the S&P stock return, and you would thus attach some degree of skepticism to this prediction.

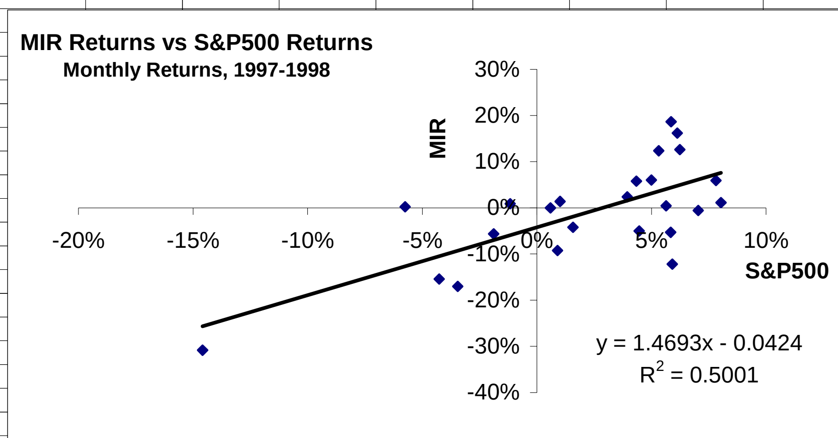
Other ways of doing a regression in Excel

As you might expect, in Excel there are other methods for calculating the slope, intercept, and R^2 of the regression equation. Excel has functions called **Slope( )**, **Intercept( )**, **Rsq( )**.

These functions are illustrated below in cells B28, B31, B34 . Note that in these functions, the MIR returns come before the S&P returns, so that we write, for example, **Slope(MIR returns,S&P returns)**.

The slope, intercept, and R^2 can be calculated directly using the **Average( )**, **Covar( )**, **Var(**), and **Correl( )** (cells B29, B32, B35 below).

	A	B	C
1	**SIMPLE REGRESSION EXAMPLE**		
2	**Date**	**S&P 500 Index SPX**	**Mirage Resorts MIR**
3	Jan-97	6.13%	16.18%
4	Feb-97	0.59%	0.00%
5	Mar-97	-4.26%	-15.42%
6	Apr-97	5.84%	-5.29%
7	May-97	5.86%	18.63%
8	Jun-97	4.35%	5.76%
9	Jul-97	7.81%	5.94%
10	Aug-97	-5.75%	0.23%
11	Sep-97	5.32%	12.35%
12	Oct-97	-3.45%	-17.01%
13	Nov-97	4.46%	-5.00%
14	Dec-97	1.57%	-4.21%
15	Jan-98	1.02%	1.37%
16	Feb-98	7.04%	-0.54%
17	Mar-98	4.99%	5.99%
18	Apr-98	0.91%	-9.25%
19	May-98	-1.88%	-5.67%
20	Jun-98	3.94%	2.40%
21	Jul-98	-1.16%	0.88%
22	Aug-98	-14.58%	-30.81%
23	Sep-98	6.24%	12.61%
24	Oct-98	8.03%	1.12%
25	Nov-98	5.91%	-12.18%
26	Dec-98	5.64%	0.42%
27			
28	Slope	1.4693	<-- =SLOPE(C3:C26,B3:B26)
29		1.4693	<-- =COVAR(C3:C26,B3:B26)/VARP(B3:B26)
30			
31	Intercept	-0.0424	<-- =INTERCEPT(C3:C26,B3:B26)
32		-0.0424	<-- =AVERAGE(C3:C26)-B28*AVERAGE(B3:B26)
33			
34	R-squared	0.5001	<-- =RSQ(C3:C26,B3:B26)
35		0.5001	<-- =CORREL(C3:C26,B3:B26)^2



Look at the alternative definitions of each of the regression variables (cells B29, B32, B35):

- The regression slope can be computed with the **Slope( )** function (cell B28), but as shown in cell B29 it is also equal to the $\dfrac{Covariance(S\&P, MIR)}{Var(S\&P)}$.

- The regression intercept can be computed with the **Intercept( )** function, but as shown in cell B32 it is also equal to $Average(MIR) - slope * Average(S\&P)$.
- The regression R^2 can be computed with the **Rsq( )** function, but as shown in cell B35 it is also equal to the squared correlation between the S&P and MIR: $\left[Correlation(S\&P, MIR)\right]^2$.

11.6. Advanced section: portfolio statistics for multiple assets

This section discusses a slightly more advanced topic which is used only in the appendix to Chapter 12. You can skip it on first reading. In section above we discussed the calculation of the portfolio mean and variance for a 2-asset portfolio. In this section we discuss the calculation for a portfolio composed of more than 2 assets.

In order to set the scene, we introduce some notation. Suppose that we have N stocks, and that for each stock i we have computed the mean $E(r_i)$ and the variance $\sigma_i^2 = Var(r_i)$ of the stock's returns. Furthermore, suppose that for each pair of stocks i and j, we have calculated the covariance of the returns $Cov(r_i, r_j)$. Here's an example with 3 stocks:

	A	B	C	D	E
1	**PORTFOLIO RETURNS FOR A 3-STOCK PORTFOLIO**				
2	**Year ending**	**General Motors GM**	**Microsoft MSFT**	**Heinz HNZ**	
3	Dec-90	-11.54%	72.99%	2.46%	
4	Dec-91	-11.35%	121.76%	14.54%	
5	Dec-92	16.54%	15.11%	16.89%	
6	Dec-93	72.64%	-5.56%	-15.95%	
7	Dec-94	-21.78%	51.63%	6.55%	
8	Dec-95	28.13%	43.56%	39.81%	
9	Dec-96	8.46%	88.32%	11.56%	
10	Dec-97	19.00%	56.43%	45.89%	
11	Dec-98	21.09%	114.60%	14.11%	
12	Dec-99	21.34%	68.36%	-27.44%	
13					
14	Average	14.25%	62.72%	10.84%	<-- =AVERAGE(D3:D12)
15	Variance	6.38%	14.43%	4.40%	<-- =VARP(D3:D12)
16	Sigma	25.25%	37.99%	20.98%	<-- =STDEVP(D3:D12)
17					
18	**Covariances**				
19	Cov(r_{GM},r_{MSFT})	-0.0552	<-- =COVAR(B3:B12,C3:C12)		
20	Cov(r_{GM},r_{HNZ})	-0.0096	<-- =COVAR(B3:B12,D3:D12)		
21	Cov(r_{MSFT},r_{HNZ})	0.0092	<-- =COVAR(C3:C12,D3:D12)		

Now suppose we form a portfolio composed of the following proportions of each of the stocks: $x_{GM} = 20\%$, $x_{MSFT} = 50\%$, $x_{HNZ} = 1 - x_{GM} - x_{MSFT} = 30\%$. Cells G3:G12 in the spreadsheet below show you the returns of this portfolio, and cells G14:G16 compute the portfolio's mean return, variance, and standard deviation:

	A	B	C	D	E	F	G	H
1	**PORTFOLIO RETURNS FOR A 3-STOCK PORTFOLIO**							
2	**Year ending**	**General Motors GM**	**Microsoft MSFT**	**Heinz HNZ**			**Portfolio return**	
3	Dec-90	-11.54%	72.99%	2.46%			34.92%	<-- =0.2*B3+0.5*C3+0.3*D3
4	Dec-91	-11.35%	121.76%	14.54%			62.97%	<-- =0.2*B4+0.5*C4+0.3*D4
5	Dec-92	16.54%	15.11%	16.89%			15.93%	
6	Dec-93	72.64%	-5.56%	-15.95%			6.96%	
7	Dec-94	-21.78%	51.63%	6.55%			23.42%	
8	Dec-95	28.13%	43.56%	39.81%			39.35%	
9	Dec-96	8.46%	88.32%	11.56%			49.32%	
10	Dec-97	19.00%	56.43%	45.89%			45.78%	
11	Dec-98	21.09%	114.60%	14.11%			65.75%	
12	Dec-99	21.34%	68.36%	-27.44%			30.22%	
13								
14	Average	14.25%	62.72%	10.84%	<-- =AVERAGE(D3:D12)	Average	37.46%	<-- =AVERAGE(G3:G12)
15	Variance	0.0638	0.1443	0.0440	<-- =VARP(D3:D12)	Variance	3.31%	<-- =VARP(G3:G12)
16	Sigma	25.25%	37.99%	20.98%	<-- =STDEVP(D3:D12)	Sigma	18.21%	<-- =STDEVP(G3:G12)
17								
18	**Covariances**						**Alternative calculation of portfolio statistics**	
19	$Cov(r_{GM}, r_{MSFT})$	-0.0552	<-- =COVAR(B3:B12,C3:C12)			**Average**	37.46%	<-- =0.2*B14+0.5*C14+0.3*D14
20	$Cov(r_{GM}, r_{HNZ})$	-0.0096	<-- =COVAR(B3:B12,D3:D12)			**Variance**	0.0331	<-- =0.2^2*B16+0.4^2*C16+0.3^2*D16 +2*0.2*0.4*B20+2*0.2*0.3*B21+2*0.4*0.3*B22
21	$Cov(r_{MSFT}, r_{HNZ})$	0.0092	<-- =COVAR(C3:C12,D3:D12)			**Sigma**	18.21%	<-- =SQRT(G20)

If you look at cells G19:G21, you'll see that there is a more efficient way of doing the same calculations, based on the following formulas:

$$Expected\ \ portfolio\ return = E\left(r_p\right) = x_{GM}E\left(r_{GM}\right) + x_{MSFT}E\left(r_{MSFT}\right) + x_{HNZ}E\left(r_{HNZ}\right)$$

$$\begin{aligned} Portfolio\ variance = Var\left(r_p\right) &= x_{GM}^2 Var\left(r_{GM}\right) + x_{MSFT}^2 Var\left(r_{MSFT}\right) + x_{HNZ}^2 Var\left(r_{HNZ}\right) \\ &+2x_{GM}x_{MSFT}Cov\left(r_{GM}, r_{MSFT}\right) + 2x_{GM}x_{HNZ}Cov\left(r_{GM}, r_{HNZ}\right) \\ &+2x_{MSFT}x_{HNZ}Cov\left(r_{MSFT}, r_{HNZ}\right) \end{aligned}$$

These formulas generalize to any number of assets: If we have a portfolio composed of N assets, and that we know all the expected returns, variances, and covariances. Then:

- The portfolio's expected return is the weighted average of the individual asset returns. Denoting the portfolio weights by $\{x_1, x_2, ..., x_N\}$, the portfolio expected return is:

$$\begin{aligned} E\left(r_p\right) &= x_1E\left(r_1\right) + x_2E\left(r_2\right) + ... + x_NE\left(r_N\right) \\ &= \sum_{i=1}^{N} x_iE\left(r_i\right) \end{aligned}$$

- The portfolio's variance of return is the sum of the following two expressions:

- The sum of each asset's variance, weighted by the *square* of the asset's portfolio proportion: $x_1^2 Var(r_1) + x_2^2 Var(r_2) + \ldots + x_N^2 Var(r_N)$.
- The sum of twice each of the covariances, weighted by the *product* of the asset proportions:

$$\begin{aligned} & 2x_1x_2Cov(r_1,r_2) + 2x_1x_3Cov(r_1,r_3) + \ldots + 2x_1x_NCov(r_1,r_N) \\ & \qquad + 2x_2x_3Cov(r_2,r_3) + \ldots + 2x_2x_NCov(r_2,r_N) \\ & \qquad \ldots \\ & \qquad\qquad + 2x_{N-1}x_NCov(r_{N-1},r_N) \end{aligned}$$

Conclusion and summary

Information about stocks—their prices, dividends, and returns—produce mounds of data. Statistics is a way of dealing with these large masses of data. This chapter has given you the necessary statistical techniques to do typical finance computations related to stocks. We've shown how to compute stock returns from basic data about stock prices, dividends, and stock splits. We've also shown how to compute the mean return (also called the average return), the variance and standard deviation of returns, and the covariance between the returns of two different stocks.

Stocks are most often combined into portfolios, and this chapter has shown you how to compute the mean and standard deviation of a portfolio's return. It also introduced you to regression analysis, which allows you to relate the returns of two stocks one to the other.

In succeeding chapters we will use these statistical techniques to do financial analysis of individual stocks and stock portfolios.

Exercises

Note: The data for these problems is included in the CD-ROM which comes with the book.

1. Here is the stock price history of "HighTech" and "LowTech" corporations.

	A	B	C
1		**"HighTech" corp. Stock share price**	**"LowTech" corp. Stock share price**
2	31-Dec-91	75.00	40.00
3	31-Dec-92	86.25	45.20
4	31-Dec-93	125.32	55.60
5	31-Dec-94	91.64	48.37
6	31-Dec-95	100.80	32.88
7	31-Dec-96	145.93	61.64
8	31-Dec-97	151.21	75.82
9	31-Dec-98	196.57	97.05
10	31-Dec-99	226.05	109.66
11	31-Dec-00	89.00	122.99

Calculate:

1.a. The annual returns for each stock.

1.b. The mean (average) return for the period of 10 years for each firm. Which stock has the higher average return?

1.c. The variance and the standard deviation of returns, for the period of 10 years for each firm. Which stock is riskier?

1.d. The covariance and correlation of the returns for each firm. Use two formulas to compute the correlation: The Excel formula **Correl** and the definition

$$Correlation\left(r_A, r_B\right) = \frac{Cov\left(r_A, r_B\right)}{\sigma_A \sigma_B}.$$

1.e. If you had to choose between the two stocks, which would you choose? Explain briefly.

2. Below you will find price data for 3 mutual funds:

	A	B	C	D
1	**DATA ON 3 MUTUAL FUNDS**			
2	**Date**	**Scudder Development Fund**	**Value Line Leveraged Growth Fund**	**Fidelity Fund**
3	4-Jan-93	24.34	17.47	9.47
4	3-Jan-94	24.2	20.32	11.39
5	3-Jan-95	20.87	19.15	11.19
6	2-Jan-96	30.35	24.45	15.25
7	2-Jan-97	30.94	26.95	18.46
8	2-Jan-98	31.28	32.08	23.44
9	4-Jan-99	33.32	47.19	31.04
10	3-Jan-00	36.06	49.12	35.36
11	2-Jan-01	33.89	47.23	33.82
12	2-Jan-02	20.01	37.31	28.46
13	2-Jan-03	13.79	26.87	21.55

2.a. Compute the annual returns on the funds for the period.

2.b. Compute the mean, variance, and standard deviation of the fund returns.

2.c. Graph the fund returns and the dates.

2.d. Calculate the correlations of the fund returns.

2.e. If the historical information correctly predicts future returns (is this reasonable?), which fund would you choose?

3. Here is the monthly stock price data for Ford corporation and GM corporation:

	A	B	C	D
1	**PRICES FOR FORD AND GM STOCK**			
2	**Date**	**Ford**	**GM**	
3	8-Nov-99	24.44	66.08	
4	1-Dec-99	25.79	65.09	
5	3-Jan-00	24.32	72.14	
6	1-Feb-00	20.35	68.54	
7	1-Mar-00	22.45	74.63	
8	3-Apr-00	27.00	84.37	
9	1-May-00	23.95	64.02	
10	1-Jun-00	22.08	52.63	
11	3-Jul-00	24.17	51.61	
12	1-Aug-00	21.95	63.97	
13	1-Sep-00	23.14	59.40	
14	2-Oct-00	23.98	56.77	
15	1-Nov-00	20.89	45.64	
16	1-Dec-00	21.52	46.96	
17	2-Jan-01	26.16	49.51	
18	1-Feb-01	25.30	51.77	

Calculate:

- Monthly returns for each firm.
- Covariance between returns of Ford corporation and GM corporation.
- Correlation between returns of Ford corporation and GM corporation.

4. By using the returns of Ford and GM corporations you calculated in the previous question, perform a regression of Ford's returns vs. GM's returns. Report:

- The slope of the regression.
- The value of the intercept.
- The r-squared of the regression.

Is the mutual impact of the two company's sales (one on the other) large or small? Explain.

5. Here is stock price and dividend data for Kellogg Co.:

	A	B	C
1	**KELLOGG PRICE AND DIVIDEND DATA**		
2		**Price**	**Dividend during year**
3	31-Dec-89	64.62	
4	31-Dec-90	78.00	1.44
5	31-Dec-91	56.75	2.15
6	31-Dec-92	62.12	1.16
7	31-Dec-93	53.75	1.32
8	31-Dec-94	55.00	1.40
9	31-Dec-95	76.62	1.50
10	31-Dec-96	69.62	1.62
11	31-Dec-97	46.38	1.28
12	31-Dec-98	40.62	0.90
13	31-Dec-99	24.25	0.98
14	31-Dec-00	26.20	1.00
15	31-Dec-01	30.86	1.00
16	31-Dec-02	33.40	1.00

5.a. Calculate the dividend-adjusted returns for each of the years, their mean and their standard deviation.

5.b. Stock analysts like to talk about the *dividend yield*—the dividend divided into the stock price. Compute the annual dividend yield for Kellogg (define it as $\frac{Dividends\ over\ the\ year}{Stock\ price\ at\ beginning\ of\ year}$) and compute its statistics (mean and standard deviation) over the period.

5.c. If you bought Kellogg stock and had no intention of ever selling it, why might you be interested in the stock's dividend yield?

6. Below you will find stock price, dividend, and split data for IBM. Calculate the dividend and split-adjusted returns for each of the years, their mean and their standard deviation.

	A	B	C	D
1	**IBM PRICE, DIVIDEND AND SPLIT DATA**			
2		**Closing price**	**Dividend during year**	**Other information**
3	31-Dec-89	98.62		
4	31-Dec-90	126.75		
5	31-Dec-91	90.00		
6	31-Dec-92	51.50		
7	31-Dec-93	56.50		
8	31-Dec-94	72.12		
9	31-Dec-95	108.50		
10	31-Dec-96	156.88		
11	31-Dec-97	98.75		2 for 1 split (May97)
12	31-Dec-98	183.25		
13	31-Dec-99	112.25		2 for 1 split (May99)
14	31-Dec-00	112.00		
15	31-Dec-01	107.89		
16	31-Dec-02	78.20	0.30	

7. Compute the covariance and correlation coefficient between IBM and Kellogg (previous two questions). Are there any advantages to diversifying between IBM and Kellogg?

8. Here is the stock price and split data for HeavySteel corporation.

	A	B	C
1	**HEAVYSTEEL CORPORATION**		
2		**Closing stock price**	**Stock splits**
3	31-Dec-90	11.24	
4	31-Dec-91	11.98	
5	31-Dec-92	10.23	
6	31-Dec-93	11.02	2 for 1
7	31-Dec-94	12.56	
8	31-Dec-95	13.45	
9	31-Dec-96	15.36	1.5 for 1
10	31-Dec-97	16.01	
11	31-Dec-98	17.23	
12	31-Dec-99	15.23	

7.a. Calculate the split-adjusted returns for each year and its statistics (mean and standard deviation).

7.b. If you bought 100 shares of this stock in the beginning of 1990 and during the period of 10 years never sold or bought additional shares, how many shares would you have by the end of 2000?

9. A *reverse split* is just like a split, but only in a reverse direction. For example, in 1 for 2 reverse split, you receive 1 shares for every 2 shares you hold. How would your answers to the previous question change if you learned that in 1999 the firm did 3 for 4 reverse split?

10. Here are two companies: Young corporation and Mature corporation. Young Corporation grows very rapidly, does not pay any dividends and retains all its profits. Mature Corporation stopped growing a long time ago, generates sizable cash flows and pays out dividends.

	A	B	C	D
1		**YOUNG Corp.**	**MATURE Corp.**	
2		Share price	Share price	Dividend per share
3	31-Dec-90	32.56	78.50	0.00
4	31-Dec-91	34.50	82.50	0.00
5	31-Dec-92	38.98	84.50	1.00
6	31-Dec-93	44.50	81.60	0.00
7	31-Dec-94	40.20	79.60	1.50
8	31-Dec-95	39.50	80.96	1.50
9	31-Dec-96	38.45	82.65	0.00
10	31-Dec-97	37.50	83.69	2.00
11	31-Dec-98	43.58	82.79	2.00
12	31-Dec-99	50.30	81.97	0.00

Calculate:

- Young's yearly returns.
- Mature's yearly returns.

- Which is the better investment of the two? Give a brief explanation

11. Chicken Feed and Poultry Delight are two stocks traded on the Farmers Stock Exchange. A statistician has determined that the returns on the two stocks are related by the equation $r_{Chicken\ Feed,t} = 50\% - 0.8 * r_{Poulty\ Delight,t}$. Show that the correlation between the two sets of returns is -1. Use the following template:

	A	B	C
2	**Year**	**Poultry Delight stock return**	**Chicken Feed stock return**
3	1990	30.73%	
4	1991	55.21%	
5	1992	15.82%	
6	1993	33.54%	
7	1994	14.93%	
8	1995	35.84%	
9	1996	48.39%	
10	1997	37.71%	
11	1998	67.85%	
12	1999	44.85%	
13			
14	Correlation		

12. Below you will find the annual returns of two assets. Fill in the blanks and graph the returns of the portfolios (rows 13-27).

	A	B	C
1		**Asset 1**	**Asset 2**
2	31-Dec-90	12.56%	7.56%
3	31-Dec-91	13.50%	8.56%
4	31-Dec-92	14.23%	4.56%
5	31-Dec-93	15.23%	2.12%
6	31-Dec-94	14.23%	1.23%
7	31-Dec-95	12.23%	0.26%
8	31-Dec-96	10.23%	3.25%
9	31-Dec-97	5.26%	4.89%
10	31-Dec-98	4.25%	5.56%
11	31-Dec-99	2.23%	6.45%
12			
13	Average return		
14	Return variance		
15	Covariance		
16	**Proportion of asset 1**	**Portfolio standard deviation**	**Portfolio mean return**
17	0		
18	0.1		
19	0.2		
20	0.3		
21	0.4		
22	0.5		
23	0.6		
24	0.7		
25	0.8		
26	0.9		
27	1		

13. Here is data on the stock prices and returns of General Electric, Boeing and S&P 500 index.

	A	B	C	D	E	F	G
1	**MONTHLY RETURNS ON GE, BOEING, S&P500, 2000**						
2	**Date**	**GE**	**GE Return**	**Boeing**	**Boeing return**	**S&P500**	**S&P return**
3	Jan-02	37.15		40.22		1130.20	
4	Feb-02	38.50	3.63%	45.33	12.71%	1106.73	-2.08%
5	Mar-02	37.40	-2.86%	47.59	4.99%	1147.39	3.67%
6	Apr-02	31.55	-15.64%	43.99	-7.56%	1076.92	-6.14%
7	May-02	31.14	-1.30%	42.23	-4.00%	1067.14	-0.91%
8	Jun-02	29.05	-6.71%	44.55	5.49%	989.82	-7.25%
9	Jul-02	32.20	10.84%	41.11	-7.72%	911.62	-7.90%
10	Aug-02	30.15	-6.37%	36.87	-10.31%	916.07	0.49%
11	Sep-02	24.65	-18.24%	33.95	-7.92%	815.28	-11.00%
12	Oct-02	25.25	2.43%	29.59	-12.84%	885.76	8.64%
13	Nov-02	27.12	7.41%	34.05	15.07%	936.31	5.71%
14	Dec-02	24.35	-10.21%	32.99	-3.11%	879.82	-6.03%
15							
16	Average return						
17	Standard deviation						
18	Covariances						
19	Cov(GE,Boeing)						
20	Cov(GE,SP)						
21	Cov(Boeing,SP)						
22							
23	Correlations						
24	Correlation(GE,Boeing)						
25	Correlation(GE,SP)						
26	Correlation(Boeing,SP)						
27							
28	Portfolio proportions						
29	GE	0.5					
30	Boeing	0.3					
31	S&P	0.2	<-- =1-B30-B29				
32							
33	Portfolio return						
34	Portfolio standard deviation						

Calculate the highlighted cells.

14. Go to http://finance.yahoo.com. Download monthly adjusted-stock price data for Oracle corporation (ORCL), Microsoft corporation (MSFT), Dell corporation (DELL) and Gateway

corporation (GTW) for 1998 and 1999. Also, download the same data for S&P500 index (^SPX) for the same period.[8] Answer the following questions:

13.a. What is the mean return, variance and standard deviation of portfolio consisting of the four stocks, where wealth is allocated equally among each stock?

13.b. On average, would you be better off by investing in this portfolio or by investing in S&P 500 index, during the period of 2years?

13.c. What is the sensitivity of your portfolio to the movements of S&P500 index? You will have to perform a regression of the portfolio returns versus S&P500 returns and report the results.

15. By using information provided in the previous problem, perform a regression of the portfolio returns vs. S&P500 index returns for the period of 24 months. Report: The slope of the regression, its intercept and r-squared. Explain what each of these numbers tell you.

16. (This is a hard question!) On the disk which comes with the book, you will find 2 years of monthly un-adjusted and adjusted stock price data for AT&T corporation (symbol: T). Calculate:

16.a. Cumulative adjustment factor for AT&T stock

16.b. What two interesting things happened in November 2002 and what happened to cumulative adjustment factor in this month? Can you explain?

Here's the data:

---

[8] Recall that when you download data from Yahoo into Excel, it is already adjusted for stock splits and dividends.

	A	B	C	D	E	F	G	H	I
1	**Date**	**Open**	**High**	**Low**	**Close**	**Volume**	**Adj. Close***	**Cumulative Adjustment factor**	
2	Dec 02			$0.19 Cash Dividend					
3	Dec 02	28.54	28.88	25.11	**26.11**	4,932,428	26.11		
4	Nov 02			$8.48 Cash Dividend					
5	Nov 02			1:5 Stock Split					
6	Nov 02	12.94	28.25	12.84	**28.04**	13,146,915	28.04		**<-- AT&T Spins Off AT&T Broadband To Shareowners And Completes AT&T Broadband Merger With Comcast**
7	Oct 02	12.1	13.64	10.45	**13.04**	14,453,869	65.2		**1 to 5 Reverse Split**
8	Sep 02			$0.04 Cash Dividend					
9	Sep 02	11.95	13.79	11.2	**12.01**	15,095,745	60.05		
10	Aug 02	10.12	12.85	8.69	**12.22**	17,147,918	61.1		
11	Jul 02	10.5	10.55	8.2	**10.18**	18,639,136	50.9		
12	Jun 02			$0.04 Cash Dividend					
13	Jun 02	11.85	12.4	9.09	**10.7**	29,520,930	53.5		
14	May 02	13.2	14.3	11.76	**11.97**	17,814,400	59.85		
15	Apr 02	15.74	15.85	12.66	**13.12**	15,936,609	65.6		
16	Mar 02			$0.04 Cash Dividend					
17	Mar 02	15.8	16.48	15	**15.7**	11,042,700	78.5		
18	Feb 02	17.55	17.91	14.18	**15.54**	16,401,442	77.7		
19	Jan 02	18.48	19.25	16.65	**17.7**	11,919,185	88.5		
20	Dec 01			$0.04 Cash Dividend					
21	Dec 01	17.35	18.75	15.8	**18.14**	14,846,490	90.7		
22	Nov 01	15.33	17.85	14.75	**17.49**	10,987,857	87.45		
23	Oct 01	19.15	20	15.17	**15.25**	15,015,643	76.25		
24	Sep 01			$0.04 Cash Dividend					
25	Sep 01	19.01	19.64	16.5	**19.3**	15,798,733	96.5		
26	Aug 01	20.32	20.95	18.66	**19.04**	7,457,491	95.2		
27	Jul 01			$5.52 Cash Dividend					
28	Jul 01	21.75	23	18.1	**20.21**	16,556,647	101.05		
29	Jun 01			$0.04 Cash Dividend					
30	Jun 01	21.16	22.16	19.82	**22**	11,332,052	110		
31	May 01	22.58	23.1	20.48	**21.17**	15,562,513	105.85		
32	Apr 01	21.3	23.27	19.85	**22.28**	12,075,000	111.4		
33	Mar 01			$0.04 Cash Dividend					
34	Mar 01	22.8	24.6	20.6	**21.3**	12,662,459	106.5		
35	Feb 01	23.95	24.53	20.2	**23**	12,220,989	115		
36	Jan 01	17.37	25.15	17.25	**23.99**	20,407,609	119.95		
37	Dec 00			$0.04 Cash Dividend					
38	Dec 00	19.44	22.69	16.5	**17.25**	23,385,210	86.25		
39	Nov 00	22.62	22.94	18.25	**19.62**	20,863,095	98.1		
40	Oct 00	29	30	21.25	**23.19**	24,254,945	115.95		
41	Sep 00			$0.22 Cash Dividend					
42	Sep 00	31.62	32.94	27.25	**29**	19,280,690	145		
43	Aug 00	30.94	32.94	29.62	**31.62**	17,828,760	158.1		
44	Jul 00	31.81	35.19	30.5	**30.94**	19,562,070	154.7		
45	Jun 00			$0.22 Cash Dividend					
46	Jun 00	34.94	37.75	31.25	**31.81**	20,312,436	159.05		
47	May 00	46.31	49	33.63	**34.94**	25,649,081	174.7		
48	Apr 00	56.69	58.81	45.88	**45.88**	12,616,194	229.4		
49	Mar 00			$0.22 Cash Dividend					
50	Mar 00	49.38	61	47.5	**56.31**	13,692,547	281.55		
51	Feb 00	52.75	53	44.31	**49.38**	10,648,485	246.9		
52	Jan 00	50.81	56	47.5	**52.75**	11,964,045	263.75		
53	Dec 99			$0.22 Cash Dividend					
54	Dec 99	55.88	58.69	49.88	**50.81**	9,812,559	254.05		
55	Nov 99	47.13	61	44.94	**55.88**	13,277,338	279.4		
56	Oct 99	43.5	49.06	41.5	**46.75**	11,850,266	233.75		
57	Sep 99			$0.22 Cash Dividend					
58	Sep 99	45.38	48.81	41.81	**43.5**	10,775,514	217.5		
59	Aug 99	52.13	52.81	44.25	**45**	12,892,813	225		
60	Jul 99	55.94	59	51.75	**52.13**	9,257,600	260.65		
61	Jun 99			$0.22 Cash Dividend					
62	Jun 99	55.5	56.88	52.38	**55.81**	10,673,172	279.05		
63	May 99	51	63	50.88	**55.5**	14,542,265	277.5		
64	Apr 99			3:2 Stock Split					
65	Apr 99	79.81	89.5	50.06	**50.5**	13,690,428	252.5		
66	Mar 99			$0.33 Cash Dividend					
67	Mar 99	82.12	89	75.87	**79.81**	9,906,500	266.03		
68	Feb 99	91.94	95.12	82.12	**82.12**	8,755,210	273.73		
69	Jan 99	76.5	96.12	76.5	**90.75**	10,024,863	302.5		

17. Explain why each of the following statements is correct or incorrect:

    17.a. Diversification reduces risk because prices of stocks do not usually move exactly together.

17.b. The expected return on a portfolio is a weighted average of the expected returns on the individual securities.

17.c. The standard deviation of returns on a portfolio is equal to the weighted average of the standard deviations on the individual securities if these returns are completely uncorrelated.

18. Suppose that the annual returns on two stocks (A and B) are perfectly negatively correlated, and that $r_A = 0.05$, $r_B = 0.15$, $\sigma_A = 0.1$, $\sigma_B = 0.4$. Assuming that there are no arbitrage opportunities, what must the one-year interest rate be?

19. Assume that an individual can either invest all of her resources in one of two securities A or B; or alternatively, she can diversify her investment between the two. The distribution of the returns are as follows:

	A	B	C	D
1	Security A		Security B	
2	Return	Probability	Return	Probability
3	-10%	0.5	-20%	0.5
4	50%	0.5	60%	0.5

Assume that the correlation between the returns from the two securities is zero.

19.a. Calculate each security's expected return, variance, and standard deviation.

19.b. Calculate the probability distribution of the returns on a *mixed portfolio* comprised of equal proportions of securities A and B. Also calculate the expected return, variance, and standard deviation.

19.c. Calculate the expected return and the variance of a mixed portfolio comprised of 75% of security A and 25% of security B.

20. The correlations between the returns of three stocks A, B, and C are given in the following table:

	A	B	C	D
1	**Stock**	A	B	C
2	A	1.00	0.80	0.10
3	B		1.00	0.15
4	C			1.00

The expected rates of return on A, B, and C are 16%, 12%, and 15%, respectively. The corresponding standard deviations of the returns are 25%, 22%, and 25%.

20.a. What is the standard deviation of a portfolio invested 25% in stock A, 25% in stock B, and 50% in stock C?

20.b. You plan to invest 50% of your money in the portfolio constructed in part a of this question and 50% in the risk-free asset. The risk-free interest rate is 5%. What is the expected return on this investment? What is the standard deviation of the return on this investment?

21. You believe that there is a 15% chance that stock A will decline by 10% and an 85% chance that it will increase by 15%. Correspondingly, there is a 30% chance that stock B will decline by 18% and a 70% chance that it will increase by 22%. The correlation coefficient between the two stocks is 0.55. Calculate the expected return, the variance, and the standard deviation for each stock. Then calculate the covariance between their returns.

22. Outdoorsy people know that the crickets chirp faster when the temperature is warmer. Some evidence for this can be found in a book published in 1948 by Harvard physics professor George

W. Pierce.[9] Pierce's book includes the table below, which relates the average number of cricket chirps per minute to the temperature at which the data was recorded. Plot the data in an Excel graph and use regression to determine the (approximate) relation between the number of chirps per second and the temperature. If you detect 19 chirps per second, what would you guess the temperature to be? What about 22 chirps a second? (We know this problem has nothing to do with finance, but it's interesting!)

	A	B
4	**Chirps per second**	**Temperature in Farenheit**
5	20.0	88.60
6	16.0	71.60
7	19.8	93.30
8	18.4	84.30
9	17.1	80.60
10	15.5	75.20
11	14.7	69.70
12	17.1	82.00
13	15.4	69.40
14	16.2	83.30
15	15.0	79.60
16	17.2	82.60
17	16.0	80.60
18	17.0	83.50
19	14.4	76.30

23. Economists have long believed that the more money printed, the higher will be long-term interest rates. Evidence for this view can be found in the table below, which gives long-term

[9] Additional facts: Cricket chirping is produced by the rapid sliding of the cricket's wings one over the other. The higher the temperature, the faster the crickets slide their wings. George W. Pierce book is called *The Songs of Insects*, and was published by Harvard University Press.

government bond rates for 31 countries and the corresponding growth rate of money supply for each country.[10]

- Plot the data and use a regression to find the relation between the money growth and the long-term bond interest rate.
- If a country has zero money growth, what is its predicted long-term bond interest rate?
- The monetary authorities in your country are considering increasing the money growth rate by 1% from its current level. Predict by how much will this increase the long-term bond interest rate.
- Do you find the evidence in the table convincing? (Discuss briefly the R^2 of the regression.)

	A	B	C	D	E	F	G
38	**MONEY GROWTH AND BOND INTEREST RATES**						
39	**Country**	**Average money growth**	**Average long-term bond interest rate**		**Country**	**Average money growth**	**Average long-term bond interest rate**
40	US	5.65%	7.40%		New Zealand	10.29%	8.81%
41	Austria	6.82%	7.80%		South Africa	14.14%	11.11%
42	Belgium	5.20%	8.22%		Honduras	16.20%	15.57%
43	Denmark	9.43%	10.36%		Jamaica	19.88%	15.35%
44	France	8.15%	8.49%		Netherlands Antilles	4.36%	9.40%
45	Germany	8.00%	7.20%		Trinidad & Tobago	12.14%	9.10%
46	Italy	12.07%	10.66%		Korea	15.12%	16.53%
47	Netherlands	7.89%	7.31%		Nepal	15.55%	8.59%
48	Norway	10.64%	8.00%		Pakistan	12.79%	7.88%
49	Switzerland	5.53%	4.54%		Thailand	10.86%	10.62%
50	Canada	8.99%	8.52%		Malawi	20.80%	17.62%
51	Japan	9.07%	6.16%		Zimbabwe	13.49%	12.01%
52	Ireland	9.43%	10.38%		Solomon Islands	15.89%	12.12%
53	Portugal	12.91%	10.79%		Western Samoa	12.90%	13.17%
54	Spain	10.38%	12.72%		Venezuela	28.47%	28.92%
55	Australia	9.15%	8.95%				

[10] The data was first presented in an article entitled “Money and Interest Rates,” by Cyril Monnet and Warren Weber in the *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 2001. My thanks to the authors for providing

24. Mabelberry Fruit and Sawyer's Jam are two competing companies. An MBA student has done a calculation and found that the return on Sawyer's Jam stock is completely predictable once the return on Mabelberry Fruit stock is known:

$r_{Sawyer's,t} = 40\% - 1.5 * r_{Mabelberry,t}$.

24.a. Given the Mabelberry Fruit stock returns below, compute the Sawyer's Jam returns

24.b. Regress Mabelberry Fruit stock returns on those Sawyer's Jam. Can you explain the R^2 ?

	A	B
2	**Year**	**Mabelberry Fruit stock return**
3	1990	30.73%
4	1991	15.00%
5	1992	-9.00%
6	1993	12.00%
7	1994	13.00%
8	1995	22.00%
9	1996	30.00%
10	1997	12.00%
11	1998	43.00%
12	1999	16.00%

me with an Excel version of their data.

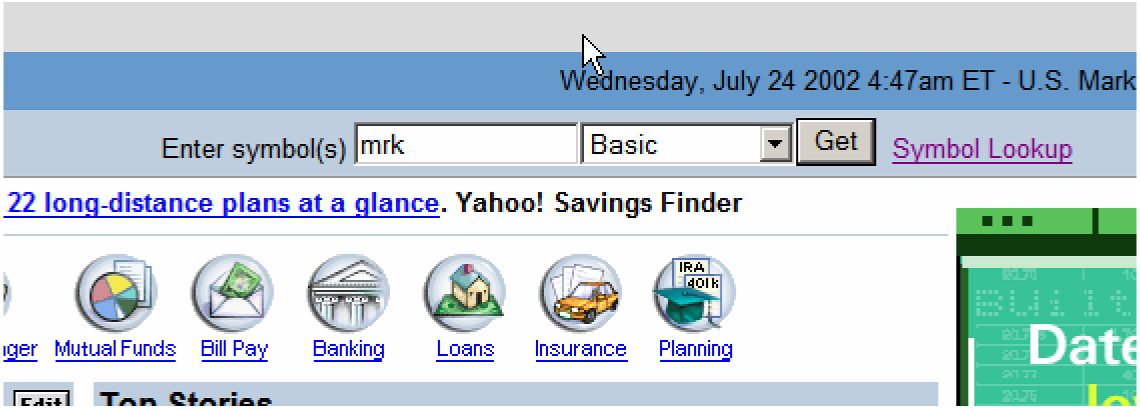
Appendix: Downloading data from Yahoo[11]

Yahoo provides free stock price and data which can be used to calculate returns. In this appendix we show you how to access this data and download it into Excel.

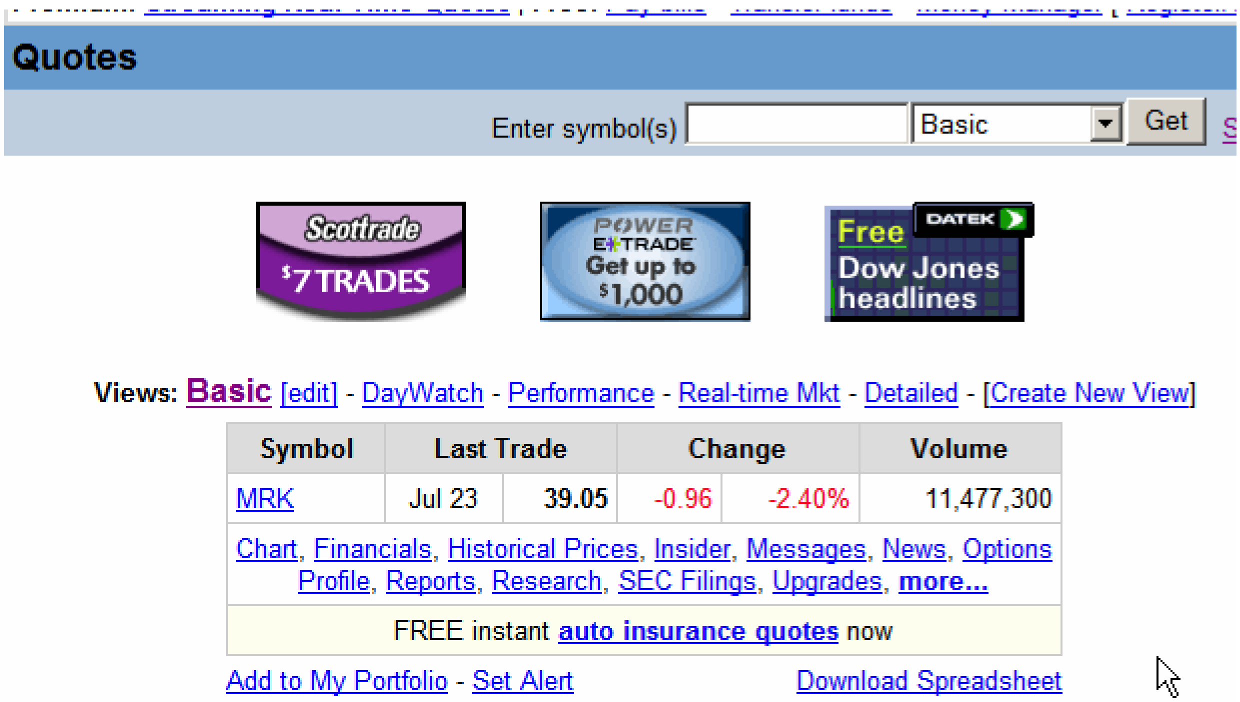
Step 1: Go to http://www.yahoo.com and click on Finance:

Step 2: In the "Enter symbol" box, put in the symbol for the stock you want to look up (we've put in MRK for Merck). You see that you can also look up symbols or put in multiple symbols. When you have put in the symbols, click on **Get**.

[11] Yahoo occasionally changes its interface; the information in this appendix is correct as of July 2003.



Step 3: This brings up the screen below. We will choose **Historical Prices** to get Merck's price history.



Step 4: In the next screen, we indicated the time period and frequency for the data we want. Yahoo provides a table with stock prices, dividends, and an **Adjusted Closing Stock Price** which accounts for dividends and stock splits:

Historical Prices - MRK (Merck & Co Inc)

More Info: Quote | Chart | News | Profile | Research

	Month	Day	Year	
Start:	Jan	01	99	○ Daily
				○ Weekly
End:	Jun	30	02	◉ Monthly
				○ Dividends

Ticker Symbol: mrk Get Data

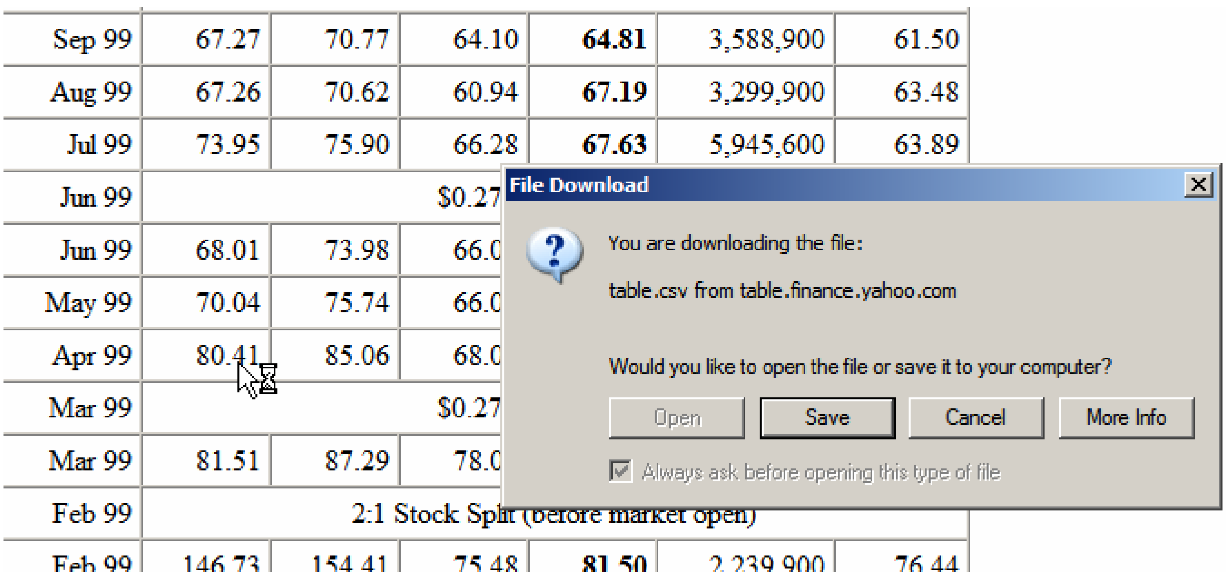
Date	Open	High	Low	Close	Volume	Adj. Close*
Jun 02	$0.35 Cash Dividend					
Jun 02	57.04	57.18	47.60	**50.64**	5,067,500	50.64
May 02	54.06	58.85	54.02	**57.10**	6,406,100	56.73
Apr 02	57.63	57.80	51.01	**54.34**	7,535,500	53.99
Mar 02	$0.35 Cash Dividend					
Mar 02	61.38	64.50	56.98	**57.58**	4,910,200	57.21
Feb 02	59.03	62.46	57.72	**61.33**	4,681,700	60.59
Jan 02	59.16	59.86	56.71	**59.18**	7,195,100	58.47
Dec 01	$0.35 Cash Dividend					
Dec 01	67.17	68.55	56.80	**58.80**	5,155,500	58.09

Step 5: The bottom of the above table allows you to download the data in spreadsheet format. In most browsers the Excel spreadsheet opens automatically (see results in Step 6):

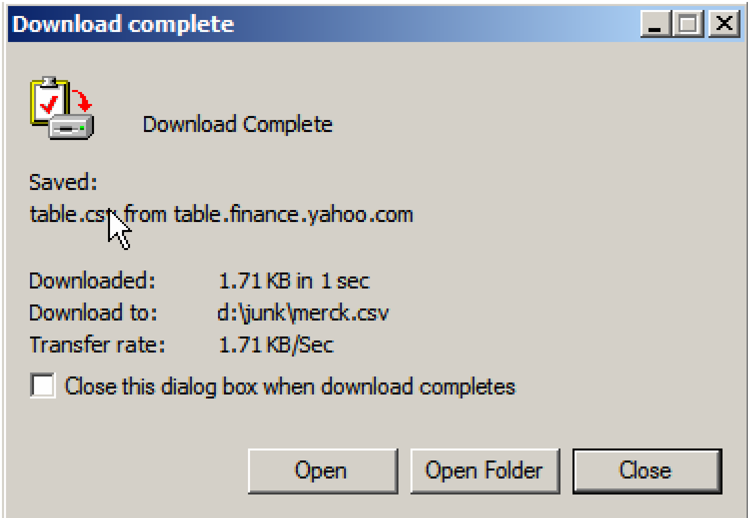
Mar 99	$0.27 Cash Dividend					
Mar 99	81.51	87.29	78.03	**80.13**	3,521,900	75.40
Feb 99	2:1 Stock Split (before market open)					
Feb 99	146.73	154.41	75.48	**81.50**	2,239,900	76.44
Jan 99	149.22	153.90	136.30	**146.75**	3,219,800	68.82

Download Spreadsheet Format
* adjusted for dividends and splits, please see FAQ.

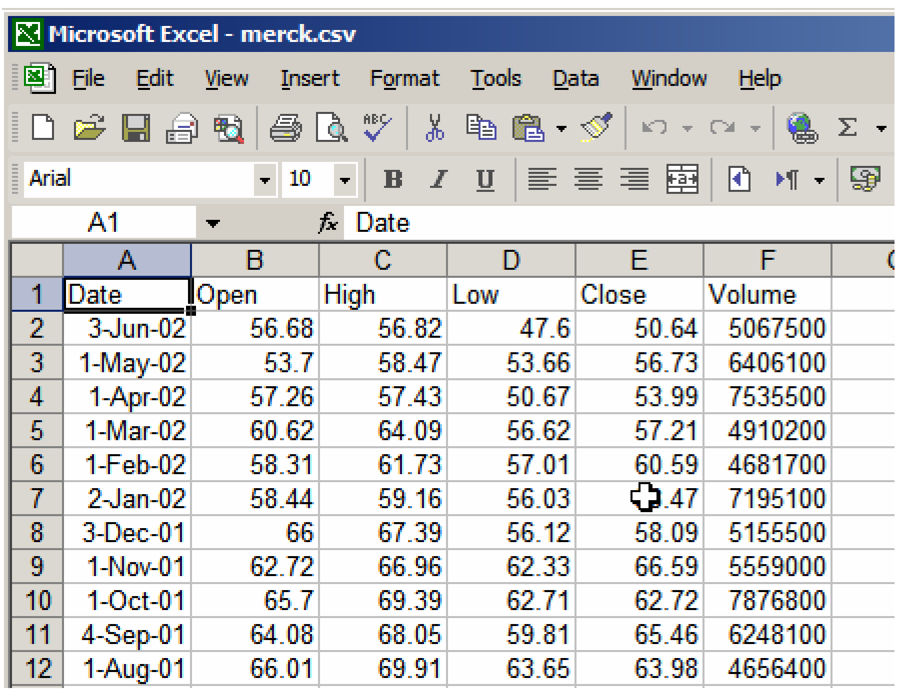
Step 6: In the author's browser Yahoo offers to save a file called **Table.csv**. We changed the name of this file to **Merck.csv** and saved it on our hard disk.



Step 7: The author's browser offered to open the file immediately (it will open as an Excel file):

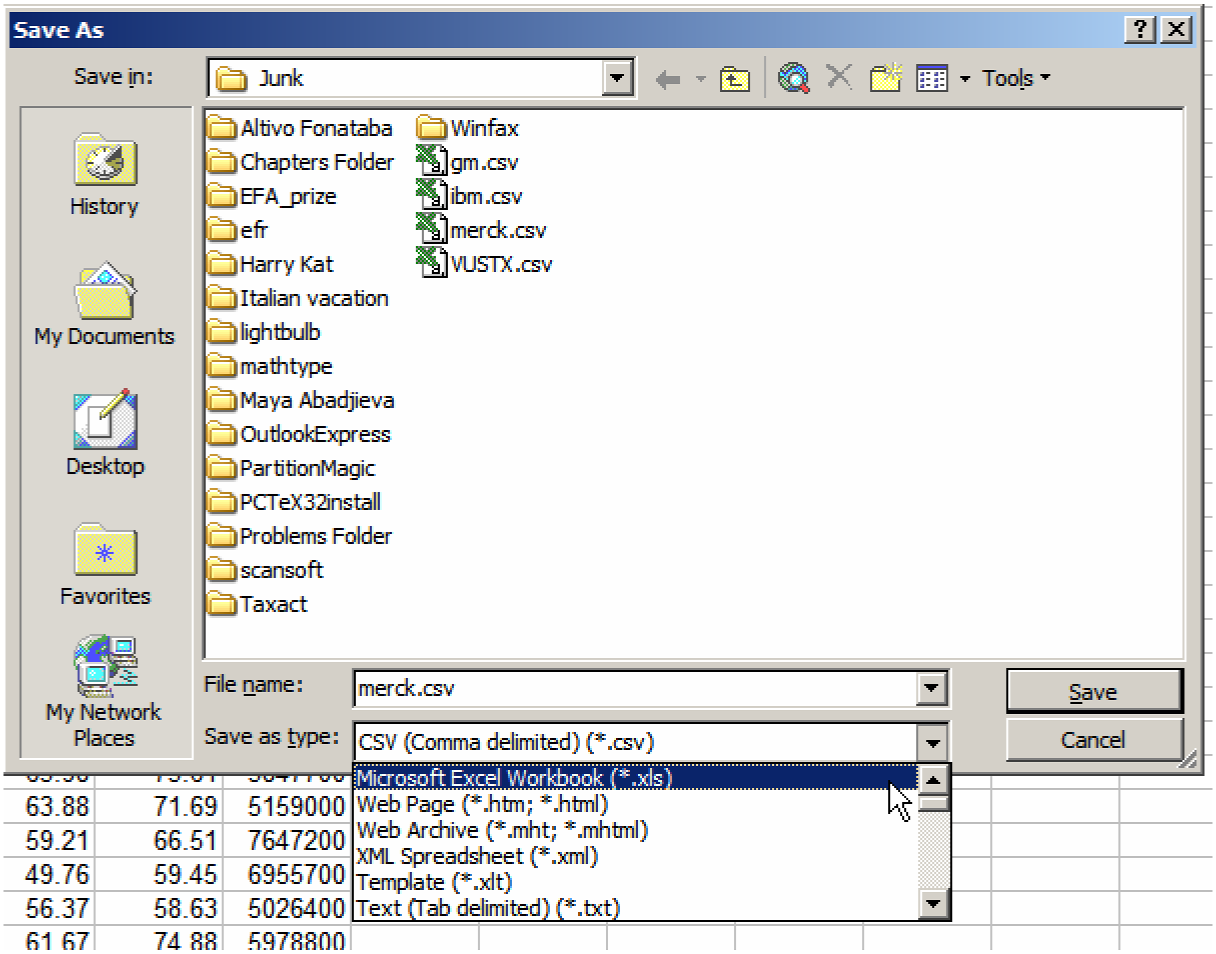


Here's the way the opened Excel file looks. Note that only the adjusted stock prices are given.



	A	B	C	D	E	F
1	Date	Open	High	Low	Close	Volume
2	3-Jun-02	56.68	56.82	47.6	50.64	5067500
3	1-May-02	53.7	58.47	53.66	56.73	6406100
4	1-Apr-02	57.26	57.43	50.67	53.99	7535500
5	1-Mar-02	60.62	64.09	56.62	57.21	4910200
6	1-Feb-02	58.31	61.73	57.01	60.59	4681700
7	2-Jan-02	58.44	59.16	56.03	[illegible].47	7195100
8	3-Dec-01	66	67.39	56.12	58.09	5155500
9	1-Nov-01	62.72	66.96	62.33	66.59	5559000
10	1-Oct-01	65.7	69.39	62.71	62.72	7876800
11	4-Sep-01	64.08	68.05	59.81	65.46	6248100
12	1-Aug-01	66.01	69.91	63.65	63.98	4656400

Step 7: It is advisable to use the Excel command **File|Save As** to save the file as a standard Excel file:


Save As
Save in:
Junk
Tools
History
My Documents
Desktop
Favorites
My Network Places
Altivo Fonataba
Chapters Folder
EFA_prize
efr
Harry Kat
Italian vacation
lightbulb
mathtype
Maya Abadjieva
OutlookExpress
PartitionMagic
PCTeX32install
Problems Folder
scansoft
Taxact
Winfax
gm.csv
ibm.csv
merck.csv
VUSTX.csv
File name:
merck.csv
Save as type:
CSV (Comma delimited) (*.csv)
Save
Cancel
Microsoft Excel Workbook (*.xls)
Web Page (*.htm; *.html)
Web Archive (*.mht; *.mhtml)
XML Spreadsheet (*.xml)
Template (*.xlt)
Text (Tab delimited) (*.txt)
63.88 71.69 5159000
59.21 66.51 7647200
49.76 59.45 6955700
56.37 58.63 5026400
61.67 74.88 5978800

CHAPTER 12: PORTFOLIO RETURNS—THE EFFICIENT FRONTIER*

This version: 15 August 2003

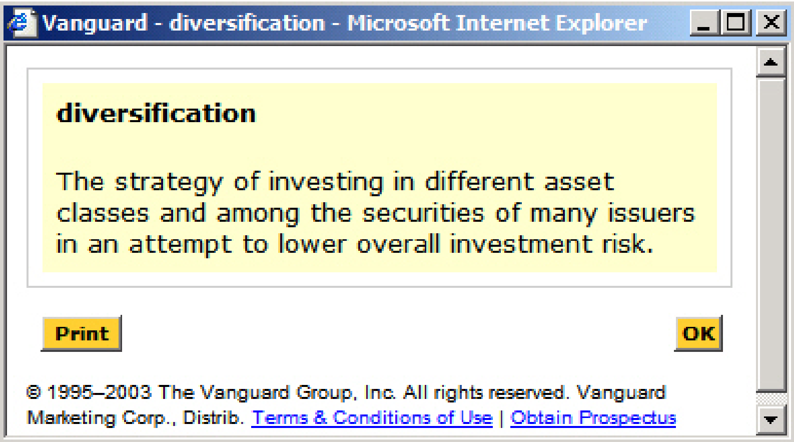
Chapter contents

* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance with Excel* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it. All the material is copyright and the rights belong to the author.

Overview

How should you invest your money? What's the best investment portfolio? How do you maximize your return without losing money? People often ask these knotty questions, and you may even be reading this book in order to answer them. However, to a large extent these questions have only partial answers. In this and the next chapter we explore some of these partial answers; you'll see that—although no one can tell you exactly how to invest—we can shed considerable light on some important general investment principles. We can also show you some rules of thumb about how *not to invest*.

Let's go back to the questions with which we started the previous paragraph:



- How should you invest your money? Finance can't tell you *in what* to invest, but it can give you some guidelines. The most important of these is: You should *diversify* your investment—spread it out among many assets in order to lower the risk. Using simple examples with only two stocks, this chapter will show you how diversification can lower investment risk.
- What is the best investment portfolio? It won't surprise you that the finance answer to this question tells you that there is no single best investment portfolio. It all depends on

your willingness to trade off *return* for *additional risk*.[1] What may surprise you, however, is that we can say a lot about how not to invest. In this chapter we develop the notion of the *efficient frontier*—this is the set of all portfolios that you would consider as investment portfolios. Inherent in the concept of the efficient frontier is that there are many portfolios that are not good investments, and that these portfolios can be somehow described (statistically).

- How do you maximize your return without losing money? To some extent the efficient frontier answers this question: It shows us which portfolios are so bad that you can improve both the return and the risk. Once we've gotten on the efficient frontier, however, the risk-return tradeoff begins to operate, and higher returns mean larger risks.[2]

In most of this chapter we examine the risk and return of portfolios composed of two financial assets. By choosing a combination of the two assets, you can achieve significant reductions in risk.[3] Much of the chapter relies on the statistics for portfolios discussed in the previous chapter. Even our main example, which considers portfolios of General Motors (GM) and Microsoft (MSFT) stock, is one we started in Chapter 11.

---

[1] As you learned in Chapter 10, nearly all the interesting finance questions involve the word "risk." Portfolio choice is no different!

[2] As the author's father used to say: "It is better to be rich and healthy than poor and sick." The investment interpretation of this is that we would all like to *have more return* and *risk less*. The efficient frontier represents the set of difficult investment choices: Once you're on the efficient frontier, it is impossible to get more return without taking on more risk.

[3] Of course in the real world there are many investment assets. We use the two-asset case to develop the requisite intuitions and ask you to take it on faith that the multi-asset case is similar.

The close links in the materials of this chapter and the materials in Chapter 11 should not blind you to their differences. Whereas Chapter 11 develops the statistical concepts necessary for portfolio choice, this chapter looks at portfolio choice as an economic choice. In this chapter we develop concepts that help us think more precisely about acceptable and unacceptable portfolios. In the next chapter we carry this line of thought further.

Finance concepts in this chapter

- Mean and standard deviation of portfolio of two assets
- Portfolio risk and return
- Minimum variance portfolio
- The efficient frontier
- Mean-variance calculations for three-asset portfolios

Excel concepts and functions used

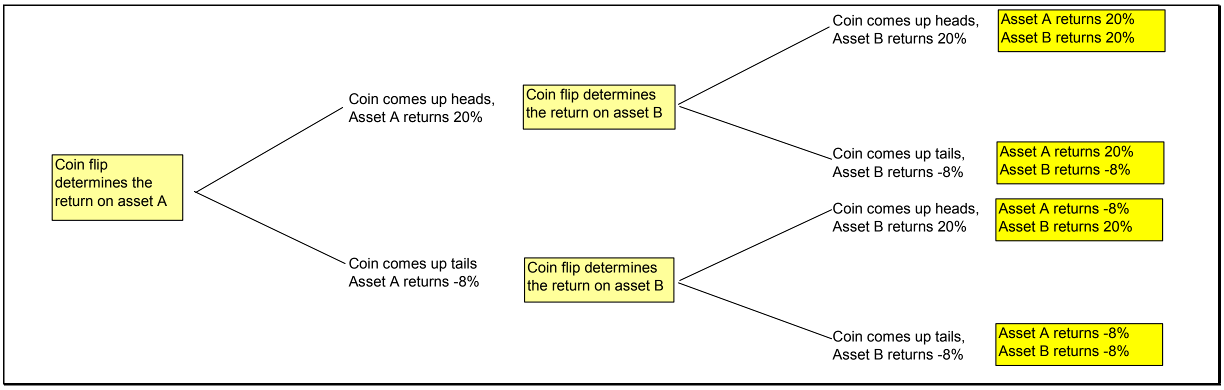
- **Average( ), Varp( ), Stdevp( )**
- Regression
- Sophisticated graphing
- **Solver**

12.1. The advantage of diversification—a simple example

In this section we give an example that illustrates the benefits of diversification. In finance jargon, diversification means investing in several different assets as opposed to putting all of your money in one single asset. In our examples you will see when diversification pays off (and when it doesn't). The examples are much simpler than the real-world examples that follow in the next sections, but they embody many of the intuitions of why investors invest in portfolios. In particular, you will see how the correlation between asset returns is important in determining the amount of risk-reduction you can get through portfolio formation.

In each of the following examples you can invest two assets. The return on each asset is uncertain and is determined by the flip of a coin: If the coin comes up heads, the asset returns 20% and if the coin comes up tails, the asset returns -8%. In dollar terms: If you invest $100 in one of the two assets, you'll get back $120 if the coin comes up heads and $92 if it comes up tails.

In terms of the sequence of coin flips, here's what the asset returns look like:



Case 1: Investing in a single risky asset

Suppose you decide to invest your $100 wholly in asset A. If the coin comes up heads, you'll earn 20% on your investment, and if it comes up tails you will lose 8%. Your $100 investment in asset A will have the following cash flow and return pattern:

	A	B	C	D	E	F	G	H	I	J
1	**CASE 1: MEAN AND STANDARD DEVIATION OF RETURN FROM A SINGLE COIN FLIP**									
2	Coin flip:									
3	heads		**Cash flow**		**Return**			**Return statistics**		
4			120		20%	<-- =C4/A6-1		Average return	6.00%	<-- =AVERAGE(E4,E8)
5								Variance	0.0196	<-- =VARP(E4,E8)
6	100							Standard deviation	14.00%	<-- =SQRT(I5)
7										
8			92		-8%	<-- =C8/A6-1				
9	Coin flip:									
10	tails									
11										

Notice the return statistics in column I: Asset A has average return of 6% and return standard deviation of 14%.[4]

Case 2: The case of the "fair" coin: splitting your investment between the assets

In Case 1 you invested only in one asset. In Cases 2-5 you will invest in both assets A and B.

In Case 2 we suppose that the coin that determines the returns on asset A and the coin that determines the returns on asset B are *uncorrelated*. In simple terms you can think of a single coin that is flipped twice—once to determine the return of A and the second time to determine the return of B. If the coin flip is "fair" then the results of the first coin flip have no influence on the results of the second coin flip.

Now here's the question we want to answer: Should you invest all your money in A? in B? Or should you split your investment between the two? The answer has to do with the effects of diversification. To examine this question more closely, let's assume that you have decided to invest $50 in each asset. Your final outcomes are given below:

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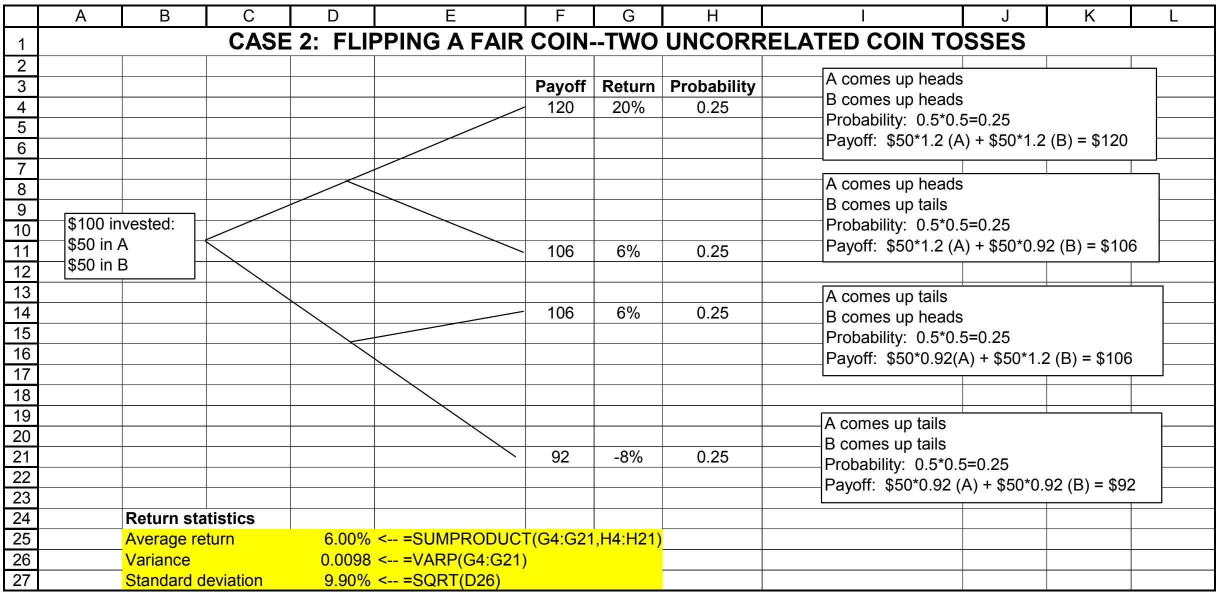
[4] You'll notice that we've used the Excel function **Varp** to compute the portfolio variance and *not* the function **Var**. The reasons for this choice—which we make throughout the book—were given in Chapter 11. Similarly we would



As you can see the average return from the investment in two assets (6 percent) is the same as the average return in case 1, where we invested in only one asset. Note, however, that the standard deviation went down from 14 percent to 9.9 percent—you earn the same but incur less risk.

Message: Diversification in uncorrelated assets improves your investment returns even if the asset returns are the same.

This message—that diversification pays off because it reduces risk—can be explored further. In the next example we explore the returns when you have *correlated* assets.

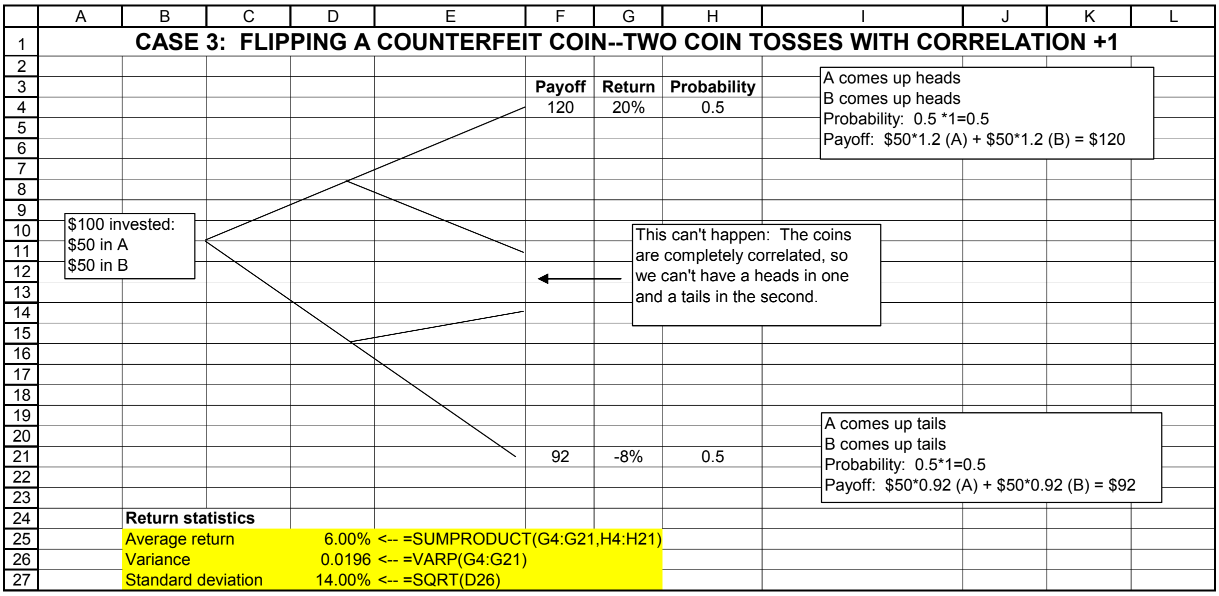
Case 3: The case of the counterfeit coin: a correlation of +1

Now suppose you have the same situation as above; only this time your coin is counterfeit. You do not know if you will get heads or tails but you do know that whatever the

---

use **StDevp** to compute the standard deviation and not **StDev** (although in this particular example, we've computed the portfolio standard deviation by taking the square root of its variance).

result of the "A" coin, the result of the "B" coin will be the same. In statistical terms this is a correlation of +1. Will diversification improve your returns in this situation?

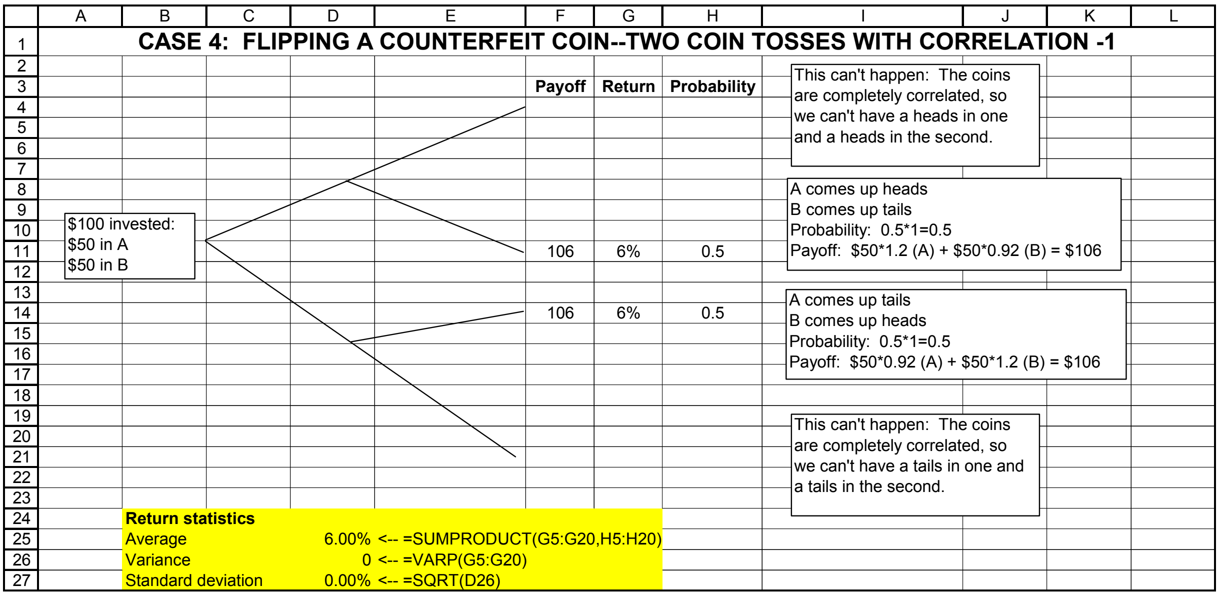


As you can see the returns from splitting your investment between two assets are identical to the return of only investing in one asset (cells D25:D27). Both the average return and the return standard deviation are the same as in case 1, where we flipped only one coin.

Message: When the asset returns are perfectly positively correlated, diversification will not reduce your risk.

Case 4: The case of the counterfeit coin—correlation of -1

We're still on the same example, and our coin is still counterfeit. But this time it's counterfeit with a perfectly negative correlation (-1): If coin "A" comes up heads, coin "B" will come up tails. In statistical terms, the correlation between the two coins is -1. For this case we can find a portfolio that completely eliminates all risk: By splitting our investment between assets A and B, we get 6 percent expected return without any standard deviation:



Message: When the asset returns are perfectly negatively correlated, diversification can completely eliminate all risk.

Case 5: The partially counterfeit coin (the real world?)

In the real world there's often a connection between the stock prices of one company and those of another. In the most general handwaving[5] way, stock prices reflect two elements:

- How well a particular business is doing: In some industries this element leads to *negative correlation*. For example if Procter & Gamble (a major manufacturer of toothpastes, laundry soaps, and so on) is gaining market share, it is likely to be at the expense of Unilever (another company in the same industry). This isn't always true, though: If Intel (a major manufacturer of computer chips) is doing well, then it may be that the computer

[5] The website http://c2.com defines "handwaving" as: "Handwaving is what people do when they don't want to tell you the details, either because they don't want to get bogged down, they don't know, nobody knows, or they have sinister ulterior motives."

industry is expanding and that AMD (another player in the same industry) is also doing well.

- How well the economy is doing: Stock prices are heavily affected by the performance of the economy. This factor tends to be an across-the-board factor, leading to *positive correlation*: When the stock market as a whole is up, most stock prices tend to be up, and vice versa. For stock prices, this factor tends to dominate the first: in general stock prices move together, though their correlation is far from complete.

Notice how careful we've been here in our language: We've used words like "tend to go up"—stock prices are only partially, not perfectly, correlated.[6]

In order to model partial correlation with our coin toss example, we'll assume that the "A" coin result influences the result of the "B" coin, but not completely. If the "A" coin comes up heads (this happens with a probability of 0.5), the probability of the "B" coin coming up heads is 0.7. If the "A" coin comes up tails (probability 0.5), the probability that the "B" coin also comes up tails is 0.7. Here's the spreadsheet that summarizes the returns:

[6] Negative correlation in stock returns can also happen: Our General Motors-Microsoft example in Chapter 11—to which we return in Section 12.2 of this chapter—is an illustration.

CASE 5: FLIPPING A PARTIALLY COUNTERFEIT COIN
TWO COIN TOSSES WITH IMPERFECT CORRELATION

$100 invested:
$50 in A
$50 in B

	Payoff	Return	Probability	
4	120	20%	0.35	A comes up heads B comes up heads Probability: 0.5*0.7=0.35 Payoff: $50*1.2 (A) + $50*1.2 (B) = $120
11	106	6%	0.15	A comes up heads B comes up tails Probability: 0.5*0.3=0.15 Payoff: $50*1.2 (A) + $50*0.92 (B) = $106
14	106	6%	0.15	A comes up tails B comes up heads Probability: 0.5*0.3=0.15 Payoff: $50*0.92(A) + $50*1.2 (B) = $106
21	92	-8%	0.35	A comes up tails B comes up tails Probability: 0.5*0.7=0.35 Payoff: $50*0.92 (A) + $50*0.92 (B) = $92

Return statistics		
Average	6.00%	<-- =SUMPRODUCT(G4:G21,H4:H21)
Variance	0.01372	<-- =H4*(G4-$D$25)^2+H11*(G11-D25)^2+H14*(G14-D25)^2+H21*(G21-D25)^2
Standard deviation	11.71%	<-- =SQRT(D26)

Message: When the asset returns are partially correlated, diversification will reduce risk but not completely eliminate it.

What's the point?

Though the two-asset, two-coin examples are simple and farfetched, the lessons you learn from these examples also apply in the "real world" cases of asset diversification:

- If the correlation between asset returns is +1 then diversification will not reduce portfolio risk.
- If the correlation between asset returns is -1 then we can create a risk-free asset—an asset with no uncertainty about its returns (a bank savings account is an example)—using a portfolio of the two assets.

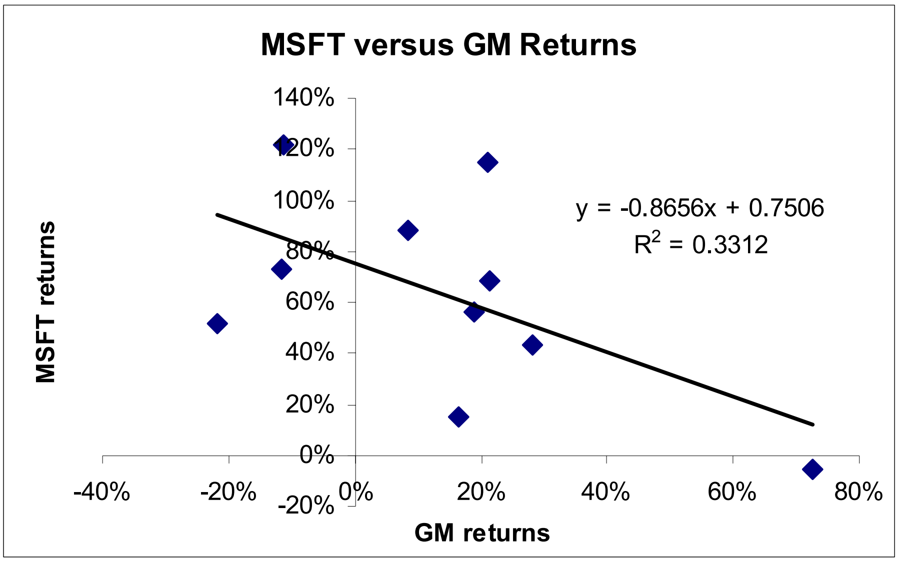
- In the real world asset returns are almost never fully correlated. When asset returns are partially, but not completely, correlated (meaning that the correlation is between -1 and +1), diversification can lower risk, though it cannot completely eliminate it.

12.2. Back to the real world—Microsoft and General Motors

In Chapter 11 we calculated the data for the annual returns on General Motors (GM) stock and on Microsoft (MSFT) stock for the 10 years between 1990 – 1999. Here are our calculations:

	A	B	C	D
1	**GM AND MSFT RETURN STATISTICS, 1990-1999**			
2	**Date**	**GM return**	**MSFT return**	
3	31-Dec-90	-11.54%	72.99%	
4	31-Dec-91	-11.35%	121.76%	
5	31-Dec-92	16.54%	15.11%	
6	31-Dec-93	72.64%	-5.56%	
7	30-Dec-94	-21.78%	51.63%	
8	29-Dec-95	28.13%	43.56%	
9	31-Dec-96	8.46%	88.32%	
10	31-Dec-97	19.00%	56.43%	
11	31-Dec-98	21.09%	114.60%	
12	31-Dec-99	21.34%	68.36%	
13				
14	Average	14.25%	62.72%	<-- =AVERAGE(C3:C12)
15	Variance	0.0638	0.1443	<-- =VARP(C3:C12)
16	Standard deviation	25.25%	37.99%	<-- =STDEVP(C3:C12)
17	Covariance of returns	-0.0552	<-- =COVAR(B3:B12,C3:C12)	
18	Correlation of returns	-0.5755	<-- =B17/(B16*C16)	

You can see that the average return of holding GM stock (14.25% per year) is much lower than the average return of holding MSFT stock (62.73%). On the other hand, the risk of holding Microsoft—measured by either the variance or by the standard deviation of the return—is higher than the risk of General Motors: This is the tradeoff we would expect—GM has lower return and lower risk than MSFT. Note also that GM and Microsoft returns are *negatively correlated* (cell B18): On average, an increase in MSFT returns was accompanied by a decrease in GM returns. If you use Excel to plot GM returns on the *x*-axis and MSFT returns on the *y*-axis, you can detect a slight "northwest to southeast" pattern in the returns.



The trendline (which illustrates the regression of MSFT on GM) shows this trend.[7]

12.3. Graphing portfolio returns

In this section we graph the returns available to the investor from an investment in a portfolio composed of GM and MSFT stock. We start by showing you several individual

[7] As explained in Chapter 11, the regression R^2 indicates the percentage MSFT's return variability explained by the variability in GM's returns. R^2 is the correlation coefficient squared: $R^2 = 0.3312 = \left[Correlation\left(Return_{GM}, Return_{MSFT}\right)\right]^2 = (-0.5755)^2$. While this R^2 may seem low, it is typical for the relation between 2 stocks.

portfolios, and end the section by graphing the curve representing all the possible portfolio returns.

Deriving the risk-return of an individual portfolio

Suppose we form a portfolio composed of 50% GM and 50% MSFT stock. Cells E8:E17 in the spreadsheet below show the annual returns of this portfolio:

	A	B	C	D	E	F
1	**A PORTFOLIO OF GM AND MSFT STOCK**					
2	**Portfolio proportions**					
3	Percentage in GM	50%				
4	Percentage in MSFT	50%	<-- =1-B3			
5						
6	**Date**	**Stock returns**			**Portfolio returns**	
7		**GM**	**MSFT**			
8	Dec-90	-11.54%	72.99%		30.73%	<-- =$B$3*B8+$B$4*C8
9	Dec-91	-11.35%	121.76%		55.21%	
10	Dec-92	16.54%	15.11%		15.82%	
11	Dec-93	72.64%	-5.56%		33.54%	
12	Dec-94	-21.78%	51.63%		14.93%	
13	Dec-95	28.13%	43.56%		35.84%	
14	Dec-96	8.46%	88.32%		48.39%	
15	Dec-97	19.00%	56.43%		37.71%	
16	Dec-98	21.09%	114.60%		67.85%	
17	Dec-99	21.34%	68.36%		44.85%	
18						
19	Average	14.25%	62.72%		38.49%	<-- =AVERAGE(E8:E17)
20	Variance	6.38%	14.43%		2.44%	<-- =VARP(E8:E17)
21	Sigma	25.25%	37.99%		15.62%	<-- =STDEVP(E8:E17)
22	Covariance of returns	-5.52%	<-- =COVAR(B8:B17,C8:C17)			

As discussed in Chapter 11, the portfolio return statistics in cells E19:E21 can be derived using formulas which involve only information about the individual asset returns, their variances, and the covariance. There's no need to do the extensive calculation in cells E19:E21:

- The average portfolio return of 38.49% is the *weighted* average of the GM and the MSFT return. Write the percentage weight of GM stock by w_{GM} and the percentage weight of

MSFT stock by w_{MSFT} ; it follows, of course, that $w_{MSFT} = 1 - w_{GM}$, since the portfolio proportions must sum to 100%. The formula for the average portfolio return is:

$$\boxed{\begin{aligned} \textit{average portfolio return},\ E(r_p) &= w_{GM}E(r_{GM}) + w_{MSFT}E(r_{MSFT}) \\ &= w_{GM}E(r_{GM}) + (1 - w_{GM})E(r_{MSFT}) \end{aligned}}$$

- The variance of the portfolio return, 2.44%, is a more complicated function of the two variances and the portfolio weights:

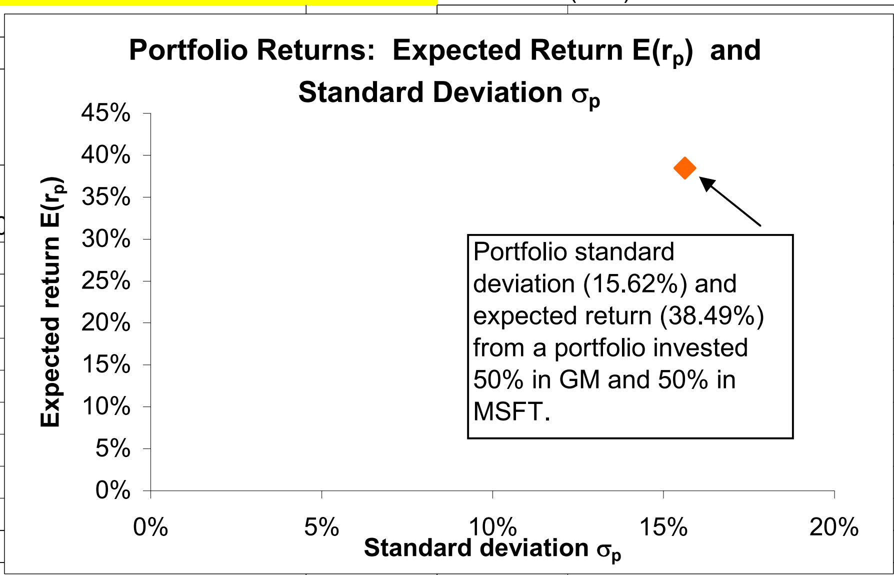
$$\textit{variance of portfolio return}$$
$$Var(r_p) = w_{GM}^2 Var(r_{GM}) + w_{MSFT}^2 Var(r_{MSFT}) + \underbrace{2w_{GM}w_{MSFT}Cov(r_{GM}, r_{MSFT})}$$

Each portfolio weight is squared and multiplied times the variance

Twice the product of the portfolio weights times the covariance

By using these two formulas, you avoid the need for the long calculation of the portfolio return, variance, and standard deviation in cells E8:E21. In the spreadsheet below we incorporate these formulas for the portfolio mean, variance and standard deviation in cells B12:B14:

	A	B	C	D
1	**PORTFOLIO STATISTICS FOR A GM-MSF PORTFOLIO**			
2		**GM**	**MSFT**	
3	Average	14.25%	62.72%	
4	Variance	6.38%	14.43%	
5	Sigma	25.25%	37.99%	
6	Covariance of returns	-5.52%	<-- =COVAR(B9:B18,C9:C18)	
7				
8	**Portfolio return and risk**			
9	Percentage in GM	50%		
10	Percentage in MSFT	50%		
11				
12	Expected portfolio return	38.49%	<-- =B9*B3+B10*C3	
13	Portfolio variance	2.44%	<-- =B9^2*B4+B10^2*C4+2*B9*B10*B6	
14	Portfolio standard deviation	15.62%	<-- =SQRT(B13)	
15				
16				
17				
18	50			%
19				
20				
21				
22				
23				
24				
25				
26				
27				
28				
29				



The point here is that you don't need to do an extensive calculation of annual portfolio returns—it's enough to know the return statistics for each stock, the portfolio proportions, and the covariance of the stock returns.

Another portfolio—increasing the weight of MSFT, decreasing GM

Now suppose we graph another portfolio—this time a portfolio invested 25% in GM and 75% in Microsoft:

	A	B	C	D
1	**PORTFOLIO STATISTICS FOR A GM-MSFT PORTFOLIO**			
2		**GM**	**MSFT**	
3	Average	14.25%	62.72%	
4	Variance	6.38%	14.43%	
5	Sigma	25.25%	37.99%	
6	Covariance of returns	-5.52%	<-- =COVAR(B9:B18,C9:C18)	
7				
8	**Portfolio return and risk**			
9	Percentage in GM	25%		
10	Percentage in MSFT	75%		
11				
12	Expected portfolio return	50.60%	<-- =B9*B3+B10*C3	
13	Portfolio variance	6.44%	<-- =B9^2*B4+B10^2*C4+2*B9*B10*B6	
14	Portfolio standard deviation	25.39%	<-- =SQRT(B13)	
15				
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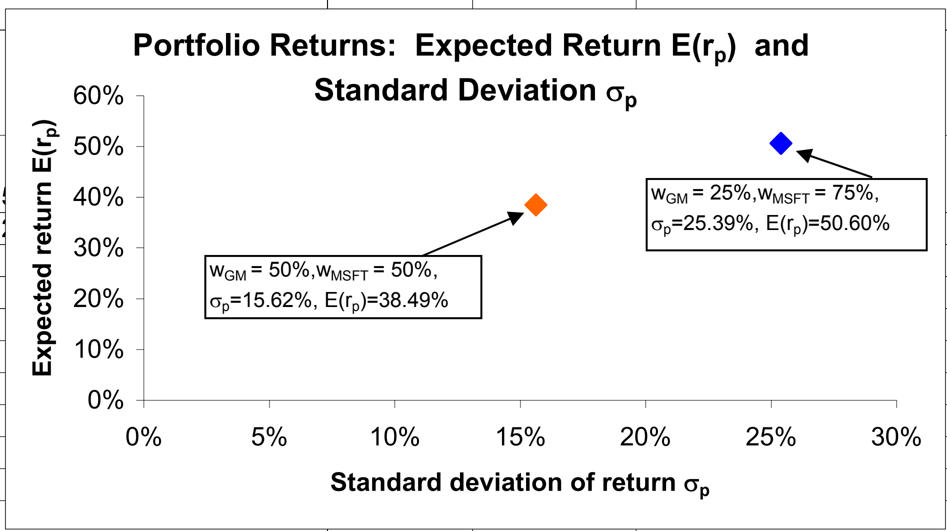
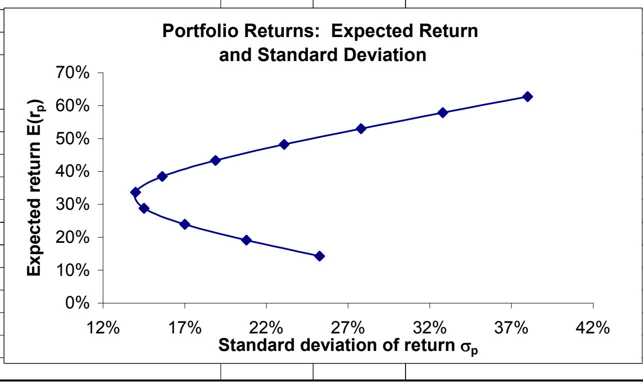


Notice that the new portfolio's performance is to the "northeast" of the first portfolio—it has both higher returns and higher standard deviation. The new portfolio gives you greater expected return, but has higher risk. This is what you would expect—higher return is achieved at the price of higher risk. As you will see in the next subsection, this may not always be the case.

Varying the portfolio composition—graphing all possible portfolios

Suppose we vary the composition of the portfolio, letting the percentage of GM vary from 0% to 100%. In cells G19:H29 below we generate a table of portfolio returns $E(r_p)$ and standard deviations σ_p.

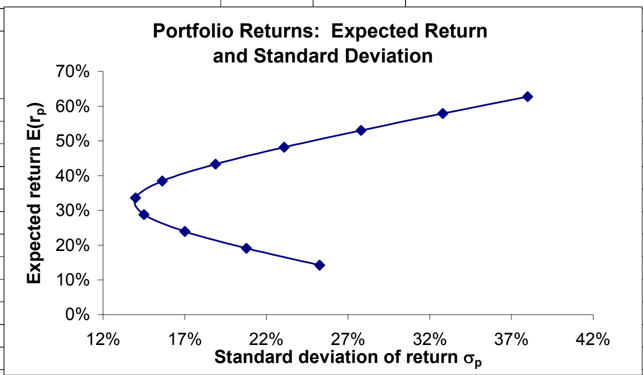
	A	B	C	D	E	F	G	H	I
1	PORTFOLIO STATISTICS FOR A GM-MSFT PORTFOLIO								
2		GM	MSFT						
3	Average	14.25%	62.72%						
4	Variance	6.38%	14.43%						
5	Sigma	25.25%	37.99%						
6	Covariance of returns	-5.52%	<-- =COVAR(B9:B18,C9:C18)						
7									
8	**Portfolio return and risk**								
9	Percentage in GM	50%							
10	Percentage in MSFT	50%							
11									
12	Expected portfolio return	38.49%	<-- =B9*B3+B10*C3						
13	Portfolio variance	2.44%	<-- =B9^2*B4+B10^2*C4+2*B9*B10*B6		=SQRT(F18^2*$B$3+(1-F18)^2*$C$3				
14	Portfolio standard deviation	15.62%	<-- =SQRT(B13)		+2*F18*(1-F18)*$B$5)				
15									
16									
17								=F18*$B$2+(1-F18)*$C$2	
18						Percentage in GM	Sigma	Expected return	
19						0.0	37.99%	62.72%	
20						0.1	32.80%	57.87%	
21						0.2	27.79%	53.03%	
22						0.3	23.08%	48.18%	
23						0.4	18.88%	43.33%	
24						0.5	15.62%	38.49%	
25						0.6	13.98%	33.64%	
26						0.7	14.51%	28.79%	
27						0.8	17.01%	23.95%	
28						0.9	20.78%	19.10%	
29						1.0	25.25%	14.25%	
30									



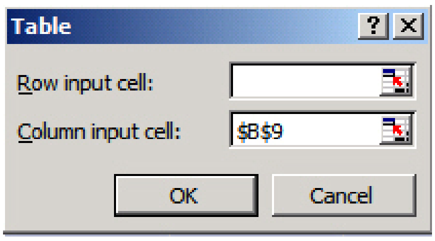
An Excel Note: Using Data Table to simplify the calculations

The table in cells G18:H28 above were generated using formulas for the standard deviation and expected return. Each cell contains a formula (note the use of absolute and relative cell references in these formulas). You can simplify the building of the table by using the **Data table** technique discussed in Chapter 30. **Data table** is not an easy technique to master, but it makes building tables much easier. Here's an example:

	A	B	C	D	E	F	G	H	I
1	**PORTFOLIO STATISTICS FOR A GM-MSFT PORTFOLIO using Data Table**								
2		**GM**	**MSFT**						
3	Average	14.25%	62.72%						
4	Variance	6.38%	14.43%						
5	Sigma	25.25%	37.99%						
6	Covariance of returns	-5.52%	<-- =COVAR(B9:B18,C9:C18)						
7									
8	**Portfolio return and risk**								
9	Percentage in GM	50%							
10	Percentage in MSFT	50%							
11									
12	Expected portfolio return	38.49%	<-- =B9*B3+B10*C3						
13	Portfolio variance	2.44%	<-- =B9^2*B4+B10^2*C4+2*B9*B10*B6						
14	Portfolio standard deviation	15.62%	<-- =SQRT(B13)						
15							Contains formula "=B14"	Contains formula "=B12"	
16									
17						**Percentage in GM**	**Sigma**	**Expected return**	
18							15.62%	38.49%	
19						0%	37.99%	62.72%	
20						10%	32.80%	57.87%	
21						20%	27.79%	53.03%	
22						30%	23.08%	48.18%	
23						40%	18.88%	43.33%	
24						50%	15.62%	38.49%	
25						60%	13.98%	33.64%	
26						70%	14.51%	28.79%	
27						80%	17.01%	23.95%	
28						90%	20.78%	19.10%	
29						100%	25.25%	14.25%	
30									

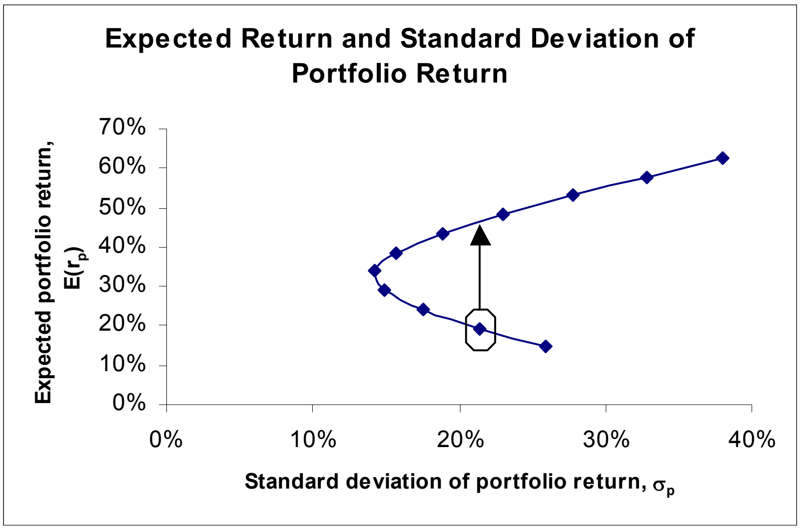


You create the data table by marking the cells G17:H29. The command **Data|Table** brings up the dialog box to which you add the appropriate cell reference:

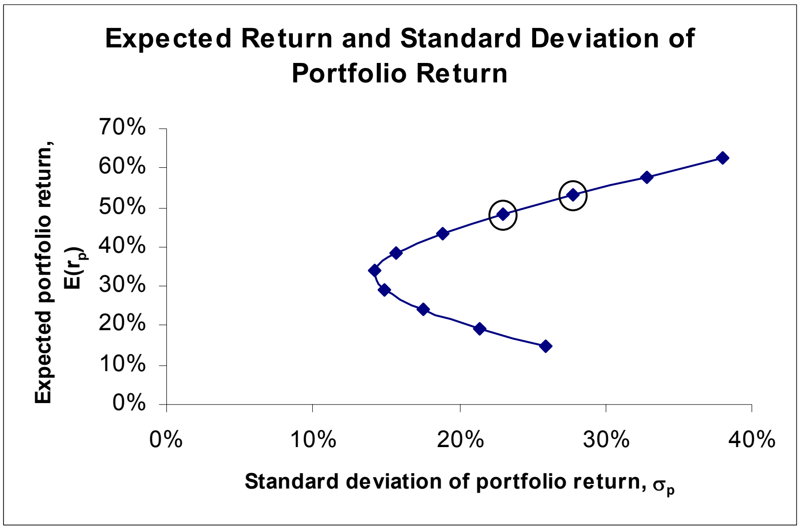


Better portfolios ... worse portfolios ...

Take a careful look at the graph in the above spreadsheet—it shows the standard deviation σ_p of the portfolio returns on the *x*-axis and the corresponding expected portfolio return $E(r_p)$ on the *y*-axis. Looking at the graph it is easy to see that some portfolios are better than others. Consider, for example, the portfolio invested 90% in GM and 10% in MSFT (this portfolio is circled in the graph below). By investing in the portfolio indicated by the arrow, you can improve the expected return without increasing the riskiness of the return. Thus the circled portfolio is not optimal. In fact none of the portfolios on the bottom part of the graph are optimal: Each is dominated by a portfolio on the top part of the graph which has the same standard deviation σ_p and higher expected return $E(r_p)$.



On the other hand, consider the two portfolios circled below:



There is a clear risk-return tradeoff between these two portfolios—it is impossible to say that one is unequivocally better than the other. The portfolio with the higher return also has the higher standard deviation of returns. All of the portfolios on the top part of the graph have this property. This top part of the graph is called the *efficient frontier*. The efficient frontier is the area of hard portfolio choices—along the efficient frontier, portfolios with greater expected return require you to undertake greater risk.

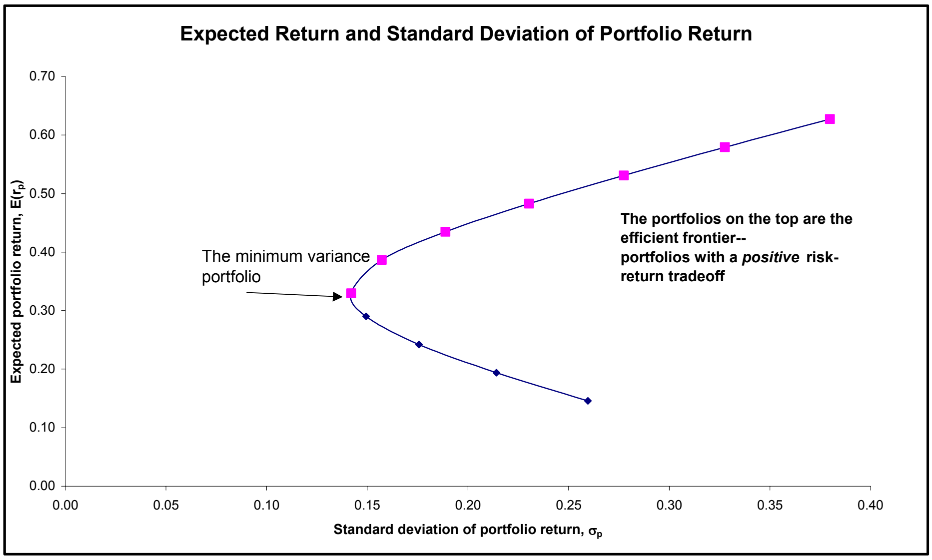
The efficient frontier slopes upward from left to right. What this means is that the choice between any two portfolios on the efficient frontier involves a tradeoff between higher expected portfolio return, $E(r_p)$, and higher risk as indicated by a higher standard deviation of the return, σ_p. An investor choosing only risky portfolios would choose a portfolio on the efficient frontier.

In the next section we investigate some of the properties of the efficient frontier.

12.4. The efficient frontier and the minimum variance portfolio

The *efficient frontier* is the set of all portfolios which are on the upward-sloping part of the graph above. "Upward-sloping" means that portfolios on the efficient frontier involve difficult choices—increasing expected portfolio return $E(r_p)$ has the cost of increasing portfolio standard deviation σ_p. If you are choosing investment portfolios that are a mix of GM and MSFT stock, then clearly the only portfolios you would be interested in are those on the efficient frontier. These portfolios are the only ones which have a "northeast" risk-return relation.

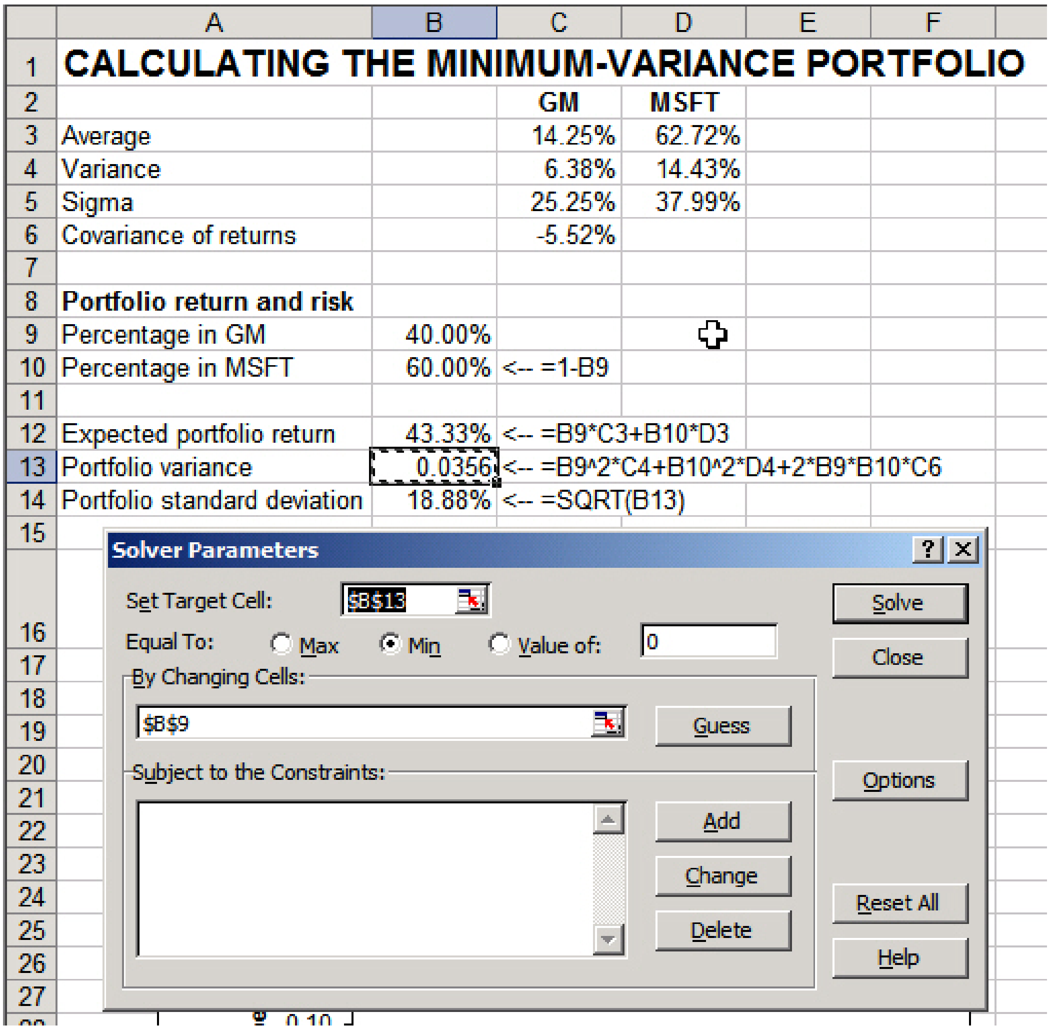
In order to calculate the efficient frontier, we have to find its starting point, the portfolio with the minimum standard deviation of returns. In the jargon of finance, this portfolio is (somewhat confusingly) called the *minimum-variance portfolio*; just recall that if the portfolio has minimum variance it also has minimum standard deviation. The minimum variance portfolio is the portfolio on the right-hand corner of the efficient frontier; the graph below indicates its approximate location:



We can find the minimum variance portfolio in two ways—either by using the **Solver** or by using a bit of mathematics. We illustrate both methods:

Using Excel's Solver

By using the **Solver** (see Chapter 32), we can calculate the percentage of GM in a portfolio which has minimum variance. The screen below shows the **Solver** dialog box. In this box, we've asked **Solver** to minimize the portfolio variance (cell B13) by changing the percentage of GM stock in the portfolio (cell B9):



	A	B	C	D
1	**CALCULATING THE MINIMUM-VARIANCE PORTFOLIO**			
2			**GM**	**MSFT**
3	Average		14.25%	62.72%
4	Variance		6.38%	14.43%
5	Sigma		25.25%	37.99%
6	Covariance of returns		-5.52%	
7				
8	**Portfolio return and risk**			
9	Percentage in GM	40.00%		
10	Percentage in MSFT	60.00%	<-- =1-B9	
11				
12	Expected portfolio return	43.33%	<-- =B9*C3+B10*D3	
13	Portfolio variance	0.0356	<-- =B9^2*C4+B10^2*D4+2*B9*B10*C6	
14	Portfolio standard deviation	18.88%	<-- =SQRT(B13)	

Pushing **Solve** gives:

	A	B	C	D	E	F	G
1	**CALCULATING THE MINIMUM-VARIANCE PORTFOLIO**						
2			**GM**	**MSFT**			
3	Average		14.25%	62.72%			
4	Variance		6.38%	14.43%			
5	Sigma		25.25%	37.99%			
6	Covariance of returns		-5.52%				
7							
8	**Portfolio return and risk**						
9	Percentage in GM	62.64%					
10	Percentage in MSFT	37.36%	<-- =1-B9				
11							
12	Expected portfolio return	32.36%	<-- =B9*C3+B10*D3				
13	Portfolio variance	0.0193	<-- =B9^2*C4+B10^2*D4+2*B9*B10*C6				
14	Portfolio standard deviation	13.90%	<-- =SQRT(B13)				

Thus the minimum variance portfolio has 62.64% in GM and 37.36% in MSFT.[8]

Minimum variance portfolios using calculus

There's actually a formula for the minimum variance portfolio:

$$w_{GM} = \frac{Var(r_{MSFT}) - Cov(r_{GM}, r_{MSFT})}{Var(r_{GM}) + Var(r_{MSFT}) - 2Cov(r_{GM}, r_{MSFT})}$$

Using this formula, which is derived in Appendix 1 (page000), is simpler than using **Solver**. Implementing the formula in Excel gives the same answer as that given by **Solver**:

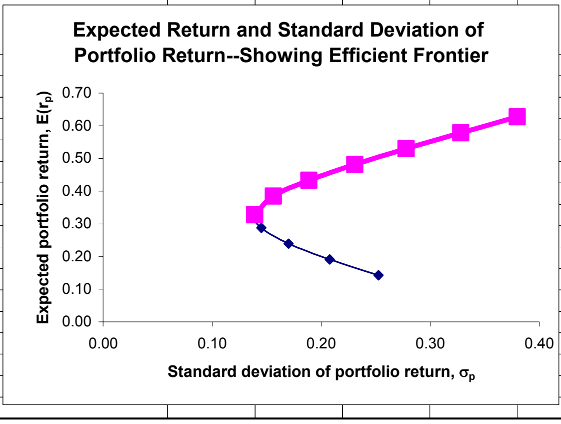
[8] Although—as explained in Chapter 32—**Solver** and **Goal Seek** are in most cases interchangeable, this is a calculation which **Solver** does easily, but which cannot be done in **Goal Seek**.

	A	B	C	D	E	F
1	**CALCULATING THE MINIMUM-VARIANCE PORTFOLIO WITH A FORMULA**					
2			**GM**	**MSFT**		
3	Average		14.25%	62.72%		
4	Variance		6.38%	14.43%		
5	Sigma		25.25%	37.99%		
6	Covariance of returns		-5.52%			
7						
8	**Minimum variance portfolio--analytic formula**					
9	Percentage in GM	62.64%	<-- =(D4-C6)/(C4+D4-2*C6)			
10	Percentage in MSFT	37.36%	<-- =1-B9			
11						
12	Expected portfolio return	32.36%	<-- =B9*C3+B10*D3			
13	Portfolio variance	0.0193	<-- =B9^2*C4+B10^2*D4+2*B9*B10*C6			
14	Portfolio standard deviation	13.90%	<-- =SQRT(B13)			

The efficient frontier and the minimum variance portfolio

Now that we know the minimum variance portfolio, we can plot the efficient frontier, the set of all the portfolios with an economically-meaningful return-risk tradeoff. "Economically-meaningful return-risk tradeoff" means that along the efficient frontier additional portfolio return $E(r_p)$ is achieved at the cost of additional portfolio standard deviation σ_p. The efficient frontier is all portfolios that are to the right of the minimum variance portfolio.

	A	B	C	D	E	F	G	H	I	J	K
1	**CALCULATING THE MINIMUM-VARIANCE PORTFOLIO WITH A FORMULA**										
2			**GM**	**MSFT**							
3	Average		14.25%	62.72%							
4	Variance		6.38%	14.43%							
5	Sigma		25.25%	37.99%							
6	Covariance of returns		-5.52%								
7											
8	**Minimum variance portfolio--analytic formula**										
9	Percentage in GM	62.64%	<-- =(D4-C6)/(C4+D4-2*C6)								
10	Percentage in MSFT	37.36%	<-- =1-B9								
11											
12	Expected portfolio return	32.36%	<-- =B9*C3+B10*D3								
13	Portfolio variance	0.0193	<-- =B9^2*C4+B10^2*D4+2*B9*B10*C6								
14	Portfolio standard deviation	13.90%	<-- =SQRT(B13)								
15								**Percentage in GM**	**Sigma**	**Expected return**	**Efficient frontier points**
16											
17								0.00%	37.99%	62.72%	62.72%
18								10.00%	32.80%	57.87%	57.87%
19								20.00%	27.79%	53.03%	53.03%
20								30.00%	23.08%	48.18%	48.18%
21								40.00%	18.88%	43.33%	43.33%
22								50.00%	15.62%	38.49%	38.49%
23								61.847400%	13.91%	32.74%	32.74%
24								70.00%	14.51%	28.79%	
25								80.00%	17.01%	23.95%	
26								90.00%	20.78%	19.10%	
27								1	25.25%	14.25%	
28											
29											
30											
31											
32											

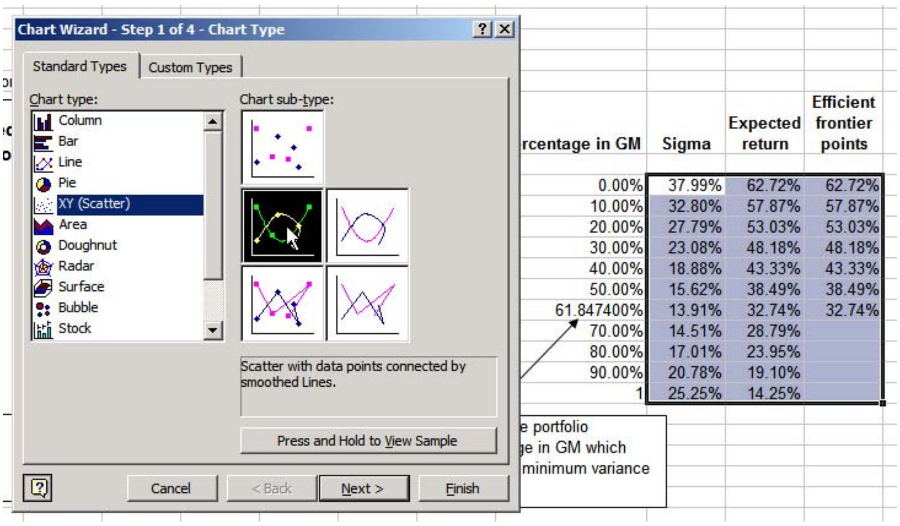


This is the portfolio percentage in GM which gives the minimum variance portfolio.

Excel trick

The graph of the efficient frontier shown above is an **XY Scatter plot**. The x-data is the data for sigma in I17:I27. Cells J17:J27 give the data for the portfolio expected returns, and cells K17:K27 give data for the expected returns *only for efficient portfolios*. The two data series, J17:J27 and K17:K27, constitute the y-data for the XY scatter plot. Where they coincide, Excel superimposes them, creating the effect seen in the graph.

To create the graph, mark the three columns I17:K27. Then go to the chart wizard and pick XY (Scatter) as shown below. Proceed from there to build the graph.



12.5. The effect of correlation on the efficient frontier

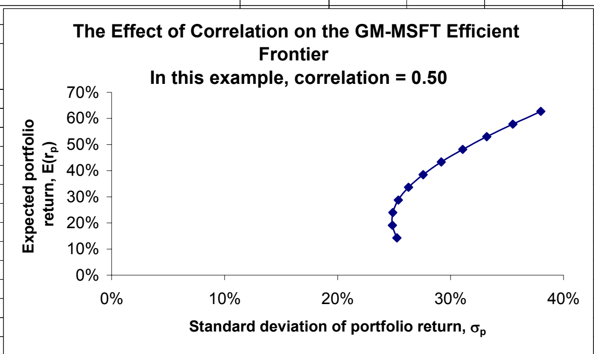
When we looked at the "coin-flip" economy of section 12.1, we concluded that the correlation between asset returns made a big difference. In this section we examine the effect of stock return correlation on portfolio returns, repeating the correlation experiment of section 12.1 for a more "real world" example.

First, recall what we concluded in section 12.1:

- When the two coins have a perfect negative correlation of -1, we can create a risk-free asset using combinations of the two assets. In this section you'll see that a similar conclusion is true for stock portfolios: Perfectly negatively correlated stock returns allow you to create a risk-free asset.
- When the two coins have a perfect positive correlation of +1, it's impossible to diversify away any risk. You will see that a similar conclusion holds for stock portfolios.
- When the two coins have correlation between -1 and +1, some of the risk can be eliminated through diversification. Again this is true for stock portfolios.

In our example we use some of the same numbers used in our GM-MSFT example, but we'll allow the correlation between the returns on the two stocks to vary. We start with the following example, in which the correlation coefficient between GM and MSFT is $\rho_{GM,MSFT}$ = 0.5.

	A	B	C	D	E	F	G	H	I	J
1	**THE EFFECT OF CORRELATION COEFFICIENT ON GM-MSFT PORTFOLIOS in this example, the correlation = 0.50**									
2		**GM**	**MSFT**							
3	Average	14.25%	62.72%							
4	Variance	6.38%	14.43%							
5	Sigma	25.25%	37.99%							
6	Correlation coefficient, $\rho_{GM,MSFT}$	0.50								
7	Covariance	0.05	<-- =B6*B5*C5							
8										
9							=SQRT(G12^2*$B$4			
10							+(1-G12)^2*$C$4+2*G12*(1-G12)*$B$7)			
11							**Percentage in GM**	**Sigma**	**Expected return**	=G12*$B$3+(1-G12)*$C$3
12							0.0	37.99%	62.72%	
13							0.1	35.52%	57.87%	
14							0.2	33.20%	53.03%	
15							0.3	31.08%	48.18%	
16							0.4	29.18%	43.33%	
17							0.5	27.57%	38.49%	
18							0.6	26.28%	33.64%	
19							0.7	25.37%	28.79%	
20							0.8	24.89%	23.95%	
21							0.9	24.85%	19.10%	
22							1.0	25.25%	14.25%	
23										
24										
25										
26										



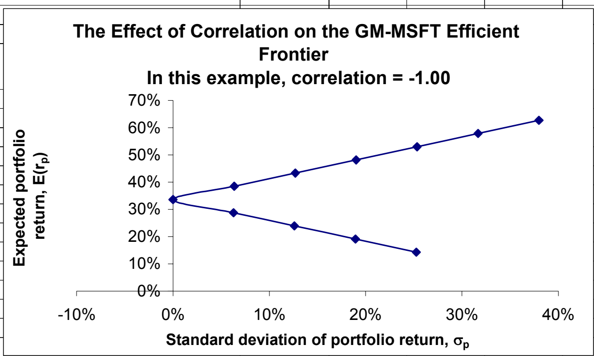
Correlation coefficient = -1—perfect negative correlation

When the correlation coefficient $\rho_{\text{GM,MSFT}}$ = -1, we can use our portfolio to create a riskless asset. This was the message in the simple "coin toss" example with which we started this chapter (Section 12.1), and it is still true here:

Perfect negative correlation between two risky assets allows the creation of a portfolio that is risk-free.

Here's our example in Excel:

	A	B	C	D
1	THE EFFECT OF CORRELATION COEFFICIENT ON GM-MSFT PORTFOLIOS in this example, the correlation = -1.00			
2		GM	MSFT	
3	Average	14.25%	62.72%	
4	Variance	6.38%	14.43%	
5	Sigma	25.25%	37.99%	
6	Correlation coefficient, $\rho_{GM,MSFT}$	-1.00		
7	Covariance	-0.10	<-- =B6*B5*C5	



=SQRT(G12^2*$B$4+(1-G12)^2*$C$4+2*G12*(1-G12)*$B$7)

=G12*$B$3+(1-G12)*$C$3

	G	H	I
11	Percentage in GM	Sigma	Expected return
12	0%	37.99%	62.72%
13	10%	31.66%	57.87%
14	20%	25.34%	53.03%
15	30%	19.01%	48.18%
16	40%	12.69%	43.33%
17	50%	6.37%	38.49%
18	60.066%	0.00%	33.61%
19	70%	6.28%	28.79%
20	80%	12.61%	23.95%
21	90%	18.93%	19.10%
22	100%	25.25%	14.25%

A little mathematics explains this result. The portfolio variance for this case can be written:

$$
\begin{aligned}
Var\left(r_p\right) &= w_{GM}^2 Var\left(r_{GM}\right) + w_{MSFT}^2 Var\left(r_{MSFT}\right) + 2 w_{GM} w_{MSFT} \rho_{GM,MSFT} \sigma_{GM} \sigma_{MSFT} \\
&= w_{GM}^2 \sigma_{GM}^2 + w_{MSFT}^2 \sigma_{MSFT}^2 - 2 w_{GM} w_{MSFT} \sigma_{GM} \sigma_{MSFT} \\
&= w_{GM}^2 \sigma_{GM}^2 + \left(1 - w_{GM}\right)^2 \sigma_{MSFT}^2 - 2 w_{GM} \left(1 - w_{GM}\right) \sigma_{GM} \sigma_{MSFT} \\
&= \left(w_{GM} \sigma_{GM} - \left(1 - w_{GM}\right) \sigma_{MSFT}\right)^2
\end{aligned}
$$

This means that we can—by choosing the appropriate weights w_{GM} and w_{MSFT}—set the portfolio variance equal to zero:

$$Var\left(r_p\right)=\left(w_{GM}\sigma_{GM}-\left(1-w_{GM}\right)\sigma_{MSFT}\right)^2=0$$

$$when\ \ w_{GM}=\frac{\sigma_{MSFT}}{\sigma_{MSFT}+\sigma_{GM}}$$

In our case, this means that

$$w_{GM}=\frac{\sigma_{MSFT}}{\sigma_{MSFT}+\sigma_{GM}}=\frac{62.72\%}{62.72\%+14.25\%}=0.60066\,.$$

This value is given in cell G18.

Correlation coefficient = +1. The case of perfect positive correlation

When the correlation coefficient $\rho_{\text{GM,MSFT}}=+1$, diversification does not reduce risk.

Perfect positive correlation between two risky assets means that risk is not reduced in a portfolio context.

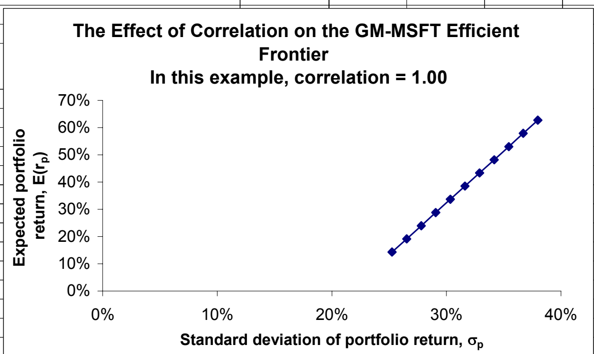
Here's our example in Excel:

	A	B	C	D
1	**THE EFFECT OF CORRELATION COEFFICIENT ON GM-MSFT PORTFOLIOS in this example, the correlation = 1.00**			
2		**GM**	**MSFT**	
3	Average	14.25%	62.72%	
4	Variance	6.38%	14.43%	
5	Sigma	25.25%	37.99%	
6	Correlation coefficient, $\rho_{GM,MSFT}$	1.00		
7	Covariance	0.10	<-- =B6*B5*C5	

=SQRT(G12^2*$B$4+(1-G12)^2*$C$4+2*G12*(1-G12)*$B$7)

=G12*$B$3+(1-G12)*$C$3

Row	G	H	I
11	**Percentage in GM**	**Sigma**	**Expected return**
12	0%	37.99%	62.72%
13	10%	36.71%	57.87%
14	20%	35.44%	53.03%
15	30%	34.17%	48.18%
16	40%	32.89%	43.33%
17	50%	31.62%	38.49%
18	60%	30.35%	33.64%
19	70%	29.07%	28.79%
20	80%	27.80%	23.95%
21	90%	26.53%	19.10%
22	100%	25.25%	14.25%



Notice what we mean by "portfolios do not reduce risk": When the correlation between the two assets' returns is +1, the standard deviation of the portfolio return for this case is the weighted average of the asset standard deviations. A little mathematics explains this result. The portfolio variance for this case can be written:

$$\begin{aligned} Var\left(r_p\right) &= w_{GM}^2 Var\left(r_{GM}\right) + w_{MSFT}^2 Var\left(r_{MSFT}\right) + 2 w_{GM} w_{MSFT} \rho_{GM,MSFT} \sigma_{GM} \sigma_{MSFT} \\ &= w_{GM}^2 \sigma_{GM}^2 + w_{MSFT}^2 \sigma_{MSFT}^2 \underbrace{+2 w_{GM} w_{MSFT} \sigma_{GM} \sigma_{MSFT}}_{\substack{\uparrow \\ \text{The correlation coefficient} \\ \rho_{GM,MSFT}=1}} \\ &= \left(w_{GM}\sigma_{GM} + \left(1 - w_{GM}\right)\sigma_{MSFT}\right)^2 \end{aligned}$$

This means that the standard deviation of the portfolio is the weighted average of the asset standard deviations:

$$\sigma\left(r_p\right) = w_{GM}\sigma_{GM} + \left(1 - w_{GM}\right)\sigma_{MSFT}$$

Thus there is no real gain from diversification.

Summary

In this chapter we have discussed the importance of diversification for portfolio returns and risks. We showed how to calculate the mean and variance and standard deviation of a portfolio's return. The *efficient frontier* is the set of those portfolios which offer the highest expected return for a given standard deviation. We discussed this frontier and how it is affected by the correlation between the asset returns.

Exercises

Note: Data for the problems is on the CD-ROM which accompanies the book.

1. The table below presents the year-end prices for the shares of Ford and PPG from 1989 to 2001:

	A	B	C
1	**PRICES FOR FORD AND PPG STOCK**		
2	**Date**	**Ford stock price**	**PPG stock price**
3	31-Dec-89	11.813	14.024
4	31-Dec-90	7.210	17.229
5	31-Dec-91	7.617	19.138
6	31-Dec-92	11.612	25.721
7	31-Dec-93	17.469	30.518
8	31-Dec-94	15.100	30.736
9	31-Dec-95	15.642	38.980
10	31-Dec-96	17.472	49.007
11	31-Dec-97	26.310	51.040
12	31-Dec-98	31.807	53.172
13	31-Dec-99	28.895	58.626
14	31-Dec-00	22.470	44.867
15	31-Dec-01	15.720	51.720

1.a. Calculate the following statistics for these two shares: average return, variance of returns, standard deviation of returns, covariance of returns and correlation coefficient.

1.b. If you invested in a portfolio composed of 50% Ford and 50% PPG, what would be the portfolio expected return? the standard deviation?

1.c. Comment on the following statement: “Ford has lower returns and higher standard deviation of returns than PPG. Therefore any rational investor would invest in PPG only and would leave Ford out of her portfolio.”

2. You invest $500 in a stock for which the return is determined by a coin flip. If the coin comes up head the stock returns 10%, and if it comes up tails the investment returns -10%. What is the average return, the return variance, and the return standard deviation of this investment, if you flip the coin one time?

3. You have $500 to invest. You decide to split it into two parts. The return on each $250 will be determined by a coin toss, and the results of the two tosses are not correlated. If the coin comes up heads, the investment will return 10% and if it comes up tails it will return -10%. What is the average return, the return variance, and the return standard deviation of this investment?

4. The previous question assumes that the correlation between the coin flips in 0. Repeat this question with the following correlations:

4.a. If the first coin flip is heads, then the second coin flip will be heads as well, and vice versa (correlation of 1).

4.b. If the first coin flip is heads, then second coin flip will be tails, and vice versa (correlation of -1).

4.c. If the first coin flip is heads, then the second coin flip will be heads with a probability of 0.8. If the first coin flip is tails, then the second coin flip will be tails with a probability of 0.6.

4.d. What can you conclude about the connection between the variance of the return from the coin flips and the correlation between the flips?

5. Calculate the average return and the variance of a portfolio composed of 30% of GM and 70% of MSFT stocks, using the data described from page000.

6. Consider the following statistics for a portfolio composed of shares of Companies A and B:

	A	B	C	D	E	F
1		**Company A stock**	**Company B stock**			
2	Average Return	25%	48%			
3	Variance	0.0800	0.1600			
4	Sigma	28.28%	40.00%			
5						
6	Covariance of Returns	0.00350				
7	Correlation of Returns	0.03094	<-- =B6/(B4*C4)			
8						
9	Portfolio					
10	Proportion of A	0.9				
11	Proportion of B	0.1				
12	Portfolio average return	27.30%	<-- =B10*B2+C2*B11			
13	Portfolio standard deviation	25.89%	<-- =SQRT(B10^2*B3+B11^2*C3+2*B10*B11*B6)			

6.a. Suggest a portfolio combination that improves return while maintaining the same level of risk.

6.b. Calculate the minimum variance portfolio for the portfolio composed of the two assets described above.

7. Consider the monthly returns for Ford and General Motors stock given below. Were there advantages to diversifying between these two stocks? Explain.

	A	B	C
1	**MONTHLY RETURNS FOR FORD AND GM STOCK**		
2	**Date**	**Ford**	**GM**
3	1-Dec-99	5.52%	-1.50%
4	3-Jan-00	-5.70%	10.83%
5	1-Feb-00	-16.32%	-4.99%
6	1-Mar-00	10.32%	8.89%
7	3-Apr-00	20.27%	13.05%
8	1-May-00	-11.30%	-24.12%
9	1-Jun-00	-7.81%	-17.79%
10	3-Jul-00	9.47%	-1.94%
11	1-Aug-00	-9.18%	23.95%
12	1-Sep-00	5.42%	-7.14%
13	2-Oct-00	3.63%	-4.43%
14	1-Nov-00	-12.89%	-19.61%
15	1-Dec-00	3.02%	2.89%
16	2-Jan-01	21.56%	5.43%
17	1-Feb-01	-3.29%	4.56%
18			
19	Average	0.85%	-0.79%
20	Standard deviation	11.23%	12.58%
21	Correlation	0.4056	

8. The following spreadsheet presents data for stocks A and B.

	A	B	C	D
1	**Return statistics of A and B stock**			
2		A	B	
3	**Average return**	34.00%	25.00%	
4	**Variance**	0.12	0.07	
5	**Sigma**	34.64%	26.46%	
6				
7	**covariance of return**	0.016		
8	**correlation of return**	0.175		

8.a. What are the return and the standard deviation of a portfolio composed of 30% of stock A and 70% of stock B?

8.b. What are the return and the standard deviation of an equally weighted portfolio of stocks A and B?

9. Suppose that the return statistics for A and B stock are given below. What is the standard deviation of the portfolio minimum variance? (The answer requires only one calculation.)

	A	B	C
1	**RETURN STATISTICS OF A AND B STOCK**		
2		**A**	**B**
3	Average return	25%	15%
4	Variance	0.1600	0.0484
5	Standard deviation	40.00%	22.00%
6			
7	Covariance of return	-0.0880	

10. ABC and XYZ are 2 stocks with the following return statistics:

	A	B	C
1		**Expected return**	**Standard deviation of return**
2	ABC	15%	33%
3	XYZ	25%	46%
4	Covariance(ABC,XYZ)	0.0865	
5	Correlation(ABC,XYZ)	0.5698	

10.a. Compute the expected return and standard deviation of a portfolio composed of 25% ABC and 75% XYZ.

10.b. Compute the returns of all portfolios that are combinations of ABC and XYZ with the proportion of ABC being 0%, 10%, ... , 90%, 100%. Graph these returns

10.c. Compute the minimum variance portfolio

11. Melissa Jones wants to invest in a portfolio composed of stocks ABC and XYX (from question 10), that will yield a return of 19%. What is the weight of each stock in such a portfolio, and what is the portfolio's standard deviation? Answer the question both by using Excel's **Goal Seek** or **Solver** and by using the mathematical formulas in the chapter (page000).

12. Your client asks you to create a two-asset portfolio having an expected return of 15% and return standard deviation of 12%. The client specifies that the portfolio include 60% of the stock 'Merlyn' (named for her beloved mother…) which has expected return is 13% and has a standard deviation of 10%.

12.a. What should be the return statistics of the second stock you'll combine in this portfolio, assuming the stocks have zero correlation?

12.b. What should be the return statistics of the second stock you'll combine in this portfolio, assuming the stocks have covariance of 0.01?

13. What will be the weights, the expected return, the variance, and the standard deviation of a minimum variance portfolio combining the stocks below, using the mathematical way:

	A	B	C	D
1	**Return statistics of X and Y stock**			
2		**X**	**Y**	
3	**Average return**	21.00%	14.00%	
4	**Variance**	0.11	0.045	
5	**Sigma**	33.17%	21.21%	
6				
7	**covariance of return**	-0.002		
8	**correlation of return**	-0.028		

14. This question relates to the data in exercise 13.

14.a. Calculate and graph the efficient frontier of the stock portfolios composed of stocks X and Y in the exercise 13.

14.b. Calculate and graph the efficient frontier of the stock portfolios composed of stocks X and Y in the exercise 13, assuming the correlation between the two stocks is -1.

15. Let's go again to the data of GM and Microsoft stocks on page000. A portfolio composed of 90% of GM and 10% Microsoft stock has expected return of 19.1% and standard deviation of 20.78%. Find another portfolio with the same standard deviation and a higher return. (You can do this by trial and error, but you can also use **Solver**.)

16. John and Mary are considering investing in a combination of ABC stock and XYZ stock. The return on ABC is determined by a coin flip: If the coin is heads, the return is 35% and if the coin is tails, the return on ABC is 10%. The return on XYZ stock is similarly determined, but by a *separate coin flip*.

16.a. Compute the mean, variance and standard deviation of the returns on ABC and XYZ.

16.b. What is the correlation of the returns? (Nothing to compute here, just think!)

16.c. John has decided to invest in a portfolio composed of 100% XYZ stock. Mary, on the other hand, is investing in a portfolio composed of 50% ABC and 50% XYZ. Whose portfolio is better? Why?

17. Elizabeth and Sandra are considering investing in a combination of ABC stock and XYZ stock. The return on both stocks is determined by a single coin flip: If the coin is heads, the return on both stocks is 35% and if the coin is tails, the return is 10%.

17.a. Compute the mean, variance and standard deviation of the returns on ABC and XYZ.

17.b. What is the correlation of the returns? (Nothing to compute here, just think!)

17.c. Elizabeth has decided to invest in a portfolio composed of 100% XYZ stock. Sandra, on the other hand, is investing in a portfolio composed of 50% ABC and 50% XYZ. Whose portfolio is better?

Appendix 1: Deriving the formula for the minimum variance portfolio

Recall the formula for variance of the portfolio:

$$Var\left(r_p\right) = w_{GM}^2 Var\left(r_{GM}\right) + w_{MSFT}^2 Var\left(r_{MSFT}\right) + 2 w_{GM} w_{MSFT} Cov\left(r_{GM}, r_{MSFT}\right)$$

Substituting in $w_{MSFT} = 1 - w_{GM}$, this equation becomes:

$$Var\left(r_p\right) = w_{GM}^2 Var\left(r_{GM}\right) + \left(1 - w_{GM}\right)^2 Var\left(r_{MSFT}\right) + 2 w_{GM}\left(1 - w_{GM}\right) Cov\left(r_{GM}, r_{MSFT}\right)$$

Setting the derivative of this equation equal to zero will give the formula for the minimum variance portfolio:

$$\frac{d\,Var\left(r_p\right)}{dw_{GM}} = 2 w_{GM} Var\left(r_{GM}\right) - 2\left(1 - w_{GM}\right) Var\left(r_{MSFT}\right) + Cov\left(r_{GM}, r_{MSFT}\right)\left(2 - 2 w_{GM}\right) = 0$$

$$\Rightarrow w_{GM} = \frac{Var\left(r_{MSFT}\right) - Cov\left(r_{GM}, r_{MSFT}\right)}{Var\left(r_{GM}\right) + Var\left(r_{MSFT}\right) - 2Cov\left(r_{GM}, r_{MSFT}\right)}$$

Appendix 2: Portfolios with three and more assets

In this appendix we look at portfolios and their efficient frontiers when there are more than 2 assets. The main points we make:

- In the multi-asset context we can still calculate the efficient frontier, and it still has its characteristic shape.
- The more risky assets there are, the more the portfolio variance is influenced by the covariances between the assets.

We start by considering a 3-asset problem. To describe 3 assets, we need to know the expected return, the variance, and all the *pairs* of covariances. This data is described below.

	A	B	C	D	E
1	**A 3-ASSET PORTFOLIO PROBLEM**				
2		**Stock A**	**Stock B**	**Stock C**	
3	Mean	10%	12%	15%	
4	Variance	15%	22%	30%	
5					
6	Cov(r_A,r_B)	0.03			
7	Cov(r_B,r_C)	-0.01			
8	Cov(r_A,r_C)	0.02			

Suppose we form a portfolio of risky assets composed of proportion x_A in asset A, x_B in asset B, and x_C in asset C. Since the portfolio is fully invested in risky assets, it follows that $x_C = 1-x_A-x_B$.

Portfolio return statistics: The expected return of the portfolio is given by:

$$E(r_p) = x_A E(r_A) + x_B E(r_B) + x_C E(r_C)$$

The calculation of the portfolio's variance of return requires both the variances and the covariances:

$$\begin{aligned} Var(r_p) &= x_A^2 Var(r_A) + x_B^2 Var(r_B) + x_C^2 Var(r_C) + \\ &2x_A x_B Cov(r_A, r_B) + 2x_A x_C Cov(r_A, r_C) + 2x_B x_C Cov(r_B, r_C) \end{aligned}$$

Notice that there are 3 *variances* and 3 *covariances*. When—at the end of this section—we show you the formula for a 4-asset problem, there will be 4 variances and 6 covariances. As the number of assets grows, so does the number of covariances (in fact their number grows much faster than the number of variances). This is the meaning of the second bullet at the beginning of this section—for multi-asset portfolio problems, the portfolio variance is increasingly influenced by the covariances.

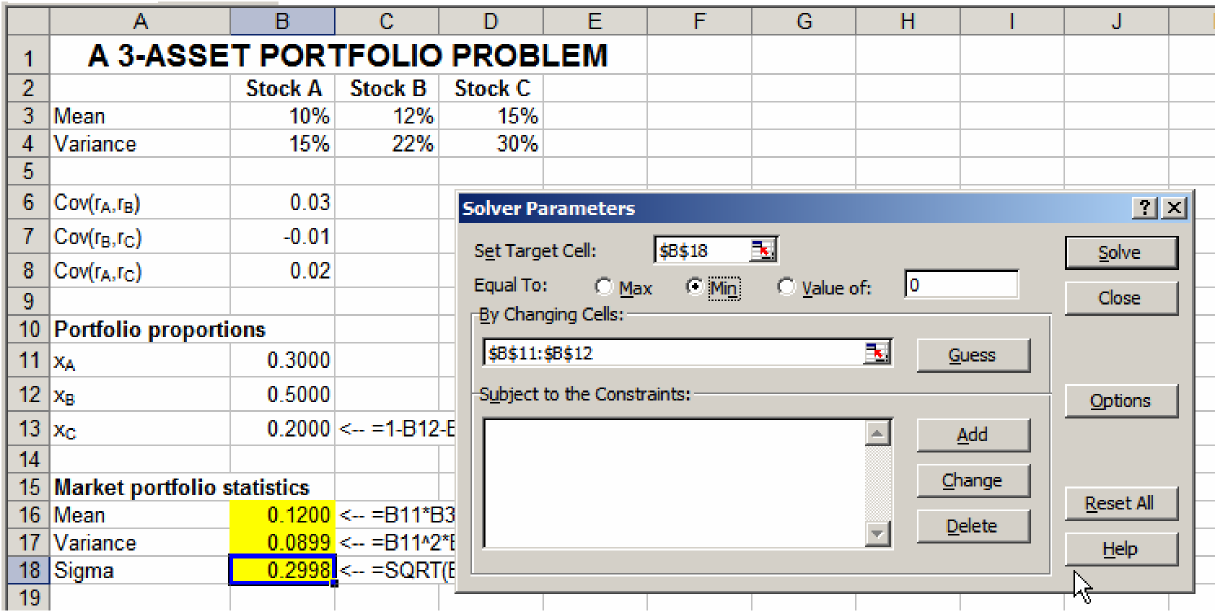
Here's an example of the mean return and variance calculation for our 3-asset portfolio: The portfolio statistics are calculated in cells B16:B18:

	A	B	C	D	E	F	G	H	I	J
1	**A 3-ASSET PORTFOLIO PROBLEM**									
2		**Stock A**	**Stock B**	**Stock C**						
3	Mean	10%	12%	15%						
4	Variance	15%	22%	30%						
5										
6	Cov(r_A,r_B)	0.03								
7	Cov(r_B,r_C)	-0.01								
8	Cov(r_A,r_C)	0.02								
9										
10	**Portfolio proportions**									
11	x_A	0.3000								
12	x_B	0.5000								
13	x_C	0.2000	<-- =1-B12-B11							
14										
15	**Market portfolio statistics**									
16	Mean	0.1200	<-- =B11*B3+B12*C3+B13*D3							
17	Variance	0.0899	<-- =B11^2*B4+B12^2*C4+B13^2*D4+2*B11*B12*B6+2*B11*B13*B8+2*B12*B13*B7							
18	Sigma	0.2998	<-- =SQRT(B17)							

Calculating the efficient frontier with 3 assets

We can use Excel to calculate and graph the efficient frontier for this case.[9] We'll make use of Excel's **Solver**.

Step 1: We use Solver to find the minimum sigma portfolio:



Here's the result:

	A	B	C	D	E	F	G	H	I	J
1	**A 3-ASSET PORTFOLIO PROBLEM**									
2		**Stock A**	**Stock B**	**Stock C**						
3	Mean	10%	12%	15%						
4	Variance	15%	22%	30%						
5										
6	Cov(r_A,r_B)	0.03								
7	Cov(r_B,r_C)	-0.01								
8	Cov(r_A,r_C)	0.02								
9										
10	**Portfolio proportions**									
11	x_A	0.4370								
12	x_B	0.3151								
13	x_C	0.2479	<-- =1-B12-B11							
14										
15	**Market portfolio statistics**									
16	Mean	0.1187	<-- =B11*B3+B12*C3+B13*D3							
17	Variance	0.0800	<-- =B11^2*B4+B12^2*C4+B13^2*D4+2*B11*B12*B6+2*B11*B13*B8+2*B12*B13*B7							
18	Sigma	0.2828	<-- =SQRT(B17)							

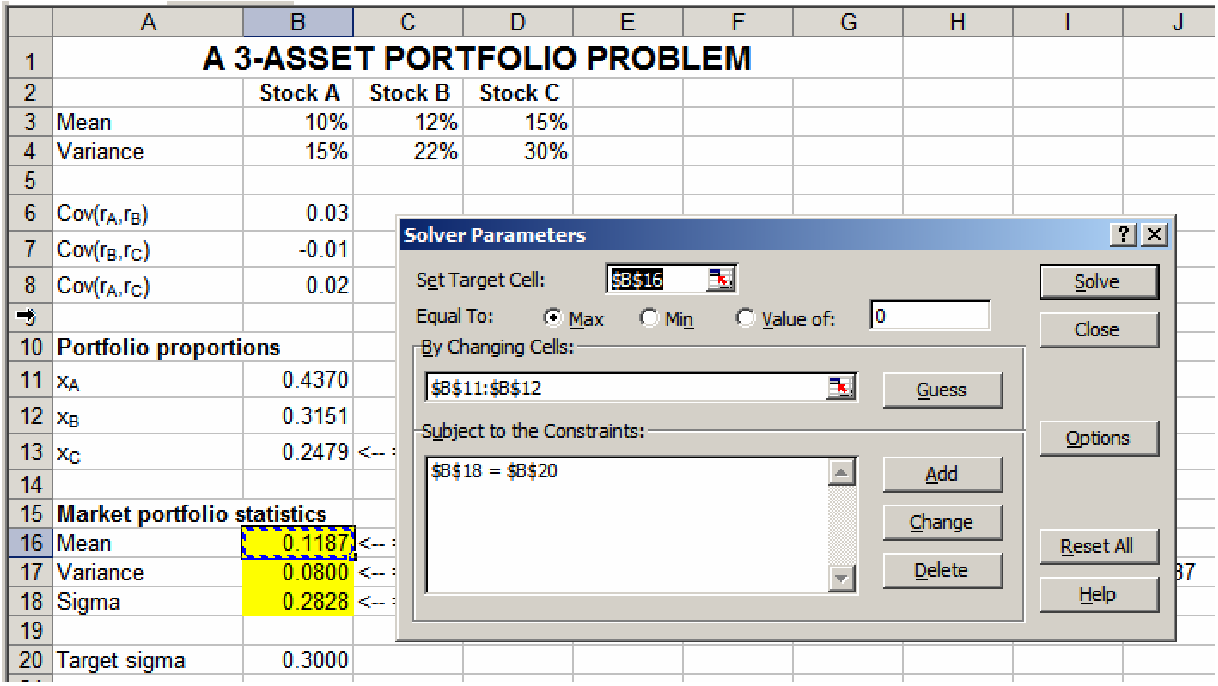
Step 2: We now specify sigma and use **Solver** to find a portfolio with the maximum return. We do this by first adding a cell ("Target sigma," cell B20) to the spreadsheet:

[9] The procedure we're about to explain is somewhat long-winded—for a much shorter and efficient procedure, see my book *Financial Modeling*.

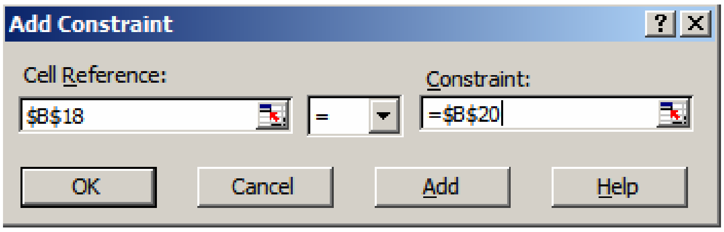
	A	B	C	D	E	F	G	H	I	J
1	A 3-ASSET PORTFOLIO PROBLEM									
2		**Stock A**	**Stock B**	**Stock C**						
3	Mean	10%	12%	15%						
4	Variance	15%	22%	30%						
5										
6	Cov(r_A,r_B)	0.03								
7	Cov(r_B,r_C)	-0.01								
8	Cov(r_A,r_C)	0.02								
9										
10	**Portfolio proportions**									
11	x_A	0.4370								
12	x_B	0.3151								
13	x_C	0.2479	<-- =1-B12-B11							
14										
15	**Market portfolio statistics**									
16	Mean	0.1187	<-- =B11*B3+B12*C3+B13*D3							
17	Variance	0.0800	<-- =B11^2*B4+B12^2*C4+B13^2*D4+2*B11*B12*B6+2*B11*B13*B8+2*B12*B13*B7							
18	Sigma	0.2828	<-- =SQRT(B17)							
19										
20	Target sigma	0.3000								
21										
22	**TABLE OF SIGMA VERSUS MEAN**									
23	**Target sigma**	**Mean**								
24	0.2828	0.1187	<-- This is the minimum sigma portfolio							

Notice that—starting from row 24—we've begun to build a table of the results. The first row of this table is the minimum sigma portfolio. Now we'll use Solver to add another row to this table.

We do this by adding a *constraint* to Solver:



The constraint was added by pressing **Add** in the lower portion of the **Solver** dialog box:

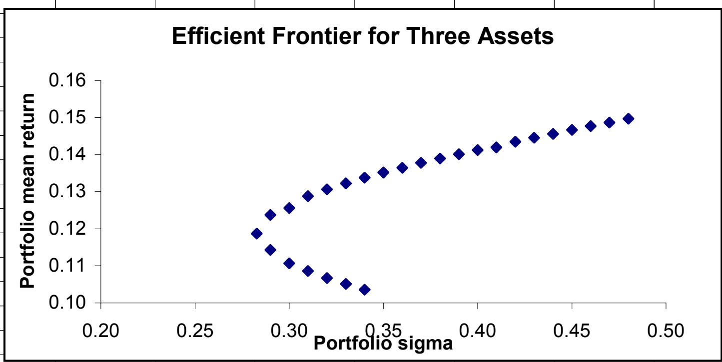


Here's the result:

	A	B	C	D	E	F	G
1	**A 3-ASSET PORTFOLIO PROBLEM**						
2		**Stock A**	**Stock B**	**Stock C**			
3	Mean	10%	12%	15%			
4	Variance	15%	22%	30%			
5							
6	Cov(r_A,r_B)	0.03					
7	Cov(r_B,r_C)	-0.01					
8	Cov(r_A,r_C)	0.02					
9							
10	**Portfolio proportions**						
11	x_A	0.2533					
12	x_B	0.3544					
13	x_C	0.3923	<-- =1-B12-B11				
14							
15	**Market portfolio statistics**						
16	Mean	0.1267	<-- =B11*B3+B12*C3+B13*D3				
17	Variance	0.0900	<-- =B11^2*B4+B12^2*C4+B13^2*D4+2*B11*B12*B6+2*				
18	Sigma	0.3000	<-- =SQRT(B17)				
19							
20	Target sigma	0.3000					

If we repeat this calculation many times for many Target sigmas, we get the efficient frontier:

	A	B	C
24	**TABLE OF SIGMA VERSUS MEAN**		
25	**Target sigma**	**Mean**	
26	0.2828	0.1187	<-- This is the minimum sigma portfolio
27	0.2900	0.1238	
28	0.3000	0.1256	
29	0.3100	0.1288	
30	0.3200	0.1307	
31	0.3300	0.1323	
32	0.3400	0.1338	
33	0.3500	0.1352	
34	0.3600	0.1365	
35	0.3700	0.1378	
36	0.3800	0.1390	
37	0.3900	0.1401	
38	0.4000	0.1413	
39	0.4100	0.1420	
40	0.4200	0.1435	
41	0.4300	0.1446	
42	0.4400	0.1456	
43	0.4500	0.1467	
44	0.4600	0.1477	
45	0.4700	0.1487	



Four assets

This section has gone into depth about how to calculate returns and variances for a 3-asset portfolio. If we have 4 assets, we can do the same kinds of calculations (we leave this as an exercise). What you have to know for this case is how to calculate the portfolio return and variance.

Call the assets A, B, C, D, and denote the portfolio weights by x_A, x_B, x_C, x_D.

Portfolio return statistics: The expected return of the portfolio is given by:

$$E(r_p) = x_A E(r_A) + x_B E(r_B) + x_C E(r_C) + x_D E(r_D)$$

The calculation of the portfolio's variance of return requires both the variances and the covariances:

$$\begin{aligned}
&Var(r_p) = x_A^2 Var(r_A) + x_B^2 Var(r_B) + x_C^2 Var(r_C) + x_D^2 Var(r_D) \\
&2x_A x_B Cov(r_A, r_B) + 2x_A x_C Cov(r_A, r_C) + 2x_A x_D Cov(r_A, r_D) \\
&+2x_B x_C Cov(r_B, r_C) + 2x_B x_D Cov(r_B, r_D) \\
&+2x_C x_D Cov(r_C, r_D)
\end{aligned}$$

Notice that there now there are 4 *variances* and 6 *covariances*.

Exercises for Appendix 2

All 3 problems relate to the following statistics for stocks ABC, QPD, and XYZ:

	A	B	C	D
1	**RETURN STATISTICS FOR 3 STOCKS**			
2		**ABC**	**QPD**	**XYZ**
3	Average return	22.00%	17.50%	30.00%
4	Variance	0.2	0.05	0.17
5	Standard deviation	44.72%	22.36%	41.23%
6				
7	Correlations			
8	Corr(ABC,QPD)	0.05		
9	Corr(ABC,XYZ)	-0.1		
10	Corr(QPD,XYZ)	0.5		

A1. Find the average return and standard deviation of a portfolio composed of 50% of stock ABC, 20% of stock QPD and 30% of stock XYZ.

A.2. Find the minimum variance portfolio and its statistics.

A.3. Find the portfolio having maximum return given that the portfolio standard deviation is 30%.

CHAPTER 13: THE CAPITAL ASSET PRICING MODEL (CAPM)*

this version: December 2002

Chapter contents

* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance with Excel* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it. All the material is copyright and the rights belong to the author.

Overview

In this chapter we add a risk-free asset to the portfolio problem discussed in Chapter 12. Adding this asset gives the investor new possibilities: She can invest either in stocks, or in the risk-free asset, or in some combination of the two. These new investment possibilities allow the investor to achieve superior returns. The addition of a risk-free asset to the portfolio of risk assets leads to four new concepts:

- The *capital market line* (CML) is the set of all optimal investment portfolios for an investor. A portfolio on the CML is a combination of the risk-free asset and the risky assets.
- The *market portfolio* (denoted by the letter *M*) is the best portfolio of risky assets available to the investor.
- The *security market line* (SML) describes the relation between the expected returns of any asset and the asset's risk.
- *Beta* (denoted by the Greek letter β) is a measure of the asset's risk used in the SML.

Finance concepts used

- Portfolios, risk-free asset
- Capital market line (CML)
- Beta, security market line (SML)
- Sharpe ratio

Excel functions used

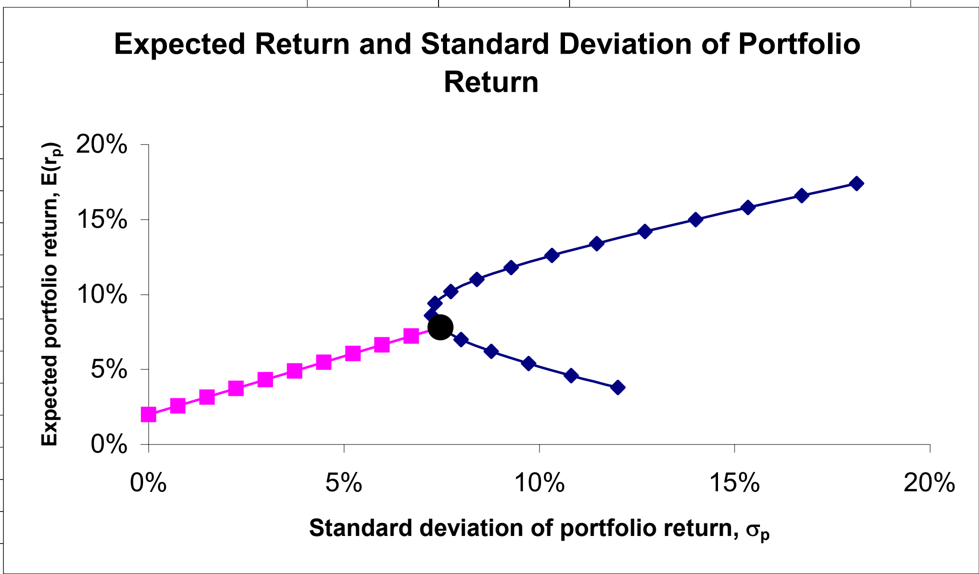
- VarP, StdevP, Sqrt

- Sophisticated graphing
- Solver

13.1. Risky portfolios and the riskless asset

We start by considering a portfolio problem of the kind dealt with in Chapter 12. There are two risky assets, Stock A and Stock B. Now suppose there exists a *risk-free asset*—an asset which gives an annual interest payment with *certainty*. You can think of this asset as being a savings account in a bank or a government bond. In the examples of this section, we'll suppose that the risk-free asset gives a 2% annual return. we'll denote the return on the risk-free asset by r_f. The first few lines of the following spreadsheet gives you all the details:

	A	B	C	D	E
1	**TWO STOCKS AND A RISK-FREE ASSET**				
2		**Stock A**	**Stock B**		
3	Average return	7.00%	15.00%		
4	Variance	0.0064	0.0196		
5	Sigma	8.00%	14.00%		
6	Covariance of returns	0.00			
7	Correlation	0.10			
8					
9	Risk-free rate, r_f	2%			



	A	B	C	D	E
27					
28	**Round dot portfolio** ●				
29	A	0.9			
30	B	0.1			
31	Mean	7.80%	<-- =B29*$B$3+(1-B29)*$C$3		
32	Sigma	7.47%	<-- =SQRT(B29^2*$B$4+(1-B29)^2*$C$4 +2*B29*(1-B29)*$B$6)		

The curved line gives the portfolio mean and standard deviation of combinations of Stock A and Stock B.[1] The straight line shows the mean and standard deviation of portfolio combinations of the risk-free asset (which returns r_f = 2%) and a specific portfolio of risky assets, denoted by round dot ●).

[1] This was illustrated in Chapters 11 and 12.

Notice that rows 29-32 give you information about the round dot portfolio ●: It is composed of 90% stock A and 10% stock B, and it has expected return 7.8% and standard deviation of return 7.47%.

Computing a point on the straight line

In the spreadsheet below we indicate two points on the straight line which connects the risk-free rate r_f and the round dot portfolio ●. Each point represents a portfolio which is partly invested in the risk free asset and partly in the portfolio ●. Take a look, and then after the spreadsheet we'll show you how to calculate the mean and standard deviation of the points on the line.

	A	B	C	D	E
1	**TWO STOCKS AND A RISK-FREE ASSET**				
2		**Stock A**	**Stock B**		
3	Average return	7.00%	15.00%		
4	Variance	0.0064	0.0196		
5	Sigma	8.00%	14.00%		
6	Covariance of returns	0.00			
7	Correlation	0.10			
8					
9	Risk-free rate	2%			
10					
11					
12					
13					
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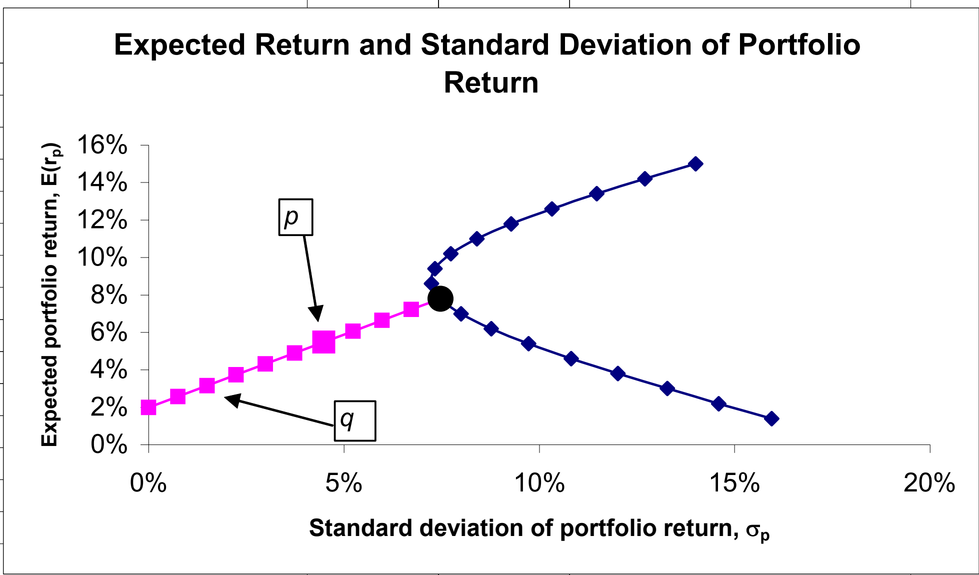
27					



The "round-dot portfolio" ● is composed of an investment 90% in A and 10% in B. What about portfolio p? p is a portfolio invested 60% in the "round-dot portfolio" and 40% in the risk-free asset. To compute the returns of this portfolio, we use the following equations:

$$E\left(r_p\right) = \underbrace{x}_{\substack{\uparrow \\ \text{Percent in} \\ \text{"round-dot"} \\ \text{portfolio}}} E\left(r_{round-dot}\right) + \underbrace{\left(1-x\right)}_{\substack{\uparrow \\ \text{Percent in} \\ \text{risk-free asset}}} * r_f = 60\% * 7.8\% + 40\% * 2\% = 5.48\%$$

$$\sigma_p = \underbrace{x}_{\substack{\uparrow \\ \text{Percent in} \\ \text{"round-dot"} \\ \text{portfolio}}} \sigma_{round-dot} = 60\% * 7.47\% = 5.48\%$$

In a similar fashion portfolio q—invested 20% in the "round-dot" portfolio and 80% in the risk-free asset—has statistics:

$$E(r_q) = \underbrace{x}_{\substack{\uparrow \\ \text{Percent in} \\ \text{"round-dot"} \\ \text{portfolio}}} E(r_{round-dot}) + \underbrace{(1-x)}_{\substack{\uparrow \\ \text{Percent in} \\ \text{risk-free asset}}} * r_f = 20\% * 7.8\% + 80\% * 2\% = 3.16\%$$

$$\sigma_q = \underbrace{x}_{\substack{\uparrow \\ \text{Percent in} \\ \text{"round-dot"} \\ \text{portfolio}}} \sigma_{round-dot} = 20\% * 7.47\% = 1.49\%$$

A statistical note

The equations used in the last calculation follow from our lessons in portfolio statistics in Chapter 11. Suppose the investor invests a percentage of her wealth x in a portfolio A of risky assets which has expected return $E(r_A)$ and standard deviation of return σ_A. Suppose she invests the rest of her wealth $1-x$ in a risk-free asset which has expected return r_f and standard deviation of return 0. By the formula given in Chapter 12, the portfolio's expected return is its weighted-average return:

$$E(r_p) = x E(r_A) + (1-x) r_f .$$

The portfolio's return variance is

$$Var(r_p) = x^2 Var(r_A) + (1-x)^2 \underbrace{Var(r_f)}_{\substack{\uparrow \\ =0\text{, since} \\ \text{the risk-free} \\ \text{asset is risk-free} \\ \text{(duh!)}}} + 2 * x * (1-x) * \underbrace{Cov(A, r_f)}_{\substack{\uparrow \\ =0\text{, since} \\ \text{the risk-free} \\ \text{asset is} \\ \text{risk-free} \\ \text{(duh!)}}} .$$

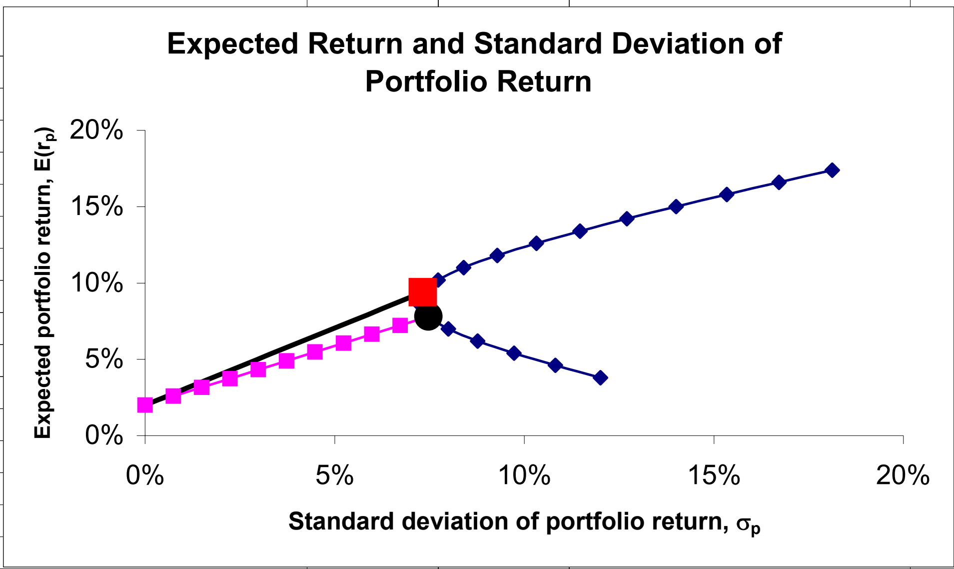
$$= x^2 Var(r_A) = x^2 \sigma_A^2$$

This means that the standard deviation of the portfolio's return is $\sigma_p = x\sigma_A$.

Improving on round dot portfolio ●

We can do better than line connecting r_f and the round dot portfolio ● by choosing another portfolio on the efficient frontier. The line connecting the risk-free asset and the "red-square" portfolio below is an improvement on the line of the previous section:

	A	B	C	D	E
1	**TWO STOCKS AND A RISK-FREE ASSET**				
2		**Stock A**	**Stock B**		
3	Average return	7.00%	15.00%		
4	Variance	0.0064	0.0196		
5	Sigma	8.00%	14.00%		
6	Covariance of returns	0.00			
7	Correlation	0.10			
8					
9	Risk-free rate	2%			

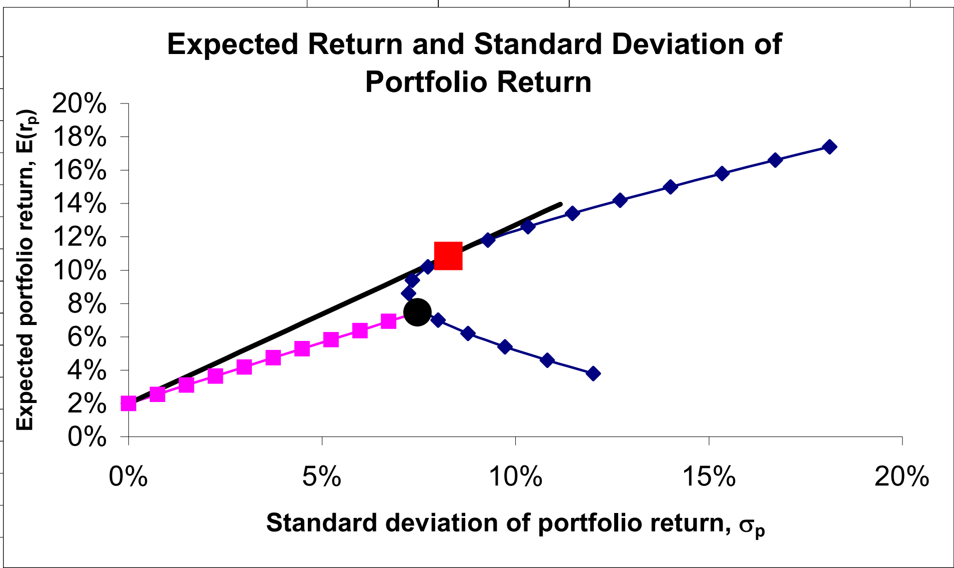


	A	B	C	D	E
27					
28	**Round dot portfolio** ●				
29	A	0.9			
30	B	0.1			
31	Mean	7.80%	<-- =B29*$B$3+(1-B29)*$C$3		
32	Sigma	7.47%	<-- =SQRT(B29^2*$B$4+(1-B29)^2*$C$4 +2*B29*(1-B29)*$B$6)		
33					
34	**Red square portfolio** ■				
35	A	0.7			
36	B	0.3			
37	Mean	9.40%	<-- =B35*$B$3+(1-B35)*$C$3		
38	Sigma	7.33%	<-- =SQRT(B35^2*$B$4+(1-B35)^2*$C$4 +2*B35*(1-B35)*$B$6)		

Since the new line is higher than the old line, all the points on the line to the red square ■ are better than the points on the line to the black circle ● . For any point on the round dot line there's always a point on the red square line which gives a higher return but has the same portfolio standard deviation σ_p.

There must be a *best* line which starts off from the point 2% on the *y*-axis. Here it is:

	A	B	C	D	E
1	**TWO STOCKS AND A RISK-FREE ASSET** **the best red square portfolio** ■				
2		**Stock A**	**Stock B**		
3	Average return	7.00%	15.00%		
4	Variance	0.0064	0.0196		
5	Sigma	8.00%	14.00%		
6	Covariance of returns	0.00			
7	Correlation	0.10			
8					
9	Risk-free rate	2%			



	A	B	C	D	E
27					
28	**Round dot portfolio** ●				
29	A	0.9			
30	B	0.1			
31	Mean	7.80%	<-- =B29*$B$3+(1-B29)*$C$3		
32	Sigma	7.47%	<-- =SQRT(B29^2*$B$4+(1-B29)^2*$C$4 +2*B29*(1-B29)*$B$6)		
33					
34	**Best red square portfolio** ■				
35	A	0.5181			
36	B	0.4819			
37	Mean	10.85%	<-- =B35*$B$3+(1-B35)*$C$3		
38	Sigma	8.26%	<-- =SQRT(B35^2*$B$4+(1-B35)^2*$C$4 +2*B35*(1-B35)*$B$6)		

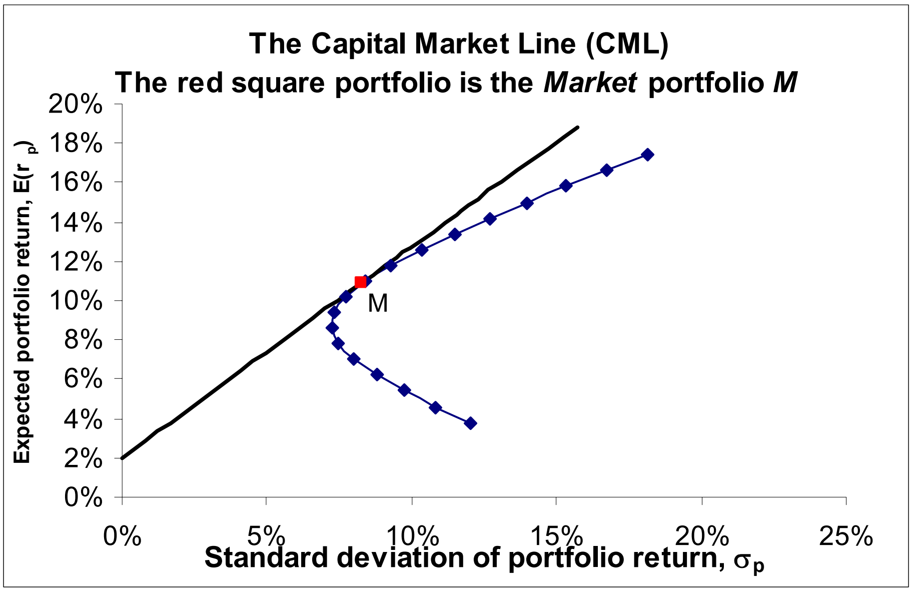
The line as drawn has several properties:

- It starts from the risk-free rate (2%) on the *y*-axis.

- It goes to (and through) a portfolio on the efficient frontier market by the red square. As you can see in cells B35:B38, this portfolio is composed 51.81% of Stock A and 48.19% of Stock B. It has expected return 10.85% and standard deviation 8.26%. In Section ??? we'll describe how we computed this portfolio.
- It is tangent to the efficient frontier—meaning, the line touches the efficient frontier only at the red square portfolio and nowhere else.
- Finally (and this is the most important point) all the best investment portfolios are on the red square line. This point is so important that we explore it in a separate subsection.

Optimal portfolios

To emphasize the optimality take another look at the "red point line":



Notice that the line is above the efficient frontier *everywhere* (except at point of tangency, which we now call the *market portfolio M*). We call this line the *capital market line* (CML):

The capital market line is the set of optimal investment portfolios. Each point on the line is:

- *A combination of some percentage invested in the risk-free asset*
- *Another percentage invested in the market portfolio M*

In Section ??? we'll show you how to compute the portfolio *M*. But first, in the next section, we explore its meaning.

13.2. Three points on the capital market line (CML)—exploring optimal investment combinations

What do portfolios on the CML—the line connecting the risk-free rate r_f and the market portfolio *M*—look like? To get a feel for this, we explore three portfolios on the CML.

First example

Suppose you have $1000 to invest. You can choose any combination of 3 assets—the risk-free asset, stock A, or stock B. Suppose you choose to invest $500 in the risk-free asset and $500 in the market portfolio *M*—the portfolio composed of 51.81% stock A and 48.19% stock B.

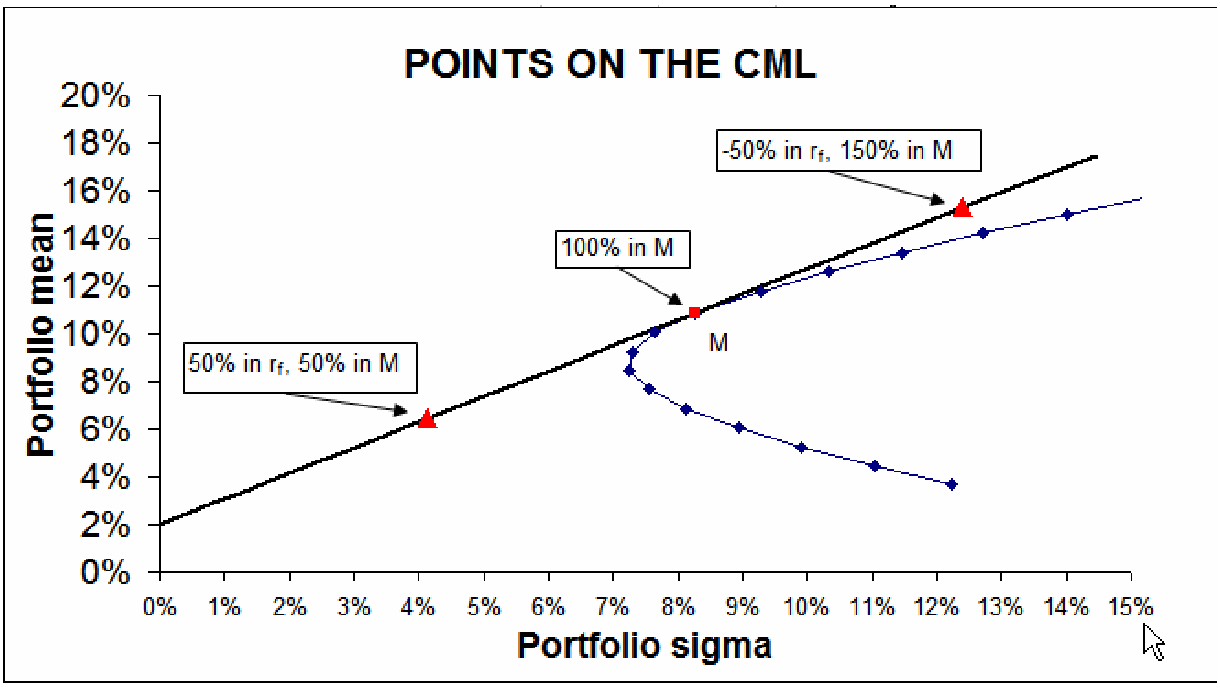
	A	B	C	D	E	F
1	**PORTFOLIO ON THE CAPITAL MARKET LINE (CML)**					
2		**Stock A**	**Stock B**	**Risk-free**		
3	Average	7.00%	15.00%	2.00%		
4	Variance	0.0064	0.0196			
5	Standard deviation (sigma)	8.00%	14.00%			
6	Covariance of returns, Stock A and Stock B	0.0011				
7						
8	Total amount to invest	$1,000				
9	Invested in risk-free	50%				
10	Invested in market portfolio M	50%				
11						
12	The % in the market portfolio M is split as follows	**Percent**	**Dollars**			
13	Stock A	51.81%	$259.07	<-- =B13*B10*$B$8		
14	Stock B	48.19%	$240.93	<-- =B14*B10*$B$8		
15	Expected return of market portfolio M	10.85%		<-- =B13*B3+B14*C3		
16	Standard deviation of market portfolio M	8.26%		<-- =SQRT(B13^2*B4+B14^2*C4+2*B13*B14*B6)		
17						
18	**Portfolio return statistics**					
19	Expected portfolio return	6.43%	<-- =B9*D3+B10*B15			
20	Portfolio standard deviation	4.13%	<-- =B10*B16			

The $500 invested in the risky assets are invested in the market portfolio *M*. The dollar investment in Stocks A and B corresponds to their proportions in *M*: $259.07 in A (51.81%) and $240.93 in B (48.19%).

This looks a little complicated, but it's really a version of the portfolio calculations we did in Chapter 12. Our investment is divided 50% into the risk-free asset and another 50% into portfolio M which has expected return 10.85% and standard deviation 8.26%. According to the formula given in Section ???, the expected return and the variance of returns are calculated by:

$$E\left(r_p\right) = xE\left(r_M\right) + \left(1 - x\right) r_f$$
$$\sigma_p = x\sigma_M$$

As you can see in cells B19:B20, this gives $E\left(r_p\right) = 6.43\%, \sigma_p = 4.13\%$. This portfolio is indicated in the graph below:



Second example

In the previous example, you split your investment of $1000 between the risk-free asset and the market portfolio *M*. This time we'll investigate an investment strategy in which you borrow money at the risk-free rate and invest *more than $1000* in the risky portfolio *M*. You do this by using borrowed funds to increase your investment in *M*.

As before, you have $1000 to invest, and as before you choose to invest some of your money in the risk-free asset and the rest in the market portfolio *M*, composed of 51.81% stock A and 48.19% stock B. However, this time you choose to *borrow* $500 at the risk-free rate and invest $1500 in the portfolio of stock A and stock B. As you can see below, this is a riskier portfolio (it has a standard deviation of 12.40%), but it also has a higher expected return (15.28%):

	A	B	C	D	E
1	**PORTFOLIO ON THE CAPITAL MARKET LINE (CML)**				
2		**Stock A**	**Stock B**	**Risk-free**	
3	Average	7.00%	15.00%	2.00%	
4	Variance	0.0064	0.0196		
5	Standard deviation (sigma)	8.00%	14.00%		
6	Covariance of returns, Stock A and Stock B	0.0011			
7					
8	Total amount to invest	$1,000			
9	Invested in risk-free	-50%			
10	Invested in market portfolio M	150%			
11					
12	The % in the market portfolio M is split as follows	**Percent**	**Dollars**		
13	Stock A	51.81%	$777.20	<-- =B13*B10*$B$8	
14	Stock B	48.19%	$722.80	<-- =B14*B10*$B$8	
15	Expected return of market portfolio M	10.85%		<-- =B13*B3+B14*C3	
16	Standard deviation of market portfolio M	8.26%		<-- =SQRT(B13^2*B4+B14^2*C4+2*B13*B14*B6)	
17					
18	**Portfolio return statistics**				
19	Expected portfolio return	15.28%	<-- =B9*D3+B10*B15		
20	Portfolio standard deviation	12.40%	<-- =B10*B16		

Generalizing

The portfolio calculations we've done in the previous two examples don't really depend on the $1,000 initial wealth. What's important is the *percentage of the investor's wealth invested in the market portfolio M* and the *percentage of the investor's wealth invested in the risk-free asset.*

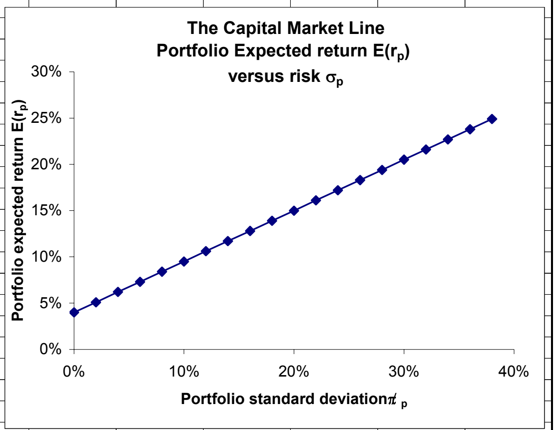
Suppose we denote these percentages by x_M and $x_{r_f} = 1 - x_M$. Then the investor's portfolio will have:

- Expected return $E\left(r_p\right) = x_M E\left(r_M\right) + \left(1 - x_M\right) r_f$
- Standard deviation of return $\sigma_p = x_M \sigma_M$

Here's an example which uses statistics which are typical for the S&P 500:

	A	B	C
1	**INVESTING IN A COMBINATION OF THE S&P500 AND THE RISK-FREE USING TYPICAL S&P500 STATISTICS**		
2	**S&P statistics**		
3	Expected return, $E(r_M)$	15%	
4	Standard deviation of return, σ_M	20%	
5			
6	Risk-free rate of interest	4%	
7			
8	**Investor's portfolio**		
9	Percentage of wealth invested in S&P	25%	
10	Percentage of wealth invested in risk-free asset	75%	<-- =1-B9
11			
12	**Investor's return and standard deviation**		
13	Expected portfolio return, $E(r_p)$	6.75%	<-- =B9*B3+B10*B6
14	Standard deviation of portfolio return, σ_p	5.00%	<-- =B9*B4
15			
16	**Table**		
17	**Percentage invested in S&P 500**	σ_p	$E(r_p)$
18	0%	0%	4.00%
19	10%	2%	5.10%
20	20%	4%	6.20%
21	30%	6%	7.30%
22	40%	8%	8.40%
23	50%	10%	9.50%
24	60%	12%	10.60%
25	70%	14%	11.70%
26	80%	16%	12.80%
27	90%	18%	13.90%
28	100%	20%	15.00%
29	110%	22%	16.10%
30	120%	24%	17.20%
31	130%	26%	18.30%
32	140%	28%	19.40%
33	150%	30%	20.50%
34	160%	32%	21.60%
35	170%	34%	22.70%
36	180%	36%	23.80%
37	190%	38%	24.90%

These are *borrowing portfolios*--the investor borrows at the risk-free rate in order to increase her investment in the market portfolio *M*.



13.3. Computing the market portfolio *M*: the Sharpe ratio

In this section we'll show how to compute the market portfolio *M*. In the process we'll introduce a concept called the *Sharpe ratio*—this is one of the standard *reward-return* measures used in capital markets. As you'll see, the portfolio *M* is the portfolio which maximizes the Sharpe ratio.

To get some intuitions, look at the spreadsheet below. It continues our example of Stocks A and B and the risk-free rate of 2%. In cells B9:B10 we're looking at a portfolio invested 30% in Stock A and 70% in Stock B. The expected return of this portfolio is 12.60% and its standard deviation is 10.32% (cells B12:B13):

	A	B	C	D	E	F
1	**PORTFOLIO RETURNS WITH A RISK-FREE ASSET THE SHARPE RATIO**					
2			**Stock A**	**Stock B**	**Risk-free**	
3	Average return		7.00%	15.00%	2.00%	
4	Variance of return		0.64%	1.96%		
5	Sigma of return		8.00%	14.00%		
6	Covariance of returns		0.0011			
7						
8	**Portfolio return and risk**					
9	Percentage in Stock A	30.00%				
10	Percentage in Stock B	70.00%				
11						
12	Expected portfolio return	12.60%	<-- =B9*C3+B10*D3			
13	Portfolio standard deviation	10.32%	<-- =SQRT(B9^2*C4+B10^2*D4+2*B9*B10*C6)			
14						
15	Excess return	10.60%	<-- =B12-E3			
16						
17	Sharpe ratio	1.0271	<-- =(B12-E3)/B13			
18						
19		**The Sharpe ratio is $[E(r_p) - r_f]/\sigma_p$. It denotes the ratio of portfolio *excess return* to portfolio *risk*.**				

The portfolio's *excess return* (sometimes called a *risk-premium*) is defined as the difference between its expected return and that of the risk-free asset:

$$\begin{aligned} \textit{portfolio risk-premium} &= \textit{portfolio expected return} - \textit{risk-free rate} \\ &= E(r_p) - r_f \\ &= 12.60\% - 2.00\% = 10.60\% \end{aligned}$$

The ratio of this risk-premium to the portfolio's standard deviation is called the *Sharpe ratio*:

$$\textit{Sharpe ratio} = \frac{E(r_p) - r_f}{\sigma_p} = \frac{12.60\% - 2.00\%}{10.32\%} = 1.0271.$$

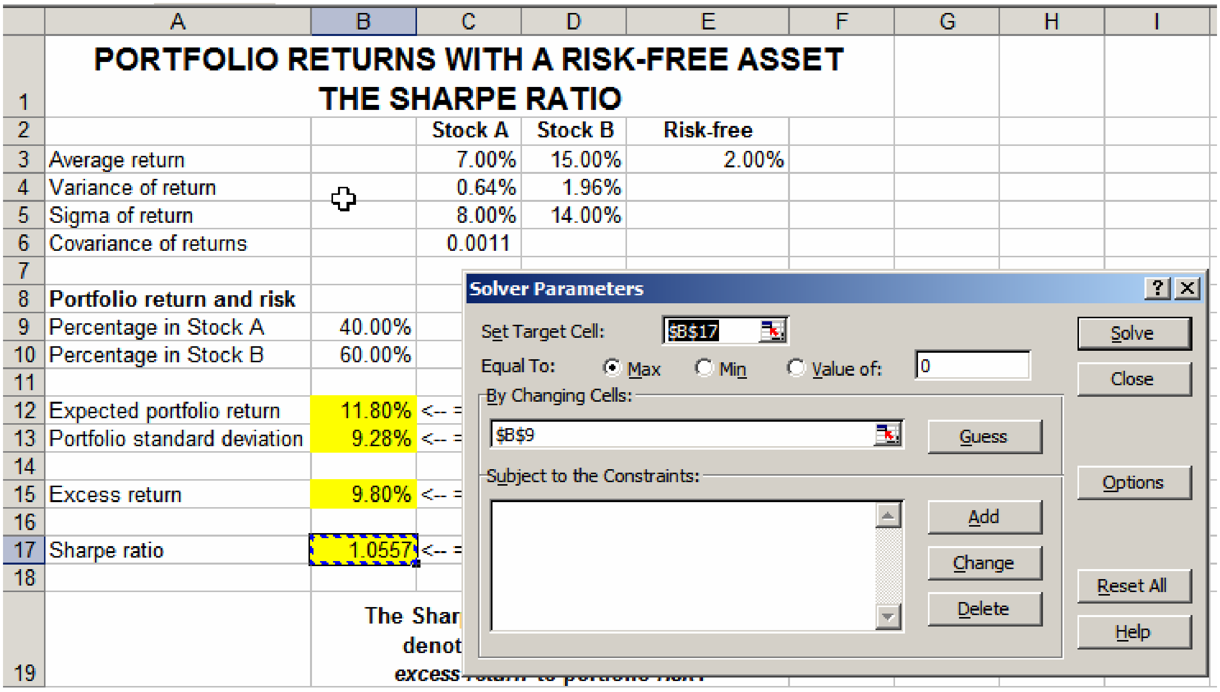
The Sharpe ratio (named after William Sharpe, one of the developers of modern portfolio theory and winner of the Nobel prize in economics in 1990) is a "reward/risk" ratio: The numerator is the extra return (over the risk-free rate) you get from your portfolio, and the denominator is the cost of this extra return—its standard deviation.

If you play a bit with the spreadsheet, you'll see that there are other portfolios with higher Sharpe ratios. Here's an example:

	A	B	C	D	E	F
8	**Portfolio return and risk**					
9	Percentage in Stock A	40.00%				
10	Percentage in Stock B	60.00%				
11						
12	Expected portfolio return	11.80%	<-- =B9*C3+B10*D3			
13	Portfolio standard deviation	9.28%	<-- =SQRT(B9^2*C4+B10^2*D4+2*B9*B10*C6)			
14						
15	Excess return	9.80%	<-- =B12-E3			
16						
17	Sharpe ratio	1.0557	<-- =(B12-E3)/B13			

Calculating the market portfolio *M*—the portfolio with the *highest attainable* Sharpe ratio

We can use Excel's Solver (see Chapter ????) to calculate the portfolio which gives the highest Sharpe ratio. This portfolio is the *market portfolio M*.



Pressing **Solve** yields the answer:

	A	B	C	D	E	F
1	**PORTFOLIO RETURNS WITH A RISK-FREE ASSET THE SHARPE RATIO**					
2			**Stock A**	**Stock B**	**Risk-free**	
3	Average return		7.00%	15.00%	2.00%	
4	Variance of return		0.64%	1.96%		
5	Sigma of return		8.00%	14.00%		
6	Covariance of returns		0.0011			
7						
8	**Portfolio return and risk**					
9	Percentage in Stock A	51.81%				
10	Percentage in Stock B	48.19%				
11						
12	Expected portfolio return	10.85%	<-- =B9*C3+B10*D3			
13	Portfolio standard deviation	8.26%	<-- =SQRT(B9^2*C4+B10^2*D4+2*B9*B10*C6)			
14						
15	Excess return	8.85%	<-- =B12-E3			
16						
17	Sharpe ratio	1.0716	<-- =(B12-E3)/B13			

From now on, we'll denote the portfolio with the maximum Sharpe ratio by M:

Given a risk-free asset and a set of risky assets (in the current example there are only 2 such assets), the market portfolio M is the portfolio that maximizes the Sharpe ratio: $\frac{E(r_M)-r_f}{\sigma_M}$ *is larger for M than for any other portfolio.*

The portfolio M is the best combination of risky assets available to the investor.

13.4. The security market line (SML): A remarkable fact

We now show a remarkable fact about expected returns. This fact, called the *security market line* (SML) states that the expected return of an asset or portfolio is determined by the asset's risk (called β), the risk-free rate, and the portfolio which maximizes the Sharpe ratio.

Summing up the SML first (then we'll explain)

The SML says that *the expected return on any portfolio of assets is related to the risk-free rate and the market risk-premium through the following relation*:

$$E(r_p) = r_f + \underbrace{\frac{Cov(r_p, r_M)}{Var(r_M)}}_{\beta_p}\left[E(r_M) - r_f\right]$$

$\uparrow$ β_p

$\uparrow$ $E(r_M)$ is the return on the portfolio which maximizes the Sharpe ratio

Note that in the above equation "portfolio" (represented by the letter "p") can be a lot of things:

- A "portfolio" can be the combination of two risky assets—60% in Stock A and 40% in Stock B.
- A "portfolio" can be just *one* risky asset—100% of your wealth invested in Stock A is a portfolio.
- A "portfolio" can be a combination of the risk-free asset and the two stocks—25% in the risk-free, 30% in Stock A, and 45% in Stock B is a portfolio.

In short: The SML defines the risk-return relation *for all assets in the market.* In the next 2 chapters we examine the uses of β for evaluating the performance of portfolio managers and for computing the cost of capital for a firm.

In order to illustrate the SML, we use a few examples.

Example 1: The SML works for a portfolio composed only of stock A

Lines 3-15 of the spreadsheet below repeat facts we've already given. In row 24 we compute the covariance between a portfolio p and the market portfolio M. If p is composed of a combination of stock A and stock B, then this covariance is given by:

$$\begin{aligned} Cov(r_p, r_M) &= Cov(x*r_A + (1-x)r_B, y*r_A + (1-y)r_B) \\ &= x*y*Cov(r_A, r_A) + (1-x)*(1-y)*Cov(r_B, r_B) \\ &\quad + x*(1-y)Cov(r_A, r_B) + (1-x)*y*Cov(r_B, r_A) \end{aligned}$$

Cell B24 computes this for our portfolio p (in this case, p is composed wholly of asset A). In cell B25 we divide $Cov(r_p,r_M)$ by $Var(r_M)$ to get the beta of the portfolio, β_p.

	A	B	C	D	E
1	**THE SECURITY MARKET LINE (SML)**				
2		**Stock A**	**Stock B**	**Risk-free**	
3	Average return	7.00%	15.00%	2.00%	
4	Variance of return	0.0064	0.0196		
5	Sigma of return	8.00%	14.00%		
6	Covariance of returns	0.0011			
7					
8	**Market portfolio M--this is the portfolio that maximizes the Sharpe ratio**				
9	Percentage in Stock A	0.5181			
10	Percentage in Stock B	0.4819			
11					
12	Expected market portfolio return, $E(r_M)$	10.85%	<-- =B9*B3+B10*C3		
13	Market portfolio return variance, σ^2_M=Var(r_M)	0.0068	<-- =B9^2*B4+B10^2*C4+2*B9*B10*B6		
14	Market portfolio standard deviation σ_M=standard deviation(r_M)	8.26%	<-- =SQRT(B13)		
15					
16	Market excess return $E(r_M)-r_f$	8.85%	<-- =B12-D3		
17					
18	**"Proof" of SML: $E(r_p) = r_f + \beta p*[E(r_M) - r_f]$**				
19	Portfolio				
20	Proportion of stock A, x	100.00%			
21	Proportion of stock B, 1-x	0.00%			
22	Expected portfolio return $E(r_p)=x*E(r_A)+(1-x)*E(r_B)$ SML, left-hand side	7.00%	<-- =B20*B3+B21*C3		
23					
24	Cov(portfolio,Market)	0.0039	<-- =B20*B9*B4+B21*B10*C4+B20*B10*B6+B21*B9*B6		
25	Beta β_p	0.5647	<-- =B24/B13		
26	$r_f+\beta_p*[E(r_M)-r_f]$ SML,right-hand side	7.00%	<-- =D3+B25*B16		

Now look at the expected portfolio return (7%) given in cell B22 and the expected portfolio return in cell B26. Though computed in different ways, they're the same. This is the SML: $E(r_p) = r_f + \underbrace{\beta_p}_{\uparrow \atop \frac{Cov(r_p,r_M)}{Var(r_M)}} \left[E(r_M) - r_f\right]$.

Notice that the beta of stock A was computed as $\beta_A = 0.5647$.

Example 2: The SML works for a portfolio composed only of stock B

Without too much bullshit, we'll repeat the calculations for stock B. This stock turns out to have a β_B = 1.4681:

	A	B	C	D	E
18	**"Proof" of SML: $E(r_p) = r_f + \beta p*[E(r_M) - r_f]$**				
19	Portfolio				
20	Proportion of stock A, x	0.00%			
21	Proportion of stock B, 1-x	100.00%			
22	Expected portfolio return $E(r_p)=x*E(r_A)+(1-x)*E(r_B)$ SML, left-hand side	15.00%	<-- =B20*B3+B21*C3		
23					
24	Cov(portfolio,Market)	0.0100	<-- =B20*B9*B4+B21*B10*C4+B20*B10*B6+B21*B9*B6		
25	Beta β_p	1.4681	<-- =B24/B13		
26	$r_f+\beta_p*[E(r_M)-r_f]$ SML,right-hand side	15.00%	<-- =D3+B25*B16		

Again notice that the SML works.

Example 3: The SML works for mixed portfolios

	A	B	C	D	E
19	Portfolio				
20	Proportion of stock A, x	80.00%			
21	Proportion of stock B, 1-x	20.00%			
22	Expected portfolio return $E(r_p)=x*E(r_A)+(1-x)*E(r_B)$ SML, left-hand side	8.60%	<-- =B20*B3+B21*C3		
23					
24	Cov(portfolio,Market)	0.0051	<-- =B20*B9*B4+B21*B10*C4+B20*B10*B6+B21*B9*B6		
25	Beta β_p	0.7453	<-- =B24/B13		
26	$r_f+\beta_p*[E(r_M)-r_f]$ SML,right-hand side	8.60%	<-- =D3+B25*B16		

A statistical note (skip this if statistics scares you)

The calculation of $Cov(r_p, r_M)$ is based on the fact that the covariance is linear and multiplicative:

$$\text{Linear:} \quad Cov(x+y,z) = Cov(x,z) + Cov(y,z)$$
$$\text{Multiplicative:} \quad Cov(ax,z) = aCov(x,z)$$

Applying this to $Cov(r_p, r_M)$, gives:

$$\begin{aligned}
Cov(r_p, r_M) &= Cov\left(x_{GM}^p r_{GM} + x_{MSFT}^p r_{MSFT}, x_{GM}^M r_{GM} + x_{MSFT}^M r_{MSFT}\right) \\
&= Cov\left(x_{GM}^p r_{GM}, x_{GM}^M r_{GM}\right) + Cov\left(x_{GM}^p r_{GM}, x_{MSFT}^M r_{GM}\right) \\
&\quad + Cov\left(x_{MSFT}^p r_{MSFT}, x_{GM}^M r_{GM}\right) + Cov\left(x_{MSFT}^p r_{MSFT}, x_{MSFT}^M r_{MSFT}\right) \\
&= x_{GM}^p x_{GM}^M Cov(r_{GM}, r_{GM}) + x_{GM}^p x_{MSFT}^M Cov(r_{GM}, r_{GM}) \\
&\quad + x_{MSFT}^p x_{GM}^M Cov(r_{MSFT}, r_{GM}) + x_{MSFT}^p x_{MSFT}^M Cov(r_{MSFT}, r_{MSFT}) \\
&= x_{GM}^p x_{GM}^M Var(r_{GM}) + x_{GM}^p x_{MSFT}^M Cov(r_{GM}, r_{GM}) \\
&\quad + x_{MSFT}^p x_{GM}^M Cov(r_{MSFT}, r_{GM}) + x_{MSFT}^p x_{MSFT}^M Var(r_{MSFT})
\end{aligned}$$

Betas add up

The portfolio beta is the weighted average of the individual betas:

$$\beta_p = x_{GM}\beta_{GM} + x_{MSFT}\beta_{MSFT}$$

For example, suppose we want to know the expected return from a portfolio invested 20% in stock A and 80% in stock B. This portfolio will have a β_p of:

$$\beta_p = x_A\beta_A + x_B\beta_B = 0.2 * 0.5647 + 0.8 * 1.4681 = 1.0164,$$

and consequently its expected return should be determined by the SML using the β_p:

	A	B	C
1	**BETAS ADD UP**		
2	β_A	0.5647	
3	β_B	1.4681	
4			
5	**Portfolio composition**		
6	Percentage A	50%	
7	Percentage B	50%	
8	Portfolio beta, β_p	1.0164	<-- =B6*B2+B7*B3

Summing up

The capital asset pricing model (CAPM) is a model of portfolio formation and asset pricing. The model shows:

- How the expected return and standard deviation of portfolios are affected by the portfolio composition.
- How the addition of a risk-free asset to the choices available to investors changes their risk-return opportunity set.
- How to compute the *market portfolio* M. This is the portfolio which maximizes the Sharpe ratio: $\frac{E(r_p)-r_f}{\sigma_p}$.
- How to choose an *optimal portfolio* when you can invest in risky and risk-free assets. This is the *capital market line* (CML) which states that all optimal portfolios are combinations of the risk-free asset and the market portfolio.

- How to compute the *beta* for a stock or portfolio. Beta (β) is a *risk-measure* for an asset. For a portfolio p, β_p is defined as: $\beta_p = \dfrac{Cov(r_p, r_M)}{\sigma_M^2}$. (Recall that "portfolio" includes the case of individual assets.)
- How the *expected return of any portfolio* is related to the risk-free rate and the portfolio's beta. This is the *security market line* (SML):

$$E(r_p) = r_f + \beta_p \left[E(r_m) - r_f \right]$$

In succeeding chapters we explore the implications of this model, using it to examine the performance of portfolio managers and to calculate a firm's cost of capital.

EXERCISES

1. Consider the following portfolio and accompanying statistics:

	Company A	Company B
Portfolio Composition	90%	10%
Average Return	21%	48%
Variance	6.15%	16%
Sigma	24.81%	39.40%
Covariance of Returns	0.00390	
Correlation of Returns	0.03986	

1.a. Is this an optimal portfolio?

1.b. If not, suggest a portfolio combination, which improves return while maintaining the same level of risk.

1.c. Calculate the minimum variance portfolio for the portfolio composed of the two assets described above.

2. Using the data provided in the previous question, calculate the market portfolio M, when the risk-free rate of return is 8%. (Recall that the M portfolio is that portfolio which maximizes the Sharpe ratio).

3. On the occasion of your birthday, your wealthy Aunt Hilda sends you a check for $5,000, under the express condition that you invest the money in either (or all) of the following: Government Bonds, Hilda's Hybrids Inc., and/or Hilda's Hubby Inc. The relevant statistics on each of these investments are provided below.

	Hilda's Hybrids	Hilda's Hubby	Government Bond
Average Return	30.00%	16.25%	10.00%
Variance	28.58%	2.30%	0.00%
Sigma	53.46%	15.17%	0.00%
Covariance of Returns	0.03425		
Correlation of Returns	0.42240		

3.a. Show the Capital Market Line, i.e. all the combinations of investment in the risk-free asset and the two companies. Provide results in both chart and graph form. Note that it will be helpful to first calculate the market portfolio M.

3.b. Supposing you decided to invest in the following proportions, 40% government bonds, 60% in the M portfolio. Calculate the expected return and variance of returns for this portfolio.

4. With reference to Question 3 above, you are feeling lucky and decide to take on a riskier portfolio. In particular, in addition to your \$5,000 gift, you are able to borrow another \$1,000 at the risk-free rate of 10%. You decide to invest this total of \$6,000 in a portfolio containing a mix of Hilda's Hybrids and Hilda's Hubby.

4.a. In what proportion will you invest your \$6,000, if your objective is to create the "best combination" of these risky assets?

4.b. What will be the expected return and the expected risk for this more daring portfolio?

5. a. Consider the data below. Compute the expected return and standard deviation of returns for a portfolio composed of 75% stock A and 25% stock B.

	Asset A	Asset B
Mean return	30%	13%
Return sigma σ	40%	10%
Correlation ρ_{AB}	0.5	

5.b. Stock C has a β_C of 1.3 and the portfolio of 75% C and 25% D has a $\beta_p = 1.8$. What is the β of stock D?

5.c. Suppose the risk-free rate is $r_f = 5\%$, $E(r_M) = 15\%$ and $\sigma_M = 25\%$.

6.

- You have $1,000 to invest. If you follow an optimal investment policy, and if you desire to invest $500 in the risk-free asset, what is the mean and standard deviation of your portfolio return?
- Your sister also has $1,000 to invest, but wants to borrow another $1,000 in order to make an investment of $2,000 in the market portfolio *M*. What will be the mean and standard deviation of her portfolio return?
- Which portfolio is better, yours or your sister's?

Appendix: The CAPM with 3 or more assets[2]

Introduction

This appendix generalizes the CAPM and SML discussion in this chapter. The first part of the appendix discusses the portfolios of 3 assets. It will then be clear how to apply this to portfolios of more than 3 assets.

In preparation for this chapter, we recall the messages of the Chapter 13. This appendix is meant to confirm that all of these "messages" still hold, even if there are more than 2 risky assets.

- Calculation of the efficient frontier
- Calculation of the Sharpe ratio.
- Calculating the *market portfolio*—the portfolio of risky assets for which the Sharpe ratio is maximized. This calculation also requires the risk-free rate r_f.
- Calculating the SML—this is a relation between the *expected return of any asset*, the *risk-free rate* r_f, and the *expected return on the market portfolio* $E(r_M)$:

$$\underbrace{E(r_i)}_{\substack{\uparrow \\ \text{the expected} \\ \text{return of some} \\ \text{asset } i\text{. This can} \\ \text{be a single asset} \\ \text{or a portfolio.}}} = r_f + \underbrace{\beta_i}_{\substack{\uparrow \\ \text{the asset's } beta \\ \text{is defined as} \\ \beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}}} \underbrace{\left[E(r_M) - r_f\right]}_{\substack{\uparrow \\ \text{the market} \\ \text{risk premium}}}$$

[2] This appendix can easily be skipped—its purpose is to demonstrate that all of the results of this chapter hold in a more general setting. If you believe this already, go on to the next chapter.

Example

We start by considering a 3-asset problem. To describe 3 assets, we need to know the expected return (or mean return—should standardize on one terminology), the variance, and all the *pairs* of covariances. This data is described below.

	A	B	C	D
1	**A 3-ASSET PORTFOLIO PROBLEM**			
2		**Stock A**	**Stock B**	**Stock C**
3	Mean	10%	12%	15%
4	Variance	15%	22%	30%
5				
6	Cov(r_A,r_B)	0.03		
7	Cov(r_B,r_C)	-0.01		
8	Cov(r_A,r_C)	0.02		
9				
10	Risk-free	6%		

Suppose we form a portfolio of risky assets composed of proportion x_A in asset A, x_B in asset B, and x_C in asset C. Since the portfolio is fully invested in risky assets, it follows that $x_C = 1-x_A-x_B$.

Portfolio return statistics: The expected return of the portfolio is given by:

$$E\left(r_p\right)=x_A E\left(r_A\right)+x_B E\left(r_B\right)+x_C E\left(r_C\right)$$

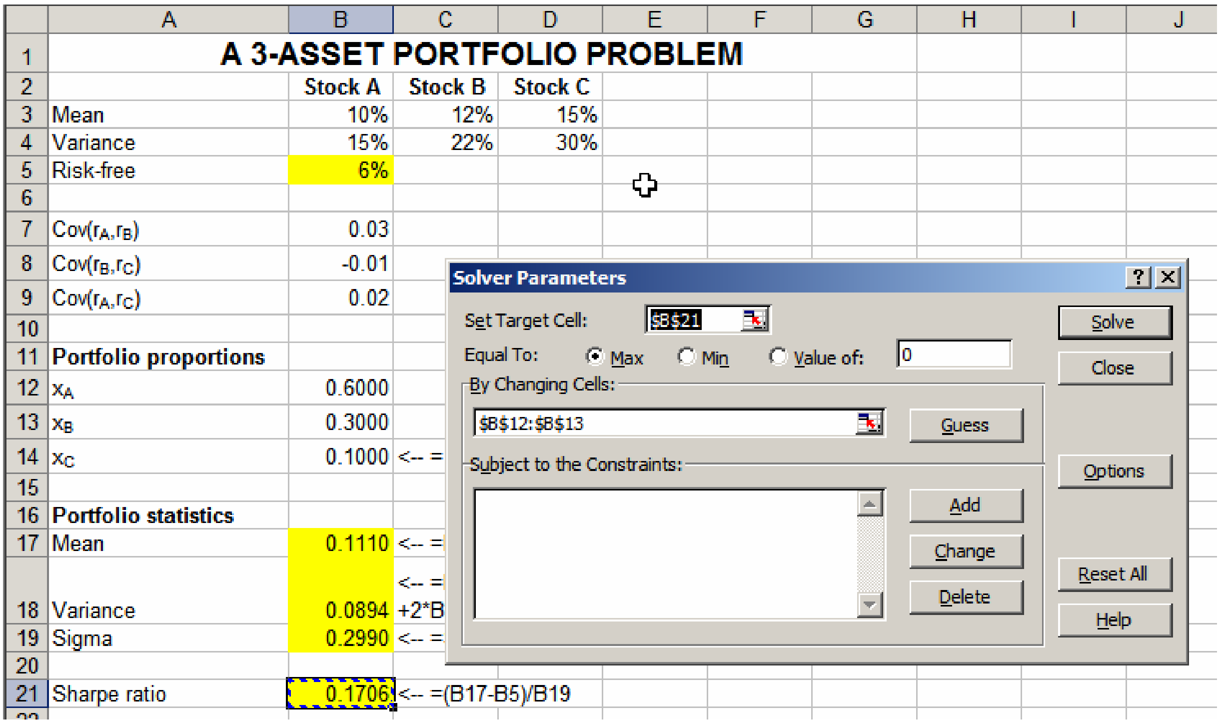
The calculation of the portfolio's variance of return requires both the variances and the covariances:

$$Var\left(r_p\right)=x_A^2 Var\left(r_A\right)+x_B^2 Var\left(r_B\right)+x_C^2 Var\left(r_C\right)+$$
$$2x_A x_B Cov\left(r_A,r_B\right)+2x_A x_C Cov\left(r_A,r_C\right)+2x_B x_C Cov\left(r_B,r_C\right)$$

Here's an example: The portfolio statistics are calculated in cells B17:B19:

	A	B	C	D	E	F	G
1	**A 3-ASSET PORTFOLIO PROBLEM**						
2		**Stock A**	**Stock B**	**Stock C**			
3	Mean	10%	12%	15%			
4	Variance	15%	22%	30%			
5	Risk-free	6%					
6							
7	$Cov(r_A,r_B)$	0.03					
8	$Cov(r_B,r_C)$	-0.01					
9	$Cov(r_A,r_C)$	0.02					
10							
11	**Portfolio proportions**						
12	x_A	0.6000					
13	x_B	0.3000					
14	x_C	0.1000	<-- =1-B13-B12				
15							
16	**Portfolio statistics**						
17	Mean	0.1110	<-- =B12*B3+B13*C3+B14*D3				
18	Variance	0.0894	<-- =B12^2*B4+B13^2*C4+B14^2*D4 +2*B12*B13*B7+2*B12*B14*B9+2*B13*B14*B8				
19	Sigma	0.2990	<-- =SQRT(B18)				
20							
21	Sharpe ratio	0.1706	<-- =(B17-B5)/B19				

Cell B21 calculates the Sharpe ratio, $\frac{E(r_p)-r_f}{\sigma_p}$, for the particular portfolio. In Chapter 13, we used Excel's **Solver** to find the portfolio with the maximum Sharpe ratio. We repeat this procedure here:



Pressing **Solve** gives the solution—the portfolio which maximizes the Sharpe ratio. Given a risk-free rate $r_f = 6\%$, this portfolio is the market portfolio M.

	A	B	C	D	E	F	G
1	**A 3-ASSET PORTFOLIO PROBLEM**						
2		**Stock A**	**Stock B**	**Stock C**			
3	Mean	10%	12%	15%			
4	Variance	15%	22%	30%			
5	Risk-free	6%					
6							
7	Cov(r_A,r_B)	0.03					
8	Cov(r_B,r_C)	-0.01					
9	Cov(r_A,r_C)	0.02					
10							
11	**Portfolio proportions**						
12	x_A	0.2378					
13	x_B	0.3575					
14	x_C	0.4047	<-- =1-B13-B12				
15							
16	**Portfolio statistics**						
17	Mean	0.1274	<-- =B12*B3+B13*C3+B14*D3				
18	Variance	0.0918	<-- =B12^2*B4+B13^2*C4+B14^2*D4 +2*B12*B13*B7+2*B12*B14*B9+2*B13*B14*B8				
19	Sigma	0.3030	<-- =SQRT(B18)				
20							
21	Sharpe ratio	0.2224	<-- =(B17-B5)/B19				

The security market line and β

In Chapter 13 we showed that the asset's β, defined as $\beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}$, relates the asset's expected return and the risk-free rate:

$$E(r_i) = r_f + \frac{Cov(r_i, r_M)}{Var(r_M)}\left[E(r_M) - r_f\right].$$

In the spreadsheet below, you can see that this is also true for our example. You need to know how to compute the covariance for a combination of assets. In the equation below, we compute the covariance between a the market portfolio, composed of proportions x_A, x_B, and x_C of stocks

A, B, C and any generic portfolio (here composed of proportions y_A, y_B, and y_C of these same stocks):

$$Cov\left(r_p, r_M\right) = x_A y_A \sigma_A^2 + x_B y_B \sigma_B^2 + x_C y_C \sigma_C^2$$
$$+\sigma_{AB}\left(x_A y_B + x_B y_A\right) + \sigma_{BC}\left(x_B y_C + x_C y_B\right) + \sigma_{AC}\left(x_A y_C + x_C y_A\right)$$

Now you can implement this, as shown in the spreadsheet:

	A	B	C	D	E	F	G
1	**THE SML WORKS FOR PORTFOLIOS OF 3 ASSETS!**						
2		**Stock A**	**Stock B**	**Stock C**			
3	Mean	10%	12%	15%			
4	Variance	15%	22%	30%			
5	Risk-free	6%					
6							
7	$Cov(r_A,r_B)$	0.03					
8	$Cov(r_B,r_C)$	-0.01					
9	$Cov(r_A,r_C)$	0.02					
10							
11	**Portfolio proportions**						
12	x_A	0.2378					
13	x_B	0.3575	<-- Now called the Market portfolio				
14	x_C	0.4047					
15							
16	**Market portfolio statistics**						
17	Mean	12.74%	<-- =B12*$B$3+B13*$C$3+B14*$D$3				
18	Variance	0.0918	<-- =B12^2*B4+B13^2*C4+B14^2*D4 +2*B12*B13*B7+2*B12*B14*B9+2*B13*B14*B8				
19	Sigma	0.3030	<-- =SQRT(B18)				
20							
21	Market risk premium, $E(r_M)-r_f$	0.0674	<-- =B17-B5				
22							
23	**"Proof" of SML**						
24	Any portfolio, p						
25	y_A	0.3					
26	y_B	0.4					
27	y_C	0.3					
28							
29	Mean, $E(r_p)$	12.30%	<-- =B25*$B$3+B26*$C$3+B27*$D$3				
30	Covariance(p,M)	0.0858	<-- =B12*B25*B4+B13*B26*C4+B14*B27*D4+B7*(B12*B				
31	Portfolio beta, Cov(p,M)/Var(M)	0.9349	<-- =B30/B18				
32							
33	$E(r_p)$ by SML $=r_f+\beta_p*[E(r_M)-r_f]$	12.30%	<-- =B5+B31*B21				
34							
35							
36							
37							
38							

When we say that the SML "works," we mean that the expected portfolio return is determined by the beta for *any* portfolio.

In cells B30:B31, we do the calculation for the β of any arbitrary portfolio. In cell B33 we show show that the $r_f + \beta[E(r_M) - r_f]$ calculates the expected return of the portfolio. Here are some other examples, which show that the SML relation always holds:

	A	B
24	Any portfolio, p	
25	y_A	0
26	y_B	1
27	y_C	0
28		
29	Mean, $E(r_p)$	12.00%
30	Covariance(p,M)	0.0817
31	Portfolio beta, Cov(p,M)/Var(M)	0.8904
32		
33	$E(r_p)$ by SML $=r_f+\beta_p*[E(r_M)-r_f]$	12.00%

	A	B
24	Any portfolio, p	
25	y_A	-0.5
26	y_B	1.3
27	y_C	0.2
28		
29	Mean, $E(r_p)$	13.60%
30	Covariance(p,M)	0.1035
31	Portfolio beta, Cov(p,M)/Var(M)	1.1278
32		
33	$E(r_p)$ by SML $=r_f+\beta_p*[E(r_M)-r_f]$	13.60%

We conclude that:

Given the market portfolio M (defined as the Sharpe ratio maximizing portfolio), then for <u>any other asset or portfolio</u> p, the following relationship holds:

$$E(r_p) = r_f + \underbrace{\frac{Cov(r_p, r_M)}{Var(r_M)}}_{\uparrow \; \beta_p}\left[E(r_M) - r_f\right].$$

Portfolios with more than 3 assets

We've repeated the calculations of this chapter for a portfolio of 3 assets. The primary result which we've demonstrated is the SML:

If M maximizes the Sharpe ratio $\frac{E(r_p)-r_f}{\sigma_p}$*, then for any asset or portfolio, the security market line—which relates the asset's expected return to its risk* β*—holds:*

$$E(r_{asset}) = r_f + \underbrace{\frac{Cov(r_{asset}, r_M)}{Var(r_M)}}_{\beta_{asset}} \left[E(r_M) - r_f \right]$$

What if there are more than 3 risky assets? Everything we've said is still true—but unfortunately the computations involved require techniques beyond the scope of this book.[3]

[3] My book *Financial Modeling* by Simon Benninga (MIT Press, 2000) contains details on how to do these calculations for the general case with many assets using matrices.

CHAPTER 15: USING THE SML TO CALCULATE A FIRM'S COST OF CAPITAL*

this version: October 17, 2003

Chapter contents

Overview

This is the second of two chapters that show the use of the security market line (SML). In the Chapter 14 we discussed the use of the SML for performance measurement, and this chapter we discuss how to use the SML to calculate the cost of capital for a firm.[1] We discussed calculating the firm's cost of capital in Chapter 6, where we used the Gordon model to calculate the cost of equity. In this chapter we use the SML to calculate the firm's cost of capital. These two models—the Gordon model and the SML—are the major approaches to computing the firm's cost of capital.

Finance concepts discussed in this chapter

- The use of security market line (SML) to calculate the cost of equity r_E for a firm.
- Calculating the firm's weighted average cost of capital (WACC). Note that the computation of the WACC was also discussed in Chapter 6, where we used the Gordon model to calculate the firm's cost of equity r_E .
- Calculating the market value of the firm's debt and equity, the firm's tax rate T_C, and the firm's cost of debt r_D . Our discussion of these issues in this chapter is in many ways a repeat of a similar discussion in Chapter 6.
- The concept of *asset beta*, β_{Assets} and its use as an alternative method to calculate the firm's WACC.

Throughout this chapter we assume that you know how to calculate the β of a stock (this issue was discussed in the previous chapter). In actual fact you often don't have to compute the

[1] If you need a lightning review of the SML, look at the first section of Chapter 13.

β of a firm's shares—the information is publicly available (in this chapter, for example, we use data on β provided by Yahoo).

Excel functions used

- **NPV**
- **Countif**

15.1. The CAPM and the firm's cost of equity—an initial example

Abracadabra Inc. is considering a new project, which has the following free cash flows.[2]

	A	B
3	**Year**	**FCF**
4	0	-1,000
5	1	1,323
6	2	1,569
7	3	3,288
8	4	1,029
9	5	1,425
10	6	622
11	7	3,800
12	8	3,800
13	9	3,800
14	10	2,700

In order to decide whether to accept or reject the project, the company needs to calculate the risk-adjusted discount rate for these cash flows. It decides that the riskiness of the new project is very much like the riskiness of Abracadabra's current activities; the financing for the project is also similar to that of the firm. In this case the appropriate discount rate is the

[2] An extended discussion of the free cash flow (FCF) is given in Chapter 6 (section 6.???). Figure 15.1 reviews the concept in tabular form.

weighted average cost of capital (WACC); this is the average cost of financing the firm's activities. Assuming that Abracadabra has both equity and debt, the formula for the WACC is given by:

$$WACC = r_E \quad * \quad \frac{E}{E+D} \quad + r_D\left(1-T_C\right) \quad * \quad \frac{D}{E+D}$$
$$= \begin{pmatrix} r_E = \\ \text{cost of} \\ \text{equity} \end{pmatrix} * \begin{pmatrix} \text{proportion} \\ \text{of firm} \\ \text{financed by} \\ \text{equity} \end{pmatrix} + \begin{pmatrix} r_D = \\ \text{cost of} \\ \text{debt} \end{pmatrix} * \begin{pmatrix} T_C = \\ 1-\text{corporate} \\ \text{tax rate} \end{pmatrix} * \begin{pmatrix} \text{proportion} \\ \text{of firm} \\ \text{financed by} \\ \text{debt} \end{pmatrix}$$

[**separate page]**

Defining the Free Cash Flow	
Profit after taxes	This is the basic measure of the profitability of the business, but it is an accounting measure that includes financing flows (such as interest), as well as non-cash expenses such as depreciation. Profit after taxes does not account for either changes in the firm's working capital or purchases of new fixed assets, both of which can be important cash drains on the firm.
+ Depreciation	This noncash expense is added back to the profit after tax.
+ after-tax interest payments (net)	FCF is an attempt to measure the cash produced by the business activity of the firm. To neutralize the effect of interest payments on the firm's profits, we: • Add back the after-tax cost of interest on debt (*after-tax* since interest payments are tax-deductible), • Subtract out the after-tax interest payments on cash and marketable securities.
- Increase in current assets	When the firm's sales increase, more investment is needed in inventories, accounts receivable, etc. This increase in current assets is not an expense for tax purposes (and is therefore ignored in the profit after taxes), but it is a cash drain on the company.
+ Increase in current liabilities	An increase in the sales often causes an increase in financing related to sales (such as accounts payable or taxes payable). This increase in current liabilities—when related to sales—provides cash to the firm. Since it is directly related to sales, we include this cash in the free cash flow calculations.
- Increase in fixed assets at cost	An increase in fixed assets (the long-term productive assets of the company) is a use of cash, which reduces the firm's free cash flow.
FCF = sum of the above	

Figure 15.1. The free cash flow (FCF) is the amount of cash generated by a firm's business activities. Discounting the FCFs at a firm's weighted average cost of capital (WACC) gives the value of the firm. The important concept of FCF was introduced in Chapter 6. It appears in several other places in this book: In the context of accounting and financial planning models, we discuss the FCF in Chapters 7, 8, 9. In Chapter 18 we return to the concept of FCF in the context of stock valuation.

We can use the SML to calculate the cost of equity for Abracadabra. Here are our assumptions for this problem:

- The firm's stock has a beta $\beta = 1.4$.
- The expected market return is $E(r_M) = 10\%$
- The risk-free rate $r_f = 4\%$.
- Abracadabra's equity has a market value $E = \$10{,}000$
- Abracadabra's debt has a market value $D = \$15{,}000$
- Abracadabra can borrow new funds at a cost of $r_D = 6\%$.
- Abracadabra's corporate tax rate is $T_C = 40\%$.

The first three assumptions mean that Abracadabra's cost of equity r_E as given by the SML is 12.4%:

$$\begin{aligned} r_E &= r_f + \beta * \left[E\left(r_M\right) - r_f \right] \\ &= 4\% + 1.4 * \left[10\% - 4\%\right] = 12.4\% \end{aligned}$$

Then Abracadabra's weighted average cost of capital (WACC) is:

$$\begin{aligned} WACC = &\quad r_E \frac{E}{E+D} \quad + \quad r_D\left(1-T_C\right)\frac{D}{E+D} \\ &= 12.4\% * \frac{10,000}{10,000+15,000} + 6\% * \left(1-40\%\right) * \frac{15,000}{10,000+15,000} \\ &= 7.12\% \end{aligned}$$

The WACC of 7.12% is the discount rate we will use to determine whether or not Abracadabra should undertake the project.

The following spreadsheet shows our calculations for the WACC (rows 20-36) and the NPV calculation for the project (rows 2-16).

	A	B	C
1	**VALUING ABRACADABRA'S INVESTMENT we calculate the WACC using the SML to compute the cost of equity r_E**		
2	Year	FCF	
3	0	-1,000	
4	1	1,323	
5	2	1,569	
6	3	3,288	
7	4	1,029	
8	5	1,425	
9	6	622	
10	7	3,800	
11	8	3,800	
12	9	3,800	
13	10	2,700	
14			
15	Weighted average cost of capital, WACC	7.12%	<-- =B36
16	Project NPV	14,424	<-- =NPV(B15,B4:B13)+B3
17			
18			
19	**Computing Abracadabra's Weighted Average Cost of Capital (WACC)**		
20	Market value of equity, E	10,000	
21	Market value of debt, D	15,000	
22	Market value of equity + debt, E+D	25,000	
23			
24	Corporate tax rate, T_C	40%	
25			
26	Abracadabra's stock beta, ≅	1.4	
27			
28	**Facts about market**		
29	$E(r_M)$	10%	
30	r_f	4%	
31			
32	**Abracadabra's cost of capital**		
33	Cost of equity using SML, r_E	12.40%	<-- =B30+B26*(B29-B30)
34	Cost of debt, r_D	6.00%	
35			
36	Weighted average cost of capital (WACC)	7.12%	<-- =B20/B22*B33+B21/B22*B34*(1-B24)

When the project free cash flows are discounted at the WACC, the net present value (NPV) is $14,424. Since the NPV is positive, Abracadabra should undertake the project.

Comparing the SML and the Gordon model for calculating the WACC

The weighted average cost of capital is the most widely used discount rate for computing the value of corporate projects and for computing the value of the firm. The WACC depends critically on the cost of equity r_E . In this chapter we compute the cost of equity using the security market line, whereas in Chapter 6 we computed the cost of equity using the Gordon dividend model.

The Gordon dividend model and the SML are only two practical ways of calculating the cost of equity.[3] Both models have their advantages and disadvantages—the Gordon model is simple to calculate but is very sensitive to assumptions about the firm's equity payout—the total dividends plus stock repurchases of the firm. The SML requires relatively more calculations, but is more widely used. The SML also requires us to make assumptions about the expected return on the market $E(r_M)$. This problem is discussed in the next section.

So which model should you use in practice? The best answer is to *use both models* and to compare the results. This way each model can serve as a "reality check" on the other. We apply this logic in Chapter 18, which discusses stock valuation. There we apply both models and compare the results to see if we have arrived at an appropriate WACC.

[3] The academic finance literature has come up with other models for calculating the cost of equity, but in practice these models are very difficult to apply and rarely used.

15.2. Using the SML to calculate the cost of capital—calculating the parameter values

The Abracadabra example of the previous section gives the broad outlines of calculating the cost of capital using the SML, but it leaves a number of questions unanswered:

- How do we calculate the market value of a firm's equity, *E*?
- How do we calculate the expected return on the market $E(r_M)$?
- How do we calculate the risk-free rate, r_f ?
- How do we calculate the market value of a firm's debt, *D*?
- How do we calculate the firm's cost of borrowing, r_D?
- How do we calculate the firm's corporate tax rate T_C?

We discuss each of these questions in turn. Although we occasionally provide an illustration, we save a full-blown example for the following section.

The market value of a firm's equity, *E*

This is easy: For a firm whose shares are sold on the stock market, the market value of the equity (*E* in our WACC equation) is the number of shares times the market value per share.

The expected return on the market $E(r_M)$

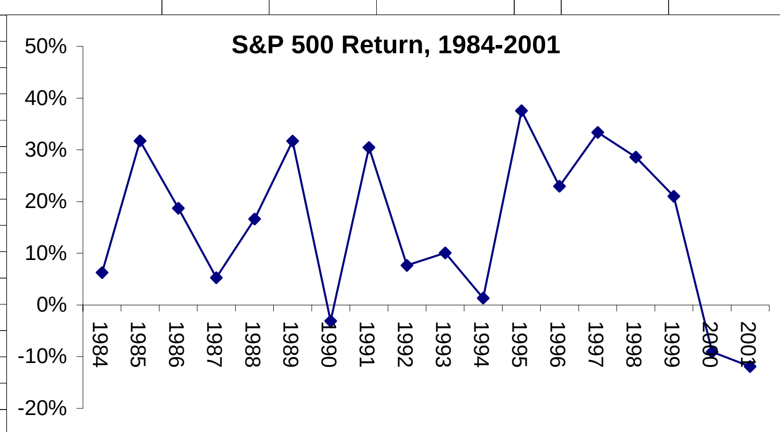
There are two ways to calculate the expected return on the market: 1) We can use the *historical* market return, or 2) We can use a version of the Gordon dividend model to derive $E(r_M)$ from current market data. Neither method is perfect, though we prefer the latter.

$E(r_M)$ **using the historic returns**: A standard technique is to use a broad-based index—usually the S&P 500 index—to proxy for the market portfolio. To do this, you need some data.

Below we show you the returns on Vanguard's 500 Index Fund. This is an index mutual fund which is invested in the S&P 500 index.[4] The average return on the S&P is around 15.51% for the period 1984-2001 (cell F23). This *historical average return* is often used as a proxy for the *expected market return* in the SML.

	A	B	C	D	E	F	G	H	I
1	**RETURNS ON THE S&P 500 INDEX, 1984-2001**								
2		**Vanguard's 500 Index fund**							
3	**Year**	**Capital return**	**Income return**	**Index 500 Total return**		**S&P 500 return**			
4	1984	1.54%	4.68%	6.22%		6.27%			
5	1985	26.09%	5.14%	31.23%		31.75%			
6	1986	14.04%	4.02%	18.06%		18.68%			
7	1987	2.27%	2.43%	4.70%		5.26%			
8	1988	11.55%	4.67%	16.22%		16.61%			
9	1989	26.67%	4.70%	31.37%		31.69%			
10	1990	-6.84%	3.52%	-3.32%		-3.10%			
11	1991	26.28%	3.94%	30.22%		30.47%			
12	1992	4.45%	2.97%	7.42%		7.62%			
13	1993	7.06%	2.84%	9.90%		10.08%			
14	1994	-1.51%	2.69%	1.18%		1.32%			
15	1995	34.35%	3.09%	37.44%		37.58%			
16	1996	20.53%	2.35%	22.88%		22.96%			
17	1997	31.11%	2.08%	33.19%		33.36%			
18	1998	27.00%	1.61%	28.61%		28.58%			
19	1999	19.70%	1.37%	21.07%		21.04%			
20	2000	-9.95%	0.90%	-9.05%		-9.10%			
21	2001	-13.11%	1.08%	-12.03%		-11.89%			
22									
23	**Average**	12.29%	3.00%	15.30%		15.51%	<-- =AVERAGE(F4:F21)		
24	**Standard deviation**	14.49%	1.28%	14.89%		14.92%	<-- =STDEVP(F4:F21)		
25									
26									
27									
28									
29									
30									
31									
32									
33									
34									
35									
36									
37									
38									
39									
40									
41									
42									

These are the S&P returns including dividends as given by Vanguard on its website. The difference between the total return on Vanguard's Index 500 portfolio and the total return on the S&P is largely due to the management fees of the Vanguard Index 500 fund.



[4] We discussed index funds in the Chapter 14, section 14.4.

Why use Vanguard instead of Yahoo for S&P Returns?

The usual data sources (for example Yahoo) give only the *price data* for the S&P 500 index. (This is somewhat strange, since Yahoo's data for individual stocks is adjusted for dividends.) Vanguard's website gives the *total return* data both for its Index 500 Fund and for the actual S&P 500 index. The Index 500 Fund's returns are slightly lower than those of the S&P 500. This is primarily due to the management fees paid by Index 500 to Vanguard.

$E(r_M)$ **using current market data**

This technique is less widely used, though we prefer it.[5] It is based on the Gordon dividend model that gives the expected return on a stock as a function of the stock's current equity payout Div_0, the current market value of the firm's equity P_0 , and the expected growth rate of g of the equity payout. The equity payout is defined as the sum of the firm's dividends and its stock repurchases (see Chapter 6, page000 for a full explanation):

Gordon Dividend Model

$$r_E = \frac{Div_0(1+g)}{P_0} + g$$

where

Div_0 = current equity payout of firm (total dividends + stock repurchases)

P_0 = current market value of equity

g = anticipated equity payout growth rate

To use the Gordon model to calculate the expected return on the market, we restate the model in terms of the price-earnings ratio: Assume that every year the firm pays out a

[5] It was first published in *Corporate Finance: A Valuation Approach* by Simon Benninga and Oded Sarig, McGraw-Hill 1997.

percentage b of its earnings to its shareholders, both in the form of dividends and stock repurchases. Then we can rewrite the formula above as:

$$r_E = \frac{Div_0\left(1+g\right)}{P_0} + g = \frac{b * EPS_0\left(1+g\right)}{P_0} + g$$

where EPS_0 is the firm's current earnings per share

Manipulating this formula a bit, we get:

$$r_E = \frac{b*\left(1+g\right)}{\underbrace{P_0 / EPS_0}_{\substack{\uparrow \\ \text{this is the firm's} \\ \text{P/E (price-earnings)} \\ \text{ratio}}}} + g$$

We now apply this logic to the market as a whole. We regard a market index such as the S&P 500 (symbolized by M) as a stock having its own payout ratio b and growth rate of equity payouts g. We then use the above formula to compute the expected market return $E\left(r_M\right)$.[6] Here's an example using data for the S&P 500 index at the end of December 2001:

	A	B	C
1	**USING THE PRICE-EARNINGS RATIO TO COMPUTE E(r_M)**		
2	S&P 500 P/E on 31dec01	46	
3	Estimated growth of equity payout, g	6%	
4	Payout ratio, b	50%	
5			
6	E(r_M)	7.15%	<-- =B4*(1+B3)/B2+B3

[6] A sensitive reader may note that there's some confusion of symbols here. The formula $r_E = \frac{b*\left(1+g\right)}{P_0 / EPS_0} + g$ uses r_E to stand for the cost of equity for a stock. Since the "cost of equity" is a synonym for the "expected return from equity," when we apply the formula to the market portfolio M (in this case, the S&P 500), we should by logic we should have called this r_M. Instead, we use $E\left(r_M\right)$. Our excuse is that the symbol $E\left(r_M\right)$ is so widely used that we cannot give it up.

The price-earnings ratio (P/E) for the S&P 500 is not that easy to find (see Figure 15.2 for the source of our data). We have had to "guesstimate" the estimated growth of dividends and the dividend payout ratio.

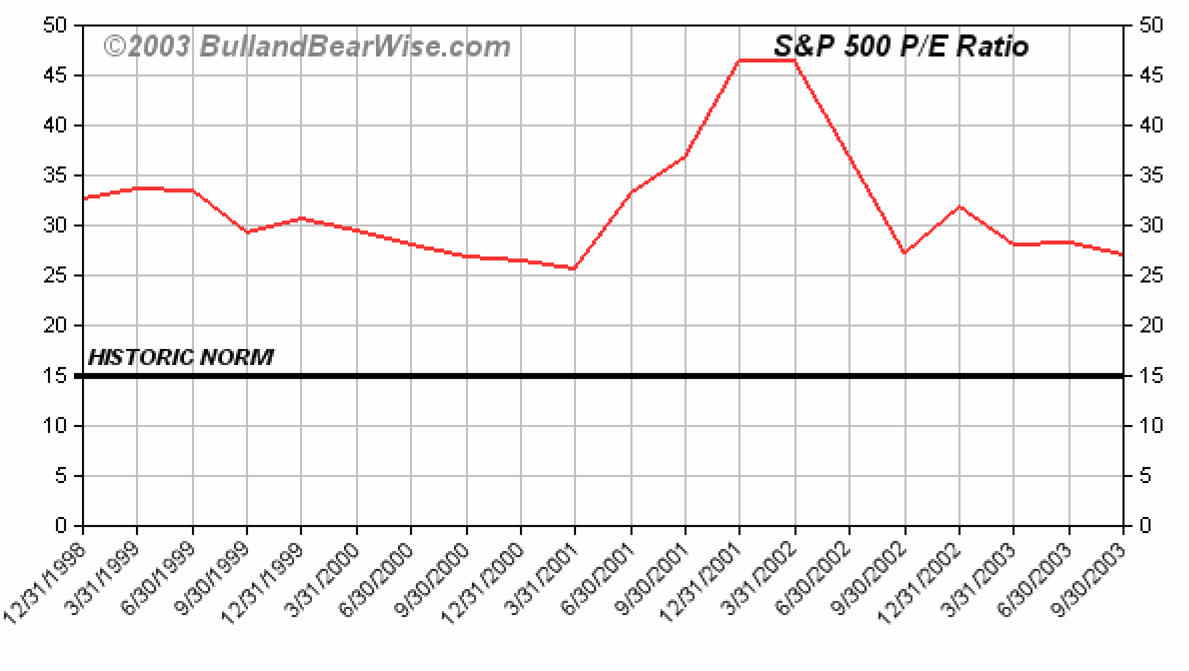


Figure 15.2. The price/earnings (P/E) ratio of the S&P 500 index is not that easy to find. Here is a website which calculates the P/E ratio. The numerical example in this section uses the P/E of 46 on 31 December 2001 to compute the expected return on the market $E(r_M)$.
Source: http://www.bullandbearwise.com

Dividend payout is defined as the total expended by firms on both cash dividends and repurchases of shares (we discussed this topic a bit in Chapter 6, when calculating the cost of equity for Courier Corporation using the Gordon model). While the cash dividends are a matter of record, the amount of repurchases is more debatable. Current estimates put the sum of dividends and repurchases at around 50% of corporate earnings. Here, for example, is a graph showing the relation between share repurchases and dividends for the Standard & Poors 500 index through 1998. Notice that by the end of the data sample, repurchases outweighed dividends:

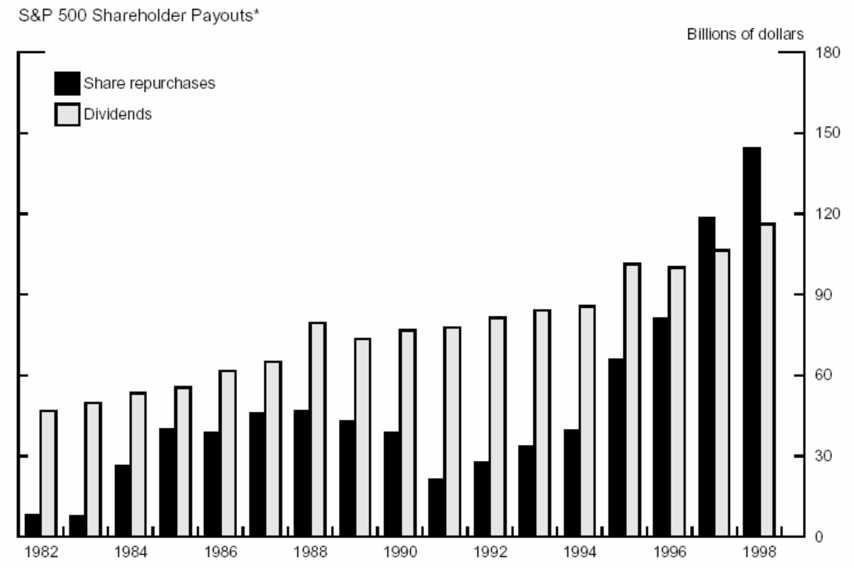


Figure 15.3: Share repurchases and dividends in the United States, 1982-1998. Whereas at the beginning of the period share repurchases were relatively small compared to dividends, by the end of the period more cash was paid out to shareholders by corporations in the form of share repurchases than as dividends.

Source: J. Nellie Liang and Steven A. Sharpe, Share Repurchases and Employee Stock Options and their Implications for S&P 500 Share Retirements and Expected Returns, Federal Reserve Bank, 1999.

Dividend growth is the market anticipation of the growth of total dividends (broadly defined as cash dividends plus repurchases) for the future. If we assume that dividends will grow at the rate of growth of the economy, 6% is a reasonable long-term estimate.

Computing the risk-free rate r_f

The risk-free rate r_f should be the short-term Treasury rate. This rate is available from a variety of places, including Yahoo (see example in next section).

Computing the value of the firm's debt *D*

In principle *D* should be the *market value* of the firm's debt. However, in practice, this value is usually very difficult to calculate. Standard practice is to use the book value of the firm's debt *minus* the value of its cash reserves; we refer to this concept as *net debt*. You should be careful even with the book value.

Now that we've defined the concepts, we use the example of Hilton Hotels to compute a firm's weighted average cost of capital.

15.3. A fully worked-out example—Hilton Hotels

We illustrate the approach to calculating the WACC by using data for Hilton Hotels Corp. (symbol: HLT). As discussed above, we need 8 parameters in order to calculate the WACC for this (or any other) company:

- E, the market value of the company's equity today. This is simply the number of shares times the current stock price.
- D, the market value of the company's debt today. We will use the *book value* (that is, the accounting value) of the company's debt as a proxy for this number.
- r_E, the cost of equity for the company. In this chapter we use the SML to calculate the cost of equity. Using the SML means that the cost of equity is dependent on:
    - The β of the firm's equity. In the previous chapter we computed this β. In practice, it is often available without computation (as in this example, read on).
    - r_f, the risk-free rate
    - $E(r_M)$, the expected return on the market

- r_D, the cost of debt for the company. In principle, this should be the marginal cost (the company's cost of obtaining new debt). In practice, we often use the company's average cost of existing debt.
- T_C the company's tax rate. In principle, this should be the company's marginal tax rate (the rate on an additional dollar of earnings). In practice we often use the company's average tax rate.

Much of the data is available on Yahoo; Figure 15.4 shows the Yahoo screen leading to Hilton Hotel's "profile," which is shown in Figure 15.5.

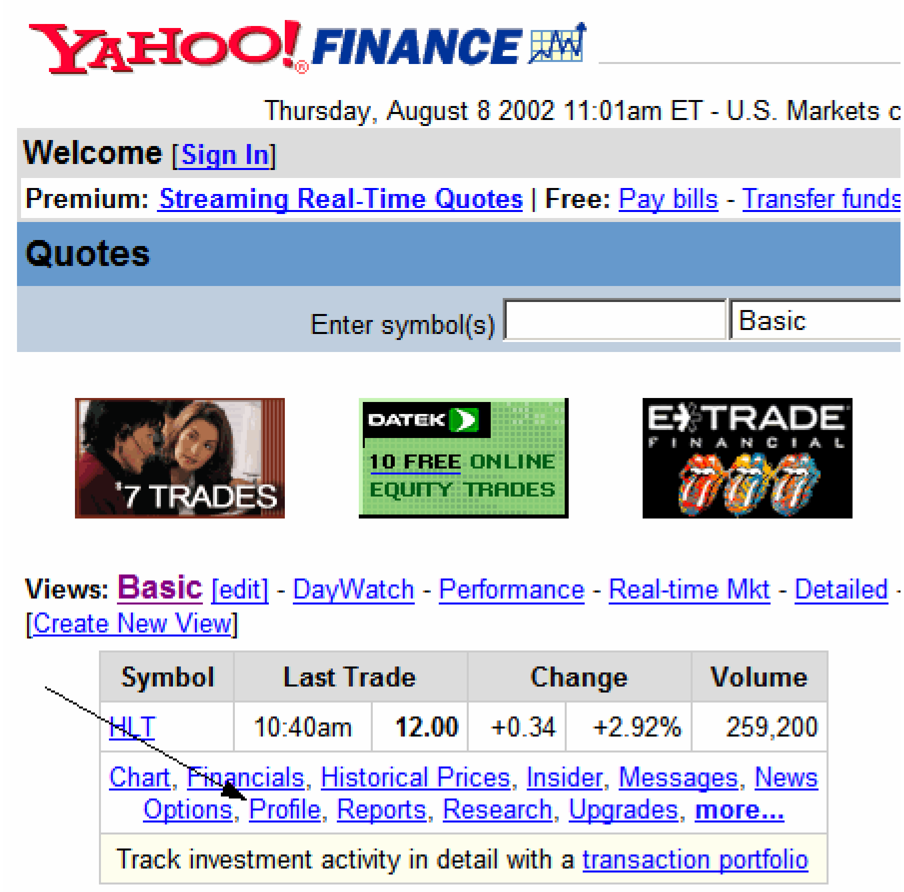


Figure 15.4. The Yahoo screen, indicating the **Profile**. This choice gives the updated financial information for the firm used below. (Note that Yahoo's presentation of financial materials changes occasionally, so that you make have to look elsewhere for the financial profile.)

From Yahoo's Profile, here are some data for Hilton as of 18 January 2002.

Statistics at a Glance -- NYSE:HLT As of 18-Jan-2002

Price and Volume

52-Week Low on 21-Sep-2001	$6.15
Recent Price	$11.32
52-Week High on 7-Jan-2002	$14.70
Beta	0.89
Daily Volume (3-month avg)	1.18M
Daily Volume (10-day avg)	1.27M

Stock Performance

big chart [1d | 5d | 3m | 6m | 1y | 2y | 5y | max]

52-Week Change	-4.7%
52-Week Change relative to S&P500	+13.5%

Share-Related Items

Market Capitalization	$4.18B
Shares Outstanding	369.3M
Float	332.4M

Dividends & Splits

Annual Dividend (indicated)	$0.08
Dividend Yield	0.71%
Last Split: factor 4 on 26-Sep-1996	

Per-Share Data

Book Value (mrq)	$4.83
Earnings (ttm)	$0.60
Earnings (mrq)	$0.06
Sales (ttm)	$8.17
Cash (mrq)	$0.14

Valuation Ratios

Price/Book (mrq)	2.35
Price/Earnings (ttm)	18.71
Price/Sales (ttm)	1.39

Income Statements

Sales (ttm)	$3.26B
EBITDA (ttm)	$1.15B
Income available to common (ttm)	$226.0M

Profitability

Profit Margin (ttm)	7.2%
Operating Margin (ttm)	23.2%

Fiscal Year

Fiscal Year Ends	Dec 31
Most recent quarter	30-Sep-2001

Management Effectiveness

Return on Assets (ttm)	2.59%
Return on Equity (ttm)	13.33%

Financial Strength

Current Ratio (mrq)	1.06
Debt/Equity (mrq)	3.03
Total Cash (mrq)	$53.0M

Short Interest
As of 10-Dec-2001

Shares Short	7.28M
Percent of Float	2.2%
Shares Short (Prior Month)	7.08M
Short Ratio	5.58
Daily Volume	1.30M

See Profile Help for a description of each item above; **M** = millions; **B** = billions; **mrq** = most-recent quarter; **ttm** = trailing twelve months; (as of 30-Sep-2001)

Figure 15.5. Yahoo's profile for Hilton Hotels. Highlighted numbers are used in the computation of Hilton's WACC.

From this data we learn:

- Hilton's equity $\beta_E = 0.89$.
- The market value of Hilton's equity is E = \$4.18 billion. As you'll see in the spreadsheet below, Yahoo has taken the number of shares times the current market value per share.
- The book value of Hilton's debt is D = \$5.352 billion. With a bit of work, this number can be calculated from Yahoo: According to Yahoo:
  - The book value of equity per share is \$4.83.
  - The debt/equity ratio of Hilton is 3.03. This is the ratio of the book value of the firm's debt to the book value of its equity.

Since Hilton has 369.3 million shares outstanding, its total book value of equity is: $369.3*4.83=1,784$. This makes the total debt of the company \$5,405. From this number we subtract the \$53 million in cash held by the company to arrive at net debt D = \$5,352.[7]

	A	B	C
1	**HILTON HOTELS CORPORATION (HLT)** **using Yahoo for much of the information**		
2	Equity beta	0.89	<-- Yahoo
3			
4	Shares outstanding (million)	369.3	<-- Yahoo
5	Market value per share	11.32	<-- Yahoo
6	Market value of equity ($ million), E	4,180	<-- =B5*B4
7			
8	Book value of equity per share	4.83	<-- Yahoo
9	Total book value of equity	1,784	<-- =B8*B4
10	Debt/Equity ratio	3.03	<-- Yahoo
11	Book value of debt	5,405	<-- =B10*B9
12	Cash on hand	53	
13	Net debt ($ million), D	5,352	<-- =B11-B12

We still need the 2 firm-related parameters (r_D, T_C) and 2 market parameters (r_f, $E(r_M)$) . For these, we'll have to work a bit.

Hilton's cost of debt, r_D is approximately 6.81%

We compute the cost of Hilton's debt r_D by taking its interest payments and dividing by the average debt over the year. Yahoo doesn't have the interest-payment information, so we have to go to Hilton's financial statements to get these figures. Figure 15.6 shows a note from the Hilton financial statements detailing the company's debt. A quick calculation shows that

$$Interest\ \ paid=\frac{402}{Average\left(6094,5716\right)}=6.81\%$$

[7] Cash is subtracted from the firm's debt because Hilton could, in principle use the cash to pay off some of its debt.

LONG-TERM DEBT

Long-term debt at December 31, 1999 and 2000 is as follows:

(in millions)	*1999*	*2000*
Industrial development revenue bonds at adjustable rates, due 2015	$ 82	82
Senior notes, with an average rate of 7.7%, due 2001 to 2017	1,057	1,052
Senior notes, with an average rate of 7.2%, due 2002 to 2004 [1]	625	625
Mortgage notes, 6.0% to 8.6%, due 2002 to 2022	308	353
7.95% Collateralized borrowings, due 2010	—	499
5% Convertible subordinated notes due 2006	494	495
Commercial paper	19	235
Revolving loans	3,506	2,372
Other	3	3
	6,094	5,716
Less current maturities	(9)	(23)
Net long-term debt	$6,085	5,693

[1] *Represents balances assumed by Park Place in connection with the December 31, 1998 spin-off of the Company's gaming business.*

Interest paid, net of amounts capitalized, was $130 million, $187 million and $402 million in 1998, 1999 and 2000, respectively.

Figure 15.6. Note from Hilton's financial statements about the company's debt.

Hilton's tax rate T_C is approximately 41%

Figure 15.7 shows Hilton's income statements, with the company's income before taxes and taxes highlighted. The average of Hilton's tax rate over the last 3 years is $T_C = 41.25\%$:

	A	B	C	D	E
1	**HILTON'S TAX RATE T_C**				
2	**Year**	**1998**	**1999**	**2000**	
3	Income before taxes	336	313	479	
4	Provision for taxes	136	130	200	
5	Tax rate	40.48%	41.53%	41.75%	<-- =D4/D3
6					
7	Average tax rate, T_C	41.25%	<-- =AVERAGE(B5:D5)		

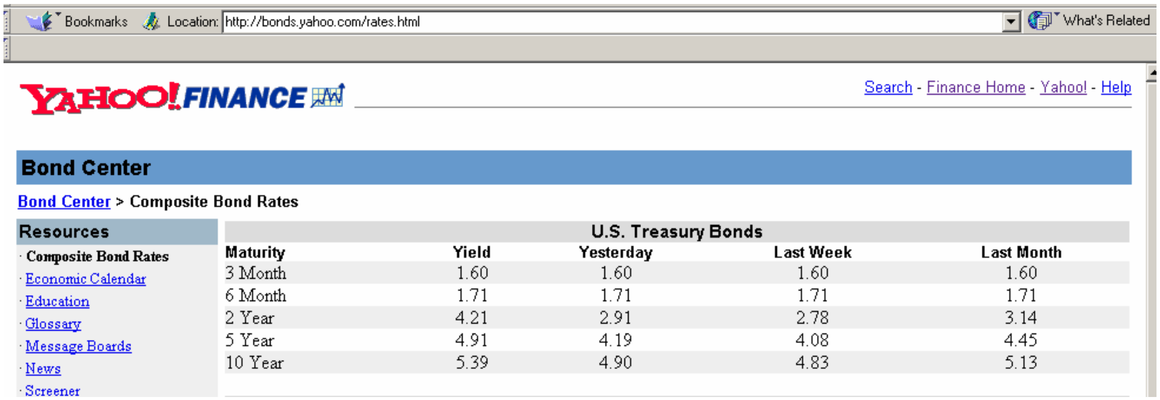
CONSOLIDATED STATEMENTS OF INCOME

(in millions, except per share amounts) *Year ended December 31,*	1998	1999	2000
Revenue			
Owned hotels	$1,485	1,813	2,429
Leased hotels	—	26	398
Management and franchise fees	104	120	350
Other fees and income	180	191	274
	1,769	2,150	3,451
Expenses			
Owned hotels	964	1,196	1,571
Leased hotels	—	26	365
Depreciation and amortization	125	187	382
Other operating expenses	152	173	241
Corporate expense, net	64	73	62
	1,305	1,655	2,621
Operating Income	464	495	830
Interest and dividend income	13	57	86
Interest expense	(137)	(237)	(453)
Interest expense, net, from unconsolidated affiliates	(4)	(2)	(16)
Net gain on asset dispositions	—	—	32
Income Before Income Taxes and Minority Interest	336	313	479
Provision for income taxes	(136)	(130)	(200)
Minority interest, net	(12)	(7)	(7)
Income from Continuing Operations	188	176	272
Income from discontinued gaming operations, net of tax provision of $111 in 1998	109	—	—
Cumulative effect of accounting change, net of tax benefit of $1 in 1999	—	(2)	—
Net Income	$ 297	174	272
Basic Earnings Per Share			
Income from continuing operations	$ .71	.66	.74
Discontinued gaming operations	.44	—	—
Cumulative effect of accounting change	—	(.01)	—
Net Income Per Share	$ 1.15	.65	.74
Diluted Earnings Per Share			
Income from continuing operations	$ .71	.66	.73
Discontinued gaming operations	.41	—	—
Cumulative effect of accounting change	—	(.01)	—
Net Income Per Share	$ 1.12	.65	.73

Figure 15.7. Hilton's income statements. The highlighted text allows us to compute the company's tax rate.

The risk-free rate in the economy r_f is 1.6%

We get this number from Yahoo, as shown in Figure 15.8.


Bookmarks Location: http://bonds.yahoo.com/rates.html What's Related

YAHOO! FINANCE

Search - Finance Home - Yahoo! - Help

Bond Center

Bond Center > Composite Bond Rates

Resources
- Composite Bond Rates
- Economic Calendar
- Education
- Glossary
- Message Boards
- News
- Screener

U.S. Treasury Bonds				
Maturity	**Yield**	**Yesterday**	**Last Week**	**Last Month**
3 Month	1.60	1.60	1.60	1.60
6 Month	1.71	1.71	1.71	1.71
2 Year	4.21	2.91	2.78	3.14
5 Year	4.91	4.19	4.08	4.45
10 Year	5.39	4.90	4.83	5.13

Figure 15.8. Yahoo screen with interest rates. The r_f for use in the SML is the short-term Treasury bond rate, 1.60%.

The expected return on the market $E(r_M)$ is approximately 7.15%

This was illustrated above in section 15.3.

So what's Hilton's WACC?

The weighted average cost of capital for Hilton is 5.11%:

	A	B	C
1	**HILTON HOTELS CORPORATION (HLT)** **using Yahoo for much of the information**		
2	Equity beta	0.89	<-- Yahoo
3			
4	Shares outstanding (million)	369.3	<-- Yahoo
5	Market value per share	11.32	<-- Yahoo
6	Market value of equity ($ million), E	4,180	<-- =B5*B4
7			
8	Book value of equity per share	4.83	<-- Yahoo
9	Total book value of equity	1,784	<-- =B8*B4
10	Debt/Equity ratio	3.03	<-- Yahoo
11	Book value of debt	5,405	<-- =B10*B9
12	Cash on hand	53	
13	Net debt ($ million), D	5,352	<-- =B11-B12
14			
15	Risk-free rate, r_f	1.60%	
16	Expected market return, $E(r_M)$	7.15%	
17			
18	**Computation of WACC**		
19	Percentage of equity, E/(E+D)	0.4386	<-- =B6/(B6+B13)
20	Percentage of debt, D/(E+D)	0.5614	<-- =1-B19
21	Cost of equity, r_E	6.54%	<-- =B15+B2*(B16-B15)
22	Cost of debt, r_D	6.81%	
23	Tax rate, T_C	41.25%	
24	WACC	5.11%	<-- =B19*B21+(1-B23)*B20*B22

15.4. Computing the WACC using an asset β_{Asset}

A somewhat different approach to computing the weighted average cost of capital (WACC) is to use the *asset beta* approach. In this approach we need both the equity beta β_E and the β_D for Hilton. The asset beta is defined as the weighted average β of the debt and equity betas:

$$\beta_{Asset} = \beta_E * \frac{E}{E+D} + \beta_D * (1-T_C) * \frac{D}{E+D}$$
$$= \begin{pmatrix}\text{equity}\\\text{beta}\end{pmatrix} * \begin{pmatrix}\text{proportion}\\\text{of equity}\\\text{in firm}\\\text{value}\end{pmatrix} + \begin{pmatrix}\text{debt}\\\text{beta}\end{pmatrix} * \begin{pmatrix}1-\text{corporate}\\\text{tax rate}\end{pmatrix} * \begin{pmatrix}\text{proportion}\\\text{of debt}\\\text{in firm}\\\text{value}\end{pmatrix}$$

Having computed the beta β_{Asset}, we now compute the WACC by using the SML:

$$WACC = r_f + \beta_{Asset} * \left[E(r_M) - r_f\right]$$

In order to illustrate this approach for Hilton, we note that all the necessary calculations have been done in the previous section—with the exception of the computation of the debt beta β_D. We compute this β by assuming that the SML holds for debt as well as equity:

$$cost\ of\ debt = r_D = r_f + \underbrace{\beta_D}_{\substack{\uparrow \\ \text{This is the} \\ \text{beta of} \\ \text{Hilton's debt}}} * \left[E(r_M) - r_f\right]$$

$$\Rightarrow \beta_D = \frac{r_D - r_f}{E(r_M) - r_f}$$

In the spreadsheet below you can see that Hilton's debt β is 0.938 (cell B8). This means that its asset beta is 0.70 (cell B15), which gives the WACC as 5.49% (cell B17).

	A	B	C
1	**HILTON HOTELS CORPORATION (HLT)** **computing the WACC with the asset beta**		
2	Equity beta, β_E	0.89	<-- Yahoo
3			
4	Risk-free rate, r_f	1.60%	
5	Expected market return, $E(r_M)$	7.15%	
6			
7	Cost of debt	6.81%	
8	Debt beta, β_D	0.938	<-- =(B7-B4)/(B5-B4)
9			
10	Corporate tax rate	41.25%	
11			
12	Percentage of equity, E/(E+D)	0.4386	
13	Percentage of debt, D/(E+D)	0.5614	
14			
15	Asset beta, β_{Asset}	0.70	<-- =B2*B12+(1-B10)*B13*B8
16			
17	WACC	5.49%	<-- =B4+B15*(B5-B4)

Why is the debt β_D so high?

Hilton's β_D is higher than as its equity beta, β_E. This may seem surprising. Before we start, in this box, with reasons why this might be so, note that the asset beta, β_{Asset} , is based on the after-tax debt beta, $(1-T_C)\beta_D$, which is of course much lower than the equity beta β_E.

Here are several other possible reasons for the size of Hilton's debt beta:

- The debt is risky because Hilton has a low credit rating. Risky debt would, of course, have a higher beta.
- The debt is risky because it has a relatively long term, and so is more sensitive to fluctuations in interest rates.
- We've overestimated the cost of the debt. In reality, r_D should be the *expected cost of Hilton's debt*. If Hilton's debt is risky, then investors in the debt expected a lower return than the promised return. So if the promised return is 6.81%, it could be that the expected return is 5%, allowing for the possibility of financial distress. This means that the debt beta β_D is lower than what we've calculated. [The actual adjustment of a debt β for financial distress is quite complicated—see my book *Financial Modeling*.]

15.5. Don't read this section!

A final question that may have occurred to you: Why is it that we get a different cost of capital using the traditional WACC approach and using the asset beta (β_{Asset}) approach? We're going to answer this question in this section, but we warn you that reading the section may be bad for your health.[8]

Still here? The answer is that for cost of capital purposes, you should adjust the SML for corporate taxes. In addition, there are two SMLs—one for equity and one for debt. Here are the appropriate formulas:

$$\text{Equity SML: } r_E = r_f * (1 - T_C) + \beta_E * \left[E(r_M) - r_f * (1 - T_C) \right]$$
$$\text{Debt SML: } r_D = r_f + \beta_D * \left[E(r_M) - r_f * (1 - T_C) \right]$$

Note that the two SMLs have the same tax-adjusted market risk premium $\left[E(r_M) - r_f * (1 - T_C) \right]$, but have different intercepts—the equity SML has intercept $r_f * (1 - T_C)$, whereas the debt SML has intercept r_f.[9]

If we apply this approach to Hilton, and if we assume that the cost of debt is $r_D = 6.81\%$, then we get the debt β_D as:

$$\beta_D = \frac{r_D - r_f}{E(r_M) - r_f * (1 - T_C)} = \frac{6.81\% - 1.60\%}{7.15\% - 1.60\%} = 0.8387$$

Now, as you can see in the spreadsheet below, the WACC is the same, whether you compute it with the traditional method or with the asset β_{Asset}:

---

[8] And—in all honesty—the difference between the two calculations in the previous part of the chapter is not big enough to make much of a difference.

[9] The two-SML model is fully explained in *Corporate Finance: A Valuation Approach* by Simon Benninga and Oded Sarig (McGraw-Hill, 1997).

	A	B	C
1	**HILTON HOTELS CORPORATION (HLT)** **using the two-SML model**		
2	Risk-free rate, r_f	1.60%	
3	Expected market return, $E(r_M)$	7.15%	
4	Corporate tax rate	41.25%	
5			
6	**WACC using traditional method**		
7	Equity beta	0.89	<-- Yahoo
8	Cost of equity	6.47%	<-- =B2*(1-B4)+B7*(B3-B2*(1-B4))
9	Cost of debt	6.81%	
10			
11	Percentage of equity, E/(E+D)	0.4386	
12	Percentage of debt, D/(E+D)	0.5614	
13			
14	WACC	5.08%	<-- =B11*B8+B12*(1-B4)*B9
15			
16	**WACC using the asset beta and the two-SML model**		
17	Equity beta, $\square_E$	0.8900	
18	Debt beta, $\square_D$	0.8387	<-- =(B9-B2)/(B3-B2*(1-B4))
19	Asset beta, $\square_{Asset}$	0.6669	<-- =B11*B17+B18*(1-B4)*B12
20	WACC	5.08%	<-- =B2*(1-B4)+B19*(B3-B2*(1-B4))

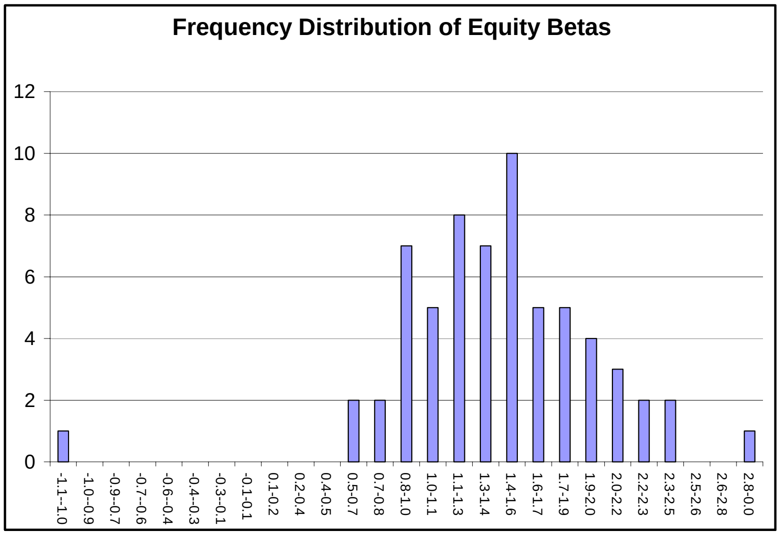
15.6. Some background information on equity betas

It helps to have information on equity betas for various industries. Here, from an article published in 1997, here are the stock betas ("equity betas") for 66 U.S. industries:

	A	B	C	D	E	F
1	**EQUITY BETAS BY INDUSTRY** Source: "Full-information Industry Betas," by Paul D. Kaplan and James D. Peterson Published in Financial Management, Summer 1998, pp. 85-93. Data is for 1996					
2	**Industry**	**Equity beta**		**Industry**	**Equity beta**	
3	Agricultural Production—Crops	1.63		Transportation by Air	1.61	
4	Agricultural Services	0.49		Measuring Instruments	1.34	
5	Livestock and Animal Specialties	1.41		Leather and Leather Products	0.83	
6	Amusement and Recreation Services	2.12		Industrial and Computer Equipment	1.49	
7	Auto Repair, Services, Parking	1.54		General Merchandise Stores	1.02	
8	Depository Institutions	1.61		Metal Mining	0.94	
9	Contractors and Builders	0.64		Mining and Quarrying	1.28	
10	Building Materials, Hardware	2.65		Miscellaneous Manufacturing	0.78	
11	Printing and Publishing	1.21		Miscellaneous Retail	1.26	
12	Business Services	1.38		Miscellaneous Repair Services	1.69	
13	Auto Dealers and Gasoline Stations	0.71		Motion Pictures	0.94	
14	Chemicals and Allied Products	1.30		Wholesale Nondurable Goods	0.95	
15	Apparel	1.00		Petroleum Refining	0.63	
16	Apparel and Accessory Stores	1.10		Oil and Gas Extraction	0.80	
17	Coal Mining	-1.16		Paper and Allied Products	1.17	
18	Nondepository Credit Institutions	1.95		Personnel Services	1.24	
19	Wholesale Trade—Durable Goods	1.38		Pipelines Except Natural Gas	0.70	
20	Educational Services	1.84		Railroad Transportation	1.24	
21	Electronic and Electrical Equipment	1.75		Eating and Drinking Places	1.34	
22	Engineering andAccounting Services	2.00		Real Estate	1.37	
23	Fabricated Metal Products	1.89		Rubber and Plastics products	1.03	
24	Security Brokers and Dealers	2.23		Hotels and Lodging	1.72	
25	Food and Kindred Products	0.66		Tobacco Products	2.21	
26	Motor Freight and Warehousing	0.69		Stone, Clay, Glass, and Concrete	1.76	
27	Furniture and Fixtures	1.04		Communications	0.95	
28	Home Furniture	1.12		Textiles	1.02	
29	Food Stores	1.02		Special Trade Contractors	1.26	
30	Health Services	1.54		Transportation Equipment	1.44	
31	Heavy Construction	2.07		Transportation Services	0.79	
32	Holding and Other Investment Offices	0.96		Electric, Gas, and Sanitary Service	0.47	
33	Insurance Carriers	1.21		Water Transportation	1.18	
34	Insurance Agents and Brokers	1.35		Lumber and Wood Products	1.60	
35						
36	Number of industries	64	<-- =COUNT(B3:B35,E3:E35)			
37						
38	Lowest beta	-1.16	<-- Coal			
39	Highest beta	2.65	<-- Building materials			
40	Average beta	1.26	<-- =AVERAGE(B3:B35,E3:E35)			
41						
42	**Note**: We would expect the weighted average equity beta					
43	to be 1; however, the *average beta* given above is not weighted.					
44						
45	How many betas < 0?	1	<-- =COUNTIF(B3:E35,"<0")			
46	How many betas > 2?	5	<-- =COUNTIF(B3:E35,">2")			
47	How many 0 < beta < 1	18	<-- =COUNTIF(B3:E35,"<1")			
48	How many 0.9 < beta < 1.1	11	<-- =COUNTIF(B3:E35,"<1.1")-COUNTIF(B3:E35,"<0.9")			
49	How many betas > 1	45	<-- =COUNTIF(B3:E35,">1")			

Note that the coal industry has a negative beta, indicating that the returns from holding coal stocks increase when the market goes down and decrease when the market goes up. To judge by its beta, therefore, the coal industry is countercyclical.

Here's the frequency distribution of the betas:



Summary

The computation of the weighted average cost of capital (WACC) is critical for corporate valuation. In this book we have already seen the importance of the WACC in Chapter 6.[10]

The WACC depends critically on our estimate of the cost of equity r_E. There are only two practical approaches for computing the cost of equity—the Gordon dividend model, discussed in Chapter 6, and the SML. This chapter has dealt in great detail with using the SML to compute the cost of equity and the resulting WACC. We've illustrated the use of the equity

[10] The issue of stock valuation is discussed in somewhat more detail in Chapter 18, which sums up the various approaches to this important topic.

β_E for computing the cost of equity r_E. We have also shown how you can use a combination of the equity beta β_E, the debt β_D, and the asset β_{Asset} to compute the WACC.

Through the use of a detailed example for Hilton Hotels, we have shown where to get the data required to make all these calculations.

Exercises (unfinished)

** The current riskfree rate is 4% and the expected rate of return on the market portfolio is 10%. The Brandywine Corporation has two divisions of equal market value. The bond to stock ratio (B/S) is 3/7. The company's bond can be assumed to present no risk of default. For the last few years, the Brandy division has been using a discount rate of 12% in capital budgeting decisions and the Wine division a discount rate of 10%. You have been asked by their managers to report on whether these discount rates are properly adjusted for the risk of the projects in the two divisions.

- What are the betas of typical projects implicit in the discount rates used by the two divisions?
- You estimate that the stock beta of Brandywine is 1.6. Is this consistent with the stock beta implicit in the discount rates used by the two divisions?
- You estimate that the stock beta of the Korbell Brandy Corp. is 1.8. This company is purely in the brandy business, its bond to stock ratio is 2/3 and its bond beta is 0.2. Based on this information (and on your estimate of Brandywine's stock beta), what discount rate

would you recommend for projects in the Brandy and in the Wine divisions of Brandywine?

** Sun Inc. has an equity beta of 0.5. Its capital structure consists of equal amounts of equity and risk-free debt. The debt has a pre-tax yield of 6% and the expected rate of return on the market index is 18%. Sun Inc. is considering expanding into Snow Inc. business. This new business is expected to generate an after-tax internal rate of return of 25%. Vacation Inc. is already in this new business, and its equity beta is 2.0 and it uses a blend of 10% (risk-free) debt and 90% equity in its capital structure. If the new project is to be funded with 50% debt, should Sun Inc. enter the Snow Inc. business? Assume that both companies have a marginal tax rate of 50%, and that the business risk of Vacation Inc. is comparable to the risk of Sun Inc.'s venture.

** A company is deciding whether to issue stock to raise money for an investment project which has the same risk as the market and an expected return of 15%. If the risk-free rate is 5%, and the expected return on the market is 12%, the company should go ahead:

a. This is false. The company should not take this project.
b. Regardless of the company's beta.
c. Unless the company's beta is greater than 1.25.
d. Unless the company's beta is less than 1.25.

** A project has the following forecasted cash flows (in thousands of dollars):

C_0	C_1	C_2	C_3
-100	60	50	40

The estimated project beta (beta of assets) is 1.6. The market return is 15% and the risk-free

rate is 7%. Estimate the opportunity cost of capital and the project's present value (using the same rate to discount each cash flow.)
b) A share of a stock with a beta of 0.75 now sells for $50. Investors expect the stock to pay a year-end dividend of $3. The T-Bill rate is 4%, and the market risk premium is perceived to be 8%. What is the investors' expectation of the price of the stock at the end of the year?
c) Reconsider the stock in question (b). Suppose investors actually believe the stock will sell for $54 at year-end. Is the stock a good or bad buy? What will investors do? At what point will

the stock reach an “equilibrium” at which it again is perceived as fairly priced?

CHAPTER 17: EFFICIENT MARKETS—SOME GENERAL PRINCIPLES OF SECURITY VALUATION*

This version: February 6, 2004

Chapter contents

Overview

Chapters 18 and 19 of *Principles of Finance with Excel* deal with the valuation of bonds (Chapter 18) and stocks (Chapter 19). We precede these asset-specific chapters with a discussion of some general principles of valuation (this chapter).

Finance often requires a lot of calculation, which is why this book concentrates on solving financial problems with a powerful tool like Excel. But sometimes understanding the way that financial markets work requires only wisdom and very little calculation. This chapter discusses some general principles of valuation which can save you from a lot of nonsense and in many cases require almost no calculation.

Here's an example of the kind of nonsense that you'll learn to avoid by reading and understanding this chapter: Your college roommate Clarence has just given you a "hot tip" on Federated Underwear (FU) stock : Clarence is sure that you should immediately buy the stock.

"It's going to go up. I know it," he says excitedly. "My pop says that FU has been fluctuating between \$15 and \$25 for the past year. Every time it gets close to \$15, it goes up, and when it gets close to \$25, it goes down again. Yesterday FU closed at \$15.05. Buy it and wait—the stock is sure to go up, and then you'll sell it at \$25 and make a killing."

After reading this chapter, you'll know to tell Clarence: "My friend, your advice is a perfect example of a *technical trading rule*. And Chapter 17 of my college finance textbook, *Principles of Finance with Excel*, explains that these rules are a clear violation of the principle of weak-form market efficiency, which almost always holds. If you want to bet your money on such foolishness, go ahead. I'm going to spend my hard-earned cash on a night out at the Efficient Markets Disco."

In the broadest sense the general principles discussed in this chapter all deal with the role of information in determining asset prices. When translated into simple language, these principles sound pretty dumb. They say things like: "Information is important." "Transactions costs matter." "One plus two equals three." When applied to asset markets the valuation principles discussed in this chapter often enable you to make surprising statements about what things are worth.

Here are four basic principles of valuation discussed in this chapter:

Efficient Markets Principle 1. *Single price for a single good*. In financial markets equivalent financial assets have the same price. Section 17.1 uses cross-listed stocks—stocks that trade in two financial, like IBM stock on the New York Stock Exchange and IBM stock on the Pacific Stock Exchange—as a non-trivial example of this principle.

Efficient Markets Principle 2. *Price additivity*: The price of a bundle of securities should be the sum of the prices of each of the securities. It is difficult to overestimate the importance of this principle. One of its predictions is that there are no "money machines"—it costs money to make money. Another prediction is that knowing the prices of the *components* of a financial asset will help you price the whole asset.

Efficient Markets Principle 3. *Information is critical*. Finding out previously-unknown information can be a very profitable exercise. Conversely, it is difficult to make money from facts that everyone knows. The more widely information is known, the less you can make money from the information. Principle 3 is usually split into three parts:

- The principle of *weak-form efficiency*: Market prices incorporate all current and past price information. If this principle holds (and almost all economists believe that it does), then it is not possible to make money based on the pattern of past prices of a traded security. This means that "money making" rules which are based on price patterns—"buy a stock if it's gone up three days in a row, sell it if it's gone down 3 days running"—are futile. The weak-form version of the efficient markets hypothesis should make you skeptical about a lot of investment strategies. An example is investment advisors who claim to be able to tell market trends from price patterns. These so-called "technical traders" are giving advice which violates the weak-form efficient markets hypothesis, and this advice should be ignored.
- The principle of *semi-strong-form efficiency*: Market prices incorporate all publicly-known information. Financial markets are awash in publicly-available information. Can you make money by carefully reading the financial statements of IBM? Probably not—IBM has many shareholders and is followed by hundreds of stock analysts. If the analysts are doing their job even moderately well, the information which can be gleaned from the IBM financial statements is already incorporated in the company's stock price. Most economists believe that markets are more-or-less semi-strong efficient (as you'll see in this chapter, it depends on how difficult it is to derive the information).
- The principle of *strong-form efficiency*: Market prices incorporate *all information* which exists (public or private) about a security. In addition to IBM's publicly-available financial statements and the analyses of stock analysts, there's also lots of *private* information about the company. For example, people working for the company know a lot about the sales, production, and costs of their individual units. Is this information also

incorporated in IBM's stock price? Almost no economists believe this. Meaning: Knowing privately-available information can provide you with profits.[1] Markets are not strong-form efficient.

Efficient Markets Principle 4. *Transactions costs are important and can screw up everything*. This is an important truth about markets. Transactions costs—by this we mean not just the costs of buying and selling securities, but also the cost of ferreting out information—make it more difficult to trade. And it's trade—the buying and selling of financial assets like stocks and bonds—which makes market prices reflect the true value of assets.

Finance concepts discussed in this chapter

- Efficiency
- Additivity
- Short sales
- Open-end and closed-end mutual funds

Excel functions used

- This chapter has some Excel, but nothing sophisticated

[1] Beware: Stock trading on the basis of insider information is also illegal.

17.1. Efficient Markets Principle 1: Competitive markets have a single price for a single good

A *competitive market* is a market with a large number of buyers and sellers, none of whom can influence the price of the goods bought and sold in the market. Financial markets are good examples of competitive markets: There are a large number of buyers and sellers for most stocks sold on major stock exchanges, there are many banks competing for your bank accounts and for your mortgage, and so on.

The principle that *competitive markets have a single price for a single good* is basic to economics and is drilled home in most introductory economics courses. Under some circumstances, this principle seems to be ridiculously obvious. For example: In the Asheville, North Carolina, farmer's market (the author's home town), there are many stands selling apples. Many of the vendors sell Granny Smith (GS) apples. The GS apples sold by the vendors are of approximately the same size and quality. The result: the price of apples of the same type is approximately the same at all the stands. Why? Suppose one vendor deviates from the equilibrium price of GS apples by selling below the price of the other vendors. Then he'll attract a lot of buyers. Being competitive, he will raise his price and the other GS vendors (also competitive) will lower their prices until, equilibrium being restored, the market price for GS apples is the same at all GS stands.

Cross-listed stocks—an application of the one-price principle

The one-good one-price principle also has applications in stock markets. Here's an example: IBM stock is traded both on the New York Stock Exchange (NYSE) and on the Pacific Stock Exchange (PSE). When both exchanges are open, the prices of IBM stock are basically the same in both exchanges. This isn't surprising: If the price of IBM in New York is $120 and its price is $118 in San Francisco, brokers would obviously try to *arbitrage* (that is: make money from unreasonable differences in prices) by buying IBM stock in San Francisco and selling it in New York. Since transactions costs are very low and since trade in stocks is instantaneous, this will drive the prices together.[2]

There's more to this than meets the eye: The NYSE opens before the PSE, but the PSE stays open later. This means that information about IBM which arrives late in the day will be incorporated in the PSE stock price but will hit the NYSE price only the next morning. In some cases this phenomenon is even more extreme—for example, there's a large group of Israeli shares which is traded both in Tel-Aviv and on the Nasdaq in the United States. The trading overlap between the two markets is only 1 hour per day (between 9:30 and 10:30 am Eastern time, both Nasdaq and Tel-Aviv are open—after this Tel-Aviv closes and all trading in the dual-listing stocks is on the Nasdaq). During this trading overlap, cross-listed stocks have the same price in both markets, but when only one market is open, this need not be so.

---

[2] Notice how we've already slipped in the importance of *transactions costs* ("Principle 4: Transactions costs can mess everything up"). This sentence suggests that transactions costs may include not only the direct cost of buying and selling (commissions, computer time, etc.), but also the time involved in transporting a good from one market to another. Luckily for this example, stock markets have pretty low transactions costs, especially for brokers and dealers.

17.2. Efficient Markets Principle 2: Bundles are priced additively

Prices are *additive* when the market price of A+B is equal to the market price of A plus the price of B. This sounds so obvious that it's hard to believe that it could be interesting (and, indeed, once you understand it, it's pretty boring!).

For an initial example, we go back to the Asheville farmer's market. Our previous example dealt with Granny Smith (GS) apples, but some of the vendors also sell Red Delicious (RD) apples. As we speak, the price of GS apples is $2 per pound and the price of RD apples is $3 per pound. Simon, a somewhat peculiar vendor, sells bags of apples containing both GS and RD apples: Each bag weighs 2 pounds and contains 1 pound of GS and 1 pound of RD apples. How should he price these bags? Obviously at $5 per bag.

Why? Not so trivial, actually. Suppose Simon prices the bags at $4.50. Then anyone wanting 1 pound of GS and 1 pound of RD will obviously buy with Simon. If Simon is sensitive to supply and demand, he'll notice the demand for his mixed bags of apples and raise the price; at the same time, other apple stands—seeing their demand weaken—will lower the prices of their apples.

Furthermore, if Simon persists in selling his bags of apples at $4.50, Sharon—a sharp cookie (or should we say "sharp apple"?)—will buy bags of apples from Simon. She'll then take the apples out of the bag and sell them at her apple stand for the market price of $2 for GS and $3 for RD. In the language of finance—Sharon is *arbitraging the price*. In the language of her grandmother, Sharon is buying cheap and selling dear.

On the other hand, suppose Simon prices the bags at $5.50. People will probably stop buying with him, even if they want bags with equal combinations of GS and RD—they can buy them cheaper elsewhere. Eventually Simon will have to lower his price. If, contrary to expectations, it turns out that Simon does a brisk business in the apple bags for \$5.50, then other smart apple stand owners will start selling their own bags of apples; since they can put together a bag for less than what Simon charges, the price of the mixed bags will go back down.

There might actually be room for Simon to sell his apple bags for \$5.05, since he's saving his customers the trouble of going to 2 apple stands. In the language of finance, he's saving them the *transactions cost* of buying the apples separately. They ought to be willing to pay him for this service.

The principle of price additivity is often summed up by the statement that *there are no money machines* in financial markets: You cannot simply make money by buying a complex financial asset (like Simon's bags of apples), taking it apart (separate bags of GS and RD), and selling the separate bags. The converse is also true: The "money machine" of combining GS and RD apples into one bag won't work.[3]

Now that you understand the principle of additivity as applied to the Asheville farmer's market, here are some non-trivial finance applications:

Additivity, example 1: The term structure prices bonds

The principle of bundle pricing is often applied to the pricing of bonds. A bond gives you a series of payments over time. Each of these payments is a separate financial package. If we can price each financial package, then we should be able to price the bond. For the moment we confine ourselves to a simple bond example, saving more complicated ones for the next chapter.

---

[3] In a broader sense, all of the efficient markets principles in this chapter say that there are no easy ways to make money on financial markets. If you want to make money, you'll have to do some meaningful work.

Here's an example. Suppose there are two bonds in the financial market, Bond A and Bond B:

- Bond A sells today for \$100 and pays off \$110 in one year. The bond's IRR is $10.00\% = \frac{110}{100} - 1$.
- Bond B also sells today for \$100. This bond has a payoff only at the end of 2 years, at which point it pays \$125. The IRR of the bond is $11.80\% = \left(\frac{125}{100}\right)^{1/2} - 1$.

Now suppose that you're trying to price a Bond C, which has a payoff of \$23 in one year and \$1023 in 2 years. The price-additivity principle says that the way to price this bond is to apply the IRRs calculated above separately to each year's bond payment. In a formula:

$$Bond\ \ price = \frac{23}{1.10} + \frac{1023}{(1.1180)^2} = 839.31$$

In this formula we've discounted the first bond payment of \$23 by the market interest rate on the one-year bonds, and we've discounted the second bond payment of \$1023 by the IRR derived from Bond B. Here's the spreadsheet:

	A	B	C	D
1	**PRICE ADDITIVITY IN BONDS**			
2	**Bond A: maturity in one year**			
3	Price today	100		
4	Payoff in one year	110		
5	IRR	10.00%	<-- =B4/B3-1	
6				
7	**Bond B: maturity in two years**			
8	Price today	100		
9	Payoff in one year	0		
10	Payoff in two years	125		
11	IRR	11.80%	<-- =(B10/B8)^(1/2)-1	
12				
13				
14	**Bond C: A bond with payments at end of year 1 and year 2**			
15	**Date**	**Payment**	**Present value of payment**	
16	**1**	23	20.91	<-- =B16/(1+B5)
17	**2**	1023	818.40	<-- =B17/(1+B11)^2
18				
19	**Bond price?**		839.31	<-- =SUM(C16:C17)

In a picture:

Pricing a bond by additivity--schematic

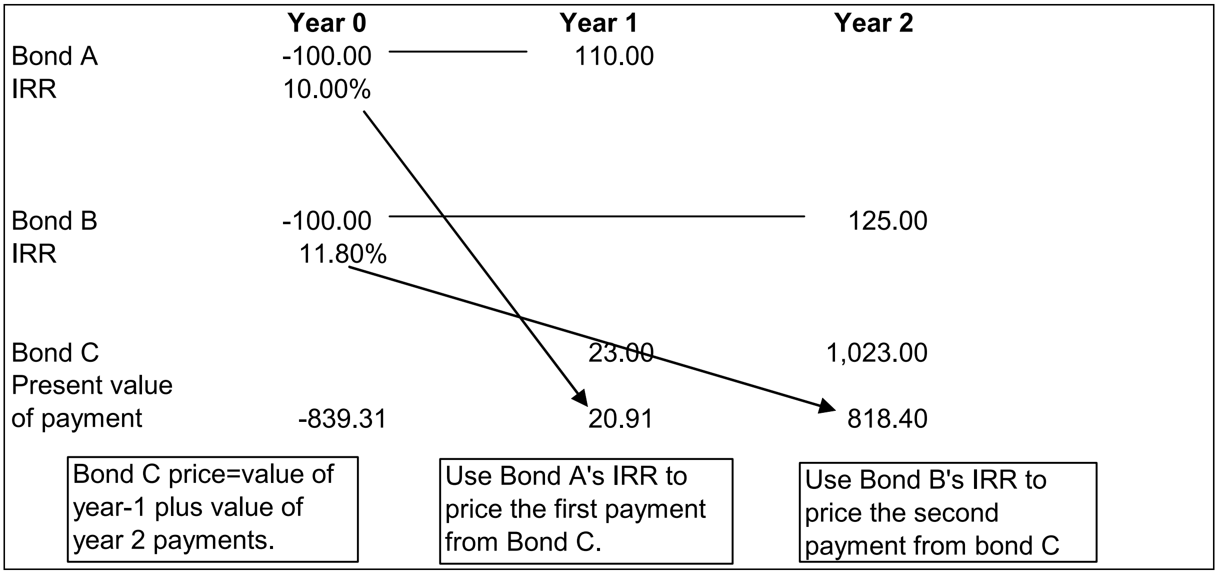


Figure 17.1

To sum up: We've used market discount rates derived from bonds with only one payment to additively price a bond with multi-year payments.

Additivity, example 2: Open-end mutual funds

The webpage of the United States Securities and Exchange Commission (SEC) defines a *mutual fund* as:

> A mutual fund is a company that brings together money from many people and invests it in stocks, bonds or other assets. The combined holdings of stocks, bonds or other assets the fund owns are known as its *portfolio*. Each investor in the fund owns shares, which represent a part of these holdings.
>
> http://www.sec.gov/investor/tools/mfcc/mutual-fund-help.htm

Figure 17.2 gives some more information from the SEC about mutual funds.

Home | Previous Page

U.S. Securities and Exchange Commission

Mutual Funds

A mutual fund is a company that pools money from many investors and invests the money in stocks, bonds, short-term money-market instruments, or other securities. Legally known as an "open-end company," a mutual fund is one of three basic types of investment company. The two other basic types are closed-end funds and Unit Investment Trusts (UITs).

Here are some of the traditional and distinguishing characteristics of mutual funds:

- Investors purchase mutual fund shares from the fund itself (or through a broker for the fund), but are not able to purchase the shares from other investors on a secondary market, such as the New York Stock Exchange or Nasdaq Stock Market. The price investors pay for mutual fund shares is the fund's per share net asset value (NAV) plus any shareholder fees that the fund imposes at purchase (such as sales loads).

- Mutual fund shares are "redeemable." This means that when mutual fund investors want to sell their fund shares, they sell them back to the fund (or to a broker acting for the fund) at their approximate NAV, minus any fees the fund imposes at that time (such as deferred sales loads or redemption fees).

- Mutual funds generally sell their shares on a continuous basis, although some funds will stop selling when, for example, they become too large.

- The investment portfolios of mutual funds typically are managed by separate entities known as "investment advisers" that are registered with the SEC.

Mutual funds come in many varieties. For example, there are index funds, stock funds, bond funds, money market funds, and more. Each of these may have a different investment objective and strategy and a different investment portfolio. Different mutual funds may also be subject to different risks, volatility, and fees and expenses.

All funds charge management fees for operating the fund. Some also charge for their distribution and service costs, commonly referred to as "12b-1" fees. Some funds may also impose sales charge or loads when you purchase or sell fund shares. In this regard, a fund may offer different "classes" of shares in the same portfolio, with each class having different fees and expenses.

Source: http://www.sec.gov/answers/mutfund.htm

Figure 17.2. Description of mutual funds from the SEC website.

Does it matter if you *bundle* securities together in a mutual fund? How should the price of such a fund be determined? The principle of pricing additivity gives us a way to handle this problem—it suggests that the price of a mutual fund should be determined by the market prices of all the fund's assets.

As a simple example, suppose you start a new company, the Super-Duper Fund, which sells a mutual fund of a very specific type.

- Super-Duper currently has 10,000 shareholders, each of who has invested $100—so that the total assets of the company are $1,000,000.
- Super-Duper's money is currently invested 50% in shares of IBM (currently trading at $100 and in shares of Intel (currently trading at $50). Thus the company currently owns 5,000 shares of IBM and 10,000 shares of Intel.
- The number of shares in the fund is *flexible*.[4] Right now, there are 10,000 shares, but this number can go up or down:
    - If a shareholder wants to sell, you promise to liquidate his proportional part of the fund's assets. So if Uncle Joe from Winona, who own one share worth $100, wants to sell his share in the company, Super-Duper will sell ½ share of IBM and 1 share of Intel and repay him his $100. Now the fund will have $999,900 in assets, still invested 50% in IBM and 50% in Intel.
    - If any new shareholders want to join, Super-Duper will buy—per $100 of new funds which come into the company—$50 of IBM and $50 of Intel.[5]

[4] In the jargon of mutual funds, this makes it an *open-end* fund. Our next example considers a closed-end fund.

Suppose that today no one sells or buys shares in the fund. The asset value of the Super-Duper fund today is $1,000,000. Now suppose that tomorrow the price of IBM is $110 and the price of Intel is $48. Then the value of a fund share is $103 (cell C14 below):

	A	B	C	D	E
1	**SUPER-DUPER OPEN END MUTUAL FUND**				
2		**Today**	**Tomorrow before new fundholders**		
3	Number of shareholders	10,000	10,000		
4					
5	**Portfolio**				
6	Price of IBM	100	110		
7	Price of Intel	50	48		
8					
9	Portfolio composition				
10	Shares of IBM	5,000	5,000		
11	Shares of Intel	10,000	10,000		
12					
13	Total fund value	1,000,000	1,030,000	<-- =C10*C6+C11*C7	
14	Value of 1 fund share	100	103	<-- =C13/C3	
15					
16	**Tomorrow: after new fundholders**				
17	Number of shareholders	10,500			
18	Total fund value	1,081,500	<-- =B17*C14		
19					
20	Portfolio composition				
21	Shares of IBM	4,915.91	<-- =B18*50%/C6		
22	Shares of Intel	11,265.63	<-- =B18*50%/C7		

New shares are created at the current fund share price, so the fund is now worth 10,500*$103=$1,081,500 .

Now suppose that at the close of the day tomorrow another 500 individuals buy shares of the fund. This means that they pay 500*$103 = $51,500 to buy shares in the fund. Assuming that the fund sticks to its current policy of splitting its investment equally between IBM and Intel, the total fund value of $1,081,500 (cell B18 above) will now be invested in 4,915.91 shares of IBM and 11,265.63 shares of Intel.

[5] Actually, Super-Duper Fund does all this at the end of the day. So if Uncle Joe wants to sell his share and Aunt Maude wants to invest an additional $100, Super-Duper has a *wash*, and it can save on the transactions costs of buying and selling. Every penny helps!

In an *open-end mutual fund* the number of shares is flexible. New shareholders buy into the fund at the per-share value of the fund, and shareholders in the fund who want to cash out cash out at the per-share value of the fund. At any point in time, the per-share value of the fund is given by the formula:

$$\begin{aligned} open\text{-}end\ \ fund\ \ per\text{-}share\ value &= \frac{fund\ \ net\ \ asset\ \ value\ (NAV)}{number\ \ of\ \ shares\ \ in\ \ fund} \\ &= \frac{market\ \ value\ \ of\ \ fund's\ \ portfolio - fund\ \ expenses}{number\ \ of\ \ shares\ \ in\ \ fund}. \end{aligned}$$

Notice that we've introduced a new bit of jargon: A mutual fund's net asset value (NAV) is the market value of the fund's portfolio minus fund expenses.

Mutual fund costs

The fund has some expenses which are charged to the fund holders and deducted from the value of the fund. These include the costs of buying and selling shares. Another fund cost is the cost of paying the managers: Typically fund managers charge their clients a percentage cost. If your fund charges 1% (in the U.S. this is typical), then this cost ($10,000 per year in our example) has to be taken out of the value of the fund.

Our Super-Duper Fund doesn't charge shareholders to buy or sell shares in the fund. However, there are also mutual funds which charge to buy shares. These so-called *front-end load* mutual funds are more expensive than *no-load* funds. Suppose, for example, that Super-Super-Duper were to charge a 7% front-end load. Then you would pay $107 (=$100 + 7% front-end load) to buy a share of the fund. Front-end loads are obviously expensive; mutual fund salespeople sometimes justify these extra charges as an appropriate price to pay for the expertise of better fund management, but there is almost no evidence to show that this is true.[6]

Example 3: Closed-end mutual funds—when additivity fails

Value-additivity doesn't always work. In this subsection we give an example of *closed-end mutual funds*. These are investment companies for which value additivity usually fails. A closed-end mutual fund is an investment company with a fixed number of shares. Like open-end mutual funds, closed-end mutual funds invest in a portfolio of stocks. As opposed to an open-end mutual fund, however, where the number of shares can be expanded or contracted, a closed-end mutual fund has a fixed number of shares which are sold on the stock market. The company issues no more new shares, and the market price fluctuates with supply and demand for the fund's shares.

Here's an example. The Chippewa Fund is a closed-end fund which looks a lot like the Super-Duper Fund. Like Super-Duper, Chippewa has 10,000 shares. Chippewa's share portfolio currently consists of $500,000 of IBM stock and $500,000 of Intel stock, and its shares are registered on the Chippewa Stock Exchange. The fund has no other assets.

What should be the price of a Chippewa Fund share? It seems that it should be equal to the per-share value of the fund's assets—in our case $100 per share (as you saw in our discussion of open-end mutual funds, the finance jargon is the *net asset value* of the Chippewa Fund is $100 per share). But checking the newspapers, you find that the share price of the

[6] Recall that in Chapter 12 we discussed a technique for judging mutual fund performance using the capital asset pricing model (CAPM). Finance researchers employing this and more sophisticated techniques find little evidence that front-load mutual funds outperform no-load mutual funds.

Chippewa Fund is $90, below its net asset value. A back-check of the prices of the Chippewa Fund shows you that Chippewa almost always sells for less than its net asset value. In fact a finance-knowledgeable friend has told you that almost all closed-end funds sell for less than their net asset value.

The reasons why closed-end funds sell at a discount are not well-understood.[7] What is well-understood, however, is that it is difficult to arbitrage a closed-end fund discount—meaning that it is difficult for investors to make money out of the discount and, by making money, cause the discount to disappear. Suppose, for example, that shares of Chippewa fund trade below $100 net asset value, say at $90. Then both existing and potential fund shareholders have a problem: On the one hand, the existing shareholders are holding $100 market-value shares worth only $90. If the closed-end fund were to break up, existing shareholders would get the net asset value of $100. So all the shareholders would in principle favor breaking up the fund, but no individual shareholder would want to sell his individual shares before such a breakup. A potential new shareholder is faced with the same problem: He gets $100 (market value) of shares for $90, but he has no guarantee that the value of the closed-end fund will ultimately get back to the market value.

[7] A readable survey of closed-end fund discounts is a paper by Elroy Dimson and Carolina Minua-Paluello, "The Closed-End Fund Discount," which is available on the Web. In their introduction, they write: "Closed-end funds are characterized by one of the most puzzling anomalies in finance: the closed-end fund discount. Shares in American funds are issued at a premium to net asset value (NAV) of up to 10 percent, while British funds are issued at a premium amounting to at least 5 percent. This premium represents the underwriting fees and start-up costs associated with the flotation. Subsequently, within a matter of months, the shares trade at a discount, which persists and fluctuates … . Upon termination (liquidation or 'open-ending') of the fund, share price rises and discounts disappear."

This whole scenario may sound somewhat improbable, but in fact there are many closed-end mutual funds. Figure 17.3 gives an actual example: Tri-Continental Corporation is a closed-end fund registered on the New York Stock Exchange. On 23 November 2001 the fund's shares were worth 11.18% less than the market value of the fund's portfolio. This *closed-end fund discount* is pervasive throughout the closed-end fund industry.

Tri-Continental Corporation—A Closed-End Fund

CLOSED-END FUND CENTER

PERFORMANCE DATA

E-MAIL / ARCHIVE / REGISTRATION / ABOUT CEFA / LEGAL INFO

Tri-Continental Corporation (TY / NYSE)
as of 11/23/2001

MORE INFO ON PDF help

Contact Information	Summary Information as of 6/30/2001
Advisor: Seligman, J.W. & Co., Inc.	**Category:** Growth & Income
Phone: (800) 221-7844	**Inception Date:** 12/31/1965
Website: http://www.tricontinental.com	**Outstanding Shares:** 131,077,105
	Admin Fees: N/A
Portfolio Mgr., Tenure	**Expense Ratio:** 0.6
Charles Smith , 12/1/1994	**Portfolio Turnover:** 38%
	Total Net Assets: $3,207.64 **(Millions)**

Description

Tri-Continental Corporation is a diversified, closed-end management investment company. The Fund seeks growth of capital while producing reasonable current income. The assets will primarily be invested in common stock. However, the Fund may invest its assets in a variety of asset classes. The Fund's capital structure includes both common and preferred stocks, as well as warrants.

Price Information as of 11/23/2001		
NAV:	22.89	Net asset value
Net chg:	0.20	
Prior Day:	22.69	
52-Week NAV Ret: (as of 11/23/2001)	-10.57%	
Mkt Price:	20.33	Market price on 23 November 2001

Average Annual Total Return as of 10/31/2001	MARKET PRICE	NAV
YTD:	-8.7%	-16.59%
1 yr:	-14.29%	-21.09%
5 yr:	9.03%	7.15%
10 yr:	9.6%	10.34%
Since Inception:	N/A	N/A

Premium / Discount as of 10/31/2001	
Current (as of 11/23/2001):	-11.18%
YTD Min:	-13.46%
YTD Max:	-10.27%
YTD Avg:	-11.22%
5-yr Avg:	-16.38%
10-yr Avg:	$\boxed{-14.57\%}$

Top 10 Holdings as of 6/30/2001	
Microsoft Corporation	4.2%
General Electric Company	3.9%
St Jude Medical Inc	3%
United Technologies Corporation	3%
Citigroup Inc	2.7%
Pitney Bowes Inc	2.6%
Baxter International Inc	2.6%
American International Group Inc	2.5%
Exxon Mobil Corporation	2.4%
Pfizer Inc	2.3%

Top 10 Sectors as of 4/30/2001	
Technology	25.3%
Financials	18.2%
Cyclicals	10.6%
Health	9.5%
Energy	8.6%
Staples	8.5%
Services	8.3%
Retail	5.7%
Utilities	3.1%
Durables	2.1%

Figure 17.3. Tri-Continental Corporation is a closed-end fund whose shares are registered on the New York Stock Exchange. On 23 November 2001, Tri-Continental's assets—the market value of the shares contained in its portfolio—totaled $3,207,840,000.
Since the fund has 131,077,105 shares, this works out to a *net asset value (NAV)* per share of:

$$NAV = \frac{3,207,840,000}{131,077,105} = \$22.89\,.$$

However, on the same date, the fund's shares sold for $20.33, *a discount* of 11.18%. The Tri-Continental discount is pervasive. In the last 10 years it has averaged 14.57%.

Summing up additivity

As long as market participants can freely arbitrage, we expect value additivity to hold: The value of a basket of goods or financial assets should equal the sum of the values of the components. Arbitrage in this case means the ability of market participants to create and sell their own bundles of goods or assets, or to break up existing bundles and sell the components. This is true whether we're discussing the cost of a bag of apples in the Asheville farmers' market or the price of an open-end mutual fund. But there are also situations, like a closed end fund, where arbitrage is difficult. For these cases, like the closed-end funds discussed above, we would not expect value additivity to hold.

We're not quite done with additivity: In the next section we discuss an interesting case where value additivity was clearly violated, but where—eventually—market prices came to reflect value additivity.

17.3. Additivity is not always instantaneous: The case of Palm and 3Com

During the 1990s, 3Com developed the Palm Pilot, a handheld personal information manager which became a raging success. In March 2000, 3Com sold 5.7% of its Palm subsidiary to the public. After this "equity carveout" there were separate stock market listings for Palm (still 94.3% owned by 3Com) and for the parent company 3Com. On March 3, 2000, the closing stock price for Palm was \$80.25 per share and the closing stock price for 3Com was \$83.06 per share. As you'll see this situation represents an interesting violation of the principle of value additivity.

In the spreadsheet below we calculate the market value of Palm (cell B5) and of 3Com (cell B10).

	A	B	C
1	**3COM AND PALM** **This spreadsheet reflects market prices on 3 March 2000, the day after the issue of 5.7% of Palm stock held by 3Com**		
2	**Palm**		
3	Price per share	80.25	
4	Number of shares outstanding	562,258,065	
5	Market value	45,121,209,716	<-- =B4*B3
6			
7	**3Com**		
8	Price per share	83.06	
9	Number of shares outstanding	349,354,000	
10	Market value	29,017,343,240	<-- =B9*B8
11			
12	Value of Palm stock held by 3Com (94.3%)	42,549,300,762	<-- =94.3%*B5
13	Value of non-Palm 3Com activities	-13,531,957,522	<-- =B10-B12

If you look at these numbers you'll see a startling *failure* of value additivity:

- 3Com owns most of Palm, but Palm's value is bigger than 3Com's! To be more precise: The 94.3% of Palm stock still owned by 3Com is worth \$42.5 billion (cell B12), but all of 3Com is worth only \$29.0 billion (cell B10).
- Using these numbers, the market seems to value all of the non-Palm activities of 3Com at a *negative* \$13.5 billion!!!! The only way this would be possible is if these activities were big money losers (which wasn't the case).

Why did additivity fail in this big way? Why didn't market participants arbitrage the 3Com and Palm stock prices so that additivity would be restored (below we explain how such an arbitrage would work)? One possible reason is that markets are (temporarily) relatively stupid: The enthusiasm for the initial public offering (IPO) of Palm at the beginning of March 2000 was so overwhelming that investors (temporarily, as you'll see below) forgot that 3Com still owned most of Palm. So they mispriced the relative values of Palm and 3Com, producing the weird case shown above. If they had thought a bit, they would have realized that a share of 3Com should be worth at least 1.52 times the price of a share of Palm:

	A	B	C
16	**Minimum logical value of 3Com shares, compared to Palm**		
17	Number of shares of Palm held by 3Com	530,209,355	<-- =94.3%*B4
18	Number of 3Com shares	349,354,000	
19	Number of Palm shares per 3Com share	1.52	<-- =B17/B18

Actually, if they knew how to read a balance sheet, they would conclude that the price of a 3Com share should be even more. In 3Com's last quarterly statement, just one week before the Palm IPO, its balance sheet showed almost $3 billion in cash and short-term investments. Assuming that these items were not needed for production of 3Com's products, they are worth $8.53 per 3Com share:

	A	B	C
22	**On 25feb00, from 3Com's balance sheet**		
23	Cash and equivalents	1,812,503,000	
24	Short-term investments	1,166,026,000	
25		2,978,529,000	<-- =B24+B23
26			
27	Cash and investments per 3Com share	8.53	<-- =B25/B9

So the minimum value for a 3Com share should have been:

$$3Com\ share\ \ price \geq 1.52 * Palm\ share\ \ price + \$8.53$$

Short-selling as a way of correcting market mispricing

Short-selling is a technique of borrowing a stock, selling it, and repaying it later.[8] Suppose you could freely short-sell Palm stock. Then you could make money from the above situation by shorting Palm stock and buying 3Com stock. We'll explain the arbitrage technique in a second, but the logic is: Palm stock is overpriced (relative to 3Com stock) and 3Com stock is underpriced (relative to Palm stock); so you should buy the cheap stock (3Com) and sell the overpriced stock (Palm).

The arbitrage with which an investor could profit from the market mis-pricing is as follows:

- Borrow a share of Palm stock and sell it. Selling borrowed stock is called *short selling*. In the example below, an arbitrageur short sells one share of Palm for $80.25.
- Buy the equivalent value of 3Com stock. At the time of the arbitrage we explore below, 3Com was selling for $83.06 per share. The arbitrageur—having just shorted Palm for $80.25, spends this money to buy 0.966 shares of 3Com (0.966*$83.06 = $80.25).

If you're right about the mispricing of the Palm versus 3Com shares, you should make money under any price scenario. In the example below the arbitrageur shorted 1 share of Palm on March 3 and used the proceeds to buy 0.966 shares of 3Com. Suppose that the arbitrageur undid his position on March 10 (meaning: he bought one share of Palm and sold 0.966 shares of

[8] The actual procedures for implementing a short sale are not simple. A well-written academic survey is a recent paper by Gene D'Avolio, "The Market for Borrowing Stock," unpublished working paper, Harvard University Economics Dept.

3Com). If on 10 March the prices of Palm and 3Com were in line with her additive valuation, then the arbitrageur would make money. In the example below, the share price of Palm on 10 March is $99 and the share price of 3Com is $159.01. As you can see, the arbitrageur makes $60.01:

	A	B	C
1	**3COM AND PALM: ARBITRAGING THE MISPRICING**		
2	March 3, 2000--short-sell 1 Palm share and buy $80.25/$83.06 = 0.9662 3Com shares		
3	Cash flow	0.00	<-- =80.25-0.9662*83.06
4			
5	March 10, 2000--buy 1 Palm share and sell $80.25/$83.06 = 0.9662 3Com shares		
6	Suppose Palm price is	99.00	
7	Logical *minimum* 3Com price	159.01	<-- =1.52*B6+8.53
8	Profit	60.01	<-- =B7-B6

If you play with the spreadsheet, you'll see that as long as you're right about the *price relation* between Palm and 3Com, you'll make money—whether the price of Palm goes up (as in the previous example) or down. For example, suppose that shares of Palm go down in price, and that they sell for $60 on the 10 March:

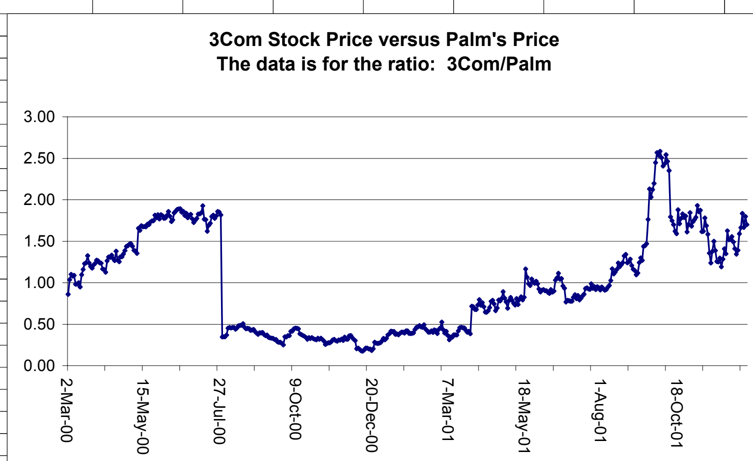
	A	B	C
1	**3COM AND PALM: ARBITRAGING THE MISPRICING**		
2	March 3, 2000--short-sell 1 Palm share and buy $80.25/$83.06 = 0.9662 3Com shares		
3	Cash flow	0.00	<-- =80.25-0.9662*83.06
4			
5	March 10, 2000--buy 1 Palm share and sell $80.25/$83.06 = 0.9662 3Com shares		
6	Suppose Palm price is	60.00	
7	Logical *minimum* 3Com price	99.73	<-- =1.52*B6+8.53
8	Profit	39.73	<-- =B7-B6

As you can see from this arbitrage example, short-selling is essential to making the prices "behave" in an additive way. Since short-selling involves selling borrowed stock, one explanation of the lack of additivity in 3Com-Palm prices is that initially there were just too few Palm shares around for arbitrageurs to sell.

What happened later?

The graph below shows the relation between Palm's stock price and 3Com's (column C of the spreadsheet calculates the ratio $\frac{3Com's\, stock\ price}{Palm's\ stock\ price}$). As you can see, the ratio climbed in the days following the Palm IPO, reaching the 1.52 point on 9 May 2000. From then on until the end of July 2000, the ratio remained above this ratio—presumably the word had gotten out and enough investors understood the intricacies the 3Com-Palm relationship to force the prices into an appropriate pattern.

	A	B	C	D	E
1	CALCULATING THE RATIO OF 3COM'S STOCK				
2	PRICE TO PALM'S STOCK PRICE				
3	Daily data, 2 March 2000 - 24 December 2001				
4	Note that remaining Palm stock was distributed to shareholders				
5	on 27 July 2000				
6					
7	Palm		3Com		
8	Date	Close	Close	3Com/Palm	
9	2-Mar-00	95.063	81.813	0.861	<-- =C9/B9
10	3-Mar-00	80.250	83.063	1.035	
11	6-Mar-00	63.125	69.563	1.102	
12	7-Mar-00	66.875	72.250	1.080	
13	8-Mar-00	64.750	70.438	1.088	
14	9-Mar-00	69.375	68.063	0.981	
15	10-Mar-00	70.000	68.938	0.985	
16	13-Mar-00	64.313	64.313	1.000	
17	14-Mar-00	57.750	54.813	0.949	
18	15-Mar-00	55.750	61.063	1.095	
19	16-Mar-00	55.563	64.500	1.161	
20	17-Mar-00	55.250	68.000	1.231	
21	20-Mar-00	55.250	68.563	1.241	
22	21-Mar-00	48.375	64.109	1.325	
23	22-Mar-00	51.563	63.875	1.239	
24	23-Mar-00	58.188	69.688	1.198	
25	24-Mar-00	57.000	67.000	1.175	
26	27-Mar-00	55.375	67.188	1.213	
27	28-Mar-00	54.813	67.625	1.234	
28	29-Mar-00	49.688	63.188	1.272	
29	30-Mar-00	46.500	58.813	1.265	
30	31-Mar-00	44.875	55.625	1.240	
31	3-Apr-00	40.313	49.750	1.234	
32	4-Apr-00	38.250	44.563	1.165	



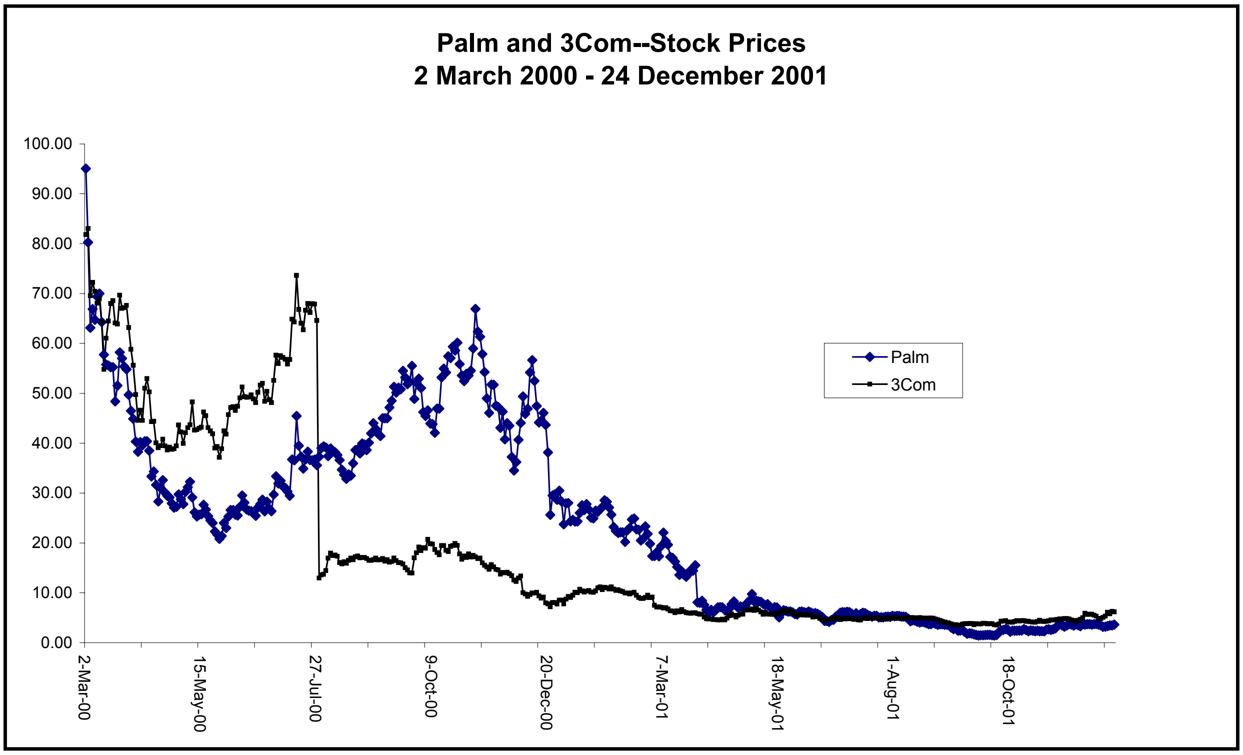
On 28 July 2000, the ratio dropped precipitously, from 1.815 to 0.347. What happened? After markets closed on July 27, 3Com *distributed all remaining Palm shares to its*

shareholders. There was no longer any compelling reason for 3Com's share price to be related to Palm's. As you can see in the graph above, since then the ratio of the prices has been all over the place.

	A	B	C	D	E	F	G
1	**CALCULATING THE RATIO OF 3COM'S STOCK PRICE TO PALM'S STOCK PRICE**						
2	Daily data, 2 March 2000 - 24 December 2001 Note that remaining Palm stock was distributed to shareholders on 27 July 2000						
3							
4	**Date**	**Palm**	**3Com**	**Ratio of 3Com/Palm**			
99	17-Jul-00	39.500	66.813	1.691	Palm's stock price		
100	18-Jul-00	37.313	64.063	1.717			
101	19-Jul-00	34.875	62.750	1.799			
102	20-Jul-00	36.750	66.625	1.813	3Com's stock price		
103	21-Jul-00	38.313	68.000	1.775			
104	24-Jul-00	36.625	66.188	1.807			
105	25-Jul-00	36.563	67.938	1.858	3Com sells for 1.815 times Palm		
106	26-Jul-00	36.688	67.875	1.850			
107	27-Jul-00	35.563	64.563	1.815	<-- =C107/B107		
108	28-Jul-00	37.250	12.938	0.347	<-- =C108/B108		
109	31-Jul-00	39.000	13.563	0.348			
110	1-Aug-00	39.375	13.688	0.348			
111	2-Aug-00	39.125	14.438	0.369			

What happened on 27 July 2000?

On 27 July 2000 (the divestiture of all Palm stock by 3Com), the price of 3Com dropped precipitously. By that date investors—well informed about the coming divestiture of Palm—realized that the divestiture of the stock by 3Com would lower 3Com's value. And so it did:



Palm and 3Com—what's the point?

Additivity is a basic efficiency feature of financial markets. As in the case of closed-end funds, it may not hold where the structural features of the funds make arbitrage difficult, or—as in the case of Palm and 3Com—it may take some time for markets to figure out what's happening and to initiate the arbitrage which will lead to additivity. Difficulties in short-selling can lead to failure of additivity.

17.4. Efficient Markets Principle 3: Cheap information is worthless

Financial markets are awash in information, and it is important for you to have some opinions about how this information affects market prices. In this section we discuss three hypotheses that relate to how information is incorporated into financial markets. The finance jargon for these hypotheses is: Weak-form efficiency, semi-strong form efficiency, and strong-form efficiency.

In one form or another all three of these hypotheses state that information is important and that cheap and easily-accessible information is likely to be worthless. The cheaper and more easily-accessible the information, the less it's worth.

Read the previous paragraph again. It sounds contradictory!

- Information is important? This seems obvious. Whether it's the cost of a bank loan or information about whether Upward Slopes Ski Site is making money—the more informed you are about a financial asset, the more you should be able to judge its worth.
- Cheap and easily-accessible information is likely to be worthless? If it's so important, why isn't it worth anything? The reason is that many people think that it's important, and so they're all trying to figure out what the information is and how it affects the value of the asset. With so much energy expended on finding out the effect of the information and with the information so cheap, you're likely to find that the whole price impact of the information has already been extracted and is already reflected in the market price.

Weak-form efficiency: almost always true

The hypothesis of weak-form efficiency says that you cannot predict the future price of a financial asset by carefully examining the asset's past prices and its current price. Since *everyone* has easy and cheap access to the past prices of IBM stock, there's nothing left to be learned from these prices—all possible information contained in these prices is already incorporated in the *current market price* of IBM. Everyone knows past prices, and therefore, if you could make a profitable prediction based on a stock's price history, so could everyone else.

In trying to implement this profitable information, you and other investors would drive its profitability out of existence. This sounds obvious (and it is), but it's a principle often overlooked by investors.

Technical analysis—do previous prices predict future prices?

Its proponents claim that *technical analysis* is the art or science of using historical stock price patterns to predict the future stock price. Finance professors think that technical analysis is neither an art nor a science, but simply voodoo. They base this belief on the weak-form efficient markets hypothesis and on tons of academic research.

Here's a simple example of technical analysis: Based on an analysis of ABC's historical stock price, you've concluded that it fluctuates in a band between $25 and $35. When the price gets close to $25, it inevitably goes up, and when the price gets close to $35, the stock price goes down. This leads you to develop the following money-making strategy:

- Buy ABC when the price gets to $25.50; since this is very close to $25, the price will have a very high probability of moving up. In any case you'll have little to lose, since the price can't go below $25.
- Sell ABC when the price gets to $34.50; since this is very close to $35, the price has a very high probability of moving down. In any case at $34.50 you have very little to gain.

This sounds like a money-making strategy, but on the other hand it's self-defeating: If all investors try to implement this strategy (and why shouldn't they, since your analysis is based on publicly-available information?), then the "price band" will narrow—no one will want to buy ABC stock when it gets close to $34.50 or $25.50.

But now everyone will try to implement a profit strategy based on the new price band. And so on and so on ...

The conclusion: There is no price band! It may be that ABC's share price has been between $25 and $35 in the past, but this says nothing about its share price in the future.

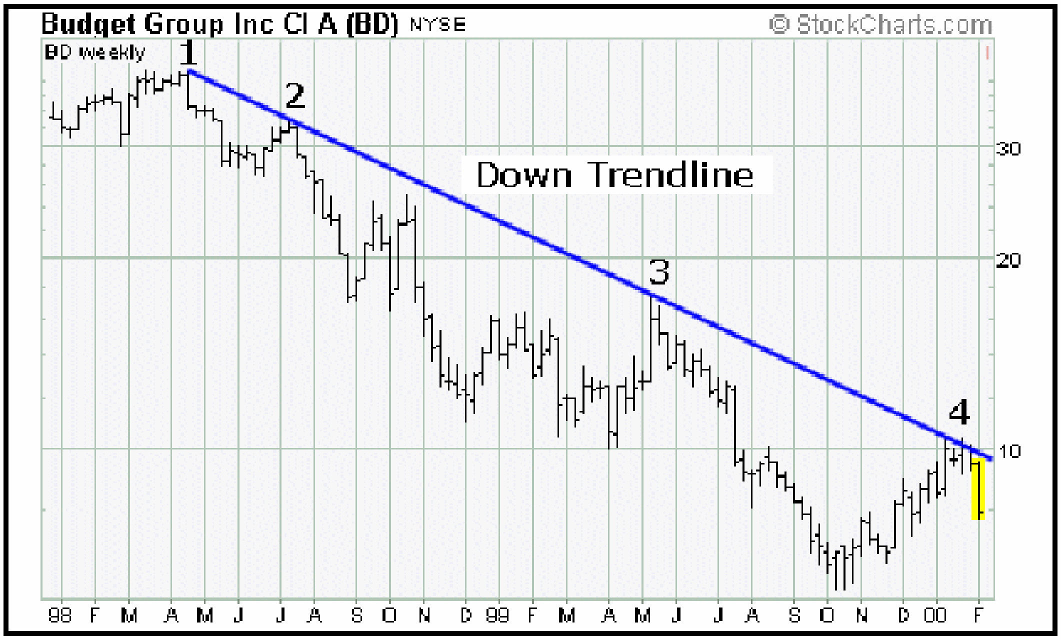
In fact you could make a broader conclusion: As long as there are many people trading in a market, a strategy based only on past and current prices cannot be profitable.

Technical trading rules—another violation of weak-form efficiency

A technical trading strategy is a rule for buying and selling a stock based on the stock's previous price movements.[9] The weak-form efficiency hypothesis says that technical trading rules won't work.

The ABC example above (where ABC's stock was assumed to trade in a band between $25 and $35) is a simple example of a technical trading rule. Figure 17.4 gives a more sophisticated example.

[9] There are lots of good websites on technical trading. Here are a few: http://technicaltrading.com/, http://www.stockcharts.com/education/What/TradingStrategies/MurphysLaws.html .



Down Trendline
A down trendline has a negative slope and is formed by connecting two or more high points. The second high must be lower than the first for the line to have a negative slope. Down trendlines act as resistance and indicate that net-supply (supply less demand) is increasing even as the price declines. A declining price combined with increasing supply is very bearish and shows the strong resolve of the sellers. As long as prices remain below the down trendline, the downtrend is considered solid and intact. A break above the down trendline indicates that net-supply is decreasing and a change of trend could be imminent.

http://www.stockcharts.com:85/education/What/ChartAnalysis/trendlines.html

Figure 17.4: Technical analysis of Budget stock.

The down trendline explains the downturn in Budget Group's stock price by connecting four "price peaks." The prediction and the associated trading rule is:

- When the stock price of Budget Group gets close to the down trendline, it will move down. To exploit this information, you should buy when the price is farther away from the trendline and sell when it is close to the line.
- *If* the stock price of Budget Group *breaks through* the down trendline "a change of trend could be imminent." This is the technical analyst's escape hatch—the information contained in the prices is true except when it's not true.

Semi-strong form efficiency: sometimes true

Semi-strong form efficiency predicts that not just past prices, but all publicly-available information, is incorporated in current security prices. This suggests, for example, that the analysis of a firm's financial statements is not going to help you make better investment decisions.

Semi-strong market efficiency seems to be true ... *occasionally*. It's a lot of work to understand *all* the publicly-available information about a stock, and it's quite common to see cases where information existed, but it wasn't incorporated into the stock price. The 3Com-Palm story discussed in Section 17.3 above is a case in point. Only after some rigorous analysis of the relation between 3Com and Palm and analysis of the cash reserves of 3Com could we conclude that Palm's price was overpriced relative to 3Com's price. There has to be a lot at stake to motivate investors to engage in this kind of research. If it's worthwhile, then we would expect semi-strong efficiency to prevail.

Strong-form efficiency: usually not true

The strong-form efficient markets hypothesis says that *all* information is incorporated into securities prices. Hardly anyone believes that this is true. In fact it's often illegal, since *all* information includes proprietary information and inside knowledge—by law, insiders are forbidden to trade on their information if it hasn't been revealed to the public.

17.5. Efficient Markets Principle 4: Transactions costs are important

Transactions costs are all the various costs of buying and selling a security and also the costs (monetary or otherwise) of *understanding* a security. When you buy a stock for $50, you pay a brokerage commission. In the United States this commission is typically ½ percent. So the purchase of a share of stock costs you $50.25 and its sale delivers you $49.75:

	A	B	C
3	Buy commission	0.50%	
4	Sell commission	0.50%	
5			
6	Stock price	$50.00	
7			
8	Purchase price	50.25	<-- =B6*(1+B3)
9	Selling price	49.75	<-- =B6*(1-B4)

The result: If you think that the stock is worth $50.15, it won't be worth your while to buy it: Even though the stock's price today is $50, less than what you think it's worth, transactions costs make it more expensive to buy the stock ($50.25) than you think it's worth.

Similarly, suppose you own a share of the stock and suppose you think it's worth only $49.80. In the absence of transactions costs, it would be logical to sell the stock, but with a ½ percent transactions cost, you would be getting less than you think the stock is worth.

Here's a more interesting example: Below are the prices of sugar in London and in New York on 25 July 2003.

	A	B	C
1	**COMPARING SUGAR PRICES IN LONDON AND NEW YORK**		
2	New York (dollars/pound)	0.0693	
3	London (dollars/tonne)	208.30	
4	pounds per tonne	2,200	
5	London (dollars/pound)	0.0947	<-- =B3/B4
6			
7	**One container of sugar**		
8	Contains 21 tons		
9	in pounds	46,200	<-- =21*B4
10	"Arbitrage profit"	1,172.64	<-- =(B5-B2)*B9

New York sugar is selling for 6.93 cents per pound, whereas sugar in London is selling for $208.30 per "tonne." Could there be an opportunity here to make money? In comparing the prices, you have to make sure the units are the same; for example—a "tonne" is a *metric ton*, 1,000 kilograms (which equals 2,200 pounds). As you can see, the London price translates to 9.47 cents per pound.

It looks like there's an arbitrage opportunity here: If we buy sugar in New York and sell it in London, we can make over 2.5 cents per pound. Since a 20-foot container can hold 21 tons of sugar (or 46,200 pounds—see cell B9 above), it looks like we could make almost $1,173 profit per container. And since a ship can hold hundreds of containers .... this must be a surefire way to get rich!

But hold on—this couldn't be. We must have forgotten the transaction costs:

- It *costs money* to ship sugar from New York to London. It costs approximately $1,000 to ship a container of sugar from New York to London. This alone would almost eliminate the arbitrage profit.
- It *takes time* to ship sugar from New York to London—somewhere between 10 days and 3 weeks, depending on the availability of shipping. So even if the freight costs are less that $1,500, this isn't an arbitrage—it's a kind of educated gamble on the price differentials between the two cities.[10]

So: There might be a profit here, but it's not certain. The transaction costs, the cost and the time needed to ship the sugar from New York to London, will eat up most of the profits. Of

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[10] What we need is a forward or a futures contract: These are contracts which enable us to fix a price today for sugar delivered in London at some point in the future. Such contracts exist, but they're beyond the scope of this book. For a good text, see John Hull, *Options, Futures, and other Derivatives*, Prentice-Hall (4th edition, 2000).

course, this is what you would expect in an efficient market: You can't make money from things which are easy to do.

Conclusion

Financial economists use the words "efficient markets" to describe a variety of rules about financial asset prices which are so simple that they almost always have to be true. In this chapter we've explored several of these asset pricing rules:

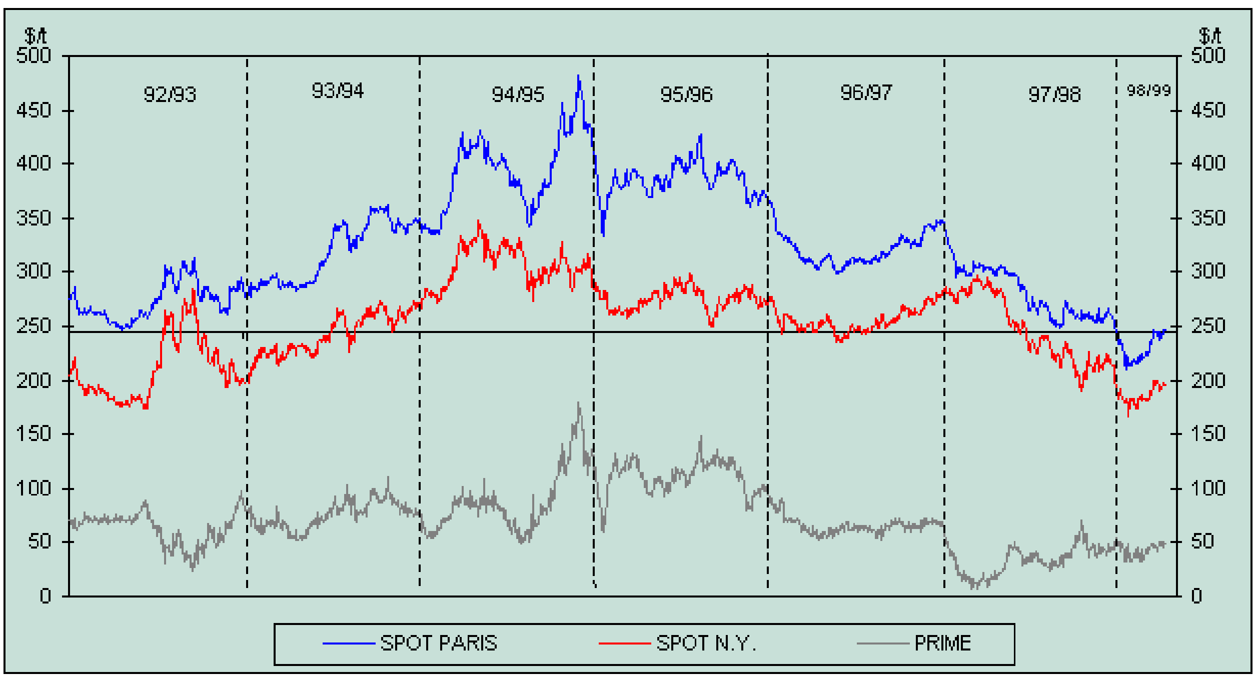
- One price for one asset. In an efficient financial market, assets which are the same ought to have the same value and price.
- Price additivity of asset bundles: In an efficient market bundling two or more assets together—whether its different kinds of apples in a bag or stocks in a mutual fund—doesn't change their value.
- Informational effects on prices: Generally-known information cannot be worth much, and the more widely the information is known, the less it is worth. We explored three versions of this principle. The weak-form efficiency principle says that the future asset price cannot be predicted from knowledge of historical asset prices and the current asset price. The semi-strong form efficiency principle says that publicly-known information—not just prices, but published accounting data and other information which can (with some work) be derived from the information—is worthless. Economists believe that semi-strong efficiency holds frequently but not always. The strong-form efficiency principle, which almost no one believes, says that *all* information—whether public or not—is worthless.

- Transactions costs: These pesky critters can screw up the previous three principles, because they interfere with arbitrage. Arbitrage, the buying and selling of assets with profit, is the mechanism by which the three above principles are forced to hold. Transactions costs, the cost of buying and selling an asset, or the cost of finding out information about the asset, can make it more difficult to arbitrage and hence cause market inefficiencies.

Exercises

Go back to the bond example of section?? Show that the annualized IRR of the bond is ???. Now note that this rate is *not* the discount rate for the cash flows of the individual bonds (the ½ year and 1-year T-bills)! So are markets inefficient?

	A	B	C
18	Chapter exercise: The YTM of the bond is		
19			
20		Date	Price/CF
21		0	-963.56
22		0.5	23
23		1	1023
24			
25		IRR	8.66%



Can you make money from this? Sugar prices (source: Bouche???)

THE NEW YORKER THE TALK OF THE TOWN

THE FINANCIAL PAGE

GET SHORTY

by James Surowiecki

Issue of 2003-12-01

A few years ago, the Finance Ministry of Malaysia suggested that a certain group of troublemakers needed to be punished. Caning, the Ministry said, would be the right penalty. And who were the malefactors being threatened with the rap of rattan? They weren't drug dealers or corrupt executives or even gum chewers. They were short sellers.

Short sellers are investors who sell assets (a company's shares, say) that they have borrowed, in the hope that the price will fall; if it does, they can buy the shares at a lower price, return them to the trader they borrowed them from, and pocket the difference. In effect, they are betting against a company's stock price. As a result, they have, historically, been regarded with great suspicion, and though the Malaysian proposal was novel, the hostility behind it was not. Shorts have been reviled since at least the seventeenth century. Napoleon deemed the short seller "an enemy of the state." England outlawed shorting for much of the eighteenth and nineteenth centuries. Just last year, Germany's Finance Minister suggested that short selling should be banned during crises. Across the world, short sellers continue to be seen as conniving sharpies, spreading false rumors and victimizing innocent companies with what House Speaker J. Dennis Hastert once called "blatant thuggery."

The United States, it's true, hasn't resorted to the rattan, but it still enforces a set of rules against short selling that have been in place since the thirties, when shorts were seen as a cause of the Great Crash. In a country as optimistic and can-do as ours, there seems to be something un-American about betting against stocks. That may be changing. The Securities and Exchange Commission is now proposing an eighteen-month experiment in which the most onerous restrictions on short selling would be lifted for three hundred big stocks. If the market for these stocks worked well, the old rules could eventually be lifted across the board. And it's about time.

"It's easy to make short sellers wear the black hats," James Chanos, the head of Kynikos Associates and one of the few pure short sellers around, says. "Short selling is always an emotional issue. Executives have their egos tied to the price of their shares, so when you take a position against them they take it personally." But give the short sellers their due: they're the canaries in the coal mine, recognizing problems before others do. In the past few years alone, shorts sounded early alarms about blow-ups like Enron, Tyco, and Boston Chicken; they also uncovered scams at lots of smaller companies that tried to cash in on the stock-market hysteria of the late nineties. In general, the companies that short sellers target deserve it. The economist Owen Lamont studied a group of companies that had clashed with short sellers—denouncing them in conference calls with investors, imploring shareholders not to lend them stock, and so forth. He found that the average stock-market return for these companies over the next three years was minus forty-two per cent, which suggests that their stock prices were as inflated as the shorts had claimed.

Even when short sellers aren't uncovering malfeasance, their presence in the market is useful. If you think of a stock price as a weighted average of the expectations of investors, restrictions on short selling skew that average by shutting out people with contrary opinions. It's a bit like setting a point spread for a football game by allowing people to bet only on one side. When a team of Yale management professors did a study of forty-seven stock markets around the world, they found that markets with active short sellers reacted to information more quickly and set prices more accurately. A traditional justification for short-selling regulations—including the rule the S.E.C. wants to repeal, which prevents short selling when prices are already falling—is that they protect markets from panics. Yet the study found no evidence of it. There's a case to be made that in the late nineties restrictions on short selling helped inflate the Internet bubble, by reducing any counterweight to the prevailing mania. This, in turn, worsened the eventual crash.

If the S.E.C. does run its experiment, corporations are hardly going to drown in a deluge of short sales. Today, only two per cent of all United States stock-market shares are shorted, and even with looser restrictions short selling is likely to remain uncommon. In part, that's because shorting stocks is simply harder than buying stocks: loans can be called in at any moment, and your potential losses are unlimited. More important, shorting demands a willingness to challenge Wall Street's foundational dogma: that stocks should, and will, go up.

"I used to think that it should be as easy to go short as it is to go long," Chanos, who was one of the first to see through Enron's hype, says. "After all, the two things seem to require the same skill set: you're evaluating whether a company's stock price reflects its fundamental value. But now I think that they aren't the same at all. Very few human beings perform well in an environment of negative reinforcement, and if you're a short, negative reinforcement is what you get all the time. When we come in every day, we know that Wall Street and the news and ten thousand public-relations departments are going to be telling us that we're idiots. You don't have that steady drumbeat of support behind you that you have if you're buying stocks. You have a steady drumbeat on your head." By lifting the regulatory sanctions on short selling, the S.E.C. might help to weaken the social sanctions. The result should be a better functioning market, which is in the interest of investors as a whole. Let corporations denounce short sellers all they want. The case against these bears is a lot of bull.

CHAPTER 18: VALUING STOCKS*

This version: October 22, 2003

Chapter contents

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it.

Overview

In chapter 17 we discussed the valuation of bonds. This chapter deals with the valuation of stocks. Whereas the valuation of bonds is a relatively straightforward matter of computing yields to maturity, the valuation of stocks is much more difficult. The difficulty lies both in the greater uncertainty about the cash flows which need to be discounted in order to arrive at a stock valuation and in the computation of the correct discount rate.

In this chapter we discuss four basic approaches to stock valuation:

- **Valuation method 1, the efficient markets approach**. In its simplest form the *efficient markets* approach states that the current stock price is correct. A somewhat more sophisticated use of the efficient markets approach to stock valuation is that a stock's value is the sum of the values of its components. We explore the implications of these statements in section 18.1.
- **Valuation method 2, discounting the future free cash flows (FCF)**. Sometimes called the *discounted cash flow* (DCF) approach to valuation, this method values the firm's debt and its equity together as the present value of the firm's future FCFs. The discount rate used is the weighted average cost of capital (WACC). This method is the valuation approach favored by most finance academics. We discuss this approach in section 18.2 and discuss the computation of the WACC in sections 18.5 and 18.6. In this chapter we do not discuss the concept or the computation of the free cash flow—this was done previously in Chapters 7-9.
- **Valuation method 3, discounting the future equity payouts**. A firm's shares can also be valued by *discounting the stream of anticipated equity payouts* at an

appropriate cost of equity r_E. The concept of equity payout (the sum of a firm's total dividends plus its stock repurchases) was previously discussed in Chapter 6.

- **Valuation method 4, multiples**. Finally we can value a firm's shares by a *comparative valuation based on multiples*. This very common method involves ratios such as the price-earnings (P/E) ratio, EBITDA multiples, and more industry specific multiples such as value per square foot of store space or value per subscriber.

With the exception of the multiple method 4, almost all of the material in this chapter is also discussed elsewhere in this book. For example, the efficient markets approach to valuation is also discussed in Chapter 15, and the Gordon dividend model (which values a firm's equity by discounting its anticipated dividend stream) is also discussed in Chapters 6 and 9. WACC computations are to be found in Chapters 5 and 15. The purpose of this chapter is to bring together these dispersed materials into a (hopefully coherent) whole.

Finance concepts discussed in this chapter

- Discounted cash flows, free cash flows (FCF)
- Cost of capital, cost of equity, cost of debt, weighted average cost of capital (WACC)
- Equity premium
- Beta, equity beta, asset beta
- Two-stage growth models

Excel functions used

- **Sum**, **NPV, If**
- Data table

18.1. Valuation method 1: The current market price of a stock is the correct price (the efficient markets approach)

The simplest stock valuation is based on the efficient markets approach (Chapter 15). This approach says that the *current market price of a stock is the correct price*. In other words: The market has already done the difficult job stock valuation, and it's done this correctly, incorporating all of the relevant information There's a lot of evidence for this approach, as you saw in Chapter 15.

This valuation method is very simple to apply:

- *Question*: "IBM looks a bit expensive to me—it's price has been going up for the last 3 months. What do you think: Is IBM's stock price currently underpriced or overpriced?"
- *Answer*: "At Podunk U., we learned that markets with a lot of trading are in general efficient, meaning that the current market price incorporates all the readily-available information about IBM. So—I don't think IBM is either underpriced or overpriced. It's actually correctly priced."

Here's another example of the use of this approach:

- *Question*: "I've been thinking of buying IBM, but I've have been putting it off. The price has gone up lately, and I'm going to wait until it comes down a bit. It seems a bit high to me right now." What do you think?
- *Answer*: "At Podunk U. we would call you a *contrarian* . You believe that if the price of a stock has gone up, it will go back down (and the opposite). But this technical approach (see Chapter 15) to stock valuation doesn't seem to work very well. So if you want to

buy IBM, go ahead and do so now. There's nothing in the price runup of the last couple of months which indicates that there will now be a price rundown."

Some more sophisticated efficient markets methods

Efficient markets valuations don't always have to be as simplistic as the above examples. In Chapter 15 we looked at *additivity*, a fundamental tenet of efficient markets. The principle of additivity says that the value of a basket of goods or financial assets should equal the sum of the values of the components. Additivity can often be used to value stocks.

Here's a very simple example: ABC Holding Corp., a publicly-traded company, owns shares in two publicly traded companies. Besides owning these subsidiaries, ABC does little else.

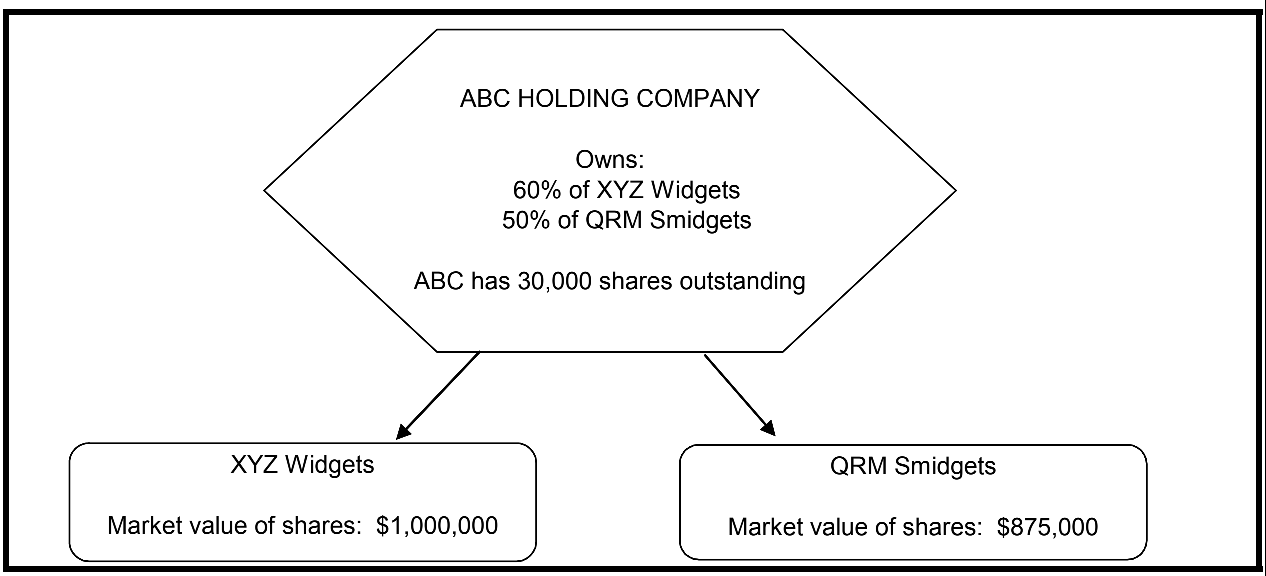

Figure 18.1. Ownership structure of ABC Holding Company

What should be the value of a share of ABC Holding? The obvious way to do this is in the following spreadsheet, which computes the share value of ABC to be $34.58:

	A	B	C	D	E
1	**ABC HOLDING COMPANY**				
2	Number of ABC shares	30,000			
3					
4	**ABC owns shares in**	**Percentage of shares owned by ABC**	**Market value**	**Market value of ABC holdings in company**	
5	XYZ Widgets	60%	1,000,000	600,000	<-- =B5*C5
6	QRM Smidgets	50%	875,000	437,500	<-- =B6*C6
7	Total value of ABC holdings			1,037,500	<-- =D6+D5
8					
9	Per share value of ABC Holdings			34.58	<-- =D7/B2

Notice what this model *is* and *is not* telling you:

- *Is* telling you: If the market values of XYZ and QRM are correct, then the market value of ABC should be $34.58. The formula works out to be:

$$\begin{matrix} ABC\ share \\ price \end{matrix} = \frac{60\%*\left[XYZ\ \ value\right]+50\%*\left[QRM\ \ value\right]}{\begin{matrix} number\ of \\ ABC\ shares \end{matrix}}$$

- *Is not* telling you: The formula tells you a relation between the 3 share prices. It tells you if the share prices are *relatively correct*, but it does not tell you if they are *absolutely correct*. Example: After doing much work and research and applying the methods of the previous section, you come to the conclusion that, while the market valuation of QRM is correct, the market value of XYZ ought to $1,600,000. Then you would conclude that the share price of ABC ought to be $46.58.

	A	B	C	D	E
1	**ABC HOLDING COMPANY**				
2	Number of ABC shares	30,000			
3					
4	**ABC owns shares in**	**Percentage of shares owned by ABC**	**Market value**	**Market value of ABC holdings in company**	
5	XYZ Widgets	60%	1,600,000	960,000	<-- =B5*C5
6	QRM Smidgets	50%	875,000	437,500	<-- =B6*C6
7	Total value of ABC holdings			1,397,500	<-- =D6+D5
8					
9	Per share value of ABC Holdings			46.58	<-- =D7/B2

Note that if ABC has some of its own overheads and if it doesn't always pass through all the dividends of its subsidiaries, its market price will be *lower* than $34.58, since the market price of ABC will reflect not only the cost of the shares of its subsidiaries, but also its own overheads. This looks a lot like the *closed-end fund* valuation problem discussed in Chapter 15.

18.2. Valuation method 2: The price of a share is the discounted value of the future anticipated free cash flows

Valuation method 1 of the previous section says that there is nothing to be gained by second-guessing market valuations. In many cases, however, the finance expert (you!) will want to do a basic valuation of a company and derive the value of a share is from the discounted value of the future anticipated free cash flows (FCF). This method, often called the *discounted cash flow* (DCF) method of valuation, was discussed and illustrated in Chapters 8 and 9. Figure 18.2 reminds you of the definition of FCF and Figure 18.3 gives a flow diagram of the FCF valuation method.

Defining the Free Cash Flow	
Profit after taxes	This is the basic measure of the profitability of the business, but it is an accounting measure that includes financing flows (such as interest), as well as non-cash expenses such as depreciation. Profit after taxes does not account for either changes in the firm's working capital or purchases of new fixed assets, both of which can be important cash drains on the firm.
+ Depreciation	This noncash expense is added back to the profit after tax.
+ after-tax interest payments (net)	FCF is an attempt to measure the cash produced by the business activity of the firm. To neutralize the effect of interest payments on the firm's profits, we: • Add back the after-tax cost of interest on debt (*after-tax* since interest payments are tax-deductible), • Subtract out the after-tax interest payments on cash and marketable securities.
- Increase in current assets	When the firm's sales increase, more investment is needed in inventories, accounts receivable, etc. This increase in current assets is not an expense for tax purposes (and is therefore ignored in the profit after taxes), but it is a cash drain on the company.
+ Increase in current liabilities	An increase in the sales often causes an increase in financing related to sales (such as accounts payable or taxes payable). This increase in current liabilities—when related to sales—provides cash to the firm. Since it is directly related to sales, we include this cash in the free cash flow calculations.
- Increase in fixed assets at cost	An increase in fixed assets (the long-term productive assets of the company) is a use of cash, which reduces the firm's free cash flow.
FCF = sum of the above	

Figure 18.2. Defining the free cash flow. We have previously discussed FCFs and their use in valuation in Chapters 7-9.

CALCULATING THE FIRM'S SHARE VALUE FROM THE FREE CASH FLOWS AND WACC

Predict firm's future free cash flows (FCF). In Chapters 8 & 9 we did this using a pro forma model.

Compute the firm's weighted average cost of capital (WACC):

$$WACC = r_E \frac{E}{E+D} + r_D (1 - T_C) \frac{D}{E+D}$$

Where r_E is the cost of equity, r_D is the cost of debt, and T_C is the firm's tax rate

Discount the all future free cash flows to get the enterprise value of the firm:

$$Enterprise\ \ value = \left[\sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t} \right] * (1+WACC)^{0.5}$$

We've multiplied by $(1+\text{WACC})^{0.5}$ because cash flows are assumed to occur in midyear.

Alternatively

Discount some of the free cash flows and the terminal value to get the enterprise value of the firm:

$$Enterprise\ \ value = \left[\sum_{t=1}^{N} \frac{FCF_t}{(1+WACC)^t} + \frac{Terminal\ Value}{(1+WACC)^N} \right] * (1+WACC)^{0.5}$$

The terminal value is what the firm will be worth on date N. We've multiplied by $(1+\text{WACC})^{0.5}$ because cash flows are assumed to occur in midyear.

Add initial cash balances to enterprise value to get total value of the firm's assets:

$$Total\ \ asset\ \ value = Enterprise\ \ value + Initial\ \ cash$$

Subtract debt value from firm value to get total equity value:

$$Equity\ \ value = Total\ \ asset\ \ value - Debt\ \ value$$

Divide equity value by the number of shares to derive the share value:

$$Share\ \ value = \frac{Total\ \ equity\ \ value}{Number\ \ of\ \ shares}$$

Figure 18.3: Flow diagram for a FCF valuation

Valuation 2: Example 1—a basic example

It is 31 December 2003 and you are trying to value Arnold Corp, which finished 2003 with a free cash flow of \$2 million. The company has debt of \$10 million and cash balances of \$1 million You estimate the following financial parameters for the company:

- The future anticipated growth rate of the FCF is 8%
- The WACC of Arnold is 15%

You can now estimate the value of Arnold:

- The *enterprise value* of Arnold is the present value of future anticipated FCFs discounted at the WACC:

$$Enterprise\ value = \underbrace{\left[\sum_{t=1}^{\infty}\frac{FCF_t}{(1+WACC)^t}\right]}_{\substack{\uparrow \\ \text{This is the PV} \\ \text{formula, assuming that} \\ \text{FCFs occur at year-end}}} * \underbrace{(1+WACC)^{0.5}}_{\substack{\uparrow \\ \text{This factor "corrects"} \\ \text{for the fact that FCFs occur} \\ \text{throughout the year.}}}$$

$$= \underbrace{\left[\sum_{t=1}^{\infty}\frac{FCF_{2003}(1+g)^t}{(1+WACC)^t}\right]}_{\substack{\uparrow \\ \text{Future FCFs are expected} \\ \text{to grow at rate } g.}} *(1+WACC)^{0.5}$$

$$= \underbrace{\left[\frac{FCF_{2003}(1+g)}{WACC-g}\right]}_{\substack{\uparrow \\ \text{This formula was} \\ \text{given in Chapter ???}}} *(1+WACC)^{0.5}$$

Doing the computations in an Excel spreadsheet shows that the enterprise value of Arnold Corp. is \$33,090,599 and that the estimated per-share value is \$24.09:

	A	B	C
1	VALUING ARNOLD CORP		
2	2003 FCF (base year)	2,000,000	
3	Future FCF growth rate	8%	
4	WACC	15%	
5	End-2003 debt	10,000,000	
6	End-2003 cash	1,000,000	
7	Number of shares outstanding	1,000,000	
8			
9	Enterprise value	33,090,599	<-- =B2*(1+B3)/(B4-B3)*(1+B4)^0.5
10	Add cash	1,000,000	<-- =B6
11	Subtract debt	-10,000,000	<-- =-B5
12	Value of equity	24,090,599	<-- =SUM(B9:B11)
13	Share value	24.09	<-- =B12/B7

Valuation method 2: Example 2—two FCF growth rates

In the valuation of Arnold Corp. in the previous subsection we assumed a FCF growth rate which is unchanging over the future. This assumption is often suitable for a mature, stable company, but it may not be appropriate for a company that is currently experiencing very high growth rates. In this subsection we show how to perform a FCF valuation of a company for which we assume *two* FCF growth rates—a high FCF growth rate for a number of years followed by a subsequent lower FCF growth rate.

Xanthum Corp. has just finished its 2003 financial year. The company's 2003 FCF was $1,000,000. Xanthum has been growing very fast; you anticipate that for the coming 5 years the FCF growth rate will be 35%. After this time, you anticipate that the FCF growth will slow to 10% per year, because the market for Xanthum's products will become mature.

Xanthum has 3,000,000 shares outstanding and a WACC of 20%. It currently has $500,000 of cash on hand which is not needed for operations; Xanthum also has $3,000,000 of debt. To value the company, we apply the same valuation scheme as before, but this time we use the two FCF growth rates:

$$Enterprise\ value = \left[\underbrace{\sum_{t=1}^{5} \frac{FCF_t}{(1+WACC)^t}}_{\substack{\uparrow \\ \text{The PV of the "high} \\ \text{growth" FCFs}}} + \underbrace{\sum_{t=6}^{\infty} \frac{FCF_t}{(1+WACC)^t}}_{\substack{\uparrow \\ \text{The PV of the "normal} \\ \text{growth" FCFS}}} \right] * \underbrace{(1+WACC)^{0.5}}_{\substack{\uparrow \\ \text{This factor "corrects"} \\ \text{for the fact that FCFs occur} \\ \text{throughout the year.}}}$$

There's a valuation formula which can be derived using techniques described in the appendix to Chapter 1:

$$Enterprise\ value =$$

$$\left[\underbrace{\frac{FCF_{2003}(1+g_{high})}{1+WACC} \left(\frac{1-\left(\frac{1+g_{high}}{1+WACC}\right)^5}{1-\frac{1+g_{high}}{1+WACC}} \right)}_{\substack{\uparrow \\ \text{In the spreadsheet this is called} \\ \text{"term 1" and } \frac{1+g_{high}}{1+WACC} \text{ is called "term1 factor"}}} + \underbrace{\frac{FCF_{2003}(1+g_{high})^5}{(1+WACC)^5} \left(\frac{1+g_{normal}}{WACC - g_{normal}} \right)}_{\substack{\uparrow \\ \text{In the spreadsheet this is called} \\ \text{"term 2"}}} \right] * (1+WACC)^{0.5}$$

The spreadsheet below shows that Xanthum's enterprise value is \$27,040,649 (cell B15) and that its per-share value is \$8.18 (cell B21):

	A	B	C
1	**VALUING XANTHUM CORP**		
2	2003 FCF (base year)	1,000,000	
3			
4	High growth rate, g_{high}	35%	
5	Normal growth rate, g_{normal}	10%	
6	Number of high growth years	5	
7	Term 1 factor: $(1+g_{high})/(1+WACC)$	113%	<-- =(1+B4)/(1+B9)
8			
9	WACC	20%	
10	End-2003 debt	3,000,000	
11	End-2003 cash	500,000	
12			
13	Term 1: PV of high-growth cash flows	7,218,292	<-- =B2*B7*(1-B7^B6)/(1-B7)
14	Term 2: PV of normal-growth cash flows	19,822,357	<-- =B2*(1+B4)^B6*(1+B5)/(B9-B5)/(1+B9)^B6
15	Enterprise value	27,040,649	<-- =SUM(B13:B14)
16	Add cash	27,540,649	<-- =B15+B11
17	Subtract debt	-3,000,000	<-- =-B10
18	Value of equity	24,540,649	<-- =SUM(B16:B17)
19			
20	Number of shares, end 2003	3,000,000	
21	Share value	8.18	<-- =B18/B20

Valuation method 2: Example 3—using the terminal value in a real-estate project

In the previous two examples we discounted an infinitely-lived stream of cash flows. Sometimes it makes more sense to discount a finite number of cash flows and then attribute a terminal value to the project.

Here's an example: Your Aunt Sarah has quite a bit of money. She's been offered a share in a partnership which is being set up by a local real estate agent. The partnership will buy an existing building, called the Station Building, for $20 million. The agent is selling 25 shares, for $800,000 each ($\$800,000 = \frac{\$20,000,000}{25}$). Aunt Sarah has asked you to do some financial analysis to determine whether this is a fair price for a partnership share in the Station Building.

Here's what you discover:

- All income from the Station Building partnership will flow through to the shareholders, who will pay taxes on the income at their personal tax rates. Aunt Sarah's tax rate is 40%.
- Station Building will be depreciated over 40 years, giving an annual depreciation of $500,000 per year.
- The building is fully rented out and brings up annual rents of $7 million. You do not anticipate that these rents will increase over the next 10 years.
- Maintenance, property taxes, and other miscellaneous expenses for Station Building cost about $1 million per year.
- The agent who is putting together the partnership has proposed selling Station Building after 10 years. He estimates that the market price of the building will not change much over this period—meaning that the market price of Station Building in year 10 is anticipated to be $20 million, like its price today.

In your valuation of the Station Building shares, you see that the annual free cash flow (FCF) to Aunt Sarah is $152,000 (cell B16 in the spreadsheet below). This FCF will be available to her in years 1-10, and is based on the building's profit before taxes of $5,500,000, which will be spread equally among the partners.

The terminal value of the building is $20,000,000, which on a per-share basis is $800,000 (cell B19). At the time the building is sold in year 10, its accumulated depreciation is $5,000,000, so that its book value is $15,000,000. To compute Aunt Sarah's cash flow from this terminal value, we deduct the per-share book value of the building ($600,000, cell B20) from the sale price to arrive at taxes of $80,000 on the profit from the sale of the building (cell B22). The cash flow from the sale is the $800,000 sale price minus the taxes--$720,000 as shown in cell B23.

	A	B	C	D	E	F	G
1	**STATION BUILDING PARTNERSHIP--SHARE VALUATION**						
2	Building cost	20,000,000					
3	Depreciable life (years)	40					
4	Annual rents	7,000,000					
5	Annual expenses	1,000,000					
6	Annual depreciation	500,000	<-- =B2/B3		**Profit and loss, Station Building as a whole**		
7	Aunt Sarah's tax rate	40%			Annual rent	7,000,000	
8	WACC	18%			Minus annual expenses	-1,000,000	
9	Shares issued	25			Minus annual depreciation	-500,000	
10	Share price	800,000			Anticipated annual building profit before taxes	5,500,000	<-- =SUM(F7:F9)
11							
12	**Profit and loss, Aunt Sarah's share**						
13	Anticipated annual building profit before taxes	220,000	<-- =F10/B9		**Terminal value, year 10, Station Building as a whole**		
14	Profit after taxes	132,000	<-- =(1-B7)*B13		Anticipated building market price	20,000,000	<-- =B2
15	Building depreciation, per share	20,000	<-- =B6/B9		Accumulated depreciation, year 10	5,000,000	<-- =B6*10
16	Free cash flow	152,000	<-- =B14+B15		Book value of building, year 10	15,000,000	<-- =B2-F15
17							
18	**Terminal value, year 10, Aunt Sarah's share**						
19	Anticipated building market price	800,000	<-- =F14/B9				
20	Book value in year 10, per share	600,000	<-- =F16/B9				
21	Profit from sale of building	200,000	<-- =B19-B20				
22	Tax on profit	80,000	<-- =B7*B21				
23	Terminal value: cash flow from sale	720,000	<-- =B19-B22				
24							
25	**Year**	**Aunt Sarah's anticipated FCF**					
26	1	152,000	<-- =$B$16				
27	2	152,000					
28	3	152,000					
29	4	152,000					
30	5	152,000					
31	6	152,000					
32	7	152,000					
33	8	152,000					
34	9	152,000					
35	10	872,000	<-- =$B$16+B23				
36							
37	Share value: Present value of Aunt Sarah's free cash flows	$820,667.53	<-- =NPV(B8,B26:B35)				

Cells B26:B35 show Aunt Sarah's anticipated free cash flows from the building partnership, including the terminal value. Discounting these cash flows at the WACC of 20% values a partnership share at $820,667.53. Conclusion: Aunt Sarah should invest in the building!

Valuation method 2: Example 4—using the terminal value to get around large FCF growth rates

Our second example of using the terminal value involves the Formanis Corporation. Formanis is in a growth industry and has had formidable FCF growth rates for the past several years, and you anticipate that these rates will continue for years 1-5. However, after year 5 you anticipate a big slowdown in Formanis's FCF growth, as its industry matures.

Here are the relevant facts about Formanis:

- The company's FCF for the current year is $1,000,000.
- You anticipate that the FCF for years 1-5 will grow at a rate of 25% per year.
- You anticipate a growth rate of FCFs of 6% per year for years 6, 7, … (termed the "long-term growth rate" in the spreadsheet below).
- The company has 5 million shares outstanding.

The valuation formula is:

$$
\begin{aligned}
Formanis\ value = {} & \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \frac{FCF_4}{(1+WACC)^4} + \frac{FCF_5}{(1+WACC)^5} \\
& + \frac{1}{(1+WACC)^5} * \underbrace{\frac{FCF_5 * (1+long\text{-}term\ growth\ rate)}{(WACC - long\text{-}term\ growth\ rate)}}_{\substack{\uparrow \\ \text{This is the terminal value:} \\ \text{an explanation is given in Chapter ??}}}
\end{aligned}
$$

To value Formanis, we first predict the FCFs for years 1-5 (cells B9:B13 of the spreadsheet). The present value of these FCFs is $6,465,787 (cell B20). The terminal value represents the year-5 present value of the Formanis cash flows for years 6, 7, … . To compute the terminal value, we assume that Formanis's cash flows for these years grow at the long-term growth rate:

$$
\begin{aligned}
Terminal\ value &= year\text{-}5\ PV\ of\ Formanis\ FCFs,\ years\ 6,7,\ldots \\
&= \frac{FCF_6}{(1+WACC)} + \frac{FCF_7}{(1+WACC)^2} + \frac{FCF_7}{(1+WACC)^2} + \ldots \\
&= \frac{FCF_5 * (1+long\text{-}term.\ growth\ rate)}{(1+WACC)} + \frac{FCF_5 * (1+long\text{-}term.\ growth\ rate)^2}{(1+WACC)^2} \\
&\qquad + \frac{FCF_5 * (1+long\text{-}term.\ growth\ rate)^3}{(1+WACC)^2} + \ldots \\
&= \frac{FCF_5 * (1+long\text{-}term\ growth\ rate)}{(WACC - long\text{-}term\ growth\ rate)}
\end{aligned}
$$

In cell B17 below the terminal value—assuming a long-term FCF growth rate of 6%—is $17,025,596.

	A	B	C
1	**FORMANIS CORPORATION**		
2	Current FCF	1,000,000	
3	Anticipated growth rate, years 1-5	25%	
4	WACC	15%	
5	Long-term growth rate, after year 5	6%	
6	Number of shares outstanding	5,000,000	
7			
8	**Year**	**Anticipated FCF**	
9	1	1,250,000	<-- =$B$2*(1+$B$3)
10	2	1,562,500	<-- =B9*(1+$B$3)
11	3	1,953,125	<-- =B10*(1+$B$3)
12	4	2,441,406	
13	5	3,051,758	
14			
15	**Terminal value calculation**		
16	FCF in year 5	3,051,758	<-- =B13
17	Terminal value	17,025,596	<-- =B16*(1+B5)/(B3-B5)
18			
19	**Valuing Formanis Corporation**		
20	Present value of FCFs, years 1-5	6,465,787	<-- =NPV(B4,B9:B13)
21	Present value of terminal value	8,464,730	<-- =B17/(1+B4)^5
22	Value of Formanis	14,930,518	<-- =B21+B20
23	Per share value	$2.99	<-- =B22/B6

The value of Formanis (cell B22) is $14,930,518. The per-share value of Formanis is $2.99 (cell B23).

The terminal value method illustrated for Formanis is often used:

- It allows the stock analyst to distinguish between short-term growth and long-term growth. Often short-term growth is a function of market performance, whereas long-term growth is determined by macro-economic factors. For example in a new and rapidly developing market, we might anticipate high short-term growth rates. But we would also anticipate that as the market matures and becomes more saturated, the long-term growth rates would approximate the growth of the economy as a whole.

- From an Excel point of view, the terminal value method allows us to do interesting sensitivity analyses. For example, here is the per-share value of Formanis for a variety of long-term growth rates and WACCs; we use the **Data Table** technique described in Chapter ???:

	A	B	C	D	E	F
26	**Sensitivity analysis: Per share value of Formanis with different WACC and long-term growth. Year 1-5 growth rate = 25%**					
27	=B23		Long-term growth rate ↓			
28		$2.99	**0%**	**2%**	**4%**	**6%**
29	WACC →	**15%**	2.51	2.64	2.80	2.99
30		**20%**	2.11	2.22	2.35	2.50
31		**25%**	1.80	1.89	1.99	2.12
32		**30%**	1.55	1.62	1.70	1.81

Varying the year 1-5 growth rate gives different values. In the table below, for example, we've assumed that year 1-5 growth is 20%:

	A	B	C	D	E	F
26	**Sensitivity analysis: Per share value of Formanis with different WACC and long-term growth. Year 1-5 growth rate = 20%**					
27	=B23		Long-term growth rate ↓			
28		$3.01	**0%**	**2%**	**4%**	**6%**
29	WACC →	**15%**	2.38	2.54	2.75	3.01
30		**20%**	2.00	2.13	2.30	2.51
31		**25%**	1.70	1.81	1.95	2.12
32		**30%**	1.46	1.55	1.66	1.81

18.3. Valuation method 3: The price of a share is the present value of its future anticipated equity cash flows discounted at the cost of equity

In the previous section we "backed into" the equity valuation of the firm, by first calculating the value of the firm's assets (the enterprise value plus initial cash balances), and then subtracting from this number the value of the firm's debts. In this section we present another method for calculating the value of the firm's equity—we directly discount the value of the firm's anticipated payouts to its shareholders.

As an example consider Haul-It Corp., which has a steady record of paying dividends and repurchasing shares. The company has 10 million shares outstanding. Here's a spreadsheet with the valuation model:

	A	B	C	D	E	F	G
1	**HAUL-IT CORPORATION--EQUITY PAYOUT HISTORY AND SHARE VALUATION**						
2		**1998**	**1999**	**2000**	**2001**	**2002**	
3	Repurchases	$1,440,000	$2,410,000	$3,500,000	$6,820,000	$4,830,000	
4	Dividends	$3,950,000	$3,997,000	$4,238,000	$4,875,000	$5,100,000	
5	Total cash paid to equity holders	$5,390,000	$6,407,000	$7,738,000	$11,695,000	$9,930,000	
6							
7	Compound annual growth, 1998-2002	16.50%	<-- =(F5/B5)^(1/4)-1				
8							
9	Haul-It's cost of equity, r_E	25.00%					
10							
11	**Valuation**						
12	Current equity payout	$9,930,000	<-- =F5				
13	Anticipated future growth	16.50%					
14							
15	Value of total equity	136,164,862	<-- =B12*(1+B13)/(B9-B13)				
16	Number of shares outstanding	10,000,000					
17	Value per share	13.62	<-- =B15/B16				
18							
19							
20							
21							
22							
23							
24							
25							
26							
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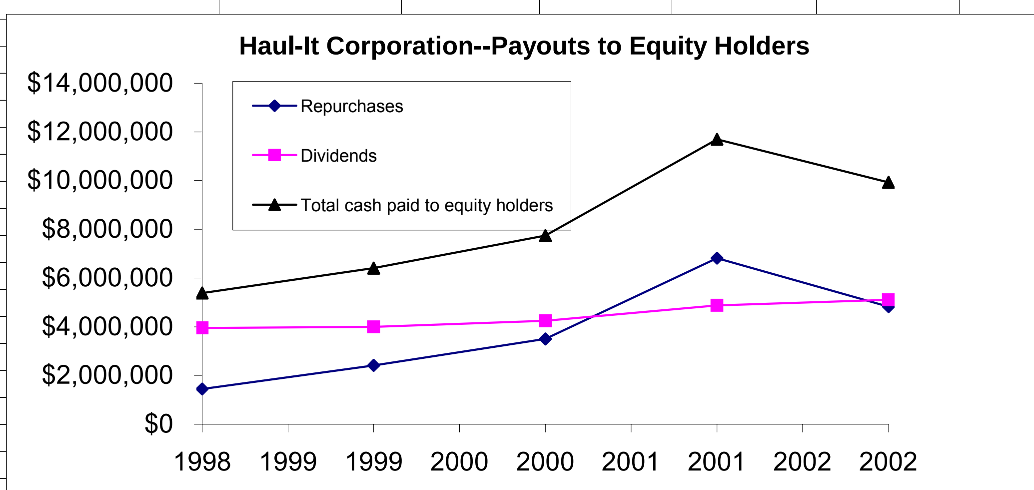


Between 1998 and 2002, Haul-It's payouts to its equity holders have increased at an impressive rate of 16.50% per year (cell B7). The company's cost of equity r_E is 25% (cell B9).[1] Assuming that future equity payout growth equals historical growth, Haul-It is valued at $136 million (cell B15), which gives a per-share value of $13.62.

The equity value of the company is the discounted value of the future anticipated equity payouts:

$$\begin{aligned}
Equity\ value &= \frac{Equity\ \ payout_{2003}}{1+r_E}+\frac{Equity\ \ payout_{2004}}{\left(1+r_E\right)^2}+\frac{Equity\ \ payout_{2004}}{\left(1+r_E\right)^3}+....\\
&= \frac{Equity\ \ payout_{2002}\left(1+g\right)}{1+r_E}+\frac{Equity\ \ payout_{2002}\left(1+g\right)^2}{\left(1+r_E\right)^2}+\frac{Equity\ \ payout_{2002}\left(1+g\right)^3}{\left(1+r_E\right)^3}+....\\
&= \frac{Equity\ \ payout_{2002}\left(1+g\right)}{r_E-g}=\frac{9,930,000\left(1.165\right)}{25.00\%-16.50\%}=136,164,862
\end{aligned}$$

Dividing the equity value by the number of shares outstanding gives the estimated value per share:

$$Value\ \ per\ \ share=\frac{Equity\ value}{Shares\ outstanding}=\frac{136,164,862}{10,000,000}=13.62$$

Why do finance professionals shun direct equity valuation?

Valuation method 3, the direct valuation of equity is so simple that it may surprise you that it is rarely used. There are several reasons for this, none of which we can fully explain at this point in the book:

- The direct equity valuation method depends on projected equity payouts (that is, dividends plus share repurchases), whereas Method 3 depends on projected free cash

[1] At this point we do not discuss how we arrived at this cost of equity. For a recapitulation of cost of capital techniques, see sections 18.??? – 18.???.

flows. Whereas a firm's equity payouts are a function of management decisions about dividends and stock repurchases, FCFs are a function of the firm's operating environment—its sales, costs, capital expenditures, and so on. Because many components of the FCFs are determined by the firm's operating environment rather than management decisions about dividends, analysts are generally more comfortable predicting FCFs.

- The FCF Method 3 discounts future FCFs at the firm's weighted average cost of capital (WACC). The equity payout method 4 discounts future equity payouts at the firm's cost of equity r_E. For reasons we will explain in Chapters 19 - 20, the cost of equity r_E is very sensitive to the firm's debt-equity ratio, whereas the WACC is not as sensitive to the debt-equity ratio.[2]

18.4. Valuation method 4, comparative valuation: Using multiples to value shares

The last valuation technique we discuss is based on a comparison of financial ratios for different companies. This valuation technique is often referred to as using "multiples." The technique is based on the logic that financial assets which are similar in nature should be priced the same way.

[2] For reasons explained in Chapter 19, the WACC may in fact be completely invariant to a firm's leverage. If this is so, we can value a firm based on Method 3 without worrying about its leverage.

A simple example: Using the price/earnings (P/E) ratio for valuation

The *price/earnings ratio* is the ratio of a firm's stock price to its earnings per share:

$$P/E = \frac{stock\ \ price}{earnings\ \ per\ \ share}.$$

When we use the P/E for valuation, we assume that similar firms should have similar P/E ratios.

Here's an example: Shoes for Less (SFL) and Lesser Shoes (LS) are both shoe stores located in similar communities. Although SFL is bigger than LS, having double the sales and double the profits, the companies are in most relevant respects similar—management, financial structure, etc. However, the market valuation of the two companies does not reflect their similarity: The P/E ratio of SFL is significantly lower than that of LS, as can be seen in the spreadsheet below:

	A	B	C	D
1	**SHOES FOR LESS (SFL) AND LESSER SHOES (LS) comparing P/E ratios**			
2		**SFL: Shoes for Less**	**LS: Lesser Shoes**	
3	**Sales**	30,000	15,000	
4	**Profits**	3,000	1,500	
5	**Number of shares**	1,000	1,000	
6	**Shareprice**	24	18	
7	**Equity value**	24,000	18,000	<-- =C6*C5
8	**EPS: Earnings per share**	3	1.5	<-- =C4/C5
9	**P/E: Price-Earnings ratio**	8.00	12.00	<-- =C6/C8

Based on the similarity between the two companies, SFL appears underpriced relative to LS—its P/E ratio is less. A market analyst might recommend that anyone interest in investing in the shoe store business invest in SFL rather than LS.[3]

[3] A more radical strategy might be to *buy* shares of SFL and to *short* shares of LS. See Chapter 10 and its discussion of Palm and 3Com shares for a discussion of this strategy.

Kroger (KR) and Safeway (SWY)

Here's a slightly more involved example. The next page gives the Yahoo profiles for these companies, both of which are in the supermarket business. Some of the data from these profiles is in the spreadsheet below, which shows 5 multiples for these two firms.

	A	B	C	D	E	F
1	**SAFEWAY (SWY) AND KROGER (KR)--COMPARISON BASED ON MULTIPLES Based on Yahoo Profiles, 12 September 2002**					
2		**KR**	**SWY**		**Who's more highly valued?**	
3	Stock price	18.09	26.91	<-- Yahoo		
4	Earnings per share (EPS)	1.37	2.60	<-- Yahoo		
5	Price/Earnings (P/E) ratio	13.20	10.35	<-- =C3/C4	Kroger	<-- =IF(B5>C5,"Kroger","Safeway")
6						
7	Book value of equity per share	4.79	11.41	<-- Yahoo		
8	Equity market to book ratio	3.78	2.36	<-- =C3/C7	Kroger	<-- =IF(B8>C8,"Kroger","Safeway")
9						
10	Number of shares outstanding (million)	788.8	466.5	<-- Yahoo		
11	Market value of equity (billion)	14.27	12.55	<-- =C10*C3/1000		
12						
13	Debt/Equity (based on book values)	2.22	1.32	<-- Yahoo		
14	Debt (billion) this number is not in Yahoo	8.39	7.03	<-- =C10*C7*C13/1000		
15	Cash (billion)	0.185	0.051	<-- Yahoo		
16	Net debt	8.20	6.98	<-- =C14-C15		
17						
18	Book value of equity + debt (billion) - cash (book value of enterprise)	11.98	12.30	<-- =C10*C7/1000+C14-C15		
19	Market value of equity + debt (billion) - cash (market value of enterprise)	22.47	19.53	<-- =C11+C14-C15		
20	Enterprise value, market to book	1.88	1.59	<-- =C19/C18	Kroger	<-- =IF(B20>C20,"Kroger","Safeway")
21						
22	Earnings before interest, taxes, depreciation and amortization (EBITDA) in billion$	3.53	2.64	<-- Yahoo		
23	Market enterprise value to EBITDA	6.37	7.40	<-- =C19/C22	Safeway	<-- =IF(B23>C23,"Kroger","Safeway")
24						
25	Sales	50.7	34.7	<-- Yahoo		
26	Market enterprise value to Sales	0.44	0.56	<-- Yahoo	Safeway	<-- =IF(B26>C26,"Kroger","Safeway")

- **Price/Earnings ratio**: This is the most common multiple used. Based on this ratio of the stock price to the earnings per share (EPS), KR is more highly valued than SWY. The problem with using this multiple is that it is influenced by many factors, including the firm's leverage. We prefer *enterprise value* ratios such as ….
- **Equity market to book ratio**: This is the ratio of the market value of the firm's equity to the book value (its accounting value). If the book value accurately measures the cost of the assets, then a higher equity market to book reflects a greater valuation of the

equity. However, the accounting numbers are heavily influenced by the age of the assets, the depreciation and other accounting policies, so that this ratio is not so accurate.

- **Enterprise market to book ratio**: The *enterprise value* is the value of the firm's equity plus its net debt (defined as book value of debt minus cash). Row 18 above measures the firm's net debt by subtracting the cash balances from the book value of the debt. The enterprise market to book ratio shows that Kroger is valued more highly than Safeway.
- **Market enterprise value to EBITDA**: Earnings before interest, taxes, depreciation, and amortization (EBITDA) is a popular Wall Street measure of the ability of a firm to produce cash. In spirit it is similar to the free cash flow concept discussed in this chapter, though it ignores changes in net working capital and capital expenditures. The market enterprise value to EBITDA ratio shows that Safeway is actually more highly valued than Kroger.
- **Market enterprise value to Sales ratio**: This one of the many other ratios we could use to compare these two firms. As a percentage of its sales, Safeway is more highly valued than Kroger; this perhaps reflects Safeway's ability to extract more cash for its shareholders from each dollar of sales. Or perhaps it reflects greater shareholder optimism about the future sales growth rate.

Using multiples to value firms—summary

The multiple method of valuation is a highly effective way of comparing the values of several companies, *as long as the companies being compared are truly comparable*.

Comparability is complicated, however, and you should be careful: Truly comparable firms will have similar operational characteristics such as sales, costs, etc. and also similar financing.[4]

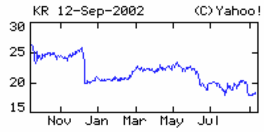
[4] We're getting ahead of ourselves, as we did in the previous footnote. The point is that it doesn't make sense to compare the stock price of two operationally similar firms if one is financed with a lot of debt and the other firm is financed primarily with equity. This point is a result of the discussion in Chapters 19-20. For more details see Chapter 10 of *Corporate Finance: A Valuation Approach* by Simon Benninga and Oded Sarig (McGraw-Hill 1997).

Statistics at a Glance -- NYSE:KR As of 12-Sep-2002

Price and Volume

52-Week Low on 29-Aug-2002	$17.45
Recent Price	$18.09
52-Week High on 18-Sep-2001	$27.33
Beta	0.23
Daily Volume (3-month avg)	3.90M
Daily Volume (10-day avg)	4.23M

Stock Performance



big chart [1d | 5d | 3m | 6m | 1y | 2y | 5y | max]

52-Week Change	-33.1%
52-Week Change relative to S&P500	-17.6%

Share-Related Items

Market Capitalization	$14.3B
Shares Outstanding	788.8M
Float	780.9M

Dividends & Splits

Annual Dividend	none
Last Split: factor 2 on 29-June-1999	

Per-Share Data

Book Value (mrq*)	$4.79
Earnings (ttm)	$1.37
Earnings (mrq)	$0.47
Sales (ttm)	$61.94
Cash (mrq*)	$0.23

Valuation Ratios

Price/Book (mrq*)	3.78
Price/Earnings (ttm)	13.18
Price/Sales (ttm)	0.29

Income Statements

Sales (ttm)	$50.7B
EBITDA (ttm*)	$3.53B
Income available to common (ttm)	$1.12B

Profitability

Profit Margin (ttm)	2.2%
Operating Margin (ttm)	4.8%

Fiscal Year

Fiscal Year Ends	Feb 2
Most recent quarter (fully updated)	25-May-2002
Most recent quarter (flash earnings)	31-May-2002

Management Effectiveness

Return on Assets (ttm)	5.92%
Return on Equity (ttm)	33.26%

Financial Strength

Current Ratio (mrq*)	0.94
Debt/Equity (mrq*)	2.22
Total Cash (mrq*)	$185.0M

Short Interest As of 8-Aug-2002

Shares Short	12.9M
Percent of Float	1.7%
Shares Short (Prior Month)	13.2M
Short Ratio	3.66
Daily Volume	3.54M

See Profile Help for a description of each item above; **K** = thousands; **M** = millions; **B** = billions; **mrq** = most-recent quarter; **ttm** = trailing twelve months;
(as of 31-May-2002, except **mrq*/ttm*** items as of 25-May-2002)

Statistics at a Glance -- NYSE:SWY As of 12-Sep-2002

Price and Volume

52-Week Low on 24-July-2002	$24.69
Recent Price	$26.91
52-Week High on 22-Mar-2002	$46.90
Beta	0.67
Daily Volume (3-month avg)	3.71M
Daily Volume (10-day avg)	3.15M

Stock Performance

big chart [1d | 5d | 3m | 6m | 1y | 2y | 5y | max]

52-Week Change	-37.5%
52-Week Change relative to S&P500	-23.0%

Share-Related Items

Market Capitalization	$12.6B
Shares Outstanding	466.5M
Float	456.2M

Dividends & Splits

Annual Dividend	none
Last Split: factor 2 on 26-Feb-1998	

Per-Share Data

Book Value (mrq*)	$11.41
Earnings (ttm)	$2.60
Earnings (mrq)	$0.63
Sales (ttm)	$69.16
Cash (mrq*)	$0.10

Valuation Ratios

Price/Book (mrq*)	2.36
Price/Earnings (ttm)	10.34
Price/Sales (ttm)	0.39

Income Statements

Sales (ttm)	$34.7B
EBITDA (ttm*)	$2.64B
Income available to common (ttm)	$1.30B

Profitability

Profit Margin (ttm)	3.8%
Operating Margin (ttm)	7.4%

Fiscal Year

Fiscal Year Ends	Dec 29
Most recent quarter (fully updated)	15-June-2002
Most recent quarter (flash earnings)	30-June-2002

Management Effectiveness

Return on Assets (ttm)	7.75%
Return on Equity (ttm)	22.49%

Financial Strength

Current Ratio (mrq*)	0.85
Debt/Equity (mrq*)	1.32
Total Cash (mrq*)	$50.5M

Short Interest As of 8-Aug-2002

Shares Short	6.66M
Percent of Float	1.5%
Shares Short (Prior Month)	7.26M
Short Ratio	1.93
Daily Volume	3.45M

See Profile Help for a description of each item above; **K** = thousands; **M** = millions; **B** = billions; **mrq** = most-recent quarter; **ttm** = trailing twelve months;
(as of 30-June-2002, except **mrq*/ttm*** items as of 15-June-2002)

Figure 18.4: Yahoo profiles for Kroger and Safeway. These profiles form the basis for the multiple valuation illustrated in section 18.4

Economics focus Taking the measure

Apart from "animal spirits", what figures excite stockmarket bulls?

AFTER shares worldwide hit their post-attack lows on September 21st, the Dow Jones Industrial Average has risen by close to 20%—in what some enthusiasts already call a new bull market. Given dismal forecasts of American growth, plunging consumer confidence and slashed estimates for corporate profits, can any of the tools that are used to measure the markets validate the bulls?

• **P/e ratios**. One common indicator the bulls seem to have forgotten, at least in America, is the price/earnings (p/e) ratio: the share price divided by earnings per share. Even when the S&P 500 index hit a three-year low just after the terrorist attacks, the average p/e ratio, at 28, was already high by historical standards; now it stands at 31. In Japan, the average p/e is around 62—which, hard to believe, is modest compared with the mid-1990s, when analysts attempted to justify p/es of over 100. In Europe, p/e ratios are now blushingly modest; they average around 16, more comfortably within historic ranges (see left-hand chart).

Adding to questions about high valuations in America is uncertainty over the "e" in the p/e ratio, the earnings that underpin share valuations. Earlier this month, Standard & Poor's, a ratings agency, complained that too many companies artificially boost their profits. A recent study by the Levy Institute estimates that operating profits for the S&P 500 have been inflated by at least 10% a year over the past two decades, thanks to a mix of one-time write-offs and other accounting tricks. Such sleights of hand mean that American shares may be even dearer than they look.

• **Yield ratios**. As soaring p/e ratios have become harder to justify in recent years, and questions about earnings have mounted, other indicators have come into fashion. One is the "earnings yield ratio", which compares returns on government bonds with an implicit earnings "yield" (in fact, the inverse of the p/e ratio) to shareholders. The theory behind this ratio, popularised by Alan Greenspan, the Fed chairman, some years ago, is that the earnings yield on shares has moved fairly closely in line with yields on government bonds, at least recently. In late September, plenty of analysts pointed to this rule of thumb as an argument that American shares were cheap.

As a relative measure, the earnings yield ratio has the virtue of comparing shares with a riskless alternative, but it is a long way from being an iron law. As Chris Johns of ABN Amro, an investment bank, points out, the relationship between bond yields and equity earnings yields is far less stable than it at first appears. In America, for most of the years since 1873, and even as recently as the 1970s, shares traded at far higher earnings yields—that is, lower p/e ratios—relative to government bonds than they do today (see the right-hand chart).

Earnings yield ratios have a problem. Traditionally, investors have looked to cash dividends as the ultimate source of share value: these are pocketable returns, after all. But as dividends have fallen out of fashion, investors have had to rely on earnings, flawed as they are, as a proxy. Shareholders face two big risks; first, that without a dividend stream they may never recoup their investment, and second, that the flaws in earnings make profits difficult to gauge. Given these, it seems a stretch to put too much faith in a fixed relationship with bond yields, much less the view that shares are fairly valued when these yields are equal.

• **Better ratios**. Some point to Tobin's Q—the ratio of a firm's market value to the replacement cost of its assets—as the best way to understand market values. This certainly has appeal, since it reflects the costs a competitor would face in re-creating a business. But replacement cost is hard to measure, and is of little help in explaining daily price movements. The next best thing, comparing market prices with the book value of assets, vastly underestimates the value of companies with intangibles such as patents and brands.

An alphabet soup of ratios is available to escape the flaws of measuring earnings: price-to-EBITDA (earnings before interest, tax, depreciation and amortisation) and price-to-cashflow, for example. These do a somewhat better job, since they measure profit in a way that, ideally, is more closely tied to a company's underlying performance. But on these measures, according to Peter Oppenheimer of HSBC, stockmarkets in America, Britain and France are still highly valued, though German shares are less so.

Of course, no single metric can unlock the secrets of share values. But the good measures are those that are useful in bear and bull markets alike. Discounted cash-flow valuation, for instance, is another metric that looks at the value of an entire firm according to the profits it expects in future. But it relies on a "risk premium"—the additional return investors require to compensate for the risks of holding shares—which is both the most important, and the most debated, figure in finance. Differing views about the risk premium can support almost any equity values. Recent weeks have shown that this slippery idea is central in the struggle between the bulls and the bears. .

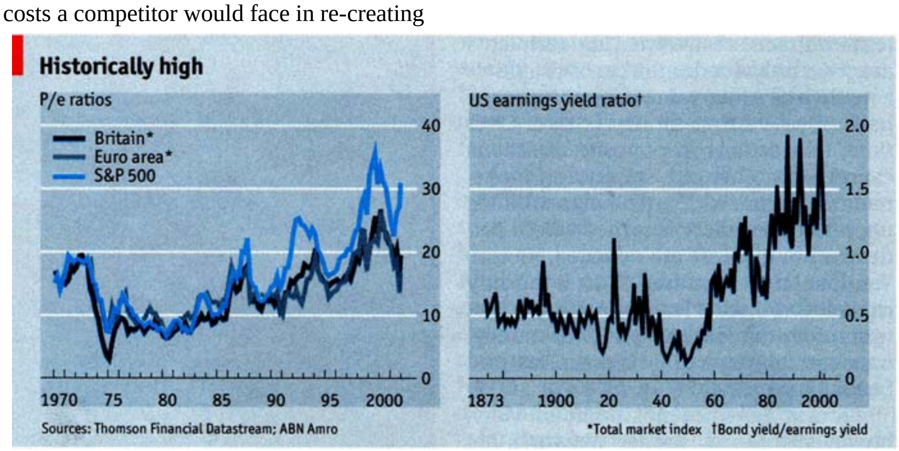


Figure 18.4: Article from the *Economist* on multiple valuation

18.5. Intermediate summary

In sections 18.1 – 18.4 we've examined 4 stock valuation methods:

- Valuation method 1, the efficient markets approach, is based on the assumption that market prices are correct.
- Valuation method 2, the free cash flow (FCF) approach, values the firm by discounting the future anticipated FCFs at the weighted average cost of capital (WACC). Sections 18.6 – 18.7 below show several methods of determining the WACC.
- Valuation method 3, the equity payout approach, values all of the firm's shares by discounting the future anticipated payouts to equity. The discount rate is the firm's cost of equity r_E.
- Valuation method 4, the multiples approach, gives a comparative valuation of firms based on ratios such as the price-earnings ratio.

In the next sections we discuss some issues related to valuation methods 2 and 3: We discuss the computation of the weighted average cost of capital (WACC) and the cost of equity r_E (sections 18.6 and 18.7).

18.6. Computing Target's WACC, the SML approach

Valuation method 2 depends on the weighted average cost of capital (WACC), which was previously discussed in Chapters 6 and 14. In this section we briefly repeat some of the things said in Chapter 14 and show how to compute the firm's WACC using the security market line (SML).

The basic WACC formula is:

$$WACC = \frac{E}{E+D} r_E + \frac{D}{D+E} r_D \left(1 - T_C\right)$$

To estimate the WACC we need to estimate the following parameters:

$r_E =$ the cost of equity
$r_D =$ the cost of the firm's debt
$E =$ market value of the firm's equity
$= number\ of\ shares * current\ market\ value\ per\ share$
$D =$ market value of the firm's debt
this is usually approximated by the *book value* of the firm's debt
$T_C =$ the firm's marginal tax rate

To illustrate the computation of the WACC, we use data for Target Corporation, a large discount retailer. Figure 18.5 gives the relevant financial information for Target. Using the Target data, we devote a short subsection to each of the WACC parameters, leaving the cost of equity r_E until last, since it is the most complicated.

	A	B	C	D	E
1	**TARGET CORPORATION**				
2	**Income statement**				
3		**2002**	**2001**		
4	Revenues	43,917	39,826		
5	Cost of sales	29,260	27,143		
6	Selling, general and administrative expenses	9,416	8,461		
7	Credit card expense	765	463		
8	Depreciation	1,212	1,079		
9	Interest expense	588	473		
10	Earnings before taxes	2,676	2,207		
11	Income taxes	1,022	839		
12	Net earnings	1,654	1,368		
13					
14	**Balance sheet**				
15	**Assets**	**2002**	**2001**		
16	Cash and cash equivalents	758	499		
17	Accounts receivable	5,565	3,831		
18	Inventory	4,760	4,449		
19	Other current assets	852	869		
20	Total current assets	11,935	9,648		
21					
22	Land, plant, property, and equipment				
23	At cost	20,936	18,442		
24	Accumulated depreciation	5,629	4,909		
25	Net land, plant, property and equipment	15,307	13,533		
26					
27	Other assets	1,361	973		
28	Total assets	28,603	24,154		
29					
30	**Liabilities and shareholder equity**				
31	Accounts payable	4,684	4,160		
32	Accrued liabilities	1,545	1,566		
33	income taxes payable	319	423		
34	Current portion of long-term debt and notes payable	975	905		
35	Total current liabilities	7,523	7,054		
36					
37	Long-term debt	10,186	8,088		
38	Deferred income taxes	1,451	1,152		
39	Shareholders equity				
40	Common stock	1,332	1,173		
41	Accumulated retained earnings	8,111	6,687		
42	Total equity	9,443	7,860		
43	Total liabilities and shareholder equity	28,603	24,154		
44					
45					
46	**Other relevant information**				
47	Shares outstanding	908,164,702			
48	Stock beta	1.16			
49	Stock price, 1 February 2003	28.21			
50					
51	**Dividends and stock repurchases**				
52	**Year**	**Dividends**	**Repurchases**	**Total equity payout**	
53	1998	165	0	165	
54	1999	178	0	178	
55	2000	190	585	775	
56	2001	203	20	223	
57	2002	218	14	232	
58					
59	Growth rate	7.21%		8.89%	<-- =(D57/D53)^(1/4)-1

Figure 18.5. Financial information for Target Corp. We use this information to determine Target's cost of equity rE and its weighted average cost of capital (WACC).

Computing the market value of Target's equity, *E*

Target has 908,164,702 shares outstanding (cell B47, Figure 18.5). On 1 February 2003, the day of the company's annual report for its 2002 financial year, the stock price of Target was \$28.21 per share. Thus the market value of the company's equity is 908,164,702*\$28.21= \$25,619,326,243. Note that in the spreadsheets all numbers appear in millions, so that Target's equity value appears as E = \$25,619.

Computing the market value of Target's debt, *D*

The Target balance sheets differentiate between short term debt ("Current portion of long-term debt and notes payable"—row 34 of Figure 18.5) and long-term debt (row 37). For purposes of computing the debt for a WACC computation, both of these numbers should be added together. This gives debt for Target as:

	A	B	C	D
6		**2002**	**2001**	
7	Current portion of long-term debt and notes payable	975	905	
8	Long-term debt in 2002 and 2001 (columns B and C)	7,523	7,054	
9	Total debt, D	8,498	7,959	<-- =C8+C7

Estimating the cost of debt r_D

A simple method to compute the cost of debt r_D is to calculate the *average interest cost* over the year. In 2002 Target paid \$588 interest (cell B9, Figure 18.5) on average debt of \$8,229. This gives

	A	B	C	D
11	Interest paid, 2002	588		
12	Average debt over 2002	8,229	<-- =AVERAGE(B9:C9)	
13	Interest cost, r_D	7.15%	<-- =B11/B12	

Target's income tax rate T_C

In 2002 Target paid taxes of $1,022 on earnings of $2,676 (cells B11 and B10 respectively of Figure 18.5). Its income tax rate was therefore 38.19%:

	A	B	C
17	Earnings before taxes, 2002	2,676	
18	Income taxes	1,022	
19	Corporate tax rate, T_C	38.19%	<-- =B18/B17

Computing Target's cost of equity r_E using the SML

The SML equation for computing Target's cost of equity r_E is given by:

$$r_E = r_f + \beta_E * \left[E(r_M) - r_f \right],$$

Yahoo gives Target's β as 1.16. In February 2003, the risk-free rate r_f was 2% and the expected return on the market $E(r_M)$ was 9.68%.[5] This gives Target's cost of equity as r_E = 10.91%:

	A	B	C	D
21	Equity beta, β_E	1.16		
22	Risk-free rate, r_f	2%		
23	Expected market return, $E(r_M)$	9.68%	<-- See discussion below	
24	Cost of equity, r_E	10.91%	<-- =B22+B21*(B23-B22)	

Putting it all together

Now that we've done all the calculations, we can compute Target's WACC:

$$\begin{aligned} WACC &= \frac{E}{E+D} r_E + \frac{D}{D+E} r_D (1-T_C) \\ &= \frac{25,619}{25,619+8,498} 10.91\% + \frac{8,498}{25,619+8,498} 7.15\% (1-38.19\%) \\ &= 9.29\% \end{aligned}$$

---

[5] To see how $E(r_M)$ was derived, see the boxed discussion on page000.

Here it is in a spreadsheet:

	A	B	C	D
1	**TARGET CORP.'S WACC USING SML FOR COST OF EQUITY**			
2	Number of shares (million)	908		
3	Market value per share, 1 February 2002	28.21		
4	Market value of equity 1 February 2002, E	25,619	<-- =B3*B2	
5				
6		**2002**	**2001**	
7	Current portion of long-term debt and notes payable	975	905	
8	Long-term debt in 2002 and 2001 (columns B and C)	7,523	7,054	
9	Total debt, D	8,498	7,959	<-- =C8+C7
10				
11	Market value of Target, E+D	34,117	<-- =B9+B4	
12				
13	Interest paid, 2002	588		
14	Average debt over 2002	8,229	<-- =AVERAGE(B9:C9)	
15	Interest cost, r_D	7.15%	<-- =B13/B14	
16				
17	Earnings before taxes, 2002	2,676		
18	Income taxes	1,022		
19	Corporate tax rate, T_C	38.19%	<-- =B18/B17	
20				
21	Equity beta, β_E	1.16		
22	Risk-free rate, r_f	2%		
23	Expected market return, $E(r_M)$	9.68%	<-- See discussion below	
24	Cost of equity, r_E	10.91%	<-- =B22+B21*(B23-B22)	
25				
26	WACC	9.29%	<-- =B4/B11*B24+(1-B19)*B9/B11*B15	

Computing the expected return on the market $E(r_M)$

The most controversial part of estimating the cost of capital using the CAPM is the estimation of the expected return on the market $E(r_M)$. We discussed this issue and some methods of estimation in Chapter 14. To recapitulate: We advocate using a P/E multiple model for estimating the equity premium. This model, presented in Chapter 14 and briefly reviewed in the box below, gives us $E(r_M)$ = 9.68%.

P/E Multiple Model for Estimating $E(r_M)$

We start with the payout form of the Gordon dividend model:

$$r_E = \underbrace{\frac{D_0(1+g)}{P_0} + g}_{\text{Gordon dividend model}} = \underbrace{\frac{b*EPS_0(1+g)}{P_0}}_{b \text{ is the dividend payout ratio, EPS}_0 \text{ is the current firm earnings per share}} + g$$

$$= \frac{b*(1+g)}{P_0 / EPS_0} + g$$

This model is now used to measure the $E(r_M)$, using current market data:

$$E(r_M) = \frac{b*(1+g)}{P_0 / EPS_0} + g$$

where

b=*market payout ratio* (in U.S. around 50%)

g=*growth rate of market earnings* (educated guess)

$P_0 / EPS_0 = \textit{market price-earnings ratio}$

Here's an Excel example:

	A	B	C
1	**ESTIMATING E(r_M) USING THE P/E RATIO**		
2	Market P/E ratio	20.00	
3	Market dividend payout ratio, *b*	50%	
4	Estimated growth of market earnings, *g*	7%	
5			
6	E(r_M)	9.68%	<-- =B3*(1+B4)/B2+B4
7	Risk-free rate, r_f	2.00%	
8	Market risk premium, E(r_M) - r_f	7.68%	<-- =B6-B7

We use these values—representative of market parameters in the U.S. in early 2003—in our determination of the Target Corp. cost of equity r_E .

	A	B	C	D	E	F
1	**ANNUALIZED REAL RETURNS ON EQUITIES, BONDS, AND BILLS, 1900-2000**					
2						
3		**Equities**	**Bonds**	**Bills**	**Equity premium**	
4	Australia	7.50%	1.10%	0.40%	7.10%	<-- =B4-D4
5	Belgium	2.50%	-0.40%	-0.30%	2.80%	<-- =B5-D5
6	Canada	6.40%	1.80%	1.70%	4.70%	
7	Denmark	4.60%	2.50%	2.80%	1.80%	
8	France	3.80%	-1.00%	-3.30%	7.10%	
9	Germany	3.60%	-2.20%	-0.60%	4.20%	
10	Ireland	4.80%	1.50%	1.30%	3.50%	
11	Italy	2.70%	-2.20%	-4.10%	6.80%	
12	Japan	4.50%	-1.60%	-2.00%	6.50%	
13	Netherland	5.80%	1.10%	0.70%	5.10%	
14	South Africe	6.80%	1.40%	0.80%	6.00%	
15	Spain	3.60%	1.20%	0.40%	3.20%	
16	Sweden	7.60%	2.40%	2.00%	5.60%	
17	Switzerland	5.00%	2.80%	1.10%	3.90%	
18	United Kingdom	5.80%	1.30%	1.00%	4.80%	
19	United States	6.70%	1.60%	0.90%	5.80%	
20	**Average**	**5.11%**	**0.71%**	**0.18%**	**4.93%**	
21						
22	*Source*: Elroy Dimson, Paul Marsh, Mike Staunton, *Triumph*					
23	*of the Optimists*, Princeton University Press 2002					
24						
25						
26						

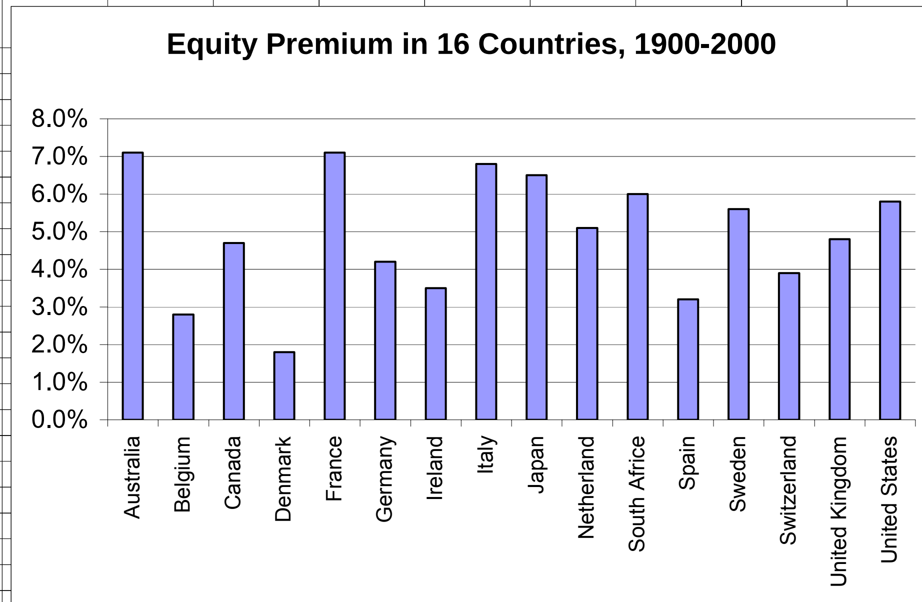


Figure 18.6. The equity premium in 16 major economies over the 20th century.

18.7. Computing Target's cost of equity r_E with the Gordon model

An alternative to the CAPM for computing the cost of equity r_E is the Gordon model, which we've previously discussed in Chapter 6. The Gordon model says that the equity value is the discounted value of future anticipated dividends. The standard version of the Gordon model is:

$$r_E = \frac{Div_0(1+g)}{P_0} + g$$

where

Div_0 = current equity payout of firm (total dividends + stock repurchases)

P_0 = current market value of equity

g = anticipated equity payout growth rate

For reasons explained in Chapter 6, we think the Gordon model should be used with the *total equity payout*, defined as total dividends plus stock repurchases. Below is the calculation for Target Corp.'s WACC using the Gordon model. The spreadsheet is the same as that of the previous section, except:

- Rows 32-36 show Target's equity payouts—the sum of its dividends and share repurchases—in each of the last five years. The compound annual growth rate of the equity payouts is 8.89% per year (cell D38).
- Rows 22-25 show the Gordon model calculation of the cost of equity r_E. This is computed as:

$$r_E = \frac{Div_0(1+g)}{P_0} + g = \frac{232*(1+8.89\%)}{25,619} + 8.89\% = 9.88\%$$

where

Div_0 = current equity payout

P_0 = current market value of equity

g = anticipated equity payout growth rate

	A	B	C	D	E
1	**TARGET CORP.'S WACC USING GORDON MODEL FOR COST OF EQUITY**				
2	Number of shares (million)	908			
3	Market value per share, 1 February 2002	28.21			
4	Market value of equity 1 February 2002, E	25,619	<-- =B3*B2		
5					
6		2002	2001		
7	Current portion of long-term debt and notes payable	975	905		
8	Long-term debt	7,523	7,054		
9	Total debt, D	8,498	7,959	<-- =C8+C7	
10					
11	Market value of Target, E+D	34,117	<-- =B9+B4		
12					
13	Interest paid, 2002	588			
14	Average debt over 2002	8,229	<-- =AVERAGE(B9:C9)		
15	Interest cost, r_D	7.15%	<-- =B13/B14		
16					
17		2002			
18	Earnings before taxes	2,676			
19	Income taxes	1,022			
20	Corporate tax rate, T_C	38.19%	<-- =B19/B18		
21					
22	Current equity value	25,619			
23	Current equity payout, Div_0	232	<-- =D36		
24	Growth rate of equity payout	8.89%	<-- =D38		
25	Cost of equity, r_E, using Gordon model	9.88%	<-- =B23*(1+B24)/B22+B24		
26					
27	WACC	8.52%	<-- =B4/B11*B25+(1-B20)*B9/B11*B15		
28					
29					
30	**Dividends and stock repurchases**				
31	**Year**	**Dividends**	**Repurchases**	**Total equity payout**	
32	1998	165	0	165	
33	1999	178	0	178	
34	2000	190	585	775	
35	2001	203	20	223	
36	2002	218	14	232	
37					
38			Growth rate	8.89%	<-- =(D36/D32)^(1/4)-1

Using the Gordon model estimate of the cost of equity, Target's WACC is 8.52% (cell B27).

Summing up

This chapter has discussed a grab-bag of share valuation methods. Three of these methods could be termed "fundamental valuations." Valuation Method 1, the simplest of the fundamental valuation methods is based on the assumption of market efficiency and says that a firm's stock is worth its current market price. Simple as it is, this approach has a lot of power and support in the academic community: If market participants have done their work, then the current price of a share reflects all publicly-available information, and there's nothing else to do.

Valuation method 2, *discounted cash flow* (DCF) valuation, is the method preferred by most finance academics and many finance practitioners. This method is based on discounting the firm's projected future free cash flows (FCF) at an appropriate weighted average cost of capital. The discounted value arrived at in this way is called the firm's *enterprise value*. To arrive at the valuation of the firm's equity, we add cash and marketable securities to the enterprise value and subtract the value of the firm's debt. Dividing by the number of shares gives the per-share valuation.

Valuation method 3, the *direct equity valuation,* discounts the projected payouts to equity holders (defined as the sum of dividends plus share repurchases) by the firm's cost of equity r_E. The resulting present value is the value of the firm's equity. Although it appears simpler and more direct than the FCF valuation, direct equity valuation is usually shunned by finance professionals. This is primarily because the cost of equity is heavily dependent on a firm's debt-equity financing mix, whereas the WACC is not nearly as dependent (and perhaps independent) of the debt-equity mix.

Valuation method 4, *multiple valuation* is widely used. This method of valuation arrives at a relative valuation of the firm by comparing a set of relevant multiples for comparable firms. When used correctly, multiple valuations can be a powerful tool, but it is often difficult to arrive at a correct "peer group" for a particular firm.

Exercises

1. Do a closed-end exercise based on ABC Holding Corp. Assume that ABC has some costs. Illustrate the closed-end fund discount.

   Thought question: Are you better off buying ABC or the proportions of its subsidiaries?

2. Go back to ABC Holdings:

$$\begin{matrix} ABC\ share \\ price \end{matrix} = \frac{60\% * \begin{bmatrix} XYZ\ share & * & number\ of \\ price & & XYZ\ shares \end{bmatrix} + 50\% * \begin{bmatrix} QRM\ share & * & number\ of \\ price & & QRM\ shares \end{bmatrix}}{\begin{matrix} number\ of \\ ABC\ shares \end{matrix}}$$

Suppose you know the share price of ABC and the share price of QRM. What should be the market price of XYZ?

CHAPTER 19: VALUING STOCKS*

This version: February 13, 2004

Chapter contents

Overview

In Chapter 17 we discussed the valuation of bonds. This chapter deals with the valuation of stocks. Whereas the valuation of bonds is a relatively straightforward matter of computing yields to maturity, the valuation of stocks is much more difficult. The difficulty lies both in the greater uncertainty about the cash flows which need to be discounted in order to arrive at a stock valuation and in the computation of the correct discount rate.

In this chapter we discuss four basic approaches to stock valuation:

- **Valuation method 1, the efficient markets approach**. In its simplest form the *efficient markets* approach states that the current stock price is correct. A somewhat more sophisticated use of the efficient markets approach to stock valuation is that a stock's value is the sum of the values of its components. We explore the implications of these statements in Section 19.1.
- **Valuation method 2, discounting the future free cash flows (FCF)**. Sometimes called the *discounted cash flow* (DCF) approach to valuation, this method values the firm's debt and its equity together as the present value of the firm's future FCFs. The discount rate used is the weighted average cost of capital (WACC). This method is the valuation approach favored by most finance academics. We discuss this approach in Section 19.2 and discuss the computation of the WACC in Sections 19.5 and 19.6. In this chapter we do not discuss the concept or the computation of the free cash flow—this was done previously in chapters 6-7.
- **Valuation method 3, discounting the future equity payouts**. A firm's shares can also be valued by *discounting the stream of anticipated equity payouts* at an

appropriate cost of equity r_E. The concept of equity payout (the sum of a firm's total dividends plus its stock repurchases) was previously discussed in Chapter 5.

- **Valuation method 4, multiples**. Finally we can value a firm's shares by a *comparative valuation based on multiples*. This very common method involves ratios such as the price-earnings (P/E) ratio, EBITDA multiples, and more industry specific multiples such as value per square foot of store space or value per subscriber.

With the exception of the multiple method 4, almost all of the material in this chapter is also discussed elsewhere in this book. For example, the efficient markets approach to valuation is also discussed in chapter 13, and the Gordon dividend model (which values a firm's equity by discounting its anticipated dividend stream) is also discussed in chapters 5 and 7. WACC computations are to be found in Chapters 5 and 13. The purpose of this chapter is to bring together these dispersed materials into a (hopefully coherent) whole.

Finance concepts discussed in this chapter

- Discounted cash flows, free cash flows (FCF)
- Cost of capital, cost of equity, cost of debt, weighted average cost of capital (WACC)
- Equity premium
- Beta, equity beta, asset beta
- Two-stage growth models

Excel functions used

- **Sum**, **NPV, If**
- Data table

19.1. Valuation method 1: The current market price of a stock is the correct price (the efficient markets approach)

The simplest stock valuation is based on the efficient markets approach (chapter 13). This approach says that the *current market price of a stock is the correct price*. In other words: The market has already done the difficult job stock valuation, and it's done this correctly, incorporating all of the relevant information There's a lot of evidence for this approach, as you saw in chapter 13.

This valuation method is very simple to apply:

- *Question*: "IBM looks a bit expensive to me—it's price has been going up for the last 3 months. What do you think: Is IBM's stock price currently underpriced or overpriced?"
- *Answer*: "At Podunk U., we learned that markets with a lot of trading are in general efficient, meaning that the current market price incorporates all the readily-available information about IBM. So—I don't think IBM is either underpriced or overpriced. It's actually correctly priced."

Here's another example of the use of this approach:

- *Question*: "I've been thinking of buying IBM, but I've have been putting it off. The price has gone up lately, and I'm going to wait until it comes down a bit. It seems a bit high to me right now." What do you think?
- *Answer*: "At Podunk U. we would call you a *contrarian* . You believe that if the price of a stock has gone up, it will go back down (and the opposite). But this technical approach (see chapter 13) to stock valuation doesn't seem to work very well. So if you want to buy

IBM, go ahead and do so now. There's nothing in the price runup of the last couple of months which indicates that there will now be a price rundown."

Some more sophisticated efficient markets methods

Efficient markets valuations don't always have to be as simplistic as the above examples. In chapter 13 we looked at *additivity*, a fundamental tenet of efficient markets. The principle of additivity says that the value of a basket of goods or financial assets should equal the sum of the values of the components. Additivity can often be used to value stocks.

Here's a very simple example: ABC Holding Corp., a publicly-traded company, owns shares in two publicly traded companies. Besides owning these subsidiaries, ABC does little else.

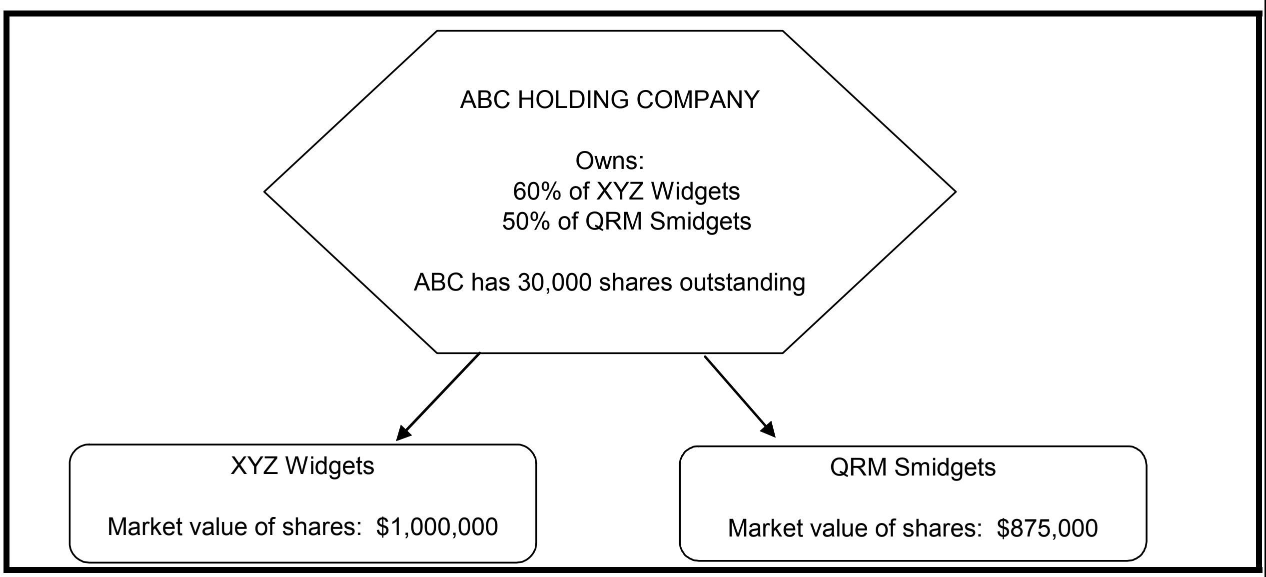


Figure 19.1. Ownership structure of ABC Holding Company

What should be the value of a share of ABC Holding? The obvious way to do this is in the following spreadsheet, which computes the share value of ABC to be $34.58:

	A	B	C	D	E
1	**ABC HOLDING COMPANY**				
2	Number of ABC shares	30,000			
3					
4	**ABC owns shares in**	**Percentage of shares owned by ABC**	**Market value**	**Market value of ABC holdings in company**	
5	XYZ Widgets	60%	1,000,000	600,000	<-- =B5*C5
6	QRM Smidgets	50%	875,000	437,500	<-- =B6*C6
7	Total value of ABC holdings			1,037,500	<-- =D6+D5
8					
9	Per share value of ABC Holdings			34.58	<-- =D7/B2

Notice what this model *is* and *is not* telling you:

- *Is* telling you: If the market values of XYZ and QRM are correct, then the market value of ABC should be $34.58. The formula works out to be:

$$\begin{matrix} ABC\ share \\ price \end{matrix} = \frac{60\% * [XYZ\ \ value] + 50\% * [QRM\ \ value]}{\begin{matrix} number\ of \\ ABC\ shares \end{matrix}}$$

- *Is not* telling you: The formula tells you a relation between the 3 share prices. It tells you if the share prices are *relatively correct*, but it does not tell you if they are *absolutely correct*. Example: After doing much work and research and applying the methods of the previous Section, you come to the conclusion that, while the market valuation of QRM is correct, the market value of XYZ ought to $1,600,000. Then you would conclude that the share price of ABC ought to be $46.58.

	A	B	C	D	E
1	**ABC HOLDING COMPANY**				
2	Number of ABC shares	30,000			
3					
4	**ABC owns shares in**	**Percentage of shares owned by ABC**	**Market value**	**Market value of ABC holdings in company**	
5	XYZ Widgets	60%	1,600,000	960,000	<-- =B5*C5
6	QRM Smidgets	50%	875,000	437,500	<-- =B6*C6
7	Total value of ABC holdings			1,397,500	<-- =D6+D5
8					
9	Per share value of ABC Holdings			46.58	<-- =D7/B2

Note that if ABC has some of its own overheads and if it doesn't always pass through all the dividends of its subsidiaries, its market price will be *lower* than $34.58, since the market price of ABC will reflect not only the cost of the shares of its subsidiaries, but also its own overheads. This looks a lot like the *closed-end fund* valuation problem discussed in chapter 13.

19.2. Valuation method 2: The price of a share is the discounted value of the future anticipated free cash flows

Valuation method 1 of the previous section says that there is nothing to be gained by second-guessing market valuations. In many cases, however, the finance expert (you!) will want to do a basic valuation of a company and derive the value of a share from the discounted value of the future anticipated free cash flows (FCF). This method, often called the *discounted cash flow* (DCF) method of valuation, was discussed and illustrated in chapter 7. Figure 19.2 reminds you of the definition of FCF and figure 19.3 gives a flow diagram of the FCF valuation method.

Defining the Free Cash Flow	
Profit after taxes	This is the basic measure of the profitability of the business, but it is an accounting measure that includes financing flows (such as interest), as well as non-cash expenses such as depreciation. Profit after taxes does not account for either changes in the firm's working capital or purchases of new fixed assets, both of which can be important cash drains on the firm.
+ Depreciation	This noncash expense is added back to the profit after tax.
+ after-tax interest payments (net)	FCF is an attempt to measure the cash produced by the business activity of the firm. To neutralize the effect of interest payments on the firm's profits, we: • Add back the after-tax cost of interest on debt (*after-tax* since interest payments are tax-deductible), • Subtract out the after-tax interest payments on cash and marketable securities.
- Increase in current assets	When the firm's sales increase, more investment is needed in inventories, accounts receivable, etc. This increase in current assets is not an expense for tax purposes (and is therefore ignored in the profit after taxes), but it is a cash drain on the company.
+ Increase in current liabilities	An increase in the sales often causes an increase in financing related to sales (such as accounts payable or taxes payable). This increase in current liabilities—when related to sales—provides cash to the firm. Since it is directly related to sales, we include this cash in the free cash flow calculations.
- Increase in fixed assets at cost	An increase in fixed assets (the long-term productive assets of the company) is a use of cash, which reduces the firm's free cash flow.
FCF = sum of the above	

Figure 19.2. Defining the free cash flow. We have previously discussed FCFs and their use in valuation in Chapters 5 - 7.

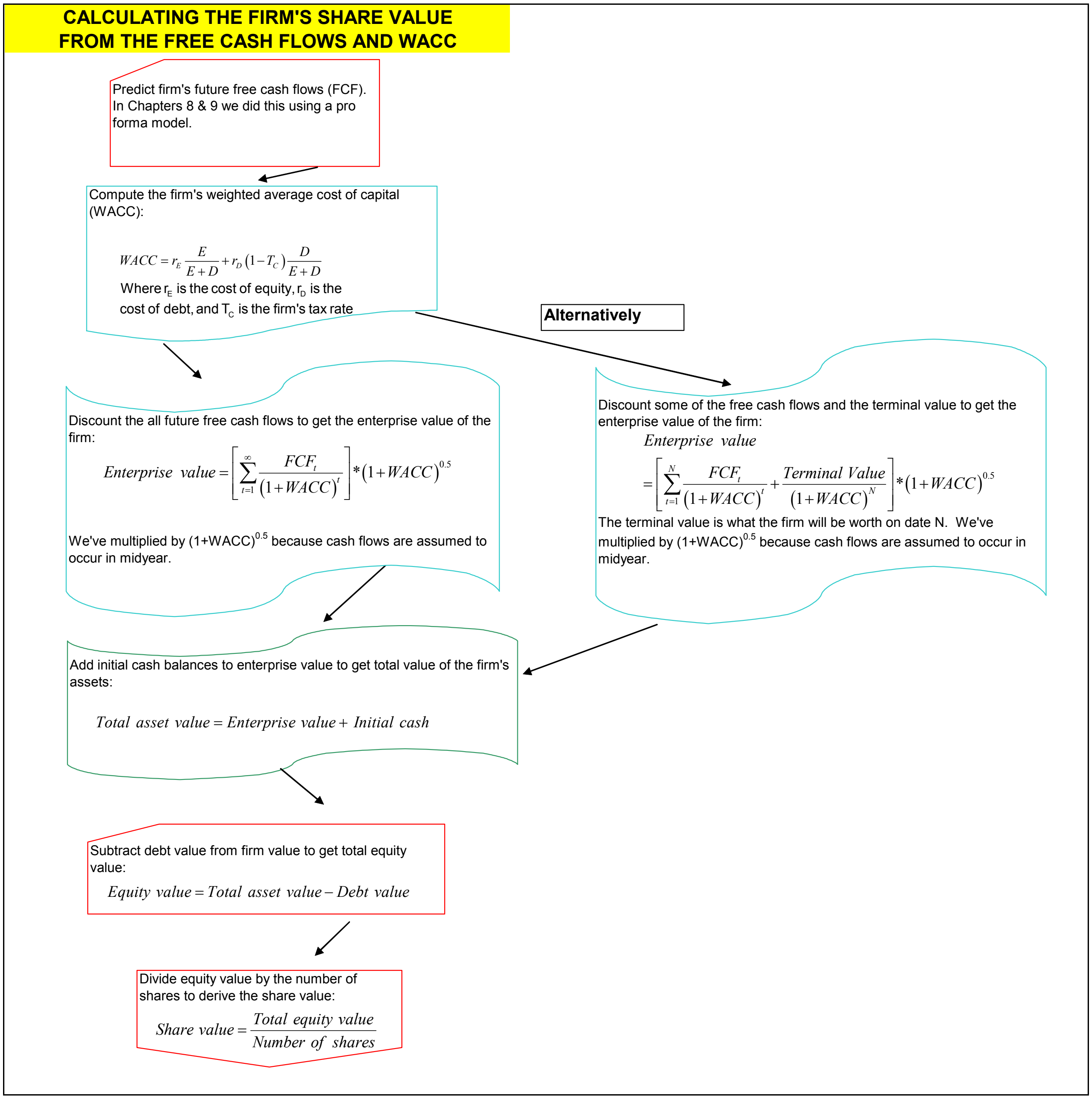


Figure 19.3: Flow diagram for a FCF valuation

Valuation 2: Example 1—a basic example

It is 31 December 2003 and you are trying to value Arnold Corp, which finished 2003 with a free cash flow of $2 million. The company has debt of $10 million and cash balances of $1 million You estimate the following financial parameters for the company:

- The future anticipated growth rate of the FCF is 8%
- The WACC of Arnold is 15%

You can now estimate the value of Arnold:

- The *enterprise value* of Arnold is the present value of future anticipated FCFs discounted at the WACC:

$$Enterprise\ value = \underbrace{\left[\sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t}\right]}_{\substack{\uparrow \\ \text{This is the PV} \\ \text{formula, assuming that} \\ \text{FCFs occur at year-end}}} * \underbrace{(1+WACC)^{0.5}}_{\substack{\uparrow \\ \text{This factor "corrects"} \\ \text{for the fact that FCFs occur} \\ \text{throughout the year.}}}$$

$$= \underbrace{\left[\sum_{t=1}^{\infty} \frac{FCF_{2003}(1+g)^t}{(1+WACC)^t}\right]}_{\substack{\uparrow \\ \text{Future FCFs are expected} \\ \text{to grow at rate } g.}} * (1+WACC)^{0.5}$$

$$= \underbrace{\left[\frac{FCF_{2003}(1+g)}{WACC-g}\right]}_{\substack{\uparrow \\ \text{This formula was} \\ \text{given in Chapter ???}}} * (1+WACC)^{0.5}$$

Doing the computations in an Excel spreadsheet shows that the enterprise value of Arnold Corp. is $33,090,599 and that the estimated per-share value is $24.09:

	A	B	C
1	**VALUING ARNOLD CORP**		
2	2003 FCF (base year)	2,000,000	
3	Future FCF growth rate	8%	
4	WACC	15%	
5	End-2003 debt	10,000,000	
6	End-2003 cash	1,000,000	
7	Number of shares outstanding	1,000,000	
8			
9	Enterprise value	33,090,599	<-- =B2*(1+B3)/(B4-B3)*(1+B4)^0.5
10	Add cash	1,000,000	<-- =B6
11	Subtract debt	-10,000,000	<-- =-B5
12	Value of equity	24,090,599	<-- =SUM(B9:B11)
13	Share value	24.09	<-- =B12/B7

Valuation method 2: Example 2—two FCF growth rates

In the valuation of Arnold Corp. in the previous subsection we assumed a FCF growth rate which is unchanging over the future. This assumption is often suitable for a mature, stable company, but it may not be appropriate for a company that is currently experiencing very high growth rates. In this subsection we show how to perform a FCF valuation of a company for which we assume *two* FCF growth rates—a high FCF growth rate for a number of years followed by a subsequent lower FCF growth rate.

Xanthum Corp. has just finished its 2003 financial year. The company's 2003 FCF was $1,000,000. Xanthum has been growing very fast; you anticipate that for the coming 5 years the FCF growth rate will be 35%. After this time, you anticipate that the FCF growth will slow to 10% per year, because the market for Xanthum's products will become mature.

Xanthum has 3,000,000 shares outstanding and a WACC of 20%. It currently has $500,000 of cash on hand which is not needed for operations; Xanthum also has $3,000,000 of debt. To value the company, we apply the same valuation scheme as before, but this time we use the two FCF growth rates:

$$Enterprise\ value = \left[\underbrace{\sum_{t=1}^{5} \frac{FCF_t}{(1+WACC)^t}}_{\substack{\uparrow \\ \text{The PV of the "high} \\ \text{growth" FCFs}}} + \underbrace{\sum_{t=6}^{\infty} \frac{FCF_t}{(1+WACC)^t}}_{\substack{\uparrow \\ \text{The PV of the "normal} \\ \text{growth" FCFS}}} \right] * \underbrace{(1+WACC)^{0.5}}_{\substack{\uparrow \\ \text{This factor "corrects"} \\ \text{for the fact that FCFs occur} \\ \text{throughout the year.}}}$$

There's a valuation formula which can be derived using techniques described in the appendix to Chapter 1:

$$Enterprise\ value =$$

$$\left[\underbrace{\frac{FCF_{2003}(1+g_{high})}{1+WACC} \left(\frac{1-\left(\frac{1+g_{high}}{1+WACC}\right)^5}{1-\frac{1+g_{high}}{1+WACC}} \right)}_{\substack{\uparrow \\ \text{In the spreadsheet this is called} \\ \text{"term 1" and } \frac{1+g_{high}}{1+WACC} \text{ is called "term1 factor"}}} + \underbrace{\frac{FCF_{2003}(1+g_{high})^5}{(1+WACC)^5} \left(\frac{1+g_{normal}}{WACC-g_{normal}} \right)}_{\substack{\uparrow \\ \text{In the spreadsheet this is called} \\ \text{"term 2"}}} \right] * (1+WACC)^{0.5}$$

The spreadsheet below shows that Xanthum's enterprise value is \$27,040,649 (cell B15) and that its per-share value is \$8.18 (cell B21):

	A	B	C
1	**VALUING XANTHUM CORP**		
2	2003 FCF (base year)	1,000,000	
3			
4	High growth rate, g_{high}	35%	
5	Normal growth rate, g_{normal}	10%	
6	Number of high growth years	5	
7	Term 1 factor: $(1+g_{high})/(1+WACC)$	113%	<-- =(1+B4)/(1+B9)
8			
9	WACC	20%	
10	End-2003 debt	3,000,000	
11	End-2003 cash	500,000	
12			
13	Term 1: PV of high-growth cash flows	7,218,292	<-- =B2*B7*(1-B7^B6)/(1-B7)
14	Term 2: PV of normal-growth cash flows	19,822,357	<-- =B2*(1+B4)^B6*(1+B5)/(B9-B5)/(1+B9)^B6
15	Enterprise value	27,040,649	<-- =SUM(B13:B14)
16	Add cash	27,540,649	<-- =B15+B11
17	Subtract debt	-3,000,000	<-- =-B10
18	Value of equity	24,540,649	<-- =SUM(B16:B17)
19			
20	Number of shares, end 2003	3,000,000	
21	Share value	8.18	<-- =B18/B20

Valuation method 2: Example 3—using the terminal value in a real-estate project

In the previous two examples we discounted an infinitely-lived stream of cash flows. Sometimes it makes more sense to discount a finite number of cash flows and then attribute a terminal value to the project.

Here's an example: Your Aunt Sarah has quite a bit of money. She's been offered a share in a partnership which is being set up by a local real estate agent. The partnership will buy an existing building, called the Station Building, for \$20 million. The agent is selling 25 shares, for \$800,000 each ($\$800,000 = \frac{\$20,000,000}{25}$). Aunt Sarah has asked you to do some financial analysis to determine whether this is a fair price for a partnership share in the Station Building.

Here's what you discover:

- All income from the Station Building partnership will flow through to the shareholders, who will pay taxes on the income at their personal tax rates. Aunt Sarah's tax rate is 40%.
- Station Building will be depreciated over 40 years, giving an annual depreciation of $500,000 per year.
- The building is fully rented out and brings up annual rents of $7 million. You do not anticipate that these rents will increase over the next 10 years.
- Maintenance, property taxes, and other miscellaneous expenses for Station Building cost about $1 million per year.
- The agent who is putting together the partnership has proposed selling Station Building after 10 years. He estimates that the market price of the building will not change much over this period—meaning that the market price of Station Building in year 10 is anticipated to be $20 million, like its price today.

In your valuation of the Station Building shares, you see that the annual free cash flow (FCF) to Aunt Sarah is $152,000 (cell B16 in the spreadsheet below). This FCF will be available to her in years 1-10, and is based on the building's profit before taxes of $5,500,000, which will be spread equally among the partners.

The terminal value of the building is $20,000,000, which on a per-share basis is $800,000 (cell B19). At the time the building is sold in year 10, its accumulated depreciation is $5,000,000, so that its book value is $15,000,000. To compute Aunt Sarah's cash flow from this terminal value, we deduct the per-share book value of the building ($600,000, cell B20) from the sale price to arrive at taxes of $80,000 on the profit from the sale of the building (cell B22). The cash flow from the sale is the $800,000 sale price minus the taxes--$720,000 as shown in cell B23.

	A	B	C	D	E	F	G
1	**STATION BUILDING PARTNERSHIP--SHARE VALUATION**						
2	Building cost	20,000,000					
3	Depreciable life (years)	40					
4	Annual rents	7,000,000					
5	Annual expenses	1,000,000					
6	Annual depreciation	500,000	<-- =B2/B3		**Profit and loss, Station Building as a whole**		
7	Aunt Sarah's tax rate	40%			Annual rent	7,000,000	
8	WACC	18%			Minus annual expenses	-1,000,000	
9	Shares issued	25			Minus annual depreciation	-500,000	
10	Share price	800,000			Anticipated annual building profit before taxes	5,500,000	<-- =SUM(F7:F9)
11							
12	**Profit and loss, Aunt Sarah's share**						
13	Anticipated annual building profit before taxes	220,000	<-- =F10/B9		**Terminal value, year 10, Station Building as a whole**		
14	Profit after taxes	132,000	<-- =(1-B7)*B13		Anticipated building market price	20,000,000	<-- =B2
15	Building depreciation, per share	20,000	<-- =B6/B9		Accumulated depreciation, year 10	5,000,000	<-- =B6*10
16	Free cash flow	152,000	<-- =B14+B15		Book value of building, year 10	15,000,000	<-- =B2-F15
17							
18	**Terminal value, year 10, Aunt Sarah's share**						
19	Anticipated building market price	800,000	<-- =F14/B9				
20	Book value in year 10, per share	600,000	<-- =F16/B9				
21	Profit from sale of building	200,000	<-- =B19-B20				
22	Tax on profit	80,000	<-- =B7*B21				
23	Terminal value: cash flow from sale	720,000	<-- =B19-B22				
24							
25	**Year**	**Aunt Sarah's anticipated FCF**					
26	1	152,000	<-- =$B$16				
27	2	152,000					
28	3	152,000					
29	4	152,000					
30	5	152,000					
31	6	152,000					
32	7	152,000					
33	8	152,000					
34	9	152,000					
35	10	872,000	<-- =$B$16+B23				
36							
37	Share value: Present value of Aunt Sarah's free cash flows	$820,667.53	<-- =NPV(B8,B26:B35)				

Cells B26:B35 show Aunt Sarah's anticipated free cash flows from the building partnership, including the terminal value. Discounting these cash flows at the WACC of 20% values a partnership share at $820,667.53. Conclusion: Aunt Sarah should invest in the building!

Valuation method 2: Example 4—using the terminal value to get around large FCF growth rates

Our second example of using the terminal value involves the Formanis Corporation. Formanis is in a growth industry and has had formidable FCF growth rates for the past several years, and you anticipate that these rates will continue for years 1-5. However, after year 5 you anticipate a big slowdown in Formanis's FCF growth, as its industry matures.

Here are the relevant facts about Formanis:

- The company's FCF for the current year is $1,000,000.
- You anticipate that the FCF for years 1-5 will grow at a rate of 25% per year.
- You anticipate a growth rate of FCFs of 6% per year for years 6, 7, … (termed the "long-term growth rate" in the spreadsheet below).
- The company has 5 million shares outstanding.

The valuation formula is:

$$Formanis\ value = \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \frac{FCF_4}{(1+WACC)^4} + \frac{FCF_5}{(1+WACC)^5} + \frac{1}{(1+WACC)^5} * \underbrace{\frac{FCF_5 * (1+long\text{-}term\ growth\ rate)}{(WACC - long\text{-}term\ growth\ rate)}}_{\substack{\uparrow \\ \text{This is the terminal value:} \\ \text{an explanation is given in Chapter ??}}}$$

To value Formanis, we first predict the FCFs for years 1-5 (cells B9:B13 of the spreadsheet). The present value of these FCFs is $6,465,787 (cell B20). The terminal value represents the year-5 present value of the Formanis cash flows for years 6, 7, … . To compute the terminal value, we assume that Formanis's cash flows for these years grow at the long-term growth rate:

$$\begin{aligned} Terminal\ value &= year\text{-}5\ PV\ of\ Formanis\ FCFs,\ years\ 6,7,... \\ &= \frac{FCF_6}{(1+WACC)} + \frac{FCF_7}{(1+WACC)^2} + \frac{FCF_7}{(1+WACC)^2} + \ldots \\ &= \frac{FCF_5 * (1+long\text{-}term.\ growth\ rate)}{(1+WACC)} + \frac{FCF_5 * (1+long\text{-}term.\ growth\ rate)^2}{(1+WACC)^2} \\ &\quad + \frac{FCF_5 * (1+long\text{-}term.\ growth\ rate)^3}{(1+WACC)^2} + \ldots \\ &= \frac{FCF_5 * (1+long\text{-}term\ growth\ rate)}{(WACC - long\text{-}term\ growth\ rate)} \end{aligned}$$

In cell B17 below the terminal value—assuming a long-term FCF growth rate of 6%—is $17,025,596.

	A	B	C
1	**FORMANIS CORPORATION**		
2	Current FCF	1,000,000	
3	Anticipated growth rate, years 1-5	25%	
4	WACC	15%	
5	Long-term growth rate, after year 5	6%	
6	Number of shares outstanding	5,000,000	
7			
8	**Year**	**Anticipated FCF**	
9	1	1,250,000	<-- =$B$2*(1+$B$3)
10	2	1,562,500	<-- =B9*(1+$B$3)
11	3	1,953,125	<-- =B10*(1+$B$3)
12	4	2,441,406	
13	5	3,051,758	
14			
15	**Terminal value calculation**		
16	FCF in year 5	3,051,758	<-- =B13
17	Terminal value	17,025,596	<-- =B16*(1+B5)/(B3-B5)
18			
19	**Valuing Formanis Corporation**		
20	Present value of FCFs, years 1-5	6,465,787	<-- =NPV(B4,B9:B13)
21	Present value of terminal value	8,464,730	<-- =B17/(1+B4)^5
22	Value of Formanis	14,930,518	<-- =B21+B20
23	Per share value	$2.99	<-- =B22/B6

The value of Formanis (cell B22) is $14,930,518. The per-share value of Formanis is $2.99 (cell B23).

The terminal value method illustrated for Formanis is often used:

- It allows the stock analyst to distinguish between short-term growth and long-term growth. Often short-term growth is a function of market performance, whereas long-term growth is determined by macro-economic factors. For example in a new and rapidly developing market, we might anticipate high short-term growth rates. But we would also anticipate that as the market matures and becomes more saturated, the long-term growth rates would approximate the growth of the economy as a whole.

- From an Excel point of view, the terminal value method allows us to do interesting sensitivity analyses. For example, here is the per-share value of Formanis for a variety of long-term growth rates and WACCs; we use the **Data Table** technique described in Chapter ???:

	A	B	C	D	E	F
26	**Sensitivity analysis: Per share value of Formanis with different WACC and long-term growth. Year 1-5 growth rate = 25%**					
27			Long-term growth rate ↓			
28	=B23	$2.99	**0%**	**2%**	**4%**	**6%**
29	WACC →	**15%**	2.51	2.64	2.80	2.99
30		**20%**	2.11	2.22	2.35	2.50
31		**25%**	1.80	1.89	1.99	2.12
32		**30%**	1.55	1.62	1.70	1.81

Varying the year 1-5 growth rate gives different values. In the table below, for example, we've assumed that year 1-5 growth is 20%:

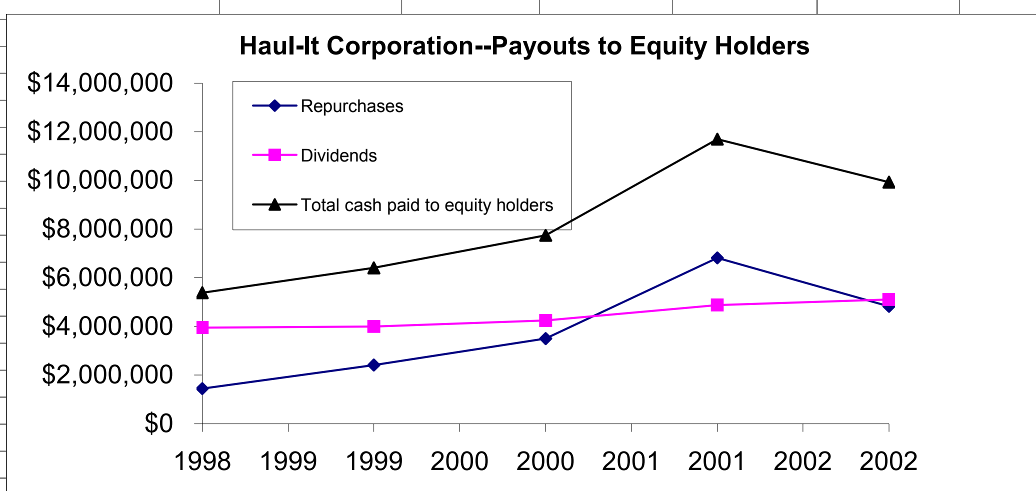
	A	B	C	D	E	F
26	**Sensitivity analysis: Per share value of Formanis with different WACC and long-term growth. Year 1-5 growth rate = 20%**					
27			Long-term growth rate ↓			
28	=B23	$3.01	**0%**	**2%**	**4%**	**6%**
29	WACC →	**15%**	2.38	2.54	2.75	3.01
30		**20%**	2.00	2.13	2.30	2.51
31		**25%**	1.70	1.81	1.95	2.12
32		**30%**	1.46	1.55	1.66	1.81

19.3. Valuation method 3: The price of a share is the present value of its future anticipated equity cash flows discounted at the cost of equity

In the previous section we "backed into" the equity valuation of the firm, by first calculating the value of the firm's assets (the enterprise value plus initial cash balances), and then subtracting from this number the value of the firm's debts. In this section we present another method for calculating the value of the firm's equity—we directly discount the value of the firm's anticipated payouts to its shareholders.

As an example consider Haul-It Corp., which has a steady record of paying dividends and repurchasing shares. The company has 10 million shares outstanding. Here's a spreadsheet with the valuation model:

	A	B	C	D	E	F	G
1	**HAUL-IT CORPORATION--EQUITY PAYOUT HISTORY AND SHARE VALUATION**						
2		**1998**	**1999**	**2000**	**2001**	**2002**	
3	Repurchases	$1,440,000	$2,410,000	$3,500,000	$6,820,000	$4,830,000	
4	Dividends	$3,950,000	$3,997,000	$4,238,000	$4,875,000	$5,100,000	
5	Total cash paid to equity holders	$5,390,000	$6,407,000	$7,738,000	$11,695,000	$9,930,000	
6							
7	Compound annual growth, 1998-2002	16.50%	<-- =(F5/B5)^(1/4)-1				
8							
9	Haul-It's cost of equity, r_E	25.00%					
10							
11	**Valuation**						
12	Current equity payout	$9,930,000	<-- =F5				
13	Anticipated future growth	16.50%					
14							
15	Value of total equity	136,164,862	<-- =B12*(1+B13)/(B9-B13)				
16	Number of shares outstanding	10,000,000					
17	Value per share	13.62	<-- =B15/B16				



Between 1998 and 2002, Haul-It's payouts to its equity holders have increased at an impressive rate of 16.50% per year (cell B7). The company's cost of equity r_E is 25% (cell B9).[1] Assuming that future equity payout growth equals historical growth, Haul-It is valued at \$136 million (cell B15), which gives a per-share value of \$13.62.

The equity value of the company is the discounted value of the future anticipated equity payouts:

$$\begin{aligned}
Equity\ value &= \frac{Equity\ payout_{2003}}{1+r_E}+\frac{Equity\ payout_{2004}}{\left(1+r_E\right)^2}+\frac{Equity\ payout_{2004}}{\left(1+r_E\right)^3}+\ldots. \\
&= \frac{Equity\ payout_{2002}\left(1+g\right)}{1+r_E}+\frac{Equity\ payout_{2002}\left(1+g\right)^2}{\left(1+r_E\right)^2}+\frac{Equity\ payout_{2002}\left(1+g\right)^3}{\left(1+r_E\right)^3}+\ldots. \\
&= \frac{Equity\ payout_{2002}\left(1+g\right)}{r_E-g}=\frac{9,930,000(1.165)}{25.00\%-16.50\%}=136,164,862
\end{aligned}$$

Dividing the equity value by the number of shares outstanding gives the estimated value per share:

$$Value\ per\ share=\frac{Equity\ value}{Shares\ outstanding}=\frac{136,164,862}{10,000,000}=13.62$$

Why do finance professionals shun direct equity valuation?

Valuation method 3, the direct valuation of equity is so simple that it may surprise you that it is rarely used. There are several reasons for this, none of which we can fully explain at this point in the book:

- The direct equity valuation method depends on projected equity payouts (that is, dividends plus share repurchases), whereas Method 2 depends on projected free cash

[1] At this point we do not discuss how we arrived at this cost of equity. For a recapitulation of cost of capital techniques, see Sections 19.??? – 19.???.

flows. Whereas a firm's equity payouts are a function of management decisions about dividends and stock repurchases, FCFs are a function of the firm's operating environment—its sales, costs, capital expenditures, and so on. Because many components of the FCFs are determined by the firm's operating environment rather than management decisions about dividends, analysts are generally more comfortable predicting FCFs.

- The FCF method 2 discounts future FCFs at the firm's weighted average cost of capital (WACC). The equity payout method 3 discounts future equity payouts at the firm's cost of equity r_E. For reasons we will explain in Chapters 19 - 20, the cost of equity r_E is very sensitive to the firm's debt-equity ratio, whereas the WACC is not as sensitive to the debt-equity ratio.[2]

19.4. Valuation method 4, comparative valuation: Using multiples to value shares

The last valuation technique we discuss is based on a comparison of financial ratios for different companies. This valuation technique is often referred to as using "multiples." The technique is based on the logic that financial assets which are similar in nature should be priced the same way.

[2] For reasons explained in Chapter 19, the WACC may in fact be completely invariant to a firm's leverage. If this is so, we can value a firm based on method 2 without worrying about its leverage.

A simple example: Using the price/earnings (P/E) ratio for valuation

The *price/earnings ratio* is the ratio of a firm's stock price to its earnings per share:

$$P/E = \frac{stock\ \ price}{earnings\ \ per\ \ share}.$$

When we use the P/E for valuation, we assume that similar firms should have similar P/E ratios.

Here's an example: Shoes for Less (SFL) and Lesser Shoes (LS) are both shoe stores located in similar communities. Although SFL is bigger than LS, having double the sales and double the profits, the companies are in most relevant respects similar—management, financial structure, etc. However, the market valuation of the two companies does not reflect their similarity: The P/E ratio of SFL is significantly lower than that of LS, as can be seen in the spreadsheet below:

	A	B	C	D
1	**SHOES FOR LESS (SFL) AND LESSER SHOES (LS) comparing P/E ratios**			
2		**SFL: Shoes for Less**	**LS: Lesser Shoes**	
3	**Sales**	30,000	15,000	
4	**Profits**	3,000	1,500	
5	**Number of shares**	1,000	1,000	
6	**Shareprice**	24	18	
7	**Equity value**	24,000	18,000	<-- =C6*C5
8	**EPS: Earnings per share**	3	1.5	<-- =C4/C5
9	**P/E: Price-Earnings ratio**	8.00	12.00	<-- =C6/C8

Based on the similarity between the two companies, SFL appears underpriced relative to LS—its P/E ratio is less. A market analyst might recommend that anyone interest in investing in the shoe store business invest in SFL rather than LS.[3]

[3] A more radical strategy might be to *buy* shares of SFL and to *short* shares of LS. See Chapter 10 and its discussion of Palm and 3Com shares for a discussion of this strategy.

Kroger (KR) and Safeway (SWY)

Here's a slightly more involved example. The next page gives the Yahoo profiles for these companies, both of which are in the supermarket business. Some of the data from these profiles is in the spreadsheet below, which shows 5 multiples for these two firms.

	A	B	C	D	E	F
1	**SAFEWAY (SWY) AND KROGER (KR)--COMPARISON BASED ON MULTIPLES Based on Yahoo Profiles, 12 September 2002**					
2		**KR**	**SWY**		**Who's more highly valued?**	
3	Stock price	18.09	26.91	<-- Yahoo		
4	Earnings per share (EPS)	1.37	2.60	<-- Yahoo		
5	Price/Earnings (P/E) ratio	13.20	10.35	<-- =C3/C4	Kroger	<-- =IF(B5>C5,"Kroger","Safeway")
6						
7	Book value of equity per share	4.79	11.41	<-- Yahoo		
8	Equity market to book ratio	3.78	2.36	<-- =C3/C7	Kroger	<-- =IF(B8>C8,"Kroger","Safeway")
9						
10	Number of shares outstanding (million)	788.8	466.5	<-- Yahoo		
11	Market value of equity (billion)	14.27	12.55	<-- =C10*C3/1000		
12						
13	Debt/Equity (based on book values)	2.22	1.32	<-- Yahoo		
14	Debt (billion) this number is not in Yahoo	8.39	7.03	<-- =C10*C7*C13/1000		
15	Cash (billion)	0.185	0.051	<-- Yahoo		
16	Net debt	8.20	6.98	<-- =C14-C15		
17						
18	Book value of equity + debt (billion) - cash (book value of enterprise)	11.98	12.30	<-- =C10*C7/1000+C14-C15		
19	Market value of equity + debt (billion) - cash (market value of enterprise)	22.47	19.53	<-- =C11+C14-C15		
20	Enterprise value, market to book	1.88	1.59	<-- =C19/C18	Kroger	<-- =IF(B20>C20,"Kroger","Safeway")
21						
22	Earnings before interest, taxes, depreciation and amortization (EBITDA) in billion$	3.53	2.64	<-- Yahoo		
23	Market enterprise value to EBITDA	6.37	7.40	<-- =C19/C22	Safeway	<-- =IF(B23>C23,"Kroger","Safeway")
24						
25	Sales	50.7	34.7	<-- Yahoo		
26	Market enterprise value to Sales	0.44	0.56	<-- Yahoo	Safeway	<-- =IF(B26>C26,"Kroger","Safeway")

- **Price/Earnings ratio**: This is the most common multiple used. Based on this ratio of the stock price to the earnings per share (EPS), KR is more highly valued than SWY. The problem with using this multiple is that it is influenced by many factors, including the firm's leverage. We prefer *enterprise value* ratios such as ….
- **Equity market to book ratio**: This is the ratio of the market value of the firm's equity to the book value (its accounting value). If the book value accurately measures the cost of the assets, then a higher equity market to book reflects a greater valuation of the

equity. However, the accounting numbers are heavily influenced by the age of the assets, the depreciation and other accounting policies, so that this ratio is not so accurate.

- **Enterprise market to book ratio**: The *enterprise value* is the value of the firm's equity plus its net debt (defined as book value of debt minus cash). Row 18 above measures the firm's net debt by subtracting the cash balances from the book value of the debt. The enterprise market to book ratio shows that Kroger is valued more highly than Safeway.
- **Market enterprise value to EBITDA**: Earnings before interest, taxes, depreciation, and amortization (EBITDA) is a popular Wall Street measure of the ability of a firm to produce cash. In spirit it is similar to the free cash flow concept discussed in this chapter, though it ignores changes in net working capital and capital expenditures. The market enterprise value to EBITDA ratio shows that Safeway is actually more highly valued than Kroger.
- **Market enterprise value to Sales ratio**: This one of the many other ratios we could use to compare these two firms. As a percentage of its sales, Safeway is more highly valued than Kroger; this perhaps reflects Safeway's ability to extract more cash for its shareholders from each dollar of sales. Or perhaps it reflects greater shareholder optimism about the future sales growth rate.

Using multiples to value firms—summary

The multiple method of valuation is a highly effective way of comparing the values of several companies, *as long as the companies being compared are truly comparable*.

Comparability is complicated, however, and you should be careful: Truly comparable firms will have similar operational characteristics such as sales, costs, etc. and also similar financing.[4]

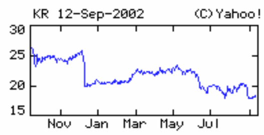
[4] We're getting ahead of ourselves, as we did in the previous footnote. The point is that it doesn't make sense to compare the stock price of two operationally similar firms if one is financed with a lot of debt and the other firm is financed primarily with equity. This point is a result of the discussion in Chapters 18-19. For more details see Chapter 10 of *Corporate Finance: A Valuation Approach* by Simon Benninga and Oded Sarig (McGraw-Hill 1997).

Statistics at a Glance -- NYSE:KR As of 12-Sep-2002

Price and Volume

52-Week Low on 29-Aug-2002	$17.45
Recent Price	$18.09
52-Week High on 18-Sep-2001	$27.33
Beta	0.23
Daily Volume (3-month avg)	3.90M
Daily Volume (10-day avg)	4.23M

Stock Performance



big chart [1d | 5d | 3m | 6m | 1y | 2y | 5y | max]

52-Week Change	-33.1%
52-Week Change relative to S&P500	-17.6%

Share-Related Items

Market Capitalization	$14.3B
Shares Outstanding	788.8M
Float	780.9M

Dividends & Splits

Annual Dividend	none
Last Split: factor 2 on 29-June-1999	

Per-Share Data

Book Value (mrq*)	$4.79
Earnings (ttm)	$1.37
Earnings (mrq)	$0.47
Sales (ttm)	$61.94
Cash (mrq*)	$0.23

Valuation Ratios

Price/Book (mrq*)	3.78
Price/Earnings (ttm)	13.18
Price/Sales (ttm)	0.29

Income Statements

Sales (ttm)	$50.7B
EBITDA (ttm*)	$3.53B
Income available to common (ttm)	$1.12B

Profitability

Profit Margin (ttm)	2.2%
Operating Margin (ttm)	4.8%

Fiscal Year

Fiscal Year Ends	Feb 2
Most recent quarter (fully updated)	25-May-2002
Most recent quarter (flash earnings)	31-May-2002

Management Effectiveness

Return on Assets (ttm)	5.92%
Return on Equity (ttm)	33.26%

Financial Strength

Current Ratio (mrq*)	0.94
Debt/Equity (mrq*)	2.22
Total Cash (mrq*)	$185.0M

Short Interest
As of 8-Aug-2002

Shares Short	12.9M
Percent of Float	1.7%
Shares Short (Prior Month)	13.2M
Short Ratio	3.66
Daily Volume	3.54M

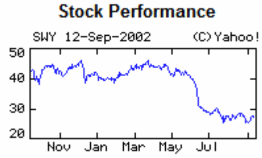
See Profile Help for a description of each item above; **K** = thousands; **M** = millions; **B** = billions; **mrq** = most-recent quarter; **ttm** = trailing twelve months;
(as of 31-May-2002, except **mrq*/ttm*** items as of 25-May-2002)

Statistics at a Glance -- NYSE:SWY As of 12-Sep-2002

Price and Volume

52-Week Low on 24-July-2002	$24.69
Recent Price	$26.91
52-Week High on 22-Mar-2002	$46.90
Beta	0.67
Daily Volume (3-month avg)	3.71M
Daily Volume (10-day avg)	3.15M

Stock Performance



big chart [1d | 5d | 3m | 6m | 1y | 2y | 5y | max]

52-Week Change	-37.5%
52-Week Change relative to S&P500	-23.0%

Share-Related Items

Market Capitalization	$12.6B
Shares Outstanding	466.5M
Float	456.2M

Dividends & Splits

Annual Dividend	none
Last Split: factor 2 on 26-Feb-1998	

Per-Share Data

Book Value (mrq*)	$11.41
Earnings (ttm)	$2.60
Earnings (mrq)	$0.63
Sales (ttm)	$69.16
Cash (mrq*)	$0.10

Valuation Ratios

Price/Book (mrq*)	2.36
Price/Earnings (ttm)	10.34
Price/Sales (ttm)	0.39

Income Statements

Sales (ttm)	$34.7B
EBITDA (ttm*)	$2.64B
Income available to common (ttm)	$1.30B

Profitability

Profit Margin (ttm)	3.8%
Operating Margin (ttm)	7.4%

Fiscal Year

Fiscal Year Ends	Dec 29
Most recent quarter (fully updated)	15-June-2002
Most recent quarter (flash earnings)	30-June-2002

Management Effectiveness

Return on Assets (ttm)	7.75%
Return on Equity (ttm)	22.49%

Financial Strength

Current Ratio (mrq*)	0.85
Debt/Equity (mrq*)	1.32
Total Cash (mrq*)	$50.5M

Short Interest
As of 8-Aug-2002

Shares Short	6.66M
Percent of Float	1.5%
Shares Short (Prior Month)	7.26M
Short Ratio	1.93
Daily Volume	3.45M

See Profile Help for a description of each item above; **K** = thousands; **M** = millions; **B** = billions; **mrq** = most-recent quarter; **ttm** = trailing twelve months;
(as of 30-June-2002, except **mrq*/ttm*** items as of 15-June-2002)

Figure 19.4: Yahoo profiles for Kroger and Safeway. These profiles form the basis for the multiple valuation illustrated in Section 19.4

Economics focus Taking the measure

Apart from "animal spirits", what figures excite stockmarket bulls?

AFTER shares worldwide hit their post-attack lows on September 21st, the Dow Jones Industrial Average has risen by close to 20%—in what some enthusiasts already call a new bull market. Given dismal forecasts of American growth, plunging consumer confidence and slashed estimates for corporate profits, can any of the tools that are used to measure the markets validate the bulls?

• **P/e ratios**. One common indicator the bulls seem to have forgotten, at least in America, is the price/earnings (p/e) ratio: the share price divided by earnings per share. Even when the S&P 500 index hit a three-year low just after the terrorist attacks, the average p/e ratio, at 28, was already high by historical standards; now it stands at 31. In Japan, the average p/e is around 62—which, hard to believe, is modest compared with the mid-1990s, when analysts attempted to justify p/es of over 100. In Europe, p/e ratios are now blushingly modest; they average around 16, more comfortably within historic ranges (see left-hand chart).

Adding to questions about high valuations in America is uncertainty over the "e" in the p/e ratio, the earnings that underpin share valuations. Earlier this month, Standard & Poor's, a ratings agency, complained that too many companies artificially boost their profits. A recent study by the Levy Institute estimates that operating profits for the S&P 500 have been inflated by at least 10% a year over the past two decades, thanks to a mix of one-time write-offs and other accounting tricks. Such sleights of hand mean that American shares may be even dearer than they look.

• **Yield ratios**. As soaring p/e ratios have become harder to justify in recent years, and questions about earnings have mounted, other indicators have come into fashion. One is the "earnings yield ratio", which compares returns on government bonds with an implicit earnings "yield" (in fact, the inverse of the p/e ratio) to shareholders. The theory behind this ratio, popularised by Alan Greenspan, the Fed chairman, some years ago, is that the earnings yield on shares has moved fairly closely in line with yields on government bonds, at least recently. In late September, plenty of analysts pointed to this rule of thumb as an argument that American shares were cheap.

As a relative measure, the earnings yield ratio has the virtue of comparing shares with a riskless alternative, but it is a long way from being an iron law. As Chris Johns of ABN Amro, an investment bank, points out, the relationship between bond yields and equity earnings yields is far less stable than it at first appears. In America, for most of the years since 1873, and even as recently as the 1970s, shares traded at far higher earnings yields—that is, lower p/e ratios—relative to government bonds than they do today (see the right-hand chart).

Earnings yield ratios have a problem. Traditionally, investors have looked to cash dividends as the ultimate source of share value: these are pocketable returns, after all. But as dividends have fallen out of fashion, investors have had to rely on earnings, flawed as they are, as a proxy. Shareholders face two big risks; first, that without a dividend stream they may never recoup their investment, and second, that the flaws in earnings make profits difficult to gauge. Given these, it seems a stretch to put too much faith in a fixed relationship with bond yields, much less the view that shares are fairly valued when these yields are equal.

• **Better ratios**. Some point to Tobin's Q—the ratio of a firm's market value to the replacement cost of its assets—as the best way to understand market values. This certainly has appeal, since it reflects the costs a competitor would face in re-creating a business. But replacement cost is hard to measure, and is of little help in explaining daily price movements. The next best thing, comparing market prices with the book value of assets, vastly underestimates the value of companies with intangibles such as patents and brands.

An alphabet soup of ratios is available to escape the flaws of measuring earnings: price-to-EBITDA (earnings before interest, tax, depreciation and amortisation) and price-to-cashflow, for example. These do a somewhat better job, since they measure profit in a way that, ideally, is more closely tied to a company's underlying performance. But on these measures, according to Peter Oppenheimer of HSBC, stockmarkets in America, Britain and France are still highly valued, though German shares are less so.

Of course, no single metric can unlock the secrets of share values. But the good measures are those that are useful in bear and bull markets alike. Discounted cash-flow valuation, for instance, is another metric that looks at the value of an entire firm according to the profits it expects in future. But it relies on a "risk premium"—the additional return investors require to compensate for the risks of holding shares—which is both the most important, and the most debated, figure in finance. Differing views about the risk premium can support almost any equity values. Recent weeks have shown that this slippery idea is central in the struggle between the bulls and the bears. .

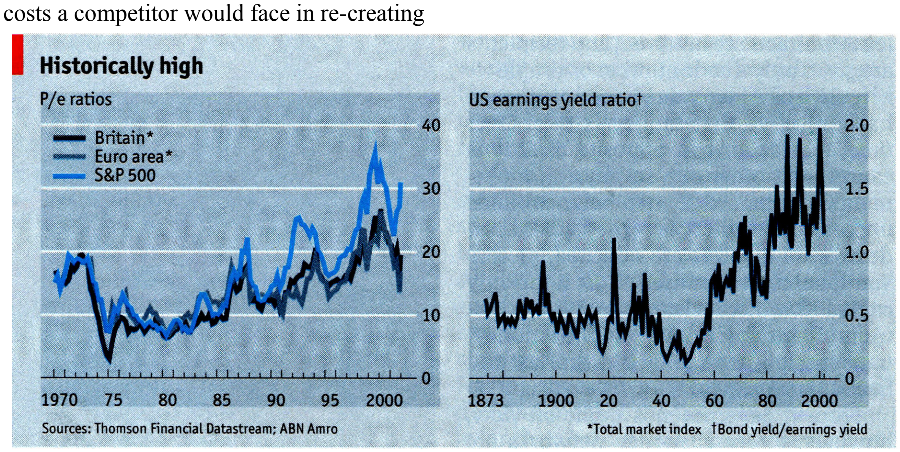


Figure 19.4: Article from the *Economist* on multiple valuation

19.5. Intermediate summary

In Sections 19.1 – 19.4 we've examined 4 stock valuation methods:

- Valuation method 1, the efficient markets approach, is based on the assumption that market prices are correct.
- Valuation method 2, the free cash flow (FCF) approach, values the firm by discounting the future anticipated FCFs at the weighted average cost of capital (WACC). Sections 19.6 – 19.7 below show several methods of determining the WACC.
- Valuation method 3, the equity payout approach, values all of the firm's shares by discounting the future anticipated payouts to equity. The discount rate is the firm's cost of equity r_E.
- Valuation method 4, the multiples approach, gives a comparative valuation of firms based on ratios such as the price-earnings ratio.

In the next sections we discuss some issues related to valuation methods 2 and 3: We discuss the computation of the weighted average cost of capital (WACC) and the cost of equity r_E (Sections 19.6 and 19.7).

19.6. Computing Target's WACC, the SML approach

Valuation method 2 depends on the weighted average cost of capital (WACC), which was previously discussed in Chapters 6 and 14. In this section we briefly repeat some of the things said in Chapter 14 and show how to compute the firm's WACC using the security market line (SML).

The basic WACC formula is:

$$WACC = \frac{E}{E+D} r_E + \frac{D}{D+E} r_D \left(1 - T_C\right)$$

To estimate the WACC we need to estimate the following parameters:

$$\begin{aligned}
r_E &= \text{ the cost of equity} \\
r_D &= \text{ the cost of the firm's debt} \\
E &= \text{market value of the firm's equity} \\
&= \textit{number of shares} * \textit{current market value per share} \\
D &= \text{market value of the firm's debt} \\
&\quad \text{this is usually approximated by the } \textit{book value} \text{ of the firm's debt} \\
T_C &= \text{ the firm's marginal tax rate}
\end{aligned}$$

To illustrate the computation of the WACC, we use data for Target Corporation, a large discount retailer. Figure 19.5 gives the relevant financial information for Target. Using the Target data, we devote a short subsection to each of the WACC parameters, leaving the cost of equity r_E until last, since it is the most complicated.

	A	B	C	D	E
1	**TARGET CORPORATION**				
2	**Income statement**				
3		**2002**	**2001**		
4	Revenues	43,917	39,826		
5	Cost of sales	29,260	27,143		
6	Selling, general and administrative expenses	9,416	8,461		
7	Credit card expense	765	463		
8	Depreciation	1,212	1,079		
9	Interest expense	588	473		
10	Earnings before taxes	2,676	2,207		
11	Income taxes	1,022	839		
12	Net earnings	1,654	1,368		
13					
14	**Balance sheet**				
15	**Assets**	**2002**	**2001**		
16	Cash and cash equivalents	758	499		
17	Accounts receivable	5,565	3,831		
18	Inventory	4,760	4,449		
19	Other current assets	852	869		
20	Total current assets	11,935	9,648		
21					
22	Land, plant, property, and equipment				
23	At cost	20,936	18,442		
24	Accumulated depreciation	5,629	4,909		
25	Net land, plant, property and equipment	15,307	13,533		
26					
27	Other assets	1,361	973		
28	Total assets	28,603	24,154		
29					
30	**Liabilities and shareholder equity**				
31	Accounts payable	4,684	4,160		
32	Accrued liabilities	1,545	1,566		
33	income taxes payable	319	423		
34	Current portion of long-term debt and notes payable	975	905		
35	Total current liabilities	7,523	7,054		
36					
37	Long-term debt	10,186	8,088		
38	Deferred income taxes	1,451	1,152		
39	Shareholders equity				
40	Common stock	1,332	1,173		
41	Accumulated retained earnings	8,111	6,687		
42	Total equity	9,443	7,860		
43	Total liabilities and shareholder equity	28,603	24,154		
44					
45					
46	**Other relevant information**				
47	Shares outstanding	908,164,702			
48	Stock beta	1.16			
49	Stock price, 1 February 2003	28.21			
50					
51	**Dividends and stock repurchases**				
52	**Year**	**Dividends**	**Repurchases**	**Total equity payout**	
53	1998	165	0	165	
54	1999	178	0	178	
55	2000	190	585	775	
56	2001	203	20	223	
57	2002	218	14	232	
58					
59	Growth rate	7.21%		8.89%	<-- =(D57/D53)^(1/4)-1

Figure 19.5. Financial information for Target Corp. We use this information to determine Target's cost of equity rE and its weighted average cost of capital (WACC).

Computing the market value of Target's equity, *E*

Target has 908,164,702 shares outstanding (cell B47, Figure 19.5). On 1 February 2003, the day of the company's annual report for its 2002 financial year, the stock price of Target was $28.21 per share. Thus the market value of the company's equity is 908,164,702*$28.21= $25,619,326,243. Note that in the spreadsheets all numbers appear in millions, so that Target's equity value appears as E = $25,619.

Computing the market value of Target's debt, *D*

The Target balance sheets differentiate between short term debt ("Current portion of long-term debt and notes payable"—row 34 of Figure 19.5) and long-term debt (row 37). For purposes of computing the debt for a WACC computation, both of these numbers should be added together. This gives debt for Target as:

	A	B	C	D
6		**2002**	**2001**	
7	Current portion of long-term debt and notes payable	975	905	
8	Long-term debt in 2002 and 2001 (columns B and C)	10,186	8,088	
9	Total debt, D	11,161	8,993	<-- =C8+C7

Estimating the cost of debt r_D

A simple method to compute the cost of debt r_D is to calculate the *average interest cost* over the year. In 2002 Target paid $588 interest (cell B9, Figure 19.5) on average debt of $10,077. This gives r_D = 5.84%:

	A	B	C	D
13	Interest paid, 2002	588		
14	Average debt over 2002	10,077	<-- =AVERAGE(B9:C9)	
15	Interest cost, r_D	5.84%	<-- =B13/B14	

Target's income tax rate T_C

In 2002 Target paid taxes of $1,022 on earnings of $2,676 (cells B11 and B10 respectively of Figure 19.5). Its income tax rate was therefore 38.19%:

	A	B	C
17	Earnings before taxes, 2002	2,676	
18	Income taxes	1,022	
19	Corporate tax rate, T_C	38.19%	<-- =B18/B17

Computing Target's cost of equity r_E using the SML

The SML equation for computing Target's cost of equity r_E is given by:

$$r_E = r_f + \beta_E * \left[E(r_M) - r_f \right],$$

Yahoo gives Target's β as 1.16. In February 2003, the risk-free rate r_f was 2% and the expected return on the market $E(r_M)$ was 9.68%.[5] This gives Target's cost of equity as r_E = 10.91%:

	A	B	C	D
21	Equity beta, β_E	1.16		
22	Risk-free rate, r_f	2%		
23	Expected market return, $E(r_M)$	9.68%	<-- See discussion below	
24	Cost of equity, r_E	10.91%	<-- =B22+B21*(B23-B22)	

Putting it all together

Now that we've done all the calculations, we can compute Target's WACC:

$$\begin{aligned} WACC &= \frac{E}{E+D} r_E + \frac{D}{D+E} r_D \left(1 - T_C\right) \\ &= \frac{25,619}{25,619 + 11,161} 10.91\% + \frac{11,161}{25,619 + 11,161} 5.84\% \left(1 - 38.19\%\right) \\ &= 8.69\% \end{aligned}$$

[5] To see how $E(r_M)$ was derived, see the boxed discussion on page000.

Here it is in a spreadsheet:

	A	B	C	D
1	**TARGET CORP.'S WACC USING SML FOR COST OF EQUITY**			
2	Number of shares (million)	908		
3	Market value per share, 1 February 2002	28.21		
4	Market value of equity 1 February 2002, E	25,619	<-- =B3*B2	
5				
6		**2002**	**2001**	
7	Current portion of long-term debt and notes payable	975	905	
8	Long-term debt in 2002 and 2001 (columns B and C)	10,186	8,088	
9	Total debt, D	11,161	8,993	<-- =C8+C7
10				
11	Market value of Target, E+D	36,780	<-- =B9+B4	
12				
13	Interest paid, 2002	588		
14	Average debt over 2002	10,077	<-- =AVERAGE(B9:C9)	
15	Interest cost, r_D	5.84%	<-- =B13/B14	
16				
17	Earnings before taxes, 2002	2,676		
18	Income taxes	1,022		
19	Corporate tax rate, T_C	38.19%	<-- =B18/B17	
20				
21	Equity beta, β_E	1.16		
22	Risk-free rate, r_f	2%		
23	Expected market return, $E(r_M)$	9.68%	<-- See discussion below	
24	Cost of equity, r_E	10.91%	<-- =B22+B21*(B23-B22)	
25				
26	WACC	8.69%	<-- =B4/B11*B24+(1-B19)*B9/B11*B15	

Computing the expected return on the market $E(r_M)$

The most controversial part of estimating the cost of capital using the CAPM is the estimation of the expected return on the market $E(r_M)$. We discussed this issue and some methods of estimation in Chapter 14. To recapitulate: We advocate using a P/E multiple model for estimating the equity premium. This model, presented in Chapter 14 and briefly reviewed in the box below, gives us $E(r_M) = 9.68\%$.

P/E Multiple Model for Estimating $E(r_M)$

We start with the payout form of the Gordon dividend model:

$$r_E = \underbrace{\frac{D_0(1+g)}{P_0} + g}_{\text{Gordon dividend model}} = \underbrace{\frac{b*EPS_0(1+g)}{P_0} + g}_{b \text{ is the dividend payout ratio, } EPS_0 \text{ is the current firm earnings per share}}$$

$$= \frac{b*(1+g)}{P_0 / EPS_0} + g$$

This model is now used to measure the $E(r_M)$, using current market data:

$$E(r_M) = \frac{b*(1+g)}{P_0 / EPS_0} + g$$

where

b=*market payout ratio* (in U.S. around 50%)

g=*growth rate of market earnings* (educated guess)

$P_0 / EPS_0 = market\ price\text{-}earnings\ ratio$

Here's an Excel example:

	A	B	C
1	**ESTIMATING E(r_M) USING THE P/E RATIO**		
2	Market P/E ratio	20.00	
3	Market dividend payout ratio, *b*	50%	
4	Estimated growth of market earnings, *g*	7%	
5			
6	E(r_M)	9.68%	<-- =B3*(1+B4)/B2+B4
7	Risk-free rate, r_f	2.00%	
8	Market risk premium, E(r_M) - r_f	7.68%	<-- =B6-B7

We use these values—representative of market parameters in the U.S. in early 2003—in our determination of the Target Corp. cost of equity r_E .

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	**ANNUALIZED REAL RETURNS ON EQUITIES, BONDS, AND BILLS, 1900-2000**														
2															
3		**Equities**	**Bonds**	**Bills**	**Equity premium**										
4	Australia	7.50%	1.10%	0.40%	7.10%	<-- =B4-D4									
5	Belgium	2.50%	-0.40%	-0.30%	2.80%	<-- =B5-D5									
6	Canada	6.40%	1.80%	1.70%	4.70%										
7	Denmark	4.60%	2.50%	2.80%	1.80%										
8	France	3.80%	-1.00%	-3.30%	7.10%										
9	Germany	3.60%	-2.20%	-0.60%	4.20%										
10	Ireland	4.80%	1.50%	1.30%	3.50%										
11	Italy	2.70%	-2.20%	-4.10%	6.80%										
12	Japan	4.50%	-1.60%	-2.00%	6.50%										
13	Netherland	5.80%	1.10%	0.70%	5.10%										
14	South Africe	6.80%	1.40%	0.80%	6.00%										
15	Spain	3.60%	1.20%	0.40%	3.20%										
16	Sweden	7.60%	2.40%	2.00%	5.60%										
17	Switzerland	5.00%	2.80%	1.10%	3.90%										
18	United Kingdom	5.80%	1.30%	1.00%	4.80%										
19	United States	6.70%	1.60%	0.90%	5.80%										
20	**Average**	**5.11%**	**0.71%**	**0.18%**	**4.93%**										
21															
22	*Source* : Elroy Dimson, Paul Marsh, Mike Staunton, *Triumph*														
23	*of the Optimists*, Princeton University Press 2002														
24															
25															
26															

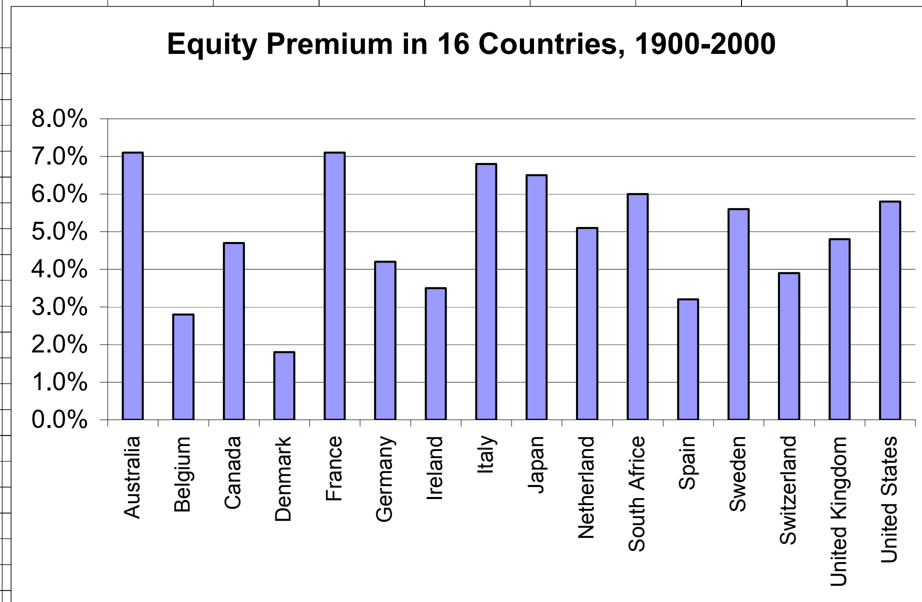


Figure 19.6. The equity premium in 16 major economies over the 20th century.

19.7. Computing Target's cost of equity r_E with the Gordon model

An alternative to the CAPM for computing the cost of equity r_E is the Gordon model, which we've previously discussed in Chapter 6. The Gordon model says that the equity value is the discounted value of future anticipated dividends. The standard version of the Gordon model is:

$$r_E = \frac{Div_0(1+g)}{P_0} + g$$

where

Div_0 = current equity payout of firm (total dividends + stock repurchases)

P_0 = current market value of equity

g = anticipated equity payout growth rate

For reasons explained in Chapter 6, we think the Gordon model should be used with the *total equity payout*, defined as total dividends plus stock repurchases. Below is the calculation for Target Corp.'s WACC using the Gordon model. The spreadsheet is the same as that of the previous section, except:

- Rows 32-36 show Target's equity payouts—the sum of its dividends and share repurchases—in each of the last five years. The compound annual growth rate of the equity payouts is 8.89% per year (cell D38).
- Rows 22-25 show the Gordon model calculation of the cost of equity r_E. This is computed as:

$$r_E = \frac{Div_0(1+g)}{P_0} + g = \frac{232*(1+8.89\%)}{25{,}619} + 8.89\% = 9.88\%$$

where

Div_0 = current equity payout

P_0 = current market value of equity

g = anticipated equity payout growth rate

	A	B	C	D	E
1	**TARGET CORP.'S WACC USING GORDON MODEL FOR COST OF EQUITY**				
2	Number of shares (million)	908			
3	Market value per share, 1 February 2002	28.21			
4	Market value of equity 1 February 2002, E	25,619	<-- =B3*B2		
5					
6		2002	2001		
7	Current portion of long-term debt and notes payable	975	905		
8	Long-term debt	7,523	7,054		
9	Total debt, D	8,498	7,959	<-- =C8+C7	
10					
11	Market value of Target, E+D	34,117	<-- =B9+B4		
12					
13	Interest paid, 2002	588			
14	Average debt over 2002	8,229	<-- =AVERAGE(B9:C9)		
15	Interest cost, r_D	7.15%	<-- =B13/B14		
16					
17		2002			
18	Earnings before taxes	2,676			
19	Income taxes	1,022			
20	Corporate tax rate, T_C	38.19%	<-- =B19/B18		
21					
22	Current equity value	25,619			
23	Current equity payout, Div_0	232	<-- =D36		
24	Growth rate of equity payout	8.89%	<-- =D38		
25	Cost of equity, r_E, using Gordon model	9.88%	<-- =B23*(1+B24)/B22+B24		
26					
27	WACC	8.52%	<-- =B4/B11*B25+(1-B20)*B9/B11*B15		
28					
29					
30	**Dividends and stock repurchases**				
31	**Year**	**Dividends**	**Repurchases**	**Total equity payout**	
32	1998	165	0	165	
33	1999	178	0	178	
34	2000	190	585	775	
35	2001	203	20	223	
36	2002	218	14	232	
37					
38			Growth rate	8.89%	<-- =(D36/D32)^(1/4)-1

Using the Gordon model estimate of the cost of equity, Target's WACC is 8.52% (cell B27).

Summing up

This chapter has discussed a grab-bag of share valuation methods. Three of these methods could be termed "fundamental valuations." Valuation Method 1, the simplest of the fundamental valuation methods is based on the assumption of market efficiency and says that a firm's stock is worth its current market price. Simple as it is, this approach has a lot of power and support in the academic community: If market participants have done their work, then the current price of a share reflects all publicly-available information, and there's nothing else to do.

Valuation method 2, *discounted cash flow* (DCF) valuation, is the method preferred by most finance academics and many finance practitioners. This method is based on discounting the firm's projected future free cash flows (FCF) at an appropriate weighted average cost of capital. The discounted value arrived at in this way is called the firm's *enterprise value*. To arrive at the valuation of the firm's equity, we add cash and marketable securities to the enterprise value and subtract the value of the firm's debt. Dividing by the number of shares gives the per-share valuation.

Valuation method 3, the *direct equity valuation*, discounts the projected payouts to equity holders (defined as the sum of dividends plus share repurchases) by the firm's cost of equity r_E. The resulting present value is the value of the firm's equity. Although it appears simpler and more direct than the FCF valuation, direct equity valuation is usually shunned by finance professionals. This is primarily because the cost of equity is heavily dependent on a firm's debt-equity financing mix, whereas the WACC is not nearly as dependent (and perhaps independent) of the debt-equity mix.

Valuation method 4, *multiple valuation* is widely used. This method of valuation arrives at a relative valuation of the firm by comparing a set of relevant multiples for comparable firms. When used correctly, multiple valuations can be a powerful tool, but it is often difficult to arrive at a correct "peer group" for a particular firm.

Exercises

CHAPTER 19 APPENDIX:

VALUING PROCTER & GAMBLE*

This version: February 8, 2004

Appendix contents

* This is a preliminary draft of a chapter of *Principles of Finance with Excel*. (benninga@wharton.upenn.edu).

Overview

This appendix implements a full-blown valuation of the stock of Procter & Gamble Corporation (PG). In doing so we illustrate some of the tricky implementation issues involved in the valuation techniques described in Chapter 16. The appendix contains advanced materials, and can easily be skipped.

In addition to the techniques described in Chapter 16, this appendix introduces two??? new techniques:

- Multiple growth rates and the Gordon model
- Asset betas

19.A.1. The Gordon model with two dividend growth rates

The Gordon model discussed in Chapter 16 (and previously in Chapter 5) assumes that there equity payouts of the firm will grow at an anticipated future growth rate *g*. Based on this assumption we showed that the cost of equity is:

$$r_E = \frac{Div_0 * (1+g)}{P_0} + g,$$

where Div_0 is the firm's current equity payout (defined as the sum of its total dividends and stock repurchases), g is the growth rate of the equity payout, and P_0 is the firm's current equity value (that is, number of shares times the current share price).

The assumption of a single future growth rate may, however, be problematic. Just as for the FCF examples in section 16.2 we concluded that there might be 2 FCF growth rates, it is often plausible to assume that there are 2 dividend growth rates. Typically, we assume that an initial period of high dividend growth is followed by normal dividend growth. In the equation below we've assume that dividends grow at a high growth rate for 5 years and that afterwards the growth rate slows down to a normal dividend growth rate. In this case the basic Gordon model equation becomes:

$$P_0 = \underbrace{\sum_{t=1}^{5} \frac{Div_0 * \left(1+g_{high}\right)^t}{\left(1+r_E\right)^t}}_{\substack{\uparrow \\ \text{The present value of the first} \\ \text{five years of dividend growth,} \\ \text{assumed to grow at a high growth} \\ \text{rate, } g_{high}.}} + \underbrace{\sum_{t=6}^{\infty} \frac{Div_0 * \left(1+g_{high}\right)^5 * \left(1+g_{normal}\right)^{t-5}}{\left(1+r_E\right)^t}}_{\substack{\uparrow \\ \text{The present value of the rest of the dividends,} \\ \text{assumed to grow at a lower rate } g_{normal}.}}$$

$$= \underbrace{\frac{Div_0\left(1+g_{high}\right)}{1+r_E}\left(\frac{1-\left(\frac{1+g_{high}}{1+r_E}\right)^5}{1-\frac{1+g_{high}}{1+r_E}}\right)}_{\substack{\uparrow \\ \text{Referred to as "term 1" in the spreadsheet below}}} + \underbrace{\frac{Div_0 * \left(1+g_{high}\right)^5}{\left(1+r_E\right)^5}\left(\frac{1+g_{normal}}{r_E - g_{normal}}\right)}_{\substack{\uparrow \\ \text{Referred to as "term 2" in the spreadsheet below}}}$$

The problem here is to solve for r_E. To show what's going on, we use **Goal Seek** or **Solver** in Excel to solve this problem.[1] Here's an illustration: ABC Corp's current share value is \$50. The firm has just paid a dividend of \$10 per share. You anticipate that this dividend will grow at 20% per year for the next 5 years, after which the dividend growth rate will slow to 5% per year. To find the cost of equity r_E, we set up the following spreadsheet:

[1] Below we also show the use of an Excel function defined by the author which automates this process.

	A	B	C
1	**TWO-STAGE GORDON DIVIDEND MODEL**		
2	P_0	50	
3	Div_0	10	
4	High growth rate, g_{high}	20%	
5	Number of high-growth years	5	
6	Term 1 factor: $(1+g_{high})/(1+r_E)$	0.9600	<-- =(1+B4)/(1+B10)
7			
8	Normal growth rate, g_{normal}	5%	
9			
10	Cost of equity, r_E	25.00%	
11			
12	Term 1: PV of high-growth dividends	44.3106	<-- =B3*B6*(1-B6^B5)/(1-B6)
13	Term 2: PV of normal-growth dividends	42.8071	<-- =B3*B6^B5*(1+B8)/(B10-B8)
14			
15	Term 1 + Term 2 - P_0	37.12	<-- =B12+B13-B2

We want to find the cost of equity r_E (cell B10) so that cell B15 equals zero. Doing this with **Goal Seek** gives:

	A	B	C
1	**TWO-STAGE GORDON DIVIDEND MODEL**		
2	P_0	50	
3	Div_0	10	
4	High growth rate, g_{high}	20%	
5	Number of high-growth years	5	
6	Term 1 factor: $(1+g_{high})/(1+r_E)$	0.9600	<-- =(1+B4)/(1+B10)
7			
8	Normal growth rate, g_{normal}	5%	
9			
10	Cost of equity, r_E	25.00%	
11			
12	Term 1: PV of high-growth dividends	44.3106	<-
13	Term 2: PV of normal-growth dividends	42.8071	<-
14			
15	Term 1 + Term 2 - P_0	37.12	<-
16			
17			
18	Cost of equity, r_E using the function **twostagegordon**	37.62%	<-- =twostagegordon(B2,B3,B4,B5,B8)
19			

Goal Seek

Set cell: $B$15

To value: 0

By changing cell: $B$10

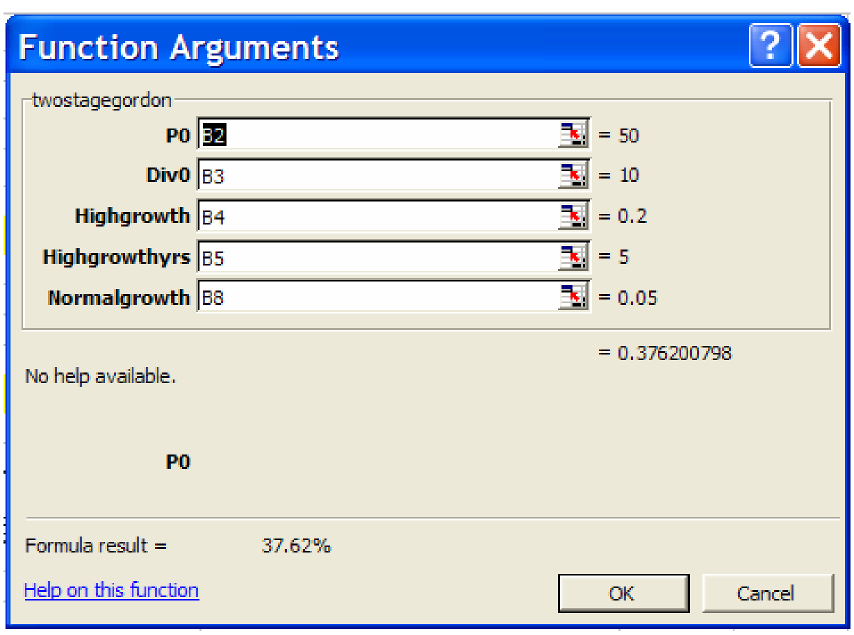
OK Cancel

Pressing **OK** gives the cost of equity as $r_E = 37.620\%$:

	A	B	C
1	**TWO-STAGE GORDON DIVIDEND MODEL**		
2	P_0	50	
3	Div_0	10	
4	High growth rate, g_{high}	20%	
5	Number of high-growth years	5	
6	Term 1 factor: $(1+g_{high})/(1+r_E)$	0.8720	<-- =(1+B4)/(1+B10)
7			
8	Normal growth rate, g_{normal}	5%	
9			
10	Cost of equity, r_E	37.62%	
11			
12	Term 1: PV of high-growth dividends	33.7744	<-- =B3*B6*(1-B6^B5)/(1-B6)
13	Term 2: PV of normal-growth dividends	16.2257	<-- =B3*B6^B5*(1+B8)/(B10-B8)
14			
15	Term 1 + Term 2 - P_0	0.00	<-- =B12+B13-B2
16			
17			
18	Cost of equity, r_E using the function **twostagegordon**	37.62%	<-- =twostagegordon(B2,B3,B4,B5,B8)

TwostageGordon—an Excel function for calculating the Gordon cost of capital

The spreadsheet for this appendix includes an Excel function defined by the author. The function **TwostageGordon** computes the cost of equity in a two-stage Gordon model. Cell B18 of the spreadsheet clip above shows this function in action. Here's the wizard for the function:



Pressing OK gives the answer which appear in cell B18.

19.A.2. Computing the FCF growth rate for Procter & Gamble

Discounting anticipated FCFs looks simple, and it is—mathematically. But reality is very complicated. We have a number of tricky issues to resolve:

- How to calculate the FCF growth rate or rates (if there is more than one)? This is illustrated in this section. We use an example for Procter & Gamble to illustrate the difficulties that can be encountered.
- How to calculate the WACC. This is illustrated in the following sections.

Build a financial model for the firm—the best way to predict FCFs

The best way to compute the FCF growth rate is to build a full-blown model of the firm. This technique was discussed in Chapters 8 and 9. The financial model method of computing the FCF growth rate is usually preferable to other methods—it takes into account the all the economic factors which drive the profitability of the firm, and it ties them together in a meaningful way (both economically and accounting). The disadvantage of this method is that building a financial model is relatively complicated and time consuming.

Projecting the FCF growth rate from the firm's past performance

You could try to project the FCFs from the firm's past performance. In this section we give some examples; we'll be frank about the difficulties and the assumptions we've had to make.

There are two main problems in computing the FCFs from historical data. The first problem has to do with the definition of the FCF. The definition given in Chapters 7-9 is:

Defining the Free Cash Flow	
Profit after taxes	This is the basic measure of the profitability of the business, but it is an accounting measure that includes financing flows (such as interest), as well as non-cash expenses such as depreciation. Profit after taxes does not account for either changes in the firm's working capital or purchases of new fixed assets, both of which can be important cash drains on the firm.
+ Depreciation	This noncash expense is added back to the profit after tax.
+ after-tax interest payments (net)	FCF is an attempt to measure the cash produced by the business activity of the firm. To neutralize the effect of interest payments on the firm's profits, we: • Add back the after-tax cost of interest on debt (*after-tax* since interest payments are tax-deductible), • Subtract out the after-tax interest payments on cash and marketable securities.
- Increase in current assets	When the firm's sales increase, more investment is needed in inventories, accounts receivable, etc. This increase in current assets is not an expense for tax purposes (and is therefore ignored in the profit after taxes), but it is a cash drain on the company.
+ Increase in current liabilities	An increase in the sales often causes an increase in financing related to sales (such as accounts payable or taxes payable). This increase in current liabilities—when related to sales—provides cash to the firm. Since it is directly related to sales, we include this cash in the free cash flow calculations.
- Increase in fixed assets at cost	An increase in fixed assets (the long-term productive assets of the company) is a use of cash, which reduces the firm's free cash flow.
FCF = sum of the above	

However, it's not always clear what constitutes a change in current assets, current liabilities, or fixed assets. In the next example, for example, we will question what items for Procter & Gamble are related to the increases in fixed assets.

A second problem we will encounter is with the growth rates. In many cases we will see growth rates which appear to us to be unnaturally large or small.

Procter & Gamble

Below we do a calculation of Procter & Gamble's FCF for the years 1993-2001.[2] Our problem in computing the historic FCF is whether to include the asset sales and the acquisitions in the cash flow or not.

	A	B	C	D	E	F	G	H	I
1	PROCTER AND GAMBLE, FCF CALCULATIONS								
2		Cash from operations	Interest expense after-tax	Capex	Asset sales	Acquisitions	FCF w/o asset sales and w/o acquisitions	FCF w/ asset sales and w/o acquisitions	FCF w/ asset sales and w/ acquisitions
3	1993	3,338,000,000	425,467,049	1,911,000,000	725,000,000	138,000,000	1,852,467,049	2,577,467,049	2,439,467,049
4	1994	3,649,000,000	318,500,299	1,841,000,000	105,000,000	295,000,000	2,126,500,299	2,231,500,299	1,936,500,299
5	1995	3,568,000,000	322,690,000	2,146,000,000	310,000,000	623,000,000	1,744,690,000	2,054,690,000	1,431,690,000
6	1996	4,158,000,000	315,755,836	2,179,000,000	402,000,000	358,000,000	2,294,755,836	2,696,755,836	2,338,755,836
7	1997	5,882,000,000	297,324,252	2,129,000,000	520,000,000	150,000,000	4,050,324,252	4,570,324,252	4,420,324,252
8	1998	4,885,000,000	362,901,191	2,559,000,000	555,000,000	3,269,000,000	2,688,901,191	3,243,901,191	-25,098,809
9	1999	5,544,000,000	650,000,000	2,828,000,000	434,000,000	137,000,000	3,366,000,000	3,800,000,000	3,663,000,000
10	2000	4,675,000,000	722,000,000	3,018,000,000	419,000,000	2,967,000,000	2,379,000,000	2,798,000,000	-169,000,000
11	2001	5,804,000,000	794,000,000	2,486,000,000	788,000,000	138,000,000	4,112,000,000	4,900,000,000	4,762,000,000
12	Growth	7.16%	<-- =(B11/B3)^(1/8)-1	3.34%	1.05%	0.00%	10.48%	8.36%	8.72%
13									
14		**Definitions**:							
15		Cash from operations includes changes in NWC and depreciation, net of interest expense							
16		"CAPEX" is changes in fixed assets, but does not include asset sales or acquisitions.							
17									
18									
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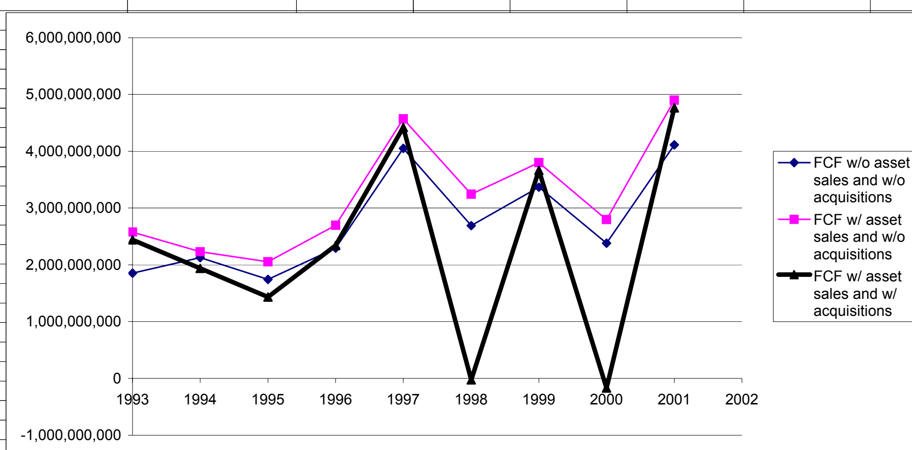

Notice that we have calculated the FCF in 3 different ways:

[2] All the data are from Edgar, courtesy of http://edgarscan.pwcglobal.com .

- Column G: FCF = Cash flow from operations + after-tax interest – CAPEX. Our natural preference is to use this definition of the FCF—in using it we will end up valuing the firm's existing asset base. This definition of FCF has grown at an annual rate of 10.48% over the period.
- Column H: FCF = Cash flow from operations + after-tax interest – CAPEX + asset sales. It could be argued that asset sales are part of PG's normal business activities. If so, this might be an appropriate definition of the free cash flows.
- Column I: FCF = Cash flow from operations + after-tax interest – CAPEX + asset sales – asset acquisitions. If both asset sales and asset acquisitions are part of PG's normal business activities, then this is the appropriate definition of FCF.

We choose the first definition (the one that does not include either asset sales or acquisitions) as our definition of Procter & Gamble's FCF. Our theoretical justification for using this definition is that the present value of the FCFs without asset sales and acquisitions gives the value of the current activities of Procter & Gamble. Assuming that asset sales and acquisitions are made at market value (that is, NPV = 0), this will also correctly value the firm.[3] (It also helps that the first definition of the FCF is relatively *smooth*, so that it is more predictable.)

Using the first definition of the FCF, we conclude that PG's future FCF growth rate is 10.48% (cell G12). This growth rate is very high; we ordinarily would not expect a consumer

[3] This "theoretical justification" is easily punctured and shouldn't be taken too seriously: If part of the ongoing business of PG is to sell and acquire assets, then we should in principle take one of the two broader definitions of the FCF.

products company to have a long-term growth rate much higher than the growth of GNP.[4] We therefore conclude that after 4 years, the FCF growth rate of Procter & Gamble will decrease to 4%. We denote the two growth rates $g_{high} = 10.48\%$ and $g_{normal} = 4\%$. If these numbers seem somewhat arbitrary, be reassured: We're going to do extensive sensitivity analysis on them using Excel's **Data Table**s.

Having made this choice, the template below gives us the format for valuing Procter & Gamble, provided we can specify the WACC and a number of other factors (highlighted below). Suppose, for example, that the WACC = 7%; then the share value of PG is computed to be $130.87 (cell B23):

	A	B	C
1	**PROCTER & GAMBLE, VALUATION**		
2	2001 FCF (base year)	4,112,000,000	<-- This is cell G11 of the FCF spreadsheet
3			
4	High FCF growth rate, g_{high}	10.48%	<-- This is cell G12 of the FCF spreadsheet
5	Number of high FCF growth years	4	<-- A guess
6	Term 1 factor: $(1+g_{high})/(1+WACC)$	103%	<-- =(1+B4)/(1+B10)
7			
8	Normal FCF growth rate, g_{normal}	4.00%	
9			
10	WACC	7.00%	
11	End-2001 debt	12,025,000,000	<-- From P&G's balance sheet
12	End-2001 cash	2,306,000,000	<-- From P&G's balance sheet
13			
14	Term 1: PV of high-growth cash flows	17,830,002,057	<-- =B2*B6*(1-B6^B5)/(1-B6)
15	Term 2: PV of normal-growth cash flows	162,024,760,076	<-- =B2*(1+B4)^B5*(1+B8)/(B10-B8)/(1+B10)^B5
16	Enterprise value	179,854,762,133	<-- =SUM(B14:B15)
17	Add cash	182,160,762,133	<-- =B16+B12
18	Subtract debt	-12,025,000,000	<-- =-B11
19	Value of equity	170,135,762,133	<-- =SUM(B17:B18)
20			
21	Number of shares, end 2001	1,300,000,000	
22	Computed value per share	130.87	<-- =B19/B21
23	End-2001 stock price	78.08	
24			
25	Analyst recommendation: Buy, Sell, Neutral	Buy	<-- =IF(B22>B23*(1.1),"Buy",IF(B22<B23/1.1,"Sell","Neutral"))

[4] A good ad-hoc model for long-term growth of a consumer products or maturity-industry is to take a *real growth factor* and add *anticipated inflation*. For Procter & Gamble, for example, we might hypothesize that long-term real sales will grow at the rate of population growth in the United States (approximately 1% - 2%). If the anticipated long-term inflation rate is about 3%, this gives nominal sales growth as 4%-5%.

Clearly the ultimate valuation depends on PG's WACC.[5] This issue is discussed in the following sections. Section 19.A.3 calculates the industry average WACC using an asset beta approach. Section 19.A.4 discusses PG's cost of debt r_D and its tax rate T_C and then computes PG's WACC using the Gordon dividend model. The Proctor & Gamble valuation is summarized in Section 19.A.5.

19.A.3. Using the industry asset beta, β_{asset}, to compute the WACC for Procter & Gamble

We use the asset beta approach (see Chapter 15) to calculate the WACC for PG. Our approach is the following:

- For a representative sample of firms in PG's industry, we compute the asset β_{asset}, defined as:

$$\beta_{asset} = \frac{E}{E+D}\beta_{equity} + \frac{D}{E+D}\beta_{debt}\left(1-T_C\right)$$

The use of the industry average asset beta is predicated on the assumption that individual firm calculations of betas include a lot of random "noise"; by calculating an industry average we eliminate much of this noise. On the other hand, the use of the industry beta assumes that PG risks are well-represented by the average risks of the industry. If you

[5] Row 25 shows how an analyst might decide on her recommendation—if the valuation of the stock exceeds the current price by more than 10%, this particular analyst gives a "buy" recommendation, and if the valuation is below the current price, she recommends "sell." Valuations within a ± 10% range of the current price lead to a "neutral" recommendation.

find this assumption difficult to swallow, you may want to use the alternative calculation in the next section, which focuses exclusively on PG data.

- The average asset beta for the industry is then used to compute the WACC of PG:

$$WACC_{PG} = r_f + \beta_{asset}^{average}\left[E\left(r_M\right) - r_f\right]$$

We use r_f = *4.1%* and a market risk premium $E\left(r_M\right) - r_f$ = *5.50%*. These values correspond reasonably to the market parameters at the end of 2001, the time of the PG valuation.

- The unknown in this analysis is the debt β_{debt} of the various companies used in the industry sample. In order to overcome this problem, we perform sensitivity analysis on this parameter. The spreadsheet below assumes a β_{debt} = 0.2.

Here is our implementation:

	A	B	C	D	E	F	G	H	I	J
1	**COMPUTING THE INDUSTRY ASSET BETA**									
2		**Equity Beta**	**Debt/Equity book**	**Market/ book**	**Debt/Equity market**	**Debt/Asset market**	**Debt beta**	**Tax rate**	**Asset beta**	
3	Alberto-Culver Company (ACV)	0.48	0.43	4.35	0.0989	0.0900	0.2	0.37	0.4482	<-- =F3*G3*(1-H3)+(1-F3)*B3
4	Allou Health & Beauty (ALU)	1.62	2.42	0.59	4.1017	0.8040	0.2	0.37	0.4188	
5	Blyth, Inc. (BTH)	0.84	0.54	3.01	0.1794	0.1521	0.2	0.37	0.7314	
6	Church & Dwight Co., Inc. (CHD)	0.48	1.48	4.35	0.3402	0.2539	0.2	0.37	0.3901	
7	Clorox Company (CLX)	0.50	0.52	6.2	0.0839	0.0774	0.2	0.37	0.4711	
8	Colgate-Palmolive Company (CL)	1.02	3.83	64.03	0.0598	0.0564	0.2	0.37	0.9695	
9	Dial Corporation (DL)	0.68	5.44	24.75	0.2198	0.1802	0.2	0.37	0.5802	
10	Elizabeth Arden, Inc. (RDEN)	1.16	4.32	2.02	2.1386	0.6814	0.2	0.37	0.4554	
11	Gillette Company (G)	0.72	2.02	17.73	0.1139	0.1023	0.2	0.37	0.6592	
12	Home Products Int'l, Inc. (HOMZ)	1.31	46.31	10.23	4.5269	0.8191	0.2	0.37	0.3402	
13	Newell Rubbermaid Inc. (NWL)	0.55	0.9	3.52	0.2557	0.2036	0.2	0.37	0.4637	
14	Procter & Gamble Co. (PG)	0.24	1.27	10.45	0.1215	0.1084	0.2	0.37	0.2276	
15	Tupperware Corporation (TUP)	0.47	2.9	11.25	0.2578	0.2049	0.2	0.37	0.3995	
16	Unilever (UL)	0.46	3.98	3.68	1.0815	0.5196	0.2	0.37	0.2865	
17										
18						Average asset beta			0.49	<-- =AVERAGE(I3:I16)
19										
20	Asset beta	0.49	<-- =I18							
21	Risk-free, r_f	4.10%								
22	Risk premium, $E(r_M) - r_f$	5.50%								
23	WACC	6.79%	<-- =B21+B20*B22							

The procedure we follow is:

- $\dfrac{Book\ \ Debt\,/\,Equity}{Market\ Equity\,/\,Book\ value\ of\ \ equity} = Market\ Debt\,/\,Equity$ (column E)

- $Market\ \ Debt\,/\,Assets = \dfrac{Market\ \ Debt\,/\,Equity}{1 + Market\ Debt\,/\,Equity}$ (column F)

$$\beta_{Asset} = \beta_{\mathrm{E}} \frac{E}{E+D} + \beta_D \left(1 - T_C\right) \frac{D}{E+D}.$$

We've been somewhat cavalier in the above table—assuming that all the companies have the same marginal tax rate and same debt beta. But a sensitivity analysis will convince you that it doesn't matter much. Here is a two-parameter data table in which we allow both the corporate tax rate and the debt beta to vary:

	A	B	C	D	E	F	G	H
26		**Data table: the effect of corporate tax rate and debt beta on WACC**						
27								
28			**Tax rate**					
29			**30%**	**33%**	**35%**	**37%**	**39%**	
30	**Debt beta-->**	**0.0**	6.58%	6.58%	6.58%	6.58%	6.58%	
31		**0.1**	6.68%	6.68%	6.68%	6.68%	6.68%	
32		**0.2**	6.79%	6.79%	6.79%	6.79%	6.79%	
33		**0.3**	6.90%	6.90%	6.90%	6.89%	6.89%	
34		**0.4**	7.01%	7.00%	7.00%	7.00%	7.00%	
35		**0.5**	7.12%	7.11%	7.11%	7.10%	7.10%	
36								
37	This data table cell (which has been hidden)							
38	refers to cell B24. In this way the data table							
39	computes the effect on the WACC in B24 given							
40	changes in the tax rate on column H and the debt							
41	beta in column G.							
42								

If you read across the rows, you will see that the tax rate barely affect the WACC. The debt beta has a bigger effect, but the maximum effect is less than 1 percent, which in the cost-of-capital literature is passable.

19.A.4. Procter & Gamble's WACC using the Gordon model and the company's cost of debt r_D and its tax rate T_C

We start by computing Procter-Gamble's cost of debt at the end of 2001. Using financial statement data results in $r_D = 7.88\%$:

	A	B	C	D
1	**COMPUTING TAX RATE T_C AND THE COST OF DEBT r_D FOR PROCTER & GAMBLE FOR 2001**			
2	**Year**	**2001**	**2000**	
3	Interest paid	$794,000,000		
4				
5	Short-term debt	$2,233,000,000	$3,241,000,000	
6	Long-term debt	$9,792,000,000	$9,012,000,000	
7	Total debt	$12,025,000,000	$12,253,000,000	
8				
9	Cash and cash equivalents	$2,306,000,000	$1,415,000,000	
10	Investment securities	$212,000,000	$185,000,000	
11				
12	Net debt	$9,507,000,000	$10,653,000,000	<-- =C7-SUM(C9:C10)
13				
14	Cost of debt, r_D	7.88%		<-- =B3/AVERAGE(B12:C12)
15				
16	Profits before taxes	$4,616,000,000		
17	Taxes	$1,694,000,000		
18	Tax rate, T_C	36.70%		<-- =B17/B16

Note that we have also calculated PG's tax rate T_C by computing:

$$T_C = \frac{Provision\ \ for\ income\, taxes}{Profits\ before\, taxes}$$

This may not be the whole story, however. PG's 2001 annual report gives the company's average interest rates at year-end 2001 as 4.5% for long-term debt and 4.1% for short-term debt. On reflection these numbers appear to be more reliable estimates of the firm's *marginal* borrowing costs. Since, wherever possible, we prefer to use marginal costs instead of historical costs, we will use $r_D = 4.3\%$.

Using the Gordon model

We've previously discussed this model in Chapter 6 and again in Chapter 16. If there's one single dividend growth rate, the Gordon formula becomes:

$$r_E = \frac{Div_0\left(1+g\right)}{P_0} + g$$

where

Div_0 = current equity payout (total dividend + stock repurchases) of firm

P_0 = current market value of equity

g = anticipated dividend growth rate

In Chapter 6 we emphasized the need to calculate the cost of equity using:

- *Total dividends + repurchases of shares*: These are the total payouts to equity holders
- *Total value of equity (number of shares * share price)*

We start with the facts—the historical data on PG's total cash dividends and stock repurchases over the years 1993-2001:

	A	B	C	D	E
1	**PROCTER & GAMBLE, DIVIDENDS AND REPURCHASES**				
2		**Cash dividends to shareholders**	**Purchases of treasury stock**	**Total**	
3	1993	850,000,000	55,000,000	905,000,000	
4	1994	949,000,000	14,000,000	963,000,000	
5	1995	1,062,000,000	114,000,000	1,176,000,000	
6	1996	1,202,000,000	432,000,000	1,634,000,000	
7	1997	1,329,000,000	1,652,000,000	2,981,000,000	
8	1998	1,462,000,000	1,929,000,000	3,391,000,000	
9	1999	1,626,000,000	2,533,000,000	4,159,000,000	
10	2000	1,796,000,000	1,766,000,000	3,562,000,000	
11	2001	1,943,000,000	1,250,000,000	3,193,000,000	
12	Growth	10.89%	47.76%	17.07%	<-- =(D11/D3)^(1/8)-1
13					
14					
15					
16					
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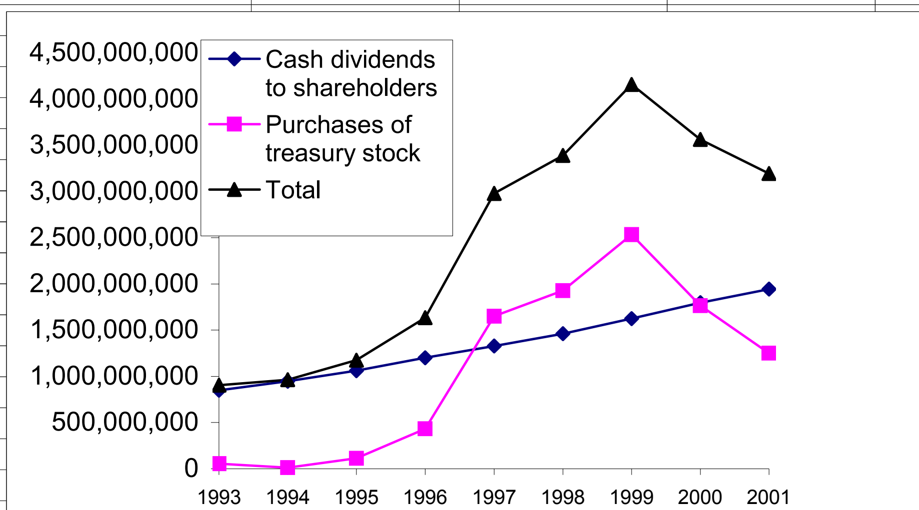


In the graph we see a pattern which is familiar to the U.S. corporate sector: The dividend growth is very smooth and hence predictable, whereas the repurchases of stock are much less predictable. The compound growth rate of dividends is 10.89% (very respectable) and the compound growth rate of the total dividends + repurchases is 17.07% (high!).

The key question for the Gordon model calculation of r_E is: What do shareholders anticipate will be future dividend growth? It is unlikely that an intelligent shareholder (like the reader of this book) will assume that the incredible growth of repurchases can continue over the foreseeable future. In the spreadsheet below we assume that the 17% dividend rate will continue for the next 4 years, after which the dividend growth will drop to 4%. This results in a cost of equity (using the two-stage Gordon model) of $r_E = 9.11\%$ and a WACC = 8.55%.

	A	B	C
1	**PROCTER & GAMBLE, CALCULATING WACC USING TWO-STAGE GORDON MODEL**		
2	End-2001 stock price	78.08	
3	Number of shares	1,300,000,000	
4	Equity value, E	101,504,000,000	<-- =B3*B2
5	Debt value (net), D	$9,719,000,000	Calculated previously
6			
7	End-2001 dividend, Div_0	3,193,000,000	Sum of dividends and repurchases
8			
9	High dividend growth rate, g_{high}	17.07%	
10	Number of high-growth years	4	
11			
12	Normal dividend growth rate, g_{normal}	4.00%	
13			
14	Cost of equity, r_E	9.11%	<-- =twostagegordon(B4,B7,B9,B10,B12)
15	Cost of debt, r_D	4.30%	Calculated previously
16	Tax rate, T_C	37%	
17			
18	WACC = r_E*E/(D+E)+r_D*(1-T_C)*D/(E+D)	8.55%	<-- =B14*B4/(B4+B5)+B15*(1-B16)*B5/(B4+B5)

19.A.5. Procter & Gamble: the bottom line on the FCF valuation

The WACC for PG is between 6.79% and 8.55%, depending on whether we calculate it using the CAPM or the Gordon model. If we assume that the company will have high FCF growth of 10.48% for the next 4 years and 4% thereafter, we can use our two-stage FCF model to value the shares:

	A	B	C
1	**PROCTER & GAMBLE, VALUATION**		
2	2001 FCF (base year)	4,112,000,000	<-- This is cell G11 of the FCF spreadsheet
3			
4	High FCF growth rate, g_{high}	10.48%	<-- This is cell G12 of the FCF spreadsheet
5	Number of high FCF growth years	4	<-- A guess
6	Term 1 factor: $(1+g_{high})/(1+WACC)$	102%	<-- =(1+B4)/(1+B10)
7			
8	Normal FCF growth rate, g_{normal}	4.00%	
9			
10	WACC	8.00%	
11	End-2001 debt	12,025,000,000	<-- From P&G's balance sheet
12	End-2001 cash	2,306,000,000	<-- From P&G's balance sheet
13			
14	Term 1: PV of high-growth cash flows	17,414,591,486	<-- =B2*B6*(1-B6^B5)/(1-B6)
15	Term 2: PV of normal-growth cash flows	117,080,006,861	<-- =B2*(1+B4)^B5*(1+B8)/(B10-B8)/(1+B10)^B5
16	Enterprise value	134,494,598,348	<-- =SUM(B14:B15)
17	Add cash	136,800,598,348	<-- =B16+B12
18	Subtract debt	-12,025,000,000	<-- =-B11
19	Value of equity	124,775,598,348	<-- =SUM(B17:B18)
20			
21	Number of shares, end 2001	1,300,000,000	
22	Computed value per share	95.98	<-- =B19/B21
23	End-2001 stock price	78.08	
24			
25	Analyst recommendation: Buy, Sell, Neutral	Buy	<-- =IF(B22>B23*(1.1),"Buy",IF(B22<B23/1.1,"Sell","Neutral"))

As you can see in cell B25, this makes P&G a strong buy, since its current (i.e., end 2001) share price is $78.08. Even at our upper estimate for the WACC, 8.55%, PG's valuation using the FCF discounting is $83.33, which is still above its current market value.

If we use **Data Table**, we can see that the current market value of $78.08 is apparently based on significantly lower expectations. In the table below we value PG stock using normal growth rates between 0% and 6% and varying the number of high-growth years from 0 to 7. The highlighted cells are those valuations which are within 10% of the current market valuation.[6]

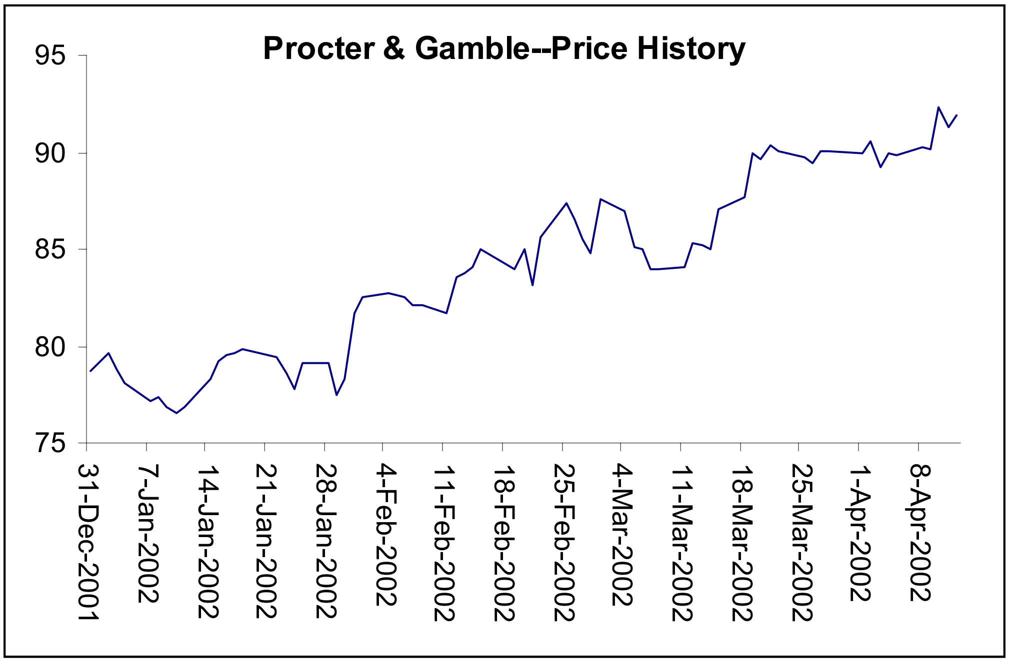
[6] The highlighting uses Excels conditional formatting tool (see Chapter 33).

	E	F	G	H	I	J	K	L	M
1	**Data table on number of high-growth years and on "normal" growth rate**								
2									
3			**Normal growth rate**						
4			**0%**	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**
5	**Number of**	**0**	32.06	38.16	46.30	57.68	74.76	103.23	160.17
6	**high-growth**	**1**	36.21	42.45	50.77	62.42	79.89	109.01	167.25
7	**years**	**2**	40.45	46.83	55.34	67.26	85.13	114.92	174.50
8		**3**	44.78	51.31	60.02	72.21	90.49	120.97	181.92
9		**4**	49.22	55.90	64.81	77.28	95.98	127.16	189.51
10		**5**	53.76	60.59	69.70	82.46	101.59	133.49	197.27
11		**6**	58.40	65.39	74.71	87.76	107.34	139.96	205.21
12		**7**	63.15	70.30	79.83	93.18	113.21	146.58	213.33
13									
14		Highlighted cells are within 10% of the current market share value.							

The highlighted cells show the expected tradeoff between high-growth years and the normal growth rate. If we assume that ultimately normal growth is in the range of 3-4%, then the market seems to be valuing PG as if it has very few years of high FCF growth.

Conclusion: Procter & Gamble—what happened?

The FCF valuation produced bullish estimates about PG: Assuming a WACC of 8%, we valued the company's stock at the beginning of 2002 at $95.98, well above the actual price of the stock of $78.08. In the event, we were also correct—in the first months of 2002, PG stock climbed steadily, reaching a price of $91.93 by mid-April 2002:



We are not, of course, claiming perfect foresight—because of the vast amounts of information which to be digested and because of the large number of assumptions the analyst has to make, stock valuation remains one of the most problematic areas of finance. However, the FCF methods illustrated in Chapter 16 and in this Appendix are powerful methods which often give significant insights into the valuation of a firm's stock.

CHAPTER 20: CAPITAL STRUCTURE AND THE VALUE OF THE FIRM*

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Chapter contents

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Overview

"Capital structure" is finance jargon for how a firm should be financed—what mixture of debt and equity should be used by the shareholders of a firm to finance the firm's activities. To start you off thinking about this tricky question, we offer the example of Mortimer and Joanna, who are competing to buy the same supermarket.

The Fair City supermarket—does financing affect the price?

Mortimer and Joanna live in Fair City. Each heads a group of investors that wants to buy a supermarket located in the center of town. Both Mortimer and Joanna have superb records as supermarket managers. As manager of a supermarket, they're pretty much the same—meaning that the supermarket they manage will have the same sales, cost of goods sold, etc. However, while the management aspect of Mortimer's group and Joanna's group is pretty much the same, there's a big financial difference between the two competing groups: Mortimer's investors want to borrow 50 percent of the money needed to purchase the supermarket, whereas Joanna's investors hate debt and have decided to put up the whole cost of purchasing the supermarket without borrowing a penny.

The question: Which group of investors—Mortimer's or Joanna's—can afford to make the higher bid for the supermarket? This is the question examined in this chapter. At this point in the chapter we offer no answers to this question, but merely want to give you an insight into how possible answers might look.

Example 1: Both groups make the same bid

Suppose that both Mortimer and Joanna's groups bid $1 million for the supermarket. In this case the balance sheets would look like this:

Mortimer's Supermarket Group Half equity (50%) and half debt (50%)			
Supermarket	$1,000,000	Debt	$500,000
		Equity	$500,000
Total assets	**$1,000,000**	**Total debt and equity**	**$1,000,000**

Joanna's Supermarket Group Only equity (100%)			
Supermarket	$1,000,000	Debt	$0
		Equity	$1,000,000
Total assets	**$1,000,000**	**Total debt and equity**	**$1,000,000**

Why would both groups make a similar bid for the supermarket? The line of reasoning which might lead to this conclusion is the following:

> *A supermarket is a supermarket is a supermarket, no matter how its financed. If Mortimer's group think the supermarket is worth $1 million, then so will Joanna's group (and vice versa). The fact that one group finances with debt and equity whereas the other group finances only with equity is irrelevant to their valuation of the supermarket.*

Example 2: Mortimer's group bids more

Is it possible that Mortimer's group should rationally decide that—because of the group's greater proportion of debt financing—the supermarket is worth more than what Joanna's group is willing to pay? One of Mortimer's investors thinks that their group can afford to bid more for the supermarket than Joanna. His line of reasoning:

The fact that we're financing with debt means that it's cheaper for us to finance the supermarket. The interest paid on debt is an expense for tax purposes, which means that debt is cheaper than equity. In addition, since equity is more risky than debt, equity holders in any case want a higher return than debt holders. So our greater use of debt means that we can afford to pay more for the supermarket.

If this logic is correct, then it's possible that Mortimer's group would bid $1,200,000 for the supermarket, whereas Joanna's group would only bid $1,000,000. In this case the two balance sheets would look like this:

Mortimer's Supermarket Group Half equity (50%) and half debt (50%)			
Supermarket	$1,200,000	Debt	$600,000
		Equity	$600,000
Total assets	**$1,200,000**	**Total debt and equity**	**$1,200,000**

Joanna's Supermarket Group Only equity (100%)			
Supermarket	$1,000,000	Debt	$0
		Equity	$1,000,000
Total assets	**$1,000,000**	**Total debt and equity**	**$1,000,000**

Of course there's no question what would happen in this case: The sellers of the supermarket would prefer to sell to Mortimer's group, which is offering a higher price.

Which example is more representative? Example 1 or Example 2?

As you'll see in the chapter, both examples could be representative of how things actually work in the world. In this chapter we frame the capital structure question (Example 1 versus Example 2) primarily in terms of the following two questions:

- Does the choice of financing affect the total cash that can be extracted from the firm? If Mortimer's group, with its higher proportion of debt financing, can extract more cash from the supermarket, then it might be logical for them to be willing to pay more for the supermarket.
- Should the choice of financing affect the discount rate the firm uses to evaluate projects? This is where *risk*, the magic word in finance, comes into play.[1] In simple words: Is the correct discount rate to be used for the supermarket by Mortimer's group different from that which should be used by Joanna's group? Does the choice of a financing mix affect the weighted average cost of capital (WACC)?

As you will see in this chapter, the answers to both these questions relate primarily to taxation. It will turn out that depending on the tax system, either Example 1 or Example 2 could be a representation of how things work.[2]

Finance concepts in this chapter

- Debt versus equity financing
- Valuation effects of leverage
- Corporate versus personal taxation
- Modigliani-Miller model
- Miller's "Debt and Taxes"

---

[1] Recall the opening words of Chapter 10: "Risk is the magic word in finance. Whenever finance people can't explain something, we try to look confident and say 'it must be the risk.' "

[2] It's even possible that another variant of Example 2 would hold in which Mortimer's group would bid less than Joanna's. This is pretty unlikely, as you'll see in the remainder of the chapter.

Excel functions used

- **If**
- **NPV**

20.1. Capital structure when there are corporate taxes—ABC Corp

We start our exploration of the effects of capital structure by examining the story of ABC Corp. This well-known company is located in Lower Fantasia. Lower Fantasia has an unusual tax code: In Lower Fantasia whereas companies are taxed on their corporate income, individuals are not taxed on their personal income.

Our hero, Arthur ABC, is trying to figure out: a) Whether to buy ABC Corp., a well-known company in Lower Fantasia, and b) If he buys the company, how to finance the purchase.

Buying ABC Corp. using only equity

This turns out to be fairly simple. ABC has an expected annual free cash flow (FCF) of $1,000 per year; this FCF is anticipated to recur, year after year, at the same level. Arthur—who has an MBA from Eastern Lower Fantasia State University (their football team is called the "elfs")—has computed the cost of capital for the purchase as $r_U = 20\%$. The symbol "U" as a subscript for the discount rate r_U stands for "unlevered" and is meant to remind you that in this case r_U is the discount rate appropriate for the case where Arthur buys ABC Corp. with only equity (meaning: his own money, without borrowing).

This r_U = *20%*. is a cost of capital that reflects only the business risks of ABC Corp. If purchased only with equity, therefore, the company is thus worth $\frac{1,000}{20\%} = 5,000$.[3] In what follows we will use the symbol V_U for the "unlevered value of the firm." V_U is what a company is worth if it is financed only with equity. In our case:

$$V_U = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+r_U)^t} = \sum_{t=1}^{\infty} \frac{\$1,000}{(1+20\%)^t} = \frac{\$1,000}{20\%} = \$5,000.$$

Buying ABC using debt

Arthur has a wonderful source of debt financing: His mother. This wealthy old lady is in fact his business partner, but their joint deals are structured so that she's always the lender and Arthur the equity owner. There's another unusual feature to the old lady's lending—she gives out *perpetual debt*—her loans require only an annual payment of interest, but no repayment of principal.[4] The cost of debt to Arthur, denoted by r_D, is the interest rate charged by his mother on her loans to him. In this case r_D = 8%.

Together, Arthur and his Mom are exploring two alternative financing arrangements:

- In Alternative A, Arthur buys ABC Corp. for cash; immediately thereafter, the company borrows $3,000 from Mom and repays it to Arthur. (Corporate finance deals in Lower Fantasia are a bit complicated!) In this case ABC Corp is a *levered* company. ("Leverage" in this context means that the company has debt on its balance sheets.)

---

[3] The FCF is already *after* corporate taxes, and the discounted FCF value of the firm is thus $\sum_{t=1}^{\infty} \frac{FCF}{(1+20\%)^t} = \frac{FCF}{20\%}$.

[4] Throughout this chapter you'll notice that we often assume that cash flows have infinite duration. This makes the valuations easier, but doesn't affect the principles.

- In Alternative B, Arthur borrows $3,000 from Mom and then buys ABC Corp. for cash. In this case, ABC Corp. is an *unlevered* "all-equity" company (no debt on its balance sheets) and Arthur is levered.

The fundamental difference between these two alternatives is that the Lower Fantasia tax code has a corporate income tax but no personal income taxes. Under the tax code, interest paid by corporations is an expense for tax purposes, but this is not true for interest paid by individuals, who aren't taxed on their personal income.

From the drawing below you can see that the *total family income* produced by Alternative A is more than that produced by Alternative B.

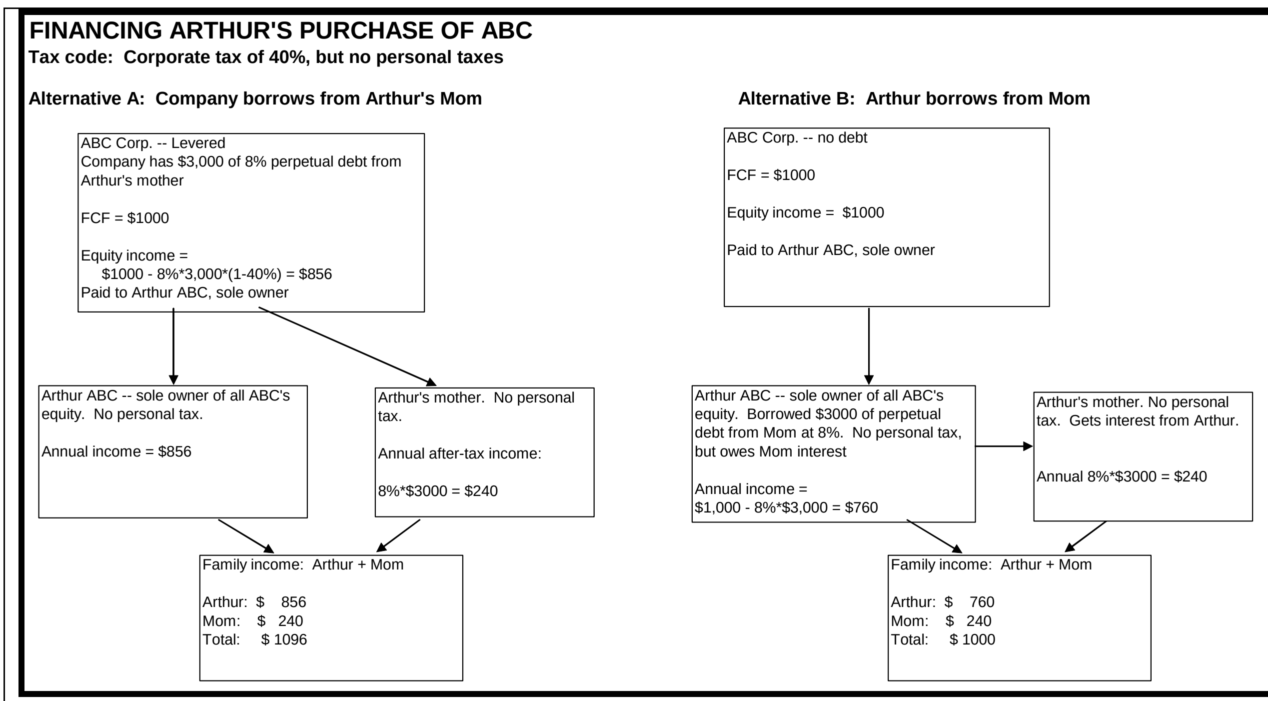


Figure 20.1. Cash flows resulting from two methods of financing the purchase of ABC Corp.

From the family point of view, it is clear that the first alternative is better than the second. In this alternative the family (Arthur + Mom) has an annual income of $1,096, as opposed to the $1,000 in the second alternative. A little thought will reveal why the first alternative is preferable—ABC Corp. has a tax advantage over Arthur with respect to borrowing. It can deduct its interest expenses from its income taxes, so that its net income taxes are only $8\%*3000*(1-40\%)=\$144$. This compares with Arthur's cost for the same loan of $8\%*3,000=240$.

To flesh this out a bit, let's write out some equations:

$$\begin{aligned}
&\textit{Total family income from ABC } (\textit{Arthur}+\textit{Mom}) \\
&= \textit{cash produced by firm} = FCF - \underbrace{r_D * Debt * (1-T_C)}_{\text{Cost of debt to ABC Corp}} + \underbrace{r_D * Debt}_{\text{Income from debt to Mom}} \\
&= FCF + r_D * Debt * T_C
\end{aligned}$$

Thus the total cash produced by the firm for its stockholders and bondholders increases with the amount of debt the firm has. Notice that the total cash produced by the firm does not increase if Arthur borrows the money from his mother.[5]

20.2. Valuing ABC Corp.—taking account of leverage and corporate taxes

Recall that we stated in section 20.1 that ABC Corp's FCFs are worth $5,000 if the company has no leverage:

[5] Looking at the drawing, it's clear why this is so—when Arthur borrows from Mom, the interest is a *wash*: Arthur has an interest expense of $240 and Mom has interest income of $240, for a net $0. When the company borrows from Mom, the company has an interest expense of *(1-40%)*8%*3000=$144*, but Mom has interest income of $240, for a net of $96.

$$
\begin{aligned}
V_U &= Unlevered\ value\ of\ \ ABC \\
&= PV\left(future\ FCFs, discounted\ @\ unlevered\ discount\ rate \right) \\
&= \sum_{t=1}^{\infty} \frac{1,000}{(1.20)^t} = \frac{Annual\ FCF}{r_U} = \frac{1,000}{20\%} = 5,000
\end{aligned}
$$

So how much is the levered version of ABC Corp worth (this is the company which borrows $3,000 from Arthur's Mom). We use the *additivity principle* explained in Chapter 16:

$$
\begin{aligned}
V_L &= Levered\ value\ of\ \ ABC \\
&= Unlevered\ value\ of\ \ ABC + PV\left(additional\ debt\text{-}related\ CFs\right) \\
&= 5,000 + \sum_{t=1}^{\infty} \frac{8\% * 3,000 * 40\%}{(1.08)^t} = 5,000 + \frac{96}{0.08} \\
&= 5,000 + 1,200 = 6,200
\end{aligned}
$$

The Additivity Principle in this context

The additivity principle (Chapter 17) says that *the value of the sum of two cash flow streams is the sum of their values*. In the context of this problem, the two cash flow streams are: 1) The stream of FCFs which derive from the firm's business activities, and 2) The stream of tax shields on the interest paid by the firm.

To value these streams using the additivity principle, we discount each at its appropriate risk-adjusted discount rate. The rate for the FCFs is r_U and the rate for the tax shields—which we assume to be riskless—is the interest rate on the debt r_D .

ABC Corp is worth more as a levered firm than as an unlevered firm because it produces more cash for its owners. The additional cash produced—generated by the fact that the company can deduct the cost of its interest payments from its taxes, whereas Arthur cannot—is worth $1,200. In symbols:

$$V_L = V_U + PV\left(additional\ \ debt\text{-}related\ \ CFs\right)$$

$$= \begin{cases} V_U = \sum_{t=1}^{\infty} \frac{FCF_t}{\left(1+r_U\right)^t} = \sum_{t=1}^{\infty} \frac{1,000}{\left(1.20\right)^t} = \frac{1,000}{20\%} = 5,000 \\ \text{The unlevered value of the firm is} \\ \text{the present value of its free cash flows} \\ \text{discounted an an appropriate (unlevered)} \\ \text{cost of capital } r_U \end{cases}$$

$$+ \begin{cases} PV\begin{pmatrix} Interest\ \ tax \\ shields \end{pmatrix} = \sum_{t=1}^{\infty} \frac{T_C * Interest_t}{\left(1+r_D\right)^t} = \sum_{t=1}^{\infty} \frac{8\% * 3,000 * 40\%}{\left(1.08\right)^t} = \frac{96}{8\%} = 1200 \\ \text{The tax shields created} \\ \text{by the debt are discounted at the interest} \\ \text{rate.} \end{cases}$$

$$= 6,200$$

The cost of equity, $r_E(L)$, and the weighted average cost of capital, WACC, with leverage

The *cost of equity* is the discount rate for the cash flows accruing to shareholders. In Chapters 9 and 16 we discussed the derivation of the cost of equity, stressing its relation to the riskiness of the equity cash flows. In this chapter we use the symbol $r_E(L)$, with the "L" showing that that the cost of equity is related to the leverage of the firm. As you will see, greater leverage leads to a larger $r_E(L)$; the reason for this being that the equity cash flows are riskier when shareholders have promised larger amounts to debtholders.

We now proceed to the computation of $r_E(L)$ for ABC Corp. The levered version of ABC Corp. is worth \$6,200, of which D = \$3,000 is debt. Thus the equity of the company is worth \$3,200. We denote the market value of the equity by E. In order to calculate the firm's cost of equity $r_E(L)$, we first compute the cash flows that the equity owners receive:

$$\begin{aligned}Annual\ equity\ cash\ \ flow &= FCF - after\text{-}tax\ interest\ \ paid\ \ by\ \ ABC\\ &= 1{,}000 - 8\% * 3{,}000 * (1-40\%) = 856\end{aligned}$$

The discounted value of this annual equity cash flow of \$856 is the value of the equity; this defines the cost of equity $r_E(L)$:

$$E = Equity\ value = \sum_{t=1}^{\infty}\frac{equity\ cash\ \ flow_t}{(1+r_E)^t}$$

$$3{,}200 = \sum_{t=1}^{\infty}\frac{856}{(1+r_E)^t} = \frac{856}{r_E}$$

$$\Rightarrow r_E(L) = \frac{856}{3{,}200} = 26.75\%$$

With a little mathematical flimflammery, we can show that:

$$\begin{aligned}r_E(L) &= r_U + [r_U - r_D]\frac{D}{E}(1-T_C)\\ &= \underbrace{20\%}_{\substack{\uparrow\\ \text{r}_\text{U}\text{ is the discount}\\ \text{rate for the FCFs,}\\ \text{which represents}\\ \text{the firm's business}\\ \text{risk}}} + \underbrace{[20\%-8\%]\frac{3{,}000}{3{,}200}(1-40\%)}_{\substack{\uparrow\\ \text{When ABC borrows, it's shareholders}\\ \text{bear an additional } \textit{financial risk}\text{. The}\\ \text{term above represents the financial risk}\\ \text{premium for the equity holders}}} = 26.75\%\end{aligned}$$

We can now compute the WACC:

$$\begin{aligned}WACC &= r_E(L)\frac{E}{E+D} + r_D(1-T_C)\frac{D}{E+D}\\ &= 26.75\%\frac{3{,}200}{3{,}200+3{,}000} + 8\%(1-40\%)\frac{3{,}000}{3{,}200+3{,}000}\\ &= 16.13\%\end{aligned}$$

With a little more “flimflammery” we can show that discounting the FCFs at the WACC gives the total value of the firm:

$$\sum_{t=1}^{\infty}\frac{FCF_t}{(1+WACC)^t} = \sum_{t=1}^{\infty}\frac{1{,}000}{(1+16.13\%)^t} = \frac{1{,}000}{16.13\%} = 6{,}200$$

Here's all of this summarized in a spreadsheet. Note from the title of the spreadsheet that we've given this model a name; we've called it the "Modigliani-Miller model with only corporate tax." To see why the name, refer to the box "Some history of finance (1)" on page000.

	A	B	C
1	**COMPUTING THE WACC IN MODIGLIANI-MILLER MODEL WITH ONLY CORPORATE TAXES**		
2	Annual FCF	1,000	
3	r_U, unlevered cost of capital	20%	
4	D, debt (perpetual)	3,000	
5	r_D, the cost of debt (interest rate)	8%	
6	T_C, corporate tax rate	40%	
7			
8	**Value of firm**		
9	V_U, unlevered value = FCF/r_U	5,000.00	<-- =B2/B3
10	Value of tax shield on interest = T_C*r_D*D/r_D = T_C*D	1,200.00	<-- =B6*B4
11	V_L, levered value of firm = V_U + T_C*D	6,200.00	<-- =B10+B9
12			
13	E, value of equity = V_L - D	3,200.00	<-- =B11-B4
14			
15	Cash flow to equity = FCF - (1-T_C)*interest	856.00	<-- =B2-(1-B6)*B5*B4
16	Return on equity, r_E(L)= [FCF - (1-T_C)*interest]/E	26.75%	<-- =B15/B13
17			
18	WACC = r_E(L)*E/(E+D) + r_D*(1-T_C)*D/(E+D)	16.13%	<-- =B16*B13/B11+(1-B6)*B5*B4/B11
19			
20	**Two checks**		
21	Return on equity, r_E(L)= r_U + (r_U - r_D)*[D/E]*(1-T_C)	26.75%	<-- =B3+(B3-B5)*B4/B13*(1-B6)
22	Value of firm, V_L = FCF/WACC	6,200.00	<-- =B2/B18

We complete this section by restating its major conclusions. If only corporate income is taxed, leverage (borrowing) increases the value of the firm. This increase in value, represented by the present value of the tax shields on the debt, increases the value of the firm's equity, increases the cost of equity r_E, and decreases the WACC.

A summary table is given below:

SUMMARY TABLE—CORPORATE VALUATION WHEN ONLY CORPORATE INCOME IS TAXED

Item	Formula	Why
V_U = Value of unlevered firm	$V_U = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+r_U)^t}$	The value of the unlevered firm is the PV of future FCFs discounted at r_U , the unlevered cost of capital
V_L = Value of the levered firm	$V_L = V_U + PV(interest\ tax\, shields)$ $= V_U + \sum_{t=1}^{N} \frac{T_C * Interest_t}{(1+r_D)^t}$	The value of the levered firm is V_U plus the present value of future interest tax shields. The cell to the left contains the formula for the value of the levered firm when there are *N* interest payments on the debt.
	$V_L = V_U + PV(interest\ tax\, shields)$ $= V_U + \sum_{t=1}^{\infty} \frac{T_C * Interest}{(1+r_D)^t}$ $= V_U + T_C * D$	The cell to the left contains the formula for the levered firm when the firm issues perpetual debt.
E = Value of equity	$V_U - (1-T_C) * D$	The equity value of the levered firm is the value of the levered firm minus the value of the firm's debt: $E = V_L - D = V_U + D * T_C - D = V_U - (1-T_C)D$
D=Value of Debt	D	The value of the debt is the value of the debt. (OK, this ain't so original!)
$r_E(L)$= Cost of equity of the levered firm	$r_E(L) = r_U + [r_U - r_D]\frac{D}{E}(1-T_C)$	The cost of equity r_E is the discount rate for equity cash flows. In a levered firm it includes a *financial risk premium*: $[r_U - r_D]\frac{D}{E}(1-T_C)$
WACC = weighted average cost of capital	$\text{WACC} = \frac{FCF}{V_L}$	You can correctly value the *whole firm* by discounting its FCFs at the WACC. This is the valuation principle we've employed in Chapters 6, 9, and 19.

Figure 20.2: Corporate value and cost of capital corporate income is taxed at rate T_C and when there are no personal taxes.

Some History of Finance (1)

The valuation model summarized in the table above is often called the *Modigliani-Miller* model, after Professors Franco Modigliani and Merton Miller, both winners of the Nobel Prize in Economics. In two path-breaking articles published in 1958 and 1963, Modigliani and Miller showed that the value of the firm would not be affected by the method in which the firm was financed, except where the tax code explicitly favors one form of financing. In the example of ABC Corp. in section 20.2, the tax code gives corporations a tax break on debt financing, whereas individuals (who are untaxed) get no such break; it is therefore optimal for the firm to finance with more debt and less equity.

Students of finance know this result as the "MM model." It has been widely studied and even more widely misunderstood.

In section ??? we consider a variation of the MM model which takes account not only of corporate taxes but also personal taxes. While the logic is the same, the conclusions are very different. This model—less widely studied and even more misunderstood—is known as the Miller model, after Merton Miller, who expounded it in a famous academic article which appeared in the *Journal of Finance* in 1977. (See the box "Some History of Finance (2)" on page000.

20.3. Why debt is valuable in Lower Fantasia—buying a turfing machine

It's easier to understand the theory of the previous section by looking at some numerical examples. In this and the following two sections we discuss several such examples. Each of these examples makes the point that under the Lower Fantasia tax regime—in which corporate income is taxed at a rate T_C, but in which there are no taxes on personal income—companies which finance with debt can increase their market value.

The tax regime in Lower Fantasia is characterized by a tax on corporate income but no other taxes. In the previous section we showed that this tax regime means that the value of companies in Lower Fantasia is increased when they lever themselves.

We start with an example that shows the effect of financing on a capital budgeting decision.

Buying a machine

Wonderturf Corp., a company in Lower Fantasia, is considering purchasing a new turfing machine. The turfing machine costs $100,000; it has a ten-year life, during which it is straight-line depreciated to zero salvage value. In each of the ten years of the machine's life, it will produce sales of $40,000. These sales will cost $15,000 to produce. The result is that the machine has an annual free cash flow of $19,000 per year (see cell B10 below):

$$\begin{aligned}
\textit{Annual Wonderturf FCF} &= \left(1-T_C\right)*\left(\textit{Sales-Expenses}-\textit{Depreciation}\right)+\textit{Depreciation} \\
&= \left(1-40\%\right)*\left(40,000-15,000-10,000\right)+10,000 \\
&= \$19,000
\end{aligned}$$

	A	B	C
1	**THE WONDERTURF TURFING MACHINE**		
2	T_C, corporate tax rate	40%	
3			
4	Machine cost, year 0	100,000	
5			
6	Free cash flow (FCF) calculation		
7	Additional sales, annually	40,000	
8	Additional annual cost of sales	15,000	
9	Annual depreciation	10,000	<-- =B4/10
10	Annual FCF, years 1-10	19,000	<-- =(1-B2)*(B7-B8-B9)+B9
11			
12	r_U, discount rate for machine FCFs	15%	
13			
14	Year	Machine FCF	
15	0	-100,000	<-- =-B4
16	1	19,000	<-- =$B$10
17	2	19,000	
18	3	19,000	
19	4	19,000	
20	5	19,000	
21	6	19,000	
22	7	19,000	
23	8	19,000	
24	9	19,000	
25	10	19,000	
26			
27	Machine NPV	-4,643	<-- =B15+NPV(B12,B16:B25)

The Wonderturf financial wizards have determined that an appropriate risk-adjusted discount rate for the turfing machine's free cash flows is r_U = 15%. Discounting the machine's FCFs at this rate shows that it has a negative NPV of -$4,643 (cell B27). Thus the conclusion is that Wonderturf should not acquire the turfing machine.

However, there's more to this story—read on!

Wonderturf gets a loan to buy the machine

Having from Wonderturf that they don't intend to buy the machine, the turfing machine's manufacturer offers the company a loan of $50,000. The loan's conditions are:

- Interest on the loan is r_D = 8%. This is also the market interest rate.

- The loan's payments in years 1-9 consist of interest only: 8%*50,000 = 4,000. This interest is an expense for tax purposes for Wonderturf, so that the after-tax cost of the interest to the company is (1-40%)*4,000 = $2,400.
- At the end of year 10, Wonderturf must repay the loan principal. In this year, the after-tax cost of the loan to the company is therefore $52,400 (the loan principal plus the after-tax interest).

The Excel table below shows that the loan to Wonderturf has a positive NPV of $10,736.

	D	E	F
12	Loan to buy machine	50,000	
13	r_D, loan interest rate	8%	
14		Loan CFs	
15		50,000	<-- =E12
16		-2,400	<-- =-(1-$B$2)*$E$13*$E$12
17		-2,400	
18		-2,400	
19		-2,400	
20		-2,400	
21		-2,400	
22		-2,400	
23		-2,400	
24		-2,400	
25		-52,400	<-- =-(1-$B$2)*$E$13*$E$12-E12
26			
27	Loan NPV	10,736	<-- =E15+NPV(E13,E16:E25)

The Wonderturf financial wizards now conclude that *it is worthwhile buying the turfing machine if Wonderturf takes the loan*. Their logic is:

$$\begin{aligned} Value(Wonderturf\ \ machine + financing) &= Value(Wonderturf\ \ machine) \ + \ Value(financing) \\ &= \quad -\$4,643 \quad + \quad \$10,736 \\ &= \$6,093 \end{aligned}$$

Here's a spreadsheet which shows their calculations:

	A	B	C	D	E	F
1	THE WONDERTURF TURFING MACHINE					
2	T_C, corporate tax rate	40%				
3						
4	Machine cost, year 0	100,000				
5						
6	Free cash flow (FCF) calculation					
7	Additional sales, annually	40,000				
8	Additional annual cost of sales	15,000				
9	Annual depreciation	10,000	<-- =B4/10			
10	Annual FCF, years 1-10	19,000	<-- =(1-B2)*(B7-B8-B9)+B9			
11						
12	r_U, discount rate for machine FCFs	15%		Loan to buy machine	50,000	
13				r_D, loan interest rate	8%	
14	Year	Machine FCF			Loan CFs	
15	0	-100,000	<-- =-B4		50,000	<-- =E12
16	1	19,000	<-- =$B$10		-2,400	<-- =-(1-$B$2)*$E$13*$E$12
17	2	19,000			-2,400	
18	3	19,000			-2,400	
19	4	19,000			-2,400	
20	5	19,000			-2,400	
21	6	19,000			-2,400	
22	7	19,000			-2,400	
23	8	19,000			-2,400	
24	9	19,000			-2,400	
25	10	19,000			-52,400	<-- =-(1-$B$2)*$E$13*$E$12-E12
26						
27	Machine NPV	-4,643	<-- =B15+NPV(B12,B16:B25)	Loan NPV	10,736	<-- =E15+NPV(E13,E16:E25)
28						
29	**NPV: Machine + Loan**	**6,093**	**<-- =B27+E27**			

As you can see in cell B29, the total value of the machine + loan combination is $6,093.

Where does the positive loan NPV come from?

The above analysis shows that the loan to Wonderturf has an NPV of $10,736. If we analyze this number, we will see that this is exactly the *PV of the tax-shields on the loan interest*:

$$NPV\left(loan\right)=50,000-\frac{\left(1-40\%\right)*4,000}{1.08}-\frac{\left(1-40\%\right)*4,000}{\left(1.08\right)^{2}}-\ldots$$
$$-\frac{\left(1-40\%\right)*4,000}{\left(1.08\right)^{9}}-\frac{\left(1-40\%\right)*4,000-50,000}{\left(1.08\right)^{10}}$$

We now split this expression into two parts:

$$NPV\left(loan\right)=50,000-\frac{4,000}{1.08}-\frac{4,000}{\left(1.08\right)^{2}}-\ldots-\frac{4,000}{\left(1.08\right)^{9}}-\frac{4,000-50,000}{\left(1.08\right)^{10}}$$
$$+\frac{40\%*4,000}{1.08}+\frac{40\%*4,000}{\left(1.08\right)^{2}}+\ldots+\frac{40\%*4,000}{\left(1.08\right)^{9}}+\frac{40\%*4,000}{\left(1.08\right)^{10}}$$

The first line above has value 0 (recall from Chapter 5 that a loan and all its repayments have zero NPV when the discount rate is the loan borrowing rate). The second line above is the PV of the tax shields on the loan interest. Their value is \$10,736:

$$NPV\left(loan\right)=10,736=\frac{40\%*4,000}{1.08}+\frac{40\%*4,000}{\left(1.08\right)^2}+...+\frac{40\%*4,000}{\left(1.08\right)^9}+\frac{40\%*4,000}{\left(1.08\right)^{10}}$$

Thus the NPV of the loan is the *present value of the tax shields on the loan interest payments.*

The Wonderturf result is not surprising!

The second line of Table 20.2 states that the value of a levered company is the sum of the value of the unlevered company *plus* the value of the debt tax shields:

$$\begin{aligned}V_L &= V_U + PV\left(interest\ tax\,shields\right)\\ &= V_U + \sum_{t=1}^{\infty}\frac{T_C * Interest_t}{\left(1+r_D\right)^t}\end{aligned}$$

This is precisely what we've done with our analysis of the Wonderturf turfing machine. For this machine:

$$\begin{aligned}V_L &= the\ value\ of\ the\ machine\ when\ purchased\ with\ a\ loan\\ &= \underbrace{V_U}_{\substack{\text{The value of}\\ \text{the machine's}\\ \text{cash flows}}} + \underbrace{\sum_{t=1}^{\infty}\frac{T_C * Interest_t}{\left(1+r_D\right)^t}}_{\substack{\text{The value of the}\\ \text{tax shields from the}\\ \text{loan interest}}}\\ &= -4,643 \quad + \quad 10,736 \quad = \quad 6,093\end{aligned}$$

20.4. Why debt is valuable in Lower Fantasia—relevering Potfooler Inc.

For our second example of the effect of financing on firm value, we use a question from a Finance 101 final exam at Eastern Lower Fantasia State University. As you'll see it's a fairly long question, with many inter-related parts.[6]

Here's the question: Potfooler, Inc. is a well-known Lower Fantasia company. Here are some facts about the company:

- Potfooler expects to have an annual free cash flow of $2 million at the end of years 1, 2, 3, … forever. Recall that the free cash flow is the after-tax amount of cash that the company generates from its business activities.
- Potfooler currently has 100,000 shares outstanding on the Lower Fantasia stock exchange. The Potfooler share price is $100 per share.
- Potfooler currently has no debt. However, a financial analyst has suggested that the company issue $3,000,000 of perpetual debt and use the proceeds to repurchase shares. The analyst explains that perpetual debt is debt which has only an annual interest payment and which has no return of principal.[7] He suggests that this would be worthwhile for the company, because of the relation $V_L = V_U + T_C D$. The current interest rate on debt in Lower Fantasia is 8%, and the interest payments on the debt will be made annually.

Students on the finance exam were asked to answer the following questions:

[6] The author's colleagues at Eastern Lower Fantasia State University love this question because it's easy to grade. If a student makes a mistake on any part of the question, then the answers on all subsequent parts of the question will also be wrong.

[7] We discussed this concept in Chapter ???. Such debt is sometimes called a *consol*.

Question 1: What is the current market value of Potfooler?

Answer: Potfooler currently has 100,000 shares outstanding, each of which is worth $100. Thus the company's equity value is currently $10,000,000 = $100*100,000. Since the company has no debt, this is also its market value. In short: $V_U = \$10,000,000$.

Question 2: After Potfooler issues $3,000,000 of debt, what will be its market value?

Answer: Since Lower Fantasia has only a corporate income tax, the relation $V_L = V_U + T_C D$ holds. This means that after the company issues its debt, its market value will be $V_L = V_U + T_C D = 10,000,000 + 40\% * 3,000,000 = 11,200,000$.

Question 3: After Potfooler issues debt of $3,000,000 and uses the proceeds to repurchase shares, what will be the company's total equity value, *E*?[8]

Answer: After Potfooler issues the debt and repurchases the shares, the total value of its equity, *E*, plus the total value of its debt, *D*, have to sum to the company's total market value V_L. In short:

$$V_L = E + D = 11,200,000$$

But $D = \$3,000,000$, and therefore:

$$E = V_L - D = 11{,}200{,}000 - 3{,}000{,}000 = 8{,}200{,}000$$

[8] Notice that up to this point in the exam, we haven't stated the price at which Potfooler repurchases the shares. This comes later.

Question 4: At what price will Potfooler repurchase its shares?

Answer: By issuing \$3 million of debt, Potfooler has raised its total market value by \$1,200,000 (from \$10 million to \$11.2 million). This increase in value belongs to all the shareholders. Since there are 100,000 shares outstanding before the share repurchase, this means that each share's price increases by $\frac{\$1,200,000}{100,000} = \$12$. Thus the answer to this question is that the share price for repurchase is \$112: Of this amount \$100 is the share price before the repurchase, and \$12 is the increase in the share price as a result of the debt issue.

Question 5: How many shares will Potfooler repurchase?

Answer: According to the previous question, Potfooler will repurchase its shares at \$112 per share. Since the company has issued \$3 million in debt to repurchase the shares, this means that it will repurchase $\frac{\$3,000,000}{\$112} = 26,785.71$.

Question 6: What was Potfooler's cost of equity before the repurchase of shares?

Answer: Potfooler has an annual free cash flow (FCF) of \$2,000,000. Thus its unlevered cost of equity, $r_E(U) = \frac{FCF}{V_U} = \frac{2,000,000}{10,000,000} = 20\%$.

Question 7: What is Potfooler's cost of equity *after* the repurchase of the shares on the open market?

Answer: Potfooler issues \$3 million in 8% debt in order to repurchase shares. Thus its annual interest bill is 8%*3,000,000 = \$240,000. Since interest is an expense for tax purposes, the company's shareholders will have an annual expected cash flow of:

$$\begin{aligned} \textit{Annual equity cash flow, after debt issuance} &= FCF - (1 - T_C) * interest \\ &= 2{,}000{,}000 - (1 - 40\%) * 240{,}000 \\ &= 1{,}856{,}000 \end{aligned}$$

The value of the equity after the share repurchase is \$8,200,000, so that the cost of equity of the levered company is

$$r_E(L) = \frac{1{,}856{,}000}{8{,}200{,}000} = 22.63\%.$$

Question 8: What is Potfooler's weighted average cost of capital (WACC) *before* the repurchase of the shares?

Answer: Recall the definition of the WACC:

$$WACC = r_E * \frac{E}{E+D} + r_D * (1 - T_C) * \frac{D}{E+D}.$$

The answer to question 8 is easy: Since Potfooler, before the share repurchase, has only equity, its WACC = r_U = *20%*.

Question 9: What is Potfooler's weighted average cost of capital (WACC) *after* the repurchase of the shares?

Answer:

$$WACC = r_E(L)*\frac{E}{E+D} + r_D*(1-T_C)*\frac{D}{E+D}$$
$$= 22.63\%*\frac{8,200,000}{8,200,000+3,000,000} + 8\%*(1-40\%)\frac{3,000,000}{8,200,000+3,000,000} = 17.86\%$$

Question 10: Why is $r_E(L) > r_U$?

Answer: Before Potfooler issued its bonds, the only risk borne by shareholders was the *business risk* inherent in the company's free cash flow. After the company issues its bonds, shareholders have to bear two kinds of risk: business risk *and* financial risk. Thus r_E (L) represents a discount rate for cash flows which are riskier than the discount rate for the FCFs, r_U. Since riskier cash flows have higher discount rates, it follows that $r_E(L) > r_U$.

Question 11: Why does the market value of Potfooler increase after the issuance of the debt and repurchase of the equity?

Answer: By issuing the debt, the shareholders of Potfooler get an additional annual cash flow—the tax shield on the debt interest. This tax shield is riskless, and its value is:

$$Present\ value\ interest\ tax\ shield = \sum_{t=1}^{\infty}\frac{T_C * Interest\ \ payment}{(1+r_D)^t}$$
$$= \frac{T_C * Interest\ \ payment}{r_D} = \frac{T_C * r_D * D}{r_D} = T_C * D$$

The present value of the tax shield accounts for the increase in Potfooler's market value:

$$V_L = \underbrace{V_U}_{\substack{\uparrow \\ \text{Potfooler's value} \\ \text{before the debt} \\ \text{issuance}}} + \underbrace{T_C D}_{\substack{\uparrow \\ \text{The PV of} \\ \text{additional} \\ \text{interest tax} \\ \text{shields}}} .$$

Question 12: Why does the WACC *decrease* after the repurchase?

Answer: After the company issues its debt, it gains an additional cash flow (the tax shield on the interest). This cash flow is riskless. Thus the *average risk* of the company's total cash flows—its FCF plus the interest tax shield—decreases. Since the WACC represents the average riskiness of the company, it decreases.

20.5. Potfooler exam question, second part

Having answered the long exam question of the previous section, students at Eastern Lower Fantasia State University were asked to put the calculations for questions 1-9 into an Excel spreadsheet. Here's the answer:

	A	B	C
1	**POTFOOLER--DEBT ISSUED TO REPURCHASE SHARES**		
2	**Unlevered company**		
3	Annual free cash flow (FCF)	$2,000,000	
4	Number of shares	100,000	
5	Price per share	$100	
6	Total equity value	$10,000,000	<-- =B5*B4
7			
8	Question 1: V_U, unlevered value of Potfooler	$10,000,000	<-- =B6
9			
10	**Levered company**		
11	Debt issued	$3,000,000	
12	Interest rate on debt	8%	
13	T_C, Lower Fantasia corporate tax rate	40%	
14	Question 2: V_L, levered value of Potfooler, $V_L = V_U + T_C$*D	$11,200,000	<-- =B8+B13*B11
15	Question 3: Equity value after share repurchase, E = V_L - D	$8,200,000	
16	Incremental firm value from exchanging equity by debt = V_L - V_U = T_C*D	$1,200,000	<-- =B13*B11
17	Incremental firm value on a per-share basis	$12	<-- =B16/B4
18	Question 4: New share value, after repurchase	$112	<-- =B5+B17
19			
20	Question 5: Number of shares repurchased = [debt used for repurchase]/[new share value]	26,785.71	<-- =B11/B18
21	Number of shares remaining after repurchase = original number of shares minus number of shares repurchased	73,214.29	<-- =B4-B20
22	**Check:** Market value of remaining shares = number of remaining shares * new share value	$8,200,000	<-- =B21*B18
23			
24	Question 6: Potfooler's cost of equity when unlevered, r_U=FCF/V_U	20.00%	
25			
26	Annual interest costs, before taxes	$240,000	<-- =B11*B12
27	Annual equity cash flow, after interest = FCF - (1-TC)*interest	$1,856,000	<-- =B3-(1-B13)*B26
28	Question 7: Potfooler's cost of equity when levered, r_E(L)=[FCF-(1-T_C)*interest]/[value of equity, E]	22.63%	<-- =B27/B22
29			
30	Question 8: Potfooler's WACC before the debt issuance = r_U	20.00%	
31			
32	Question 9: Potfooler's WACC after the debt issuance = r_E(L)*E/(E+D)+r_D*(1-T_C)*D/(E+D)		
33	Percentage of equity in Potfooler = E/(E+D)	73.21%	<-- =B22/B14
34	Percentage of debt in Potfooler = D/(E+D)	26.79%	<-- =B11/B14
35	WACC = r_E(L)*E/(E+D)+r_D*(1-T_C)*D/(E+D)	17.86%	<-- =B28*B33+B12*(1-B13)*B34

This spreadsheet enables us to do some interesting analysis:

What happens if the corporate tax rate T_C = 0%?

When T_C = 0, leverage doesn't change the value of the firm. If you put T_C = 0% into cell B13 of the previous spreadsheet, you'll get a demonstration of this. The spreadsheet is given below, and the analysis follows after the spreadsheet:

	A	B	C
1	**POTFOOLER--DEBT ISSUED TO REPURCHASE SHARES, corporate tax rate = 0%**		
2	**Unlevered company**		
3	Annual free cash flow (FCF)	$2,000,000	
4	Number of shares	100,000	
5	Price per share	$100	
6	Total equity value	$10,000,000	<-- =B5*B4
7			
8	Question 1: V_U, unlevered value of Potfooler	$10,000,000	<-- =B6
9			
10	**Levered company**		
11	Debt issued	$3,000,000	
12	Interest rate on debt	8%	
13	T_C, Lower Fantasia corporate tax rate	0%	
14	Question 2: V_L, levered value of Potfooler, $V_L = V_U + T_C$*D	$10,000,000	<-- =B8+B13*B11
15	Question 3: Equity value after share repurchase, E = V_L - D	$7,000,000	
16	Incremental firm value from exchanging equity by debt = V_L - V_U = T_C*D	$0	<-- =B13*B11
17	Incremental firm value on a per-share basis	$0	<-- =B16/B4
18	Question 4: New share value, after repurchase	$100	<-- =B5+B17
19			
20	Question 5: Number of shares repurchased = [debt used for repurchase]/[new share value]	30,000.00	<-- =B11/B18
21	Number of shares remaining after repurchase = original number of shares minus number of shares repurchased	70,000.00	<-- =B4-B20
22	**Check:** Market value of remaining shares = number of remaining shares * new share value	$7,000,000	<-- =B21*B18
23			
24	Question 6: Potfooler's cost of equity when unlevered, r_U=FCF/V_U	20.00%	
25			
26	Annual interest costs, before taxes	$240,000	<-- =B11*B12
27	Annual equity cash flow, after interest = FCF - (1-TC)*interest	$1,760,000	<-- =B3-(1-B13)*B26
28	Question 7: Potfooler's cost of equity when levered, r_E(L)=[FCF-(1-T_C)*interest]/[value of equity, E]	25.14%	<-- =B27/B22
29			
30	Question 8: Potfooler's WACC before the debt issuance = r_U	20.00%	
31			
32	Question 9: Potfooler's WACC after the debt issuance = r_E(L)*E/(E+D)+r_D*(1-T_C)*D/(E+D)		
33	Percentage of equity in Potfooler = E/(E+D)	70.00%	<-- =B22/B14
34	Percentage of debt in Potfooler = D/(E+D)	30.00%	<-- =B11/B14
35	WACC = r_E(L)*E/(E+D)+r_D*(1-T_C)*D/(E+D)	20.00%	<-- =B28*B33+B12*(1-B13)*B34

- The total value of the company doesn't change (cell B14) when the amount debt (cell B11) changes. In a formula:

$$V_L = V_U + \underbrace{T_C D}_{\substack{\uparrow \\ \text{When } T_C=0\%, \\ \text{this term is zero}}} = V_U$$

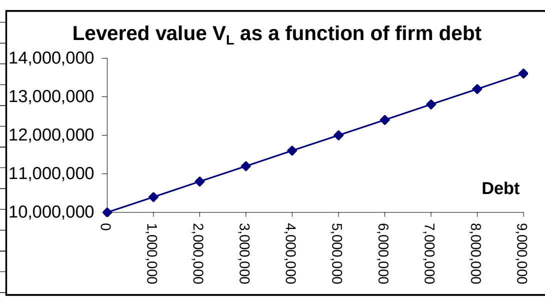
- The company's equity becomes more risky. That is: $r_E(L) > r_U$. You can see this in cell B28: r_E(L) = 25.14% after the debt is issued as opposed to r_U = 20%.

- The company's share price doesn't change. After the issuance of the debt and the repurchase of the equity, the share price is still $100 (cell B18).
- The company's WACC doesn't change. The *average riskiness* of the company's cash flows remains the same:

$$
\begin{aligned}
WACC &= r_E\left(L\right)*\frac{E}{E+D}+r_D*\left(1-T_C\right)*\frac{D}{E+D} \\
&= 25.14\%*\frac{7,000,000}{7,000,000+3,000,000}+8\%*\underbrace{\left(1-0\%\right)}_{\substack{\uparrow \\ \text{Remember that} \\ \text{in this version of the} \\ \text{question } T_C=0\%}}\frac{3,000,000}{7,000,000+3,000,000} \\
&= 20\% = r_U
\end{aligned}
$$

Relate the company's value to different levels of debt

By making a **Data Table** (see Chapter ???), we can make the following table and graph:



	B	C	D	E
38	Debt issued	Value of levered firm, $V_L = V_U + T_C$*D	Cost of equity r_E(L)	WACC
39				
40	0	10,000,000	20.00%	20.00%
41	1,000,000	10,400,000	20.77%	19.23%
42	2,000,000	10,800,000	21.64%	18.52%
43	3,000,000	11,200,000	22.63%	17.86%
44	4,000,000	11,600,000	23.79%	17.24%
45	5,000,000	12,000,000	25.14%	16.67%
46	6,000,000	12,400,000	26.75%	16.13%
47	7,000,000	12,800,000	28.69%	15.63%
48	8,000,000	13,200,000	31.08%	15.15%
49	9,000,000	13,600,000	34.09%	14.71%
50				
51				
52				

20.6. Considering personal as well as corporate taxes—the case of XYZ Corp.

In our story about ABC Corp (Arthur and Mom), the capital structure decision mattered because Lower Fantasia taxes corporations but not individuals. The result is that shareholders (like Arthur) benefit from having companies borrow instead of doing the borrowing themselves.

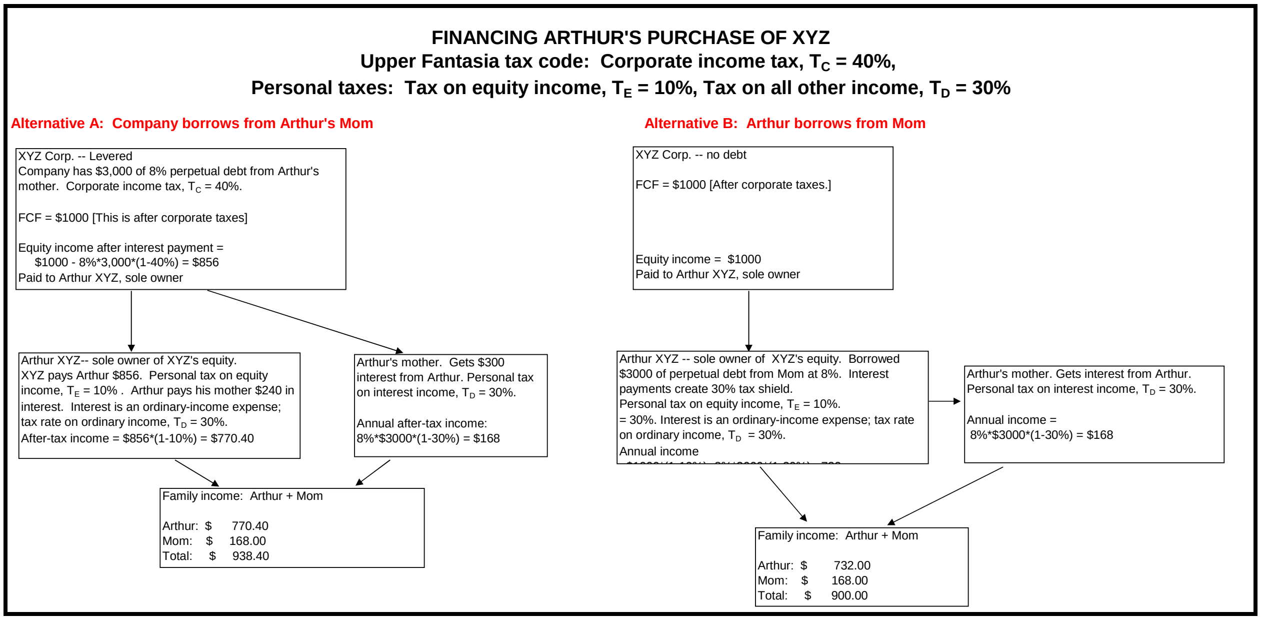
In this section we tell the story of Upper Fantasia, a country very much like Lower Fantasia, but with a somewhat different tax system. Upper Fantasia has 3 kinds of taxes:

- Corporations are subject to a 40% corporate tax rate. We denote this tax rate by T_C.
- Individual income derived from shares (this refers to dividends and capital gains on shares—in the jargon of the Upper Fantasia tax code, this is called "equity income") is subject to a 10% tax rate. The equity tax rate is denoted by T_E .
- All *ordinary income* (this term includes individual income derived from bonds; however it does not include equity income) is subject to a 30% tax rate. We denote this tax rate by T_D . When individuals *pay interest*, they get to deduct the interest payments from their ordinary income.

As before, our mythical entrepreneur, Arthur XYZ, is trying to figure out how to finance his purchase of XYZ Corp. His Mom (bless her!) is always available to lend him money. The questions about the debt are the same as before:

- Should the purchase of the company be financed with debt?
- If so, who should borrow—the company or Arthur?

The diagram below explains the cash flows:



When the company borrows the money, the total family income is $938.40. This compares to the total income of $900 when Arthur borrows the money from Mom. So it's better in this case for the company to borrow the money.

In order to understand what's happening, we create a spreadsheet. We'll have more to say about this spreadsheet (and the economics underlying it) below, but in the meantime, we stress its final conclusion

- Since the total family income (the combined income of Arthur XYZ and his Mom) is larger when the company borrows than when Arthur borrows (cell B27 versus cell C27), the company should lever itself, and not Arthur.
- The advantages of corporate borrowing are considerably less in this case than in the previous case of ABC Corp. In the previous case corporate leverage of $3,000 added $96 to the family cash flows each year; in the current case it adds only $38.40. The difference is, of course, the fact that we now have taxes on personal income, which were absent in the ABC Corp. example.

	A	B	C	D
1	**FINANCING ARTHUR'S PURCHASE OF XYZ** **Upper Fantasia tax code: Corporate income tax, T_C = 40%,** **Personal taxes: Tax on equity income, T_E = 10%, Tax on all other income, T_D = 30%**			
2	**Computing the family income**			
3	T_C, corporate tax rate	40%		
4	T_E, personal equity tax rate	10%		
5	T_D, personal debt tax rate on ordinary income	30%		
6	r_D, interest rate	8%		
7	D, Debt	3,000		
8	FCF, free cash flow (already after corporate taxes)	1,000		
9				
10		**Company borrows**	**Arthur borrows**	
11	FCF, after personal tax	1,000.00	1,000.00	
12	Corporate debt	3,000.00	0.00	
13	Corporate pre-tax interest payment	240.00	0.00	
14	Corporate after-tax interest payment	144.00	0.00	<-- =C13*(1-$B$3)
15	Payout to equity owners	856.00	1,000.00	<-- =C11-C14
16				
17	Arthur's income			
18	Pre-tax equity income from XYZ	856.00	1,000.00	<-- =C15
19	Post-tax equity income from XYZ	770.40	900.00	<-- =C18*(1-$B$4)
20	Arthur's debt	0.00	3,000.00	
21	Arthur's pre-tax interest payment	0.00	240.00	<-- =$B$6*C20
22	Arthur's after-tax interest payment	0.00	168.00	
23	Arthur's post-tax income	770.40	732.00	<-- =C19-C22
24				
25	Mom's pre-tax income	240.00	240.00	<-- =C20*B6
26	Mom's post-tax income	168.00	168.00	<-- =C25*(1-$B$5)
27	Total family income	938.40	900.00	<-- =C23+C26
28				
29	Who should borrow--Arthur or company?	Company	<-- =IF(B27>C27,"Company",IF(B27<C27,"Arthur","Indifferent"))	
30				
31	**Net advantage of corporate debt**			
32	$(1-T_D)-(1-T_E)*(1-T_C)$	0.16		

In order to understand this better, we need some equations:

$$Total\ cash\ produced\ by\ firm = \underbrace{\underbrace{FCF - r_D * Debt * (1-T_C)}_{\text{Dividend to Arthur}} * (1-T_E)}_{\text{Arthur's after-tax dividend}} + \underbrace{r_D * Debt * (1-T_D)}_{\text{Income from debt to Mom}}$$

$$= FCF + r_D * Debt * \underbrace{\left[(1-T_D)-(1-T_E)*(1-T_C)\right]}_{\text{Net tax corporate tax-advantage of debt}}$$

$$In\ this\ case = (1-T_D)-(1-T_C)*(1-T_E) = (1-30\%)-(1-10\%)*(1-40\%) = 16\%$$

The term which makes all the difference is

$$\underbrace{\left(1-T_D\right)}_{\substack{\uparrow \\ \text{After-tax} \\ \text{ordinary income} \\ \text{(including interest)}}} - \underbrace{\underbrace{\left(1-T_E\right)}_{\substack{\uparrow \\ \text{After-tax} \\ \text{equity income}}}\underbrace{\left(1-T_C\right)}_{\substack{\uparrow \\ \text{After-tax} \\ \text{corporate income}}}}_{\substack{\uparrow \\ \text{Net after-tax personal} \\ \text{income from pre-tax corporate} \\ \text{cash flows}}}$$

If this term is positive, as in the previous example (see cell B32), then XYZ corporation should borrow; if it's negative—as in the next example (in which the corporate tax rate is $T_C = 20\%$), then Arthur should borrow and not the firm:

	A	B	C	D
1	**FINANCING ARTHUR'S PURCHASE OF XYZ** **Upper Fantasia tax code: Corporate income tax, T_C = 20%** **(instead of 40% in previous example)** **Personal taxes: Tax on equity income, T_E = 10%,** **Tax on all other income, T_D = 30%**			
2	**Computing the family income**			
3	T_C, corporate tax rate	20%		
4	T_E, personal equity tax rate	10%		
5	T_D, personal debt tax rate on ordinary income	30%		
6	r_D, interest rate	8%		
7	D, Debt	3,000		
8	FCF, free cash flow (already after corporate taxes)	1,000		
9				
10		**Company borrows**	**Arthur borrows**	
11	FCF, after personal tax	1,000.00	1,000.00	
12	Corporate debt	3,000.00	0.00	
13	Corporate pre-tax interest payment	240.00	0.00	
14	Corporate after-tax interest payment	192.00	0.00	<-- =C13*(1-$B$3)
15	Payout to equity owners	808.00	1,000.00	<-- =C11-C14
16				
17	Arthur's income			
18	Pre-tax equity income from XYZ	808.00	1,000.00	<-- =C15
19	Post-tax equity income from XYZ	727.20	900.00	<-- =C18*(1-$B$4)
20	Arthur's debt	0.00	3,000.00	
21	Arthur's pre-tax interest payment	0.00	240.00	<-- =$B$6*C20
22	Arthur's after-tax interest payment	0.00	168.00	
23	Arthur's post-tax income	727.20	732.00	<-- =C19-C22
24				
25	Mom's pre-tax income	240.00	240.00	<-- =C20*B6
26	Mom's post-tax income	168.00	168.00	<-- =C25*(1-$B$5)
27	Total family income	895.20	900.00	<-- =C23+C26
28				
29	Who should borrow--Arthur or company?	Arthur	<-- =IF(B27>C27,"Company",IF(B27<C27,"Arthur","Indifferent"))	
30				
31	**Net advantage of corporate debt**			
32	$(1-T_D)-(1-T_E)*(1-T_C)$	-0.02		

Some Finance History (2)

The Modigliani-Miller model dates from two articles published in 1958 and 1963. In 1977 Merton Miller (half of the MM team), reconsidered the problem of capital structure. He still focused on taxation, but this time considered the case where both corporate and personal income was taxed.

Miller's reasoning, incorporated in our example of XYZ Corp, was that the corporate tax rate T_C gives an advantage to corporations wishing to finance with debt. On the other hand, for individuals equity income is generally taxed at a lower rate T_E than the tax rate T_D on debt income. The primary reason for this is that the major part of income from equity is received by shareholders as capital gains; these are not only taxed at a lower tax rate, but the taxes on capital gains are also *postponable* (as a shareholder, you can decide when to sell your shares and realize your capital gains). This postponability lowers the T_E below the statutory rate (see some discussion in Chapter 21). Thus, Miller reasoned, there is a tradeoff:

- On the corporate level, the deductibility of interest means that corporations produce higher before-personal-tax payouts to stakeholders (bondholders and shareholders) when they have more debt financing.
- On the personal level, giving stakeholders (bond and shareholders) more interest income instead of equity income means taxing them at higher personal rates.

This tradeoff is summarized in the expression $(1-T_D)-(1-T_C)(1-T_E)$:

$$\underbrace{\underbrace{(1-T_D)}_{\substack{\text{Payments to debtholders are}\\ \text{only taxed at the personal tax of}\\ \text{the debtholder, since the firm makes these}\\ \text{payments out of pre-tax income}}} \quad - \quad \underbrace{(1-T_C)*(1-T_E)}_{\substack{\text{Equity income is taxed twice:}\\ \text{once at the firm level (since}\\ \text{payments to shareholders are paid}\\ \text{out of after-tax earnings), and then again at the}\\ \text{personal level}}}}_{\text{On the other hand, } T_E < T_D \text{, so that there is a tradeoff ...}}$$

XYZ CORP--ARTHUR AND MOM'S INCOME

XYZ Corp

After-tax cost of debt: $(1-T_C)r_D$

Each $ of before tax interest paid:
1) Decreases shareholder income by $(1-T_C)$
2) Increases Arthur's Mom's interest payments by $1.

Shareholder Arthur

Equity income taxed at T_E

Each $ of before tax interest paid by XYZ decreases Arthur's income from the company by $(1-T_C)*(1-T_E)$

Bondholder Mom

Interest income taxed at T_D

Each $ of interest paid by the company increases Mom's income by $(1-T_D)$

Family income: Arthur + Mom

Each $ of before-tax interest paid by XYZ increases Mom's income by $(1-T_D)$ and decreases Arthur's by $(1-T_C)*(1-T_E)$.

Net effect: $(1-T_D) - (1-T_E)*(1-T_C)$.

If this is positive, it's good for the family and the firm should increase its borrowing; if its negative, it's bad for the family and the firm should decrease its borrowing.

Miller suggested that *in equilibrium*
$(1-T_D) - (1-T_E)*(1-T_C) = 0$

Figure 20.3. Cash flows of Arthur + Mom's family income. In this flow diagram, Arthur is the shareholder of XYZ Corp., and Mom is the bondholder of XYZ (meaning: she lends the company money). Each $1 of interest income paid to Mom by XYZ Corp. changes the family income by $(1-T_D)-(1-T_E)*(1-T_C)$. If this term is positive, then XYZ Corp.'s borrowing from Mom adds to the family income; if it is negative, then XYZ Corp.'s borrowing detracts from the family income.

20.7. Valuing XYZ Corp.—taking account of leverage and taxes

We redo the calculations in section 20.2, but this time use all the taxes—the corporate tax rate T_C , the personal tax rate on equity income T_E , and the personal tax rate on ordinary income (including interest) T_D. Without leverage XYZ Corp's FCFs are worth $5,000:

$$\begin{aligned} V_U &= \text{Unlevered value of XYZ} \\ &= PV\left(\text{future FCFs, discounted @ unlevered discount rate}\right) \\ &= \sum_{t=1}^{\infty} \frac{1{,}000}{(1.20)^t} = \frac{\text{Annual FCF}}{r_U} = \frac{1{,}000}{20\%} = 5{,}000 \end{aligned}$$

We use the additivity principle to value the levered version of XYZ Corp:

$$V_L = V_U + PV\left(\text{additional debt-related CFs}\right)$$

$$= \left\{ \begin{array}{l} V_U = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+r_U)^t} = \sum_{t=1}^{\infty} \frac{1{,}000}{(1.20)^t} = \frac{1{,}000}{20\%} = 5{,}000 \\ \text{The unlevered value of the firm is} \\ \text{the present value of its free cash flows} \\ \text{discounted an an appropriate (unlevered)} \\ \text{cost of capital } r_U \end{array} \right.$$

$$+ \left\{ \begin{array}{l} PV\begin{pmatrix} \text{Interest tax} \\ \text{shields} \end{pmatrix} = \sum_{t=1}^{\infty} \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]*Interest_t}{\left(1+(1-T_D)r_D\right)^t} \\ \qquad = \sum_{t=1}^{\infty} \frac{8\%*3{,}000*16\%}{\left(1+8\%*(1-30\%)\right)^t} = \frac{38.4}{5.6\%} = 685.71 \\ \text{The tax shields created} \\ \text{by the debt are discounted at the} \\ \text{consumer's after-tax interest rate.} \end{array} \right.$$

$$= 5{,}685.71$$

XYZ Corp is worth more as a levered firm than as an unlevered firm because it produces more cash for its owners when it is levered. The additional cash produced—generated by the fact that the company has a cheaper cost of debt than Arthur—is worth $685.71, which is the present value of the future tax shields on the interest:

$$
\begin{aligned}
PV\begin{pmatrix} Interest\ tax \\ shields \end{pmatrix} &= \sum_{t=1}^{\infty} \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]*Interest}{\left(1+(1-T_D)r_D\right)^t} \\
&= \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]}{(1-T_D)r_D}*Interest \\
&= \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]}{(1-T_D)}*\frac{Interest}{r_D} \\
&= \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]}{(1-T_D)}*D
\end{aligned}
$$

We use the letter T to denote the *debt-valuation factor:* $T = \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]}{(1-T_D)}$. T is the capitalized advantage of debt.[9]

What about the cost of capital—r_E and WACC with leverage?

The levered version of XYZ Corp. is worth $5,685.71, of which $3,000 is debt. Subtracting the value of the debt from the total worth of the company, we see that the equity of the company is worth $2,685.71. In order to calculate the firm's cost of equity r_E , we first compute the after-tax cash flows accruing to the equity owners:

$$
\begin{aligned}
annual\ after\text{-}corporate\text{-}tax\ equity\ cash\ \ flow &= \left[FCF - after\text{-}tax\ interest\ \ paid\ by\ \ XYZ\right] \\
&= \left[1{,}000 - 8\%*3{,}000*(1-40\%)\right] = 856.00
\end{aligned}
$$

The discounted value of this annual equity cash flow of $856.00 is the value of the equity; this defines the cost of equity r_E :

[9] To relate this to the previous case with only corporate taxes, note that when $T_E = T_D = 0$, $T = T_C$.

$$
\begin{aligned}
&Equity\ value = \sum_{t=1}^{\infty} \frac{equity\ cash\ \ flow_t}{\left(1+r_E\right)^t} \\
&2,685.71 = \sum_{t=1}^{\infty} \frac{856.00}{\left(1+r_E\right)^t} = \frac{856.00}{r_E} \\
&\Rightarrow r_E = \frac{856.00}{2685.71} = 31.87\%
\end{aligned}
$$

With a little mathematical flimflammery, we can show that:

$$
\begin{aligned}
r_E &= r_U + \left[r_U * \left(1-T\right) - r_D * \left(1-T_C\right)\right]\frac{D}{E} \\
&= \underbrace{20\%}_{\substack{\uparrow \\ \text{r_U is the discount} \\ \text{rate for the FCFs,} \\ \text{which represents} \\ \text{the firm's business} \\ \text{risk}}} + \underbrace{\left[20\%\left(1-22.86\%\right) - 8\%\left(1-40\%\right)\right]\frac{3,000}{2,685.71}}_{\substack{\uparrow \\ \text{When XYZ borrows, it's shareholders} \\ \text{bear an additional } \textit{financial risk}\text{. The} \\ \text{term above represents the financial risk} \\ \text{premium for the equity holders}}} = 31.87\%
\end{aligned}
$$

We can now compute the WACC:

$$
\begin{aligned}
WACC &= r_E \frac{E}{E+D} + r_D\left(1-T_C\right)\frac{D}{E+D} \\
&= 31.87\% \frac{2,685.71}{2,685.71+3,000} + 8\%\left(1-40\%\right)\frac{3,000}{2,685.71+3,000} \\
&= 17.59\%
\end{aligned}
$$

With a little more “flimflammery” we can show that discounting the FCFs at the WACC gives the total value of the firm:

$$
\sum_{t=1}^{\infty} \frac{FCF_t}{\left(1+WACC\right)^t} = \sum_{t=1}^{\infty} \frac{1,000}{\left(1+17.59\%\right)^t} = \frac{1,000}{17.59\%} = 5,685.71
$$

Here’s all of this summarized in a spreadsheet:

	A	B	C
1	**COMPUTING THE WACC IN THE MILLER MODEL with corporate and personal taxes**		
2	FCF, annual free cash flow (already after corporate taxes)	1,000	
3	r_U, unlevered cost of capital	20%	
4	D, Debt	3,000	
5	r_D, interest rate	8%	
6	T_C, corporate tax rate	40%	
7	T_E, personal equity tax rate	10%	
8	T_D, personal debt tax rate on ordinary income	30%	
9			
10	Tax advantage of debt, $(1-T_D)-(1-T_C)*(1-T_E)$	16.00%	<-- =(1-B8)-(1-B6)*(1-B7)
11	$T= [(1-T_D)-(1-T_C)*(1-T_E)]/(1-T_D)$, tax factor	22.86%	<-- =B10/(1-B8)
12			
13	**Value of firm**		
14	V_U, unlevered value	5,000.00	<-- =B2/B3
15	Value of tax shield on interest	685.71	<-- =B10*B5*B4/((1-B8)*B5)
16	V_L, levered value of firm	5,685.71	<-- =B15+B14
17			
18	E, value of equity	2,685.71	<-- =B16-B4
19			
20	Cash flow to equity	856.00	<-- =B2-(1-B6)*B5*B4
21	Return on equity, $r_E(L)$	31.87%	<-- =B20/B18
22			
23	WACC	17.59%	<-- =B21*B18/B16+(1-B6)*B5*B4/B16
24			
25	**Three checks**		
26	Return on equity, $r_E(L) = r_U + [r_U*(1-T) - r_D*(1-T_C)]*D/E$	31.87%	<-- =B3+(B3*(1-B11)-B5*(1-B6))*B4/B18
27	Value of firm, V_L = FCF/WACC	5,685.71	<-- =B2/B23
28	Value of firm, $V_L = V_U + T*D$	5,685.71	<-- =B14+B11*B4

Summarizing this section

We complete this section by restating its major conclusions. If corporate income is taxed, and if the tax system differentiates between income derived from equity and ordinary income, then leverage (borrowing) may increase or decrease the value of the firm, depending on the sign of the tax factor $(1-T_D)-(1-T_E)*(1-T_C)$.

A summary table is given below:

SUMMARY TABLE—CHANGING LEVERAGE WHEN CORPORATE AND PERSONAL INCOME ARE TAXED

Symbols: Corporate tax rate: T_C, personal tax rate on equity income T_E, personal tax rate on ordinary income T_D

Tax advantage of debt = $(1-T_D)-(1-T_C)*(1-T_E)$, Tax factor: $T=\frac{(1-T_D)-(1-T_C)*(1-T_E)}{(1-T_D)}$

Item	Formula	Why
V_U = Value of unlevered firm	$V_U=\sum_{t=1}^{\infty}\frac{FCF_t}{(1+r_U)^t}$	The value of the unlevered firm is the PV of future FCFs discounted at r_U, the unlevered cost of capital
V_L = Value of the levered firm	$V_L=V_U+PV(net\ interest\ tax\,shields)$ $=V_U+\sum_{t=1}^{N}\frac{[(1-T_D)-(1-T_E)(1-T_C)]*Interest_t}{(1+r_D(1-T_D))^t}$ Another way to write this is $V_L=V_U+T*D$, where	The value of the levered firm is V_U plus the present value of future interest tax shields. When there are both corporate and personal taxes, the PV of the tax shields is given by: $\sum_{t=1}^{N}\frac{[(1-T_D)-(1-T_E)(1-T_C)]*Interest_t}{(1+r_D(1-T_D))^t}$
	$V_L=V_U+PV(net\ interest\ tax\,shields)$ $=V_U+\sum_{t=1}^{\infty}\frac{[(1-T_D)-(1-T_E)(1-T_C)]*Interest}{(1+r_D(1-T_D))^t}$ $=V_U+T*D,$ $where\ T=\frac{(1-T_D)-(1-T_E)(1-T_C)}{(1-T_D)}$	The cell to the left contains the formula for the value of the levered firm when the firm issues perpetual debt. This formula is the same as the parallel formula in Figure 20.2 for the case where $T_E=T_D=0$. In the general case where personal taxes are perhaps not zero, $T=\frac{(1-T_D)-(1-T_E)(1-T_C)}{(1-T_D)}$ can be positive, negative, or zero.
E = Value of equity	$E=V_U-(1-T)D$	The equity value of the levered firm = $E=V_L-D=V_U-(1-T)D$
D=Value of Debt	D	
$r_E(L)$= Cost of equity of the levered firm	$r_E(L)=r_U+[r_U(1-T)-r_D(1-T_C)]\frac{D}{E}$	
WACC = weighted average cost of capital	WACC=$\frac{FCF}{V_L}$	

Figure 20.4 . Corporate value and cost of capital when corporate income is taxed at rate T_C, personal ordinary income is taxed at rate T_D, and personal equity income is taxed at rate T_E .

20.8. Buying a sturfing machine in Upper Fantasia

In this section and the next we return to the examples of sections 20.3 and 20.4. This time we do these examples for a company in Upper Fantasia, where, as you will recall there are three tax rates:

- In Upper Fantasia corporate income is taxed at the rate T_C = 40%
- Personal income from equity (meaning: dividends and capital gains) is taxed at rate T_E = 10%
- Personal income from all other sources is taxed at rate T_D = 30%

Sonderturf considers buying a sturfing machine

Sonderturf Corp., a company is Upper Fantasia, is considering purchasing a new sturfing machine. The sturfing machine costs \$100,000; it has a ten-year life, during which it is straight-line depreciated to zero salvage value. In each of the ten years of the machine's life, it will produce sales of \$40,000. These sales will cost \$15,000 to produce. The result is that the machine has an annual free cash flow of \$19,000 per year.

The Sonderturf financial wizards have determined that an appropriate risk-adjusted discount rate for the sturfing machine's free cash flows is r_U = 15%. Discounting the machine's FCFs at this rate shows that it has a negative NPV of -\$4,643. Thus the conclusion is that Sonderturf should not acquire the sturfing machine. (For details of these calculations, refer to section 20.3, page000.)

Sonderturf gets a loan to buy the machine

Having heard the bad news from Sonderturf, the sturfing machine's manufacturer offers the company a loan of $50,000. The loan's conditions are exactly the same as those of the loan in section 20.??? which was offered to Sonderturf in Lower Fantasia: In years 1-9, Sonderturf will pay only interest ($4,000), and in year 10 it will pay interest of $4,000 as well as repay the loan principal.

It follows from Figure 20.4 that the value of the loan is

$$PV\begin{pmatrix} loan\ in\ Upper\ Fantasia,\ where\ there\ are\ corporate\ income \\ taxes\ T_C, taxes\ on\ equity\ income\ T_E, \\ and\ taxes\ on\ ordinary\ income\ T_D \end{pmatrix} =$$

$$PV\left(net\ interest\ tax\ shields\right) = \sum_{t=1}^{10} \frac{\left[\left(1-T_D\right)-\left(1-T_E\right)\left(1-T_C\right)\right]*Interest_t}{\left(1+r_D\left(1-T_D\right)\right)^t}$$

$$= \sum_{t=1}^{10} \frac{\left[\left(1-30\%\right)-\left(1-10\%\right)\left(1-40\%\right)\right]*\$4,000}{\left(1+r_D\left(1-30\%\right)\right)^t} = \$4,801$$

The Sonderturf financial wizards conclude that the company should now purchase the machine, taking the loan to finance part of the purchase. They calculate that:

$$\begin{aligned} NPV\left(machine+loan\right) &= NPV\left(machine\right)+NPV\left(loan\right) \\ &= NPV\left(machine\right)+PV\underbrace{\left(loan\ interest\ tax\ shields\right)}_{\substack{\uparrow \\ \text{In Upper Fantasia the tax shield} \\ \text{takes account of corporate as well as} \\ \text{personal taxes:} \\ \sum_{t=1}^{10}\frac{\left[(1-T_D)-(1-T_E)(1-T_C)\right]*Interest_t}{\left(1+(1-T_D)*r_D\right)^t}}} \\ &= -\$4,643 \qquad + \qquad \$4,801 \\ &= \$158 \end{aligned}$$

The calculations are shown in the following Excel spreadsheet:

	A	B	C	D	E	F
1			THE SONDERTURF STURFING MACHINE			
2	T_C, corporate tax rate	40%				
3	T_E, personal tax rate on equity	10%				
4	T_D, personal tax rate on debt	30%				
5						
6	Machine cost, year 0	100,000				
7						
8	Free cash flow (FCF) calculation					
9	Additional sales, annually	40,000				
10	Additional annual cost of sales	15,000				
11	Annual depreciation	10,000	<-- =B6/10			
12	Annual FCF, years 1-10	19,000	<-- =(1-B2)*(B9-B10-B11)+B11			
13						
14	Discount rate for machine FCFs	15%		Loan to buy machine	50,000	
15				r_D, loan interest rate	8%	
16				Net annual advantage of debt financing, $(1-T_D)-(1-T_E)*(1-T_C)$	16%	<-- =(1-B4)-(1-B3)*(1-B2)
17						
18	Year	Machine FCF			Tax advantage of interest	
19	0	-100,000	<-- =-B6			
20	1	19,000	<-- =$B$12		640	<-- =$E$16*$E$15*$E$14
21	2	19,000			640	<-- =$E$16*$E$15*$E$14
22	3	19,000			640	
23	4	19,000			640	
24	5	19,000			640	
25	6	19,000			640	
26	7	19,000			640	
27	8	19,000			640	
28	9	19,000			640	
29	10	19,000			640	
30						
31	Machine NPV	-4,643	<-- =B19+NPV(B14,B20:B29)	Loan NPV	4,801	<-- =E19+NPV(E15*(1-B4),E20:E29)
32						
33	**NPV: Machine + Loan**	**158**	**<-- =B31+E31**			

In Upper Fantasia debt is not always valuable!

The Lower Fantasia tax system—which has only a corporate tax T_C but no other taxes on personal income—*always makes it more valuable to finance with debt.* You can see this from the following formula drawn from Figure 20.2, which holds in Lower Fantasia:

$$V_L^{Lower\ Fantasia} = V_U + PV\left(interest\ \ tax\,shields\right) = V_U + \sum_{t=1}^{\infty} \frac{T_C * Interest_t}{\left(1+r_D\right)^t} > V_U \ .$$

The same formula in Upper Fantasia—with its more complicated (but more realistic) tax system which combines a corporate income tax T_C with a personal tax on equity income T_E and a personal tax on ordinary income T_D—is given by:

$$V_L^{Upper\ Fantasia} = V_U + PV\left(interest\ \ tax\,shields\right) = V_U + \sum_{t=1}^{N} \frac{\left[\left(1-T_D\right)-\left(1-T_E\right)*\left(1-T_C\right)\right]*Interest_t}{\left(1+\left(1-T_D\right)r_D\right)^t}$$

The last expression need not always be positive. For example:

$$\sum_{t=1}^{N} \frac{\left[(1-T_D)-(1-T_E)*(1-T_C)\right]*Interest_t}{\left(1+(1-T_D)r_D\right)^t} > 0 \quad if \quad (1-T_D)-(1-T_E)*(1-T_C) > 0$$

$$\sum_{t=1}^{N} \frac{\left[(1-T_D)-(1-T_E)*(1-T_C)\right]*Interest_t}{\left(1+(1-T_D)r_D\right)^t} = 0 \quad if \quad (1-T_D)-(1-T_E)*(1-T_C) = 0$$

$$\sum_{t=1}^{N} \frac{\left[(1-T_D)-(1-T_E)*(1-T_C)\right]*Interest_t}{\left(1+(1-T_D)r_D\right)^t} < 0 \quad if \quad (1-T_D)-(1-T_E)*(1-T_C) < 0$$

The conclusion is that in Upper Fantasia, financing with debt need not make a project more valuable. Suppose, for example, that T_C = 40%, T_E = 3%, and T_D = 50%. Then the spreadsheet below shows that financing the sturfing machine with debt *decreases* the NPV:

	A	B	C	D	E	F
1	**THE SONDERTURF STURFING MACHINE** **different taxes make debt disadvantageous!**					
2	T_C, corporate tax rate	40%				
3	T_E, personal tax rate on equity	3%				
4	T_D, personal tax rate on debt	50%				
5						
6	Machine cost, year 0	100,000				
7						
8	Free cash flow (FCF) calculation					
9	Additional sales, annually	40,000				
10	Additional annual cost of sales	15,000				
11	Annual depreciation	10,000	<-- =B6/10			
12	Annual FCF, years 1-10	19,000	<-- =(1-B2)*(B9-B10-B11)+B11			
13						
14	Discount rate for machine FCFs	15%		Loan to buy machine	50,000	
15				r_D, loan interest rate	8%	
16				Net annual advantage of debt financing, $(1-T_D)-(1-T_E)*(1-T_C)$	-8%	<-- =(1-B4)-(1-B3)*(1-B2)
17						
18	Year	Machine FCF			Tax advantage of interest	
19	0	-100,000	<-- =-B6			
20	1	19,000	<-- =$B$12		-328	<-- =$E$16*$E$15*$E$14
21	2	19,000			-328	<-- =$E$16*$E$15*$E$14
22	3	19,000			-328	
23	4	19,000			-328	
24	5	19,000			-328	
25	6	19,000			-328	
26	7	19,000			-328	
27	8	19,000			-328	
28	9	19,000			-328	
29	10	19,000			-328	
30						
31	Machine NPV	-4,643	<-- =B19+NPV(B14,B20:B29)	Loan NPV	-2,660	<-- =E19+NPV(E15*(1-B4),E20:E29)
32						
33	**NPV: Machine + Loan**	**-7,304**	**<-- =B31+E31**			

20.9. Relevering Smotfooler Inc., an Upper Fantasia company

In Section 20.4 we offered a question from a Finance 101 exam at Eastern Lower Fantasia State University. This section offers a similar question from an exam at Upper Fantasia University (their football team is called the Ufus).

Here's the question: Smotfooler, Inc. is a well-known Upper Fantasia company. Here are some facts about the company:

- Smotfooler expects to have an annual free cash flow of $2 million at the end of years 1, 2, 3, … forever. Recall that the free cash flow is the after-tax amount of cash that the company generates from its business activities.
- Smotfooler currently has 100,000 shares outstanding on the Upper Fantasia stock exchange. The Smotfooler share price is $100 per share.
- Smotfooler currently has no debt. However, a financial analyst has suggested that the company issue $3,000,000 of perpetual debt and use the proceeds to repurchase shares. The current interest rate on debt in Upper Fantasia is 8%, and the interest payments on the debt will be made annually.
- Tax rates in Upper Fantasia are: $T_C = 40\%$, $T_D = 30\%$, $T_E = 10\%$.

Students on the finance exam were asked to answer the following questions:

Question 1: What is the current market value of Smotfooler?

Answer: Smotfooler currently has 100,000 shares outstanding, each of which is worth $100. Thus the company's equity value is currently $10,000,000 = $100*100,000. Since the company has no debt, this is also its market value. In short: $V_U = \$10,000,000$.

Question 2: After Smotfooler issues $3,000,000 of debt, what will be its market value?

Answer: Since Upper Fantasia has only a corporate income tax, the relation $V_L = V_U + T\,D$ holds, where:

$$T = \frac{(1-T_D)-(1-T_C)*(1-T_E)}{(1-T_D)} = \frac{(1-30\%)-(1-40\%)(1-10\%)}{(1-30\%)} = 22.86\%.$$

(See also cell B7 on the spreadsheet below).

This means that after the company issues its debt, its market value will be

$V_L = V_U + T\,D = 10,000,000 + 22.86\% * 3,000,000 = 10,685,714$.

Question 3: After Smotfooler issues debt of $3,000,000 and uses the proceeds to repurchase shares, what will be the company's *total equity value, E?*

Answer: After Potfooler issues the debt and repurchases the shares, the total value of its equity, *E*, plus the total value of its debt, *D*, have to sum to the company's total market value V_L. In short:

$$V_L = 10,685,714 = E + D$$

But $D = \$3,000,000$, and therefore:

$$E = 10,685,714 - 3,000,000 = 7,685,714$$

Question 4: At what price will Smotfooler repurchase its shares?

Answer: By issuing $3 million of debt, Smotfooler has raised its total market value by $685,714 (from $10 million to $10,685,714). This increase in value belongs to all the shareholders. Since there are 100,000 shares outstanding before the share repurchase, this means that each share's price increases by $\frac{\$685,714}{100,000} = \$6.86$. Thus the answer to this question is that the share price for repurchase is $106.86. Of this amount $100 is the share price before the repurchase, and $6.86 is the increase in the share price as a result of the debt issue.

Question 5: How many shares will Smotfooler repurchase?

Answer: According to the previous question, Smotfooler will repurchase its shares at $106.86 per share. Since the company has issued $3 million in debt to repurchase the shares, this means that it will repurchase $\frac{\$3,000,000}{\$106.86} = 28,074.87$.

Question 6: What was Smotfooler's cost of equity before the repurchase of shares?

Answer: Smotfooler has an annual free cash flow (FCF) of $2,000,000. Thus its unlevered cost of equity, $r_E(U) = r_U = \frac{FCF}{V_U} = \frac{2,000,000}{10,000,000} = 20\%$.

Question 7: What is Smotfooler's cost of equity *after* the repurchase of the shares on the open market?

Answer: Smotfooler issues $3 million in 8% debt in order to repurchase shares. Thus its annual interest bill is 8%*8,000,000 = $240,000. Since interest is an expense for tax purposes, the company's shareholders will have an annual expected cash flow of:

$$\begin{aligned} \textit{Annual equity cash flow, after debt issuance} &= FCF - (1 - T_C) * \textit{interest} \\ &= 2,000,000 - (1 - 40\%) * 240,000 \\ &= 1,856,000 \end{aligned}$$

The value of the equity after the share repurchase is $7,685,714, so that the cost of equity of the levered company is

$$r_E(L) = \frac{1,856,000}{7,685,714} = 24.15\% .$$

Note from Figure 20.4. that there's another way to do this calculation:

$$r_E(L) = r_U + \left[r_U (1-T) - r_D (1-T_C) \right] \frac{D}{E} =$$

$$= 20\% + \left[20\% (1-22.86\%) - 8\% (1-40\%) \right] \frac{3,000,000}{7,685,714} = 24.15\%$$

Question 8: What is Smotfooler's weighted average cost of capital (WACC) *before* the repurchase of the shares?

Answer: Recall the definition of the WACC:

$$WACC = r_E(L) * \frac{E}{E+D} + r_D * (1-T_C) * \frac{D}{E+D} .$$

The answer to question 8 is easy: Since Smotfooler, before the share repurchase, has only equity, its WACC = r_U = *20%*.

Question 9: What is Smotfooler's weighted average cost of capital (WACC) *after* the repurchase of the shares?

Answer:

$$WACC = r_E(L) * \frac{E}{E+D} + r_D * (1-T_C) * \frac{D}{E+D}$$

$$= 24.15\% * \frac{7,685,714}{7,685,714 + 3,000,000} + 8\% * (1-40\%) \frac{3,000,000}{7,685,714 + 3,000,000} = 18.72\%$$

Question 10: Why is $r_E(L) > r_U$?

Answer: Before Smotfooler issued its bonds, the only risk borne by shareholders was the *business risk* inherent in the company's free cash flow. After the company issues its bonds, shareholders have to bear two kinds of risk: business risk *and* financial risk. Thus r_E(L) represents a discount rate for cash flows which are riskier than the discount rate for the FCFs, r_U. Since riskier cash flows have higher discount rates, it follows that $r_E(L) > r_U$.

Question 11: Why does the market value of Smotfooler increase after the issuance of the debt and repurchase of the equity?

Answer: By issuing the debt, Smotfooler increases the amount of cash it produces by $\left[(1-T_D)-(1-T_C)*(1-T_E)\right]*Interest\ \ payment$ for every year which it has debt. This additional cash flow is riskless. Since the holders of riskless cash flows in Upper Fantasia use a discount rate of $(1-T_D)*r_D$ to value the cash flows, it follows that:

$$\begin{aligned}
Value\ of\ \ additional\ \ debt\text{-}related\ \ cash\ \ flows &= \sum_{t=1}^{\infty} \frac{\left[(1-T_D)-(1-T_C)*(1-T_E)\right]*Interest\ \ payment}{\left(1+(1-T_D)r_D\right)^t} \\
&= \frac{(1-T_D)-(1-T_C)*(1-T_E)}{(1-T_D)r_D} Interest\ \ payment \\
&= \frac{(1-T_D)-(1-T_C)*(1-T_E)}{(1-T_D)r_D}*r_D D = \underbrace{T}_{T=\frac{(1-T_D)-(1-T_C)*(1-T_E)}{(1-T_D)}} *D
\end{aligned}$$

The present value of the tax shield accounts for the increase in Smotfooler's market value:

$$V_L = \underbrace{V_U}_{\text{Smotfooler's value before the debt issuance}} + \underbrace{T\,D}_{\text{The PV of additional debt-related cash flows}} .$$

Question 12: Does debt always increase corporate value in Upper Fantasia?

Answer: No. It depends on the sizes of the three tax rates T_C, T_D, and T_E. In the example below, there is a net *tax disadvantage* to debt—by issuing debt, Smotfooler *lowers* its market value and *raises* its WACC:

	A	B	C
1	**SMOTFOOLER--DEBT ISSUED TO REPURCHASE SHARES** **Smotfooler is located in Upper Fantasia**		
2	**Upper Fantasia tax system**		
3	T_C, Upper Fantasia corporate tax rate	40%	
4	T_E, Upper Fantasia personal tax rate on equity income	10%	
5	T_D, Upper Fantasia personal tax rate on ordinary income	30%	
6	Annual debt advantage: $(1-T_D)-(1-T_E)*(1-T_C)$	16%	<-- =(1-B5)-(1-B4)*(1-B3)
7	PV of debt advantage: $T = [(1-T_D)-(1-T_E)*(1-T_C)]/(1-T_D)$	22.86%	<-- =B6/(1-B5)
8			
9	**Unlevered company**		
10	Annual free cash flow (FCF)	$2,000,000	
11	Number of shares	100,000	
12	Price per share	$100	
13	Total equity value	$10,000,000	<-- =B12*B11
14			
15	Question 1: V_U, unlevered value of Smotfooler	$10,000,000	<-- =B13
16			
17	**Levered company**		
18	Debt issued	$3,000,000	
19	Interest rate on debt	8%	
20	Question 2: V_L, levered value of Smotfooler, $V_L = V_U + T*D$	$10,685,714	<-- =B15+B7*B18
21	Question 3: Equity value after share repurchase, $E = V_L - D$	$7,685,714	
22	Incremental firm value from exchanging equity by debt = $V_L - V_U = T*D$	$685,714	<-- =B20-B15
23	Incremental firm value on a per-share basis	$7	<-- =B22/B11
24	Question 4: New share value, after repurchase	$106.86	<-- =B12+B23
25			
26	Question 5: Number of shares repurchased = [debt used for repurchase]/[new share value]	28,074.87	<-- =B18/B24
27	Number of shares remaining after repurchase = original number of shares minus number of shares repurchased	71,925.13	<-- =B11-B26
28	**Check:** Market value of remaining shares = number of remaining shares * new share value	$7,685,714	<-- =B27*B24
29			
30	Question 6: Smotfooler's cost of equity when unlevered, $r_U=FCF/V_U$	20.00%	
31			
32	Annual interest costs, before taxes	$240,000	<-- =B18*B19
33	Annual equity cash flow, after interest = FCF - $(1-T_C)$*interest	$1,856,000	<-- =B10-(1-B3)*B32
34	Question 7: Smotfooler's cost of equity when levered, $r_E(L)=[FCF-(1-T_C)*interest]/[value\ of\ equity,\ E]$	24.15%	<-- =B33/B28
35	Note: See formula in row 44 below for another way to compute the levered cost of equity		
36			
37	Question 8: Smotfooler's WACC before the debt issuance = rU	20.00%	
38			
39	Question 9: Smotfooler's WACC after the debt issuance $= r_E(L)*E/(E+D)+r_D*(1-TC)*D/(E+D)$		
40	Percentage of equity in Smotfooler = E/(E+D)	71.93%	<-- =B28/B20
41	Percentage of debt in Smotfooler = D/(E+D)	28.07%	<-- =B18/B20
42	$WACC = r_E(L)*E/(E+D)+r_D*(1-T_C)*D/(E+D)$	18.72%	<-- =B34*B40+B19*(1-B3)*B41
43			
44	Additional formula: $r_E(L)=r_U+[r_U*(1-T)-r_D*(1-T_C))*D/E$	24.15%	<-- =B30+(B30*(1-B7)-B19*(1-B3))*B18/B21

20.10. Is there really an advantage to debt?

In this chapter we've laid out the theory of capital structure. We can answer the question of the importance of capital structure in several ways:

Method 1: What are the relevant tax rates T_C, T_D, T_E?

As you can see, the value of XYZ Corp. is critically dependent on 2 factors:

- r_U, the risk-adjusted rate of return for the free cash flows. This rate is unaffected by the capital structure, since the free cash flows are operating cash flows and do not depend on the financing of the firm.
- $(1-T_D)-(1-T_C)(1-T_E)$--the relative after-tax costs of debt versus equity income.

Looking at this second parameter, we examine several cases. In the case below, the anticipated dividend yield of 2% is taxed at 40% while the anticipated capital gains yield of 6% is taxed at 10%. The equity tax rate is 17.5%, and the net tax advantage of debt over equity is 8.02%:

	A	B	C
1	**WHAT ARE THE RELATIVE TAX EFFECTS**		
2	Corporate tax rate, T_C	37%	
3			
4	Anticipated equity tax		**Tax rate**
5	Dividend yield	2.00%	40%
6	Capital gains yield	6.00%	10%
7			
8	Net after-tax yield	6.60%	<-- =B5*(1-C5)+B6*(1-C6)
9	Before tax yield	8.00%	<-- =B5+B6
10			
11	Personal tax rate on equity income, T_E	17.50%	<-- =1-B8/B9
12	Personal tax rate on ordinary income, T_D	40.00%	
13			
14	Tax advantage of debt over equity: $(1-T_D)-(1-T_C)*(1-T_E)$	8.02%	<-- =(1-B12)-(1-B2)*(1-B11)

With a somewhat different yield and tax configuration there is actually a net tax *disadvantage* to debt:

	A	B	C
1	**WHAT ARE THE RELATIVE TAX EFFECTS**		
2	Corporate tax rate, T_C	37%	
3			
4	Anticipated equity tax		**Tax rate**
5	Dividend yield	0.00%	40%
6	Capital gains yield	6.00%	0%
7			
8	Net after-tax yield	6.00%	<-- =B5*(1-C5)+B6*(1-C6)
9	Before tax yield	6.00%	<-- =B5+B6
10			
11	Personal tax rate on equity income, T_E	0.00%	<-- =1-B8/B9
12	Personal tax rate on ordinary income, T_D	40.00%	
13			
14	Tax advantage of debt over equity: $(1-T_D)-(1-T_C)*(1-T_E)$	-3.00%	<-- =(1-B12)-(1-B2)*(1-B11)

Below you will see a third case in which only corporate income is taxed. In this case there is an overwhelming advantage to debt financing:

	A	B	C
1	**WHAT ARE THE RELATIVE TAX EFFECTS**		
2	Corporate tax rate, T_C	37%	
3			
4	Anticipated equity tax		**Tax rate**
5	Dividend yield	5.00%	0%
6	Capital gains yield	0.00%	0%
7			
8	Net after-tax yield	5.00%	<-- =B5*(1-C5)+B6*(1-C6)
9	Before tax yield	5.00%	<-- =B5+B6
10			
11	Personal tax rate on equity income, T_E	0.00%	<-- =1-B8/B9
12	Personal tax rate on ordinary income, T_D	0.00%	
13			
14	Tax advantage of debt over equity: $(1-T_D)-(1-T_C)*(1-T_E)$	37.00%	<-- =(1-B12)-(1-B2)*(1-B11)

Method 2: What's the evidence in firm behavior?

Instead of asking whether tax rates support a net tax advantage, we can also look at different firms. We can ask whether in a particular industry there is a consistent behavior towards debt. The answer is no, as you will see in Chapter 20. As you will see in Chapter 20, we interpret this "inconsistent" behavior as evidence in favor of the argument that there is no net tax advantage to debt—that is, that firm financial policy does not affect its market value.

Method 3: What does sophisticated finance research say?

Chapter 20 looks at the latest academic research on the capital structure question. Our reading of this research is that the importance of debt over equity financing has been heavily overemphasized in finance textbooks. There may be a small advantage of debt over equity, but it is overwhelmed by the overall uncertainty of valuing a firm.

Summary and conclusion—United Widgets Corporation

United Widgets is a new company set up by John and Cindy, who are pondering the effect of the equity/debt financing mix. The question they have in mind is—does it matter whether the company is financed with share capital (equity) or with money borrowed from a bank (debt)? The risk-return tradeoff between the two financing alternatives is complex:

- The providers of equity financing are promised a share of the firm's profits (if there are any). If there are no profits, then shareholders will not get any dividends; although they will surely be disappointed, they cannot use the non-payment of dividends to force the firm into bankruptcy.
- The providers of debt financing are promised a series of fixed payments. If United Widgets cannot keep the commitment of making the fixed payments, then the company may become insolvent. Bankruptcy will affect the shareholders of the company, denying them their share in United Widgets.

- Debt financing is generally cheaper than equity financing: The riskiness of the interest payments promised by United Widgets to its lenders is less than the riskiness of the dividend payments promised by the company to its shareholders. In addition interest is a tax-deductible expense for United Widgets, whereas dividends have to be paid out of after-tax income. Shareholders, being at greater risk than lenders, will therefore demand a *higher expected return* than debtholders. The relative cheapness of debt versus equity appears to make debt preferable as a financing mechanism. But:
- Debt financing makes equity financing even more risky. The risky dividend stream which comes from the company is endangered even further when shareholders promise debtholders a series of future payments. The higher the amount of debt the firm has, the more risky the equity financing becomes.[10]

Realizing all these factors, John and Cindy ask themselves the following questions:

- Does the debt/equity mix affect the amount of cash that can be extracted from United Widgets?
- Does the mix of equity and debt affect the discount rate that United Widgets should use for discounting project cash flows? As we have seen in Chapters 6, 14, and 19, the relevant discount rate is the weighted average cost of capital (WACC).
- Does the debt/equity mix affect the cost of equity?

The next pages give schematic answers to these questions. The remainder of the chapter explains these answers (on first reading you may want to skip this and go on to the body of the

[10] John and Cindy briefly considered financing their firm with *only debt*. But this is impossible!

chapter). Chapter 20 explores some empirical results and tries to give you a “take” on how to apply the theoretical answers developed in this chapter.

Financing United Widgets—Capital Structure and Its Effects on Cost of Capital and Firm Valuation

UNITED WIDGETS

John and Cindy set up a new company--United Widgets, Inc. They decide to buy a widget machine because financial analysis shows that the NPV of the machine's cash flows is positive.

United Widgets is financed with equity (meaning: money put up by John and Cindy and their friends) and debt (money borrowed from the bank).

Does the equity/debt financing mix change the discount rate used to evaluate widget machines?

Does the equity/debt financing mix change the *total cash* extracted from the company?

EFFECT OF DEBT/EQUITY MIX ON WEIGHTED AVERAGE COST OF CAPITAL (WACC)

1. If there are no taxes, the debt/equity mix does not affect the widget-machine discount rate.

2. If there are only corporate taxes and no personal taxes, then more debt means that the widget discount rate decreases.

3. If both personal and corporate incomes are taxed, widget machine discount rates can increase/decrease/stay same when the debt/equity mix changes.

EFFECT OF DEBT/EQUITY MIX ON TOTAL CASH EXTRACTED FROM COMPANY

1. If there are no taxes, the debt/equity mix does not affect the total amount of cash extracted from the company.

2. If there are only corporate taxes and no personal taxes, then more debt means more cash extracted from the company; happens because the tax system subsidizes debt (interest is an expense for tax purposes).

3. If both personal and corporate incomes are taxed, the cash extracted from the company can go up or down: Companies enjoy a tax subsidy on their interest payments (since interest is an expense for tax purposes). But shareholders pay lower taxes on earnings from equity (because of an advantageous capital gains tax) than on interest earnings from debt.

EFFECT OF DEBT/EQUITY MIX ON COST OF COST OF EQUITY AND WACC

More debt in the debt/equity mix <u>*always*</u> makes equity riskier! The equity owners have to pay debt holders before they pay themselves and this increases their risk.

The effect of capital structure on WACC depends on the mix of corporate and personal taxes:

1. If there are no taxes, WACC is unaffected by capital structure: the increase in the cost of equity as the debt/equity mix increases exactly offsets the savings of cheaper debt.

2. If there are only corporate taxes, WACC decreases when more debt is used to finance the firm.

3. If there are both corporate and personal taxes, WACC can increase/decrease/stay same. Empirical evidence (Chapter 21) seems to indicate that it doesn't change much.

$$WACC = r_E(L)\frac{E}{E+D} + r_D(1-T_C)\frac{D}{E+D}$$

where:

$r_E(L) =$ cost of equity (increases when debt/equity ratio $\uparrow$)

$r_D =$ cost of debt

$E =$ market value of firm's equity

$D =$ market value of firm's debt

$T_C =$ corporate tax rate

Figure 20.5

Exercises

1. Go back to the supermarket example from the beginning of the chapter. Assume that the supermarket after-tax operating income is $120,000 each year. If Mortimer's Group took a $500,000 loan in 9% annual interest rate and its tax rate is 30%, what will be the return on equity (ROE) for Mortimer's group and Joanne's group. $\left(ROE = \frac{Profit\ after\ tax}{Equity} \right)$?

Mortimer's Supermarket Group Half equity (50%) and half debt (50%)			
Supermarket	$1,000,000	Debt	$500,000
		Equity	$500,000
Total assets	**$1,000,000**	**Total debt and equity**	**$1,000,000**
Joanna's Supermarket Group Only equity (100%)			
Supermarket	$1,000,000	Debt	$0
		Equity	$1,000,000
Total assets	**$1,000,000**	**Total debt and equity**	**$1,000,000**

2.

2.a. Repeat Exercise 1 with the following balance sheets (assume that the debt still bears a 9% interest rate):

Half equity (50%) and half debt (50%)			
Supermarket	$1,200,000	Debt	$600,000
		Equity	$600,000
Total assets	**$1,200,000**	**Total debt and equity**	**$1,200,000**
Joanna's Supermarket Group Only equity (100%)			
Supermarket	$1,000,000	Debt	$0
		Equity	$1,000,000
Total assets	**$1,000,000**	**Total debt and equity**	**$1,000,000**

2.b. Show in a **Data Table** and a Excel chart the sensitivity of the ROE to the equity/debt ratio.

3. You are interested in buying a warehouse for your firm. The warehouse costs $350,000 and using it will save the firm $50,000 annually forever. The firm can borrow any amount of money at an 8% annual interest rate; all money borrowed is "perpetual debt"—meaning that the firm pays only the annual interest payment and never returns the debt principal. The firm's tax rate is 40%.

What will be the firm's additional annual income and its return on equity (ROE) on the investment in the following four cases?

3.a. The firm finances the purchase with equity only.

3.b. The firm finances the purchase with 75% equity and 25% debt.

3.c. The firm finances the purchase with 50% equity and 50% debt.

3.d. The firm finances the purchase with 20% equity and 80% debt.

4.

4.a. Repeat Exercise 3 and show the total annual amounts received by the firm's share holders and debt-holders.

4.b Show in a **Data Table** and an Excel chart the change in the total amount received by the firm's share holders and debt holders as a function of the equity invested in the project.

5. Eddy is the sole owner of his firm. He now wishes to purchase the company next door for $600,000. His calculations show that the annual income before tax from the purchase is $80,000.

He is considering two financing alternatives: The first is to ask for a personal loan of $300,000 and pay the remaining amount from his savings. The second alternative is to finance the purchase by having his firm take the $300,000 loan. Assuming the interest rate on the loan is 9% (for infinite duration) and the corporate tax rate is 40%, what will be the total amount received by the firm's share holders and debt holders in each scenario, assuming that only the interest paid by Eddy's firm is an expense for tax purposes.

6. Returning to the previous exercise: What is the value of the firm Eddy wishes to buy under the two financing alternatives?

7. Annie owns a "shell firm"—this is a firm which is incorporated but has no activity whatsoever. Annie's shell firm is about to buy another firm for $900,000. The firm she is purchasing has an annual free cash flow (FCF) of $120,000 each year.

7.a. Annie's bank is willing to give her a perpetual loan equal to half of the purchase amount at 8% interest. Assuming Annie's firm has no debt and its tax rate is T_C = 30%, what will be her firm's value after the purchase:

- In case it will finance the purchase with equity only.
- In case it takes the loan.

7.b. What will be the firm's value in case the loan is repaid in 20 equal repayments?

8. Section 20.3 gives two formulas for the cost of equity $r_E(L)$ of a levered firm for the case when there are only corporate taxes:

$$r_E(L) = \frac{Annual\ equity\ cash\ flow}{Value\ of\ equity}$$

$$r_E(L) = r_U + [r_U - r_D]\frac{D}{E}(1 - T_C)$$

Use both these formulas to find the cost of equity $r_E(L)$ for the following cases:

8.a. The cost of equity $r_E(L)$ for the firm Eddy is buying in Exercise 5.

8.b. The cost of equity $r_E(L)$ of Amadeus Supermarket in Exercise 1.

8.c. The cost of Equity $r_E(L)$ of Annie's firm from Exercise 7.

9. Section 20.3 gives two formulas for the weighted average cost of capital (WACC) of a levered firm for the case when there are only corporate taxes:

$$WACC = r_E(L)\frac{E}{E+D} + r_D(1 - T_C)\frac{D}{E+D}$$

$$WACC = \frac{FCF}{V_L}$$

Use both these formulas to find the WACC for the following cases:

8.a. The WACC for the firm Eddy is buying in Exercise 5.

8.b. The WACC of Amadeus Supermarket in Exercise 1.

8.c. The WACC of Annie's firm from Exercise 7.

10. "Sandy-Candy," a hot new chewing gum company is for sale for $2,000,000. Henry is interested in buying the company and is exploring various financing alternatives. He knows that the interest rate on debt is r_D =9%, corporate tax rate is T_C = 36% and the cost

of capital of the purchase is r_U = 12%. Henry estimates that 'Sandy-Candy' has a free cash flow (FCF) of $300,000 each year.

10.a. What will be the market value of Sandy Candy if Henry does not take a loan?

10.b. What will be the market value of Sandy Candy if Henry takes a $1,200,000 loan. Assume that the loan is paid by out of Sandy Candy's earnings, and that the interest is an expense for tax purposes.

10.c. What will be Sandy Candy's cost of equity r_E for the two cases above?

10.d. What will be the Sandy Candy's WACC for the two cases above?

11. Debby, the owner of Oxford Corporation, has decided that it's time to make some changes to the firm's capital structure. She estimates that Oxford's FCF is $150,000 each year and that this FCF can be expected to recur annually forever. The company has not debt and has 30,000 shares outstanding, each of which is currently worth $50.

Debby wants Oxford to borrow $600,000 of perpetual debt and to use the proceeds to repurchase shares. Assuming the interest rate on debt is r_D = 6% and the corporate tax rate is T_C = 30%, calculate the following changes:

11.a. What is Oxford's market value before it issued debt?

11.b. What is Oxford's market value after it issued debt?

11.c. What will be Oxford share price after the debt issuance?

11.d. How many shares will be repurchased?

11.e. What is Oxford's equity value after the repurchase of the shares?

11.f. What is Oxford's cost of equity after the repurchase and dividend payment?

11.g. What is Oxford WACC after the repurchase and dividend payment?

12. XYZ Corp. is about to borrow $100,000. The terms of the loan specify an annual equal repayment of principal in each of the next 8 years. The loan rate is r_D = 8%, and XYZ has a corporate tax rate of T_C = 40%. If the loan interest is an expense for tax purposes for XYZ, and if there are no other taxes besides corporate taxes, what will be the increase in XYZ's market value?

13. Go back to the exercise of buying the turfing machine (Section 20.3). Repeat the exercise assuming the loan is repaid in 10 equal payments. What is the NPV of the investment now?

14.

14.a. According to a recent tax reform in Lower Fantasia, the personal tax rate on all ordinary income except capital gains from stocks was changed from 0% to 25%. Capital gains will henceforth be taxed at 15%. The Lower Fantasia corporate tax rate remains unchanged at 40%. Assuming you plan to take a loan, what will be better – to borrow using your firm or take a personal loan? Show the net advantage of corporate debt in this case.

14.b. Will your answer to 14.a. change if the corporate tax rate becomes 20%?

15.

15.a. Eddy, from Exercise 5, needs your help again. He didn't purchase the firm since the bank didn't approved him the loan, but now his dad is willing to step in and help him by loaning him the same amount ($300,000). In addition, after the recent elections he's now facing a personal tax rate of 40% (equal to the corporate tax rate) and a 15% tax on equity income. What should he do – finance the purchase using a firm or take a personal loan? Calculate the total amount received by the stakeholders (shareholders and debt holders).

15.b. Assuming Eddy purchases the firm next door using his own firm, calculate the value of the firm, his cost of equity and the WACC (assume his unlevered discount rate is 12%).

16. Assume that the corporate tax rate is T_C = 30%, the equity income tax rate is T_E = 10%. What is the ordinary income tax rate T_D for which an investor will be indifferent between choosing a personal loan or a loan using a firm?

17.

17.a. Repeat exercise 11 (Oxford Corporation) assuming the ordinary income tax rate is T_D = 34% and the personal equity tax rate is T_E = 15%.

17.b. For this case calculate the "net advantage of corporate debt" and calculate the expression $T = \dfrac{(1-T_D)-(1-T_E)(1-T_C)}{(1-T_D)}$.

18. You are interested in buying a machine that will produce sales of $50,000 in each of the next six years. The machine costs $120,000 and has a six year life. It is straight line depreciated to a zero salvage value. In addition, the machine activity costs $18,000 annually. The discount rate you decided to use for the machine's FCF is 12%.

You are considering taking a 9%, six-year, loan to finance the purchase of the machine. The loan amount will be $70,000. The loan terms specify annual payments of interest only in years 1-5 and the repayment of the whole principal in year 6. Assuming that the corporate tax rate is T_C = 40%, the personal tax rate (on ordinary income) is T_D = 22% and the equity tax rate is T_E = 15%, answer the following:

18.a. What is the machine FCF?

18.b. What is the NPV of the machine if it is financed with equity only?

18.b. Calculate the "net advantage of corporate debt," *T*.

18.c. What is the NPV of the machine if it is financed with a mix of equity and debt?

19.

19.a. Fill in the following Excel sheet:

	A	B	C
1	**FILL IN THE TAX EFFECTS**		
2	Corporate tax rate, T_C	36%	
3			
4	Anticipated equity tax		**Tax rate**
5	Dividend yield	2.50%	40%
6	Capital gains yield	5.00%	10%
7			
8	Net after-tax yield	**??**	
9	Before tax yield	**??**	
10			
11	Personal tax rate on equity income, T_E	**??**	
12	Personal tax rate on ordinary income, T_D	**??**	
13			
14	Tax advantage of debt over equity: $(1-T_D)-(1-T_C)*(1-T_E)$	**??**	

19.b. Show in a graph the change in "net advantage of corporate debt" as a function of the personal tax rate.

CHAPTER 21: THE EVIDENCE ON CAPITAL STRUCTURE*

this version: February 8, 2004

Chapter contents

Overview

Chapter 17 discussed the theory of capital structure, which concerns itself with the effects of financing on the valuation of assets. Capital structure theory asks whether, all other factors being the same, firms which are more highly leveraged are worth more than firms with less leverage.

In Chapter 17 we suggested that the importance of capital structure depends on how it affects the ability of the corporation to extract cash from its operating and its financial activities. If, by increasing its leverage, a corporation can increase the total amount of cash it pays to its shareholders and bondholders, then it should do so. If, on the other hand, increasing leverage does not change the amount of cash paid to shareholders and bondholders, then increased leverage is not worthwhile.

In Chapter 17 we related the corporate ability to extract cash from a corporation's activities to the trade-off between personal and corporate taxation: Corporate borrowing is tax deductible (since interest is an expense for tax purposes); this tends to favor corporations with more rather than less debt in their capital structures. On the other hand, a corporation with more debt in its capital structure channels more of its income to bondholders rather than to shareholders, and bondholders have a higher tax rate on their interest income than do shareholders on their equity income.

To see why the Chapter 17 discussion of leverage is important, suppose for a moment that firms with more debt are worth more than similar but less-levered firms. Then we would suggest to corporate managers the following steps:

- Corporate managers should strive to increase the amount of debt used in financing corporate activities. If, for example, a firm builds a new plant, then it should try to borrow the maximal amount it can to build the plant.
- Corporate managers should minimize the amount of cash they have on hand (subject, of course, to operational and safety considerations). If leverage (that is, paying interest on debt) adds to value, then holding cash (that is, having an asset which earns interest) is a detriment to value.
- Corporate managers should increase the corporate dividend payments. By paying out dividends, managers decrease the amount of cash on hand and thus increase the effective leverage of the firm.
- For the same reason corporate managers should increase share repurchases, which decrease the amount of cash on hand and thus increase effective leverage.

The bullets above tell a manager how she should operate if leverage is a positive value driver. If, on the other hand, leverage is a negative value driver—meaning that more leverage decreases corporate value—then the manager should take the opposite actions. And if—as we suggested at the end of Chapter 17—leverage is a neutral value drive because the tax benefits of corporate leverage are offset by the tax disadvantages of leverage at the personal taxation level, then none of the above matters.

As you can see, leverage theory can have significant operative implications.

What do we do in this chapter?

Chapter 17 was largely theoretical. In this chapter, on the other hand, we discuss the market evidence on capital structure. We ask whether we see—in market prices, cost of capital, and market risk measures—evidence for or against the positive effects of more debt on the value of firms.

In section 21.1 we summarize the results of Chapter 17. The upshot of these results is that the effects of financing on valuation depend largely on the tax system. Roughly speaking, if firms, by borrowing, can increase the total cash flow available to shareholders and bondholders, then the firms should move towards a more leveraged capital structure.

The remaining sections of the chapter ????

Finance concepts discussed

- What are some facts about capital structure (how do firms capitalize?)
- Does capital structure affect the value of the firm?
- Does capital structure affect the cost of capital?
- Are there other important considerations? Bankruptcy costs, control, etc.
- How do you measure the firm's unlevered cost of capital r_U ?
- How do you compute the WACC for an *industry*?

Excel functions used

- **Average**
- **Stdev**
- **Regression (trendline)**

21.1. Summarizing the theory

The theory of capital structure outlined in the previous chapter says that the effect of capital structure on the value of the firm is primarily due to tax considerations. Very roughly speaking, if firms enjoy interest tax deductibility which is unavailable to their shareholders, then firms should borrow and increase their debt/equity ratios. This theory—the "Modigliani-Miller" theory (Chapter 17, sections ???-???)—should be contrasted with the "Miller model" (Chapter 17, sections ??? - ???) which postulates that the advantage of corporate debt is to some extent offset by the tax advantage of equity to investors.

These are complex concepts which we illustrated with two simple examples (Arthur ABC and Arthur XYX) in the previous chapter. We sum up:

1. Leverage adds value to a firm if the *capitalized value of the interest tax shields* is positive:

$$\begin{aligned} V_L &= V_U + PV\left(Capitalized\ interest\ tax\ shields\right) \\ &= PV\left(FCFs, discounted\ at\ r_U\right) + \sum_{t=1}^{\infty} \frac{\left[\left(1-T_D\right)-\left(1-T_E\right)*\left(1-T_C\right)\right]*Interest_t}{1+\left(1-T_D\right)r_D} \end{aligned}$$

Here:

$$\begin{aligned} T_C &= the\ corporate\ tax\ rate \\ T_E &= the\ personal\ tax\ rate\ on\ equity\ income \\ T_D &= the\ personal\ tax\ rate\ on\ ordinary\ income\ (including\ interest) \end{aligned}$$

2. Assuming that a firm is contemplating a permanent change in its capital structure,

$$PV\left(Capitalized\ interest\ tax\ shields\right)=\sum_{t=1}^{\infty}\frac{\left[\left(1-T_D\right)-\left(1-T_E\right)*\left(1-T_C\right)\right]*Interest}{1+\left(1-T_D\right)r_D}$$
$$=\frac{\left[\left(1-T_D\right)-\left(1-T_E\right)*\left(1-T_C\right)\right]*r_D*\Delta Debt}{\left(1-T_D\right)r_D}$$
$$=\frac{\left[\left(1-T_D\right)-\left(1-T_E\right)*\left(1-T_C\right)\right]*\Delta Debt}{\left(1-T_D\right)}=T*\Delta Debt$$
$$where\ T=\frac{\left[\left(1-T_D\right)-\left(1-T_E\right)*\left(1-T_C\right)\right]}{\left(1-T_D\right)}$$

3. In the classic Modigliani-Miller theory, which invokes only corporate taxes, $T=T_C$, so that debt always adds to value. In Miller's more complex model, which takes into account both personal and corporate taxes, T can be positive, negative, or zero, depending on the sign of $\left(1-T_D\right)-\left(1-T_E\right)\left(1-T_D\right)$. Miller hypothesized that $\left(1-T_D\right)-\left(1-T_E\right)\left(1-T_D\right)=0$; if this is so, then there would be no advantage to debt over equity financing.

4. Leverage affects both the weighted average cost of capital (WACC) and the cost of equity r_E:

Weighted average cost of capital, *WACC*	$WACC=\frac{E+D*\left(1-T\right)}{E+D}*r_U$	If debt adds value (i.e., $T > 0$), leverage decreases the WACC
Cost of equity of a levered firm, r_E	$r_E=r_U+\left[r_U*\left(1-T\right)-r_D*\left(1-T_C\right)\right]\frac{D}{E}$	More debt *always* makes equity more risky and increases the cost of equity r_E. The amount by which the equity becomes more risk depends on the relative sizes of T and T_C.
Cost of unlevered capital, r_U	$r_U=\frac{r_D*D*\left(1-T_C\right)+r_E*E}{E+D*\left(1-T\right)}$	Often we estimate a firm's cost of equity; this formula lets you back out the unlevered cost of capital from r_E.

5. Contrary to the formula in 2 above, the value of debt interest tax shields is not the only factor in determining the effect on firm value of a change in debt. Three other prominent factors

discussed by academics and practitioners are: bankruptcy costs, the costs of financial control (change name), and the option effects associated with debt. These costs are difficult to quantify, but they certainly exist:

5.a. Costs of financial distress ("bankruptcy costs"): Increasing a firm's leverage also makes it more likely that a firm will have a greater future probability of getting into financial trouble. The present value of the costs of getting out of this trouble (they should be called "costs of financial distress," but they are usually call termed "bankruptcy costs") should be deducted from the benefits of additional leverage.[1]

5.b. Costs of financial control. Borrowers will usually lend the firm more money only if they can exercise more control. Often this control involves debt covenants. These are restrictions imposed by the lender on the firm. For example, the Giant Industries bond issue discussed in section 15.4 (page000) has the following covenants:

> "The Indentures . . . contain restrictive covenants that, among other things, restrict the ability of the Company and its subsidiaries to create liens, to incur or guarantee debt, to pay dividends, to repurchase shares of the Company's common stock, to sell certain assets or subsidiary stock, to engage in certain mergers, to engage in certain transactions with affiliates or to alter the Company's current line of business."

---

[1] Empirical research in finance estimates bankruptcy costs as generally less than 10% of the face value of debt at the time of bankruptcy. If the Modigliani-Miller full tax shield on debt were to hold, it is unlikely that bankruptcy costs of this magnitude would retard corporate desires for more leverage. A recent paper (Timothy Fisher and M. Jocelyn Martel, "On Direct Bankruptcy Costs and the Firm's Bankruptcy Decision" (January 2001). http://ssrn.com/abstract=256128) gives interesting information of the size of bankruptcy and liquidation costs in Canada.

5.b. Option effects of debt: The shareholders in a heavily indebted firm have less to lose than those in a low-leverage firm. They may thus feel free to take more risks. Increased leverage may thus affect the riskiness of the firm's free cash flows (FCF). An example: Bob and Jerry each own a similar building; the market value of each of their buildings is $100,000. The buildings are in need of a very expensive repair. Bob owns his building outright, whereas Jerry has a $99,000 mortgage on his building. Bob is much more likely to do the repairs, since he has more to lose; Jerry might well reason that in the worst case if something happens to his building, he'll default on his mortgage and let the bank take care of the problems.[2]

6. Finally, it may that firms are limited in their borrowing by the kinds of assets they own. If lenders require loan collateral, then firms with many fixed assets may be more easily able to borrow than firms with more "ephemeral" assets. Thus, even if Modigliani-Miller are right, and firms want to borrow as much as possible, it may be that software firms (with fewer tangible assets) are less able to borrow than real estate firms.

21.2. How do firms capitalize?

One way to think about capital structure is to look the actual capital structures for different companies and industries. As an example consider Abbott Laboratories, a major American pharmaceutical firm: On 20 March 2002, Abbott's balance sheets showed debt of

[2] Lenders know all about option effects. It causes them to restrict their lending and also to impose covenants on the borrowers.

approximately \$8.7 billion and equity of \$10.7 billion. Taking these book values debt and equity, Abbott had a book value debt/equity ratio of 0.81:

$$\textit{Abbott Labs, book value, debt-equity ratio} = \frac{Debt}{Equity} = \frac{8.7}{10.7} = 0.81$$

The book value of Abbott’s equity understates its market value. On 20 March 2002, Abbott had 1,563,436,372 shares outstanding; the market price per share was \$51.80. Multiplying these two numbers together gives the market value of Abbott’s equity as \$81 billion, so that Abbott had a market value debt/equity ratio of 0.108:

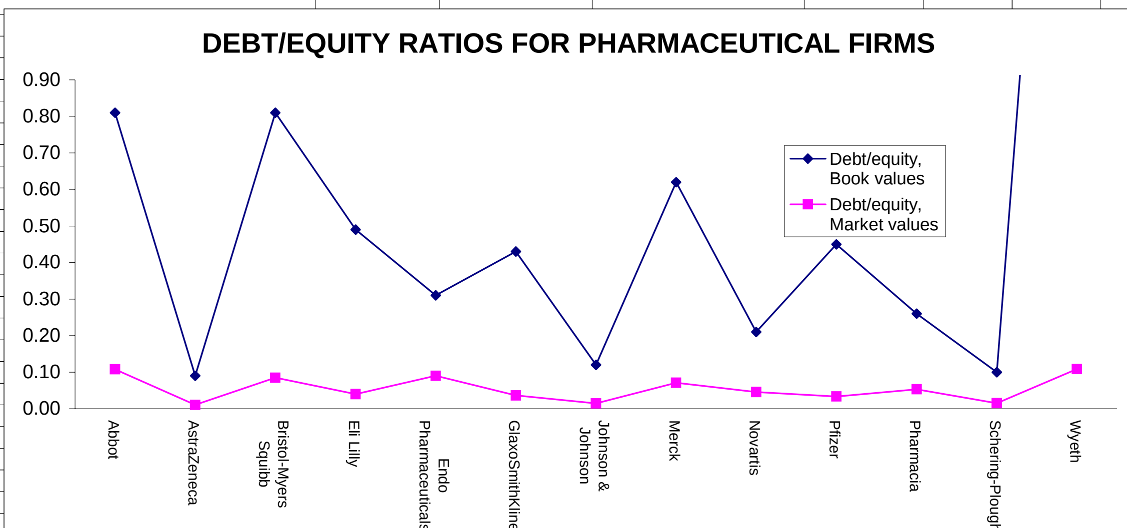
$$\textit{Abbott Labs, market value, debt-equity ratio} = \frac{Debt}{Equity} = \frac{8.7}{81.0} = 0.11$$

The debt-equity ratio of pharmaceutical firms

In the spreadsheet below we calculate the debt/equity ratio in both book and market values for major pharmaceutical companies.

DEBT/EQUITY RATIOS FOR MAJOR DRUG COMPANIES
Source: Yahoo, 20mar02

	Debt/equity, Book values	Debt/equity, Market values
Abbot	0.81	0.11
AstraZeneca	0.09	0.01
Bristol-Myers Squibb	0.81	0.08
Eli Lilly	0.49	0.04
Endo Pharmaceuticals	0.31	0.09
GlaxoSmithKline	0.43	0.04
Johnson & Johnson	0.12	0.01
Merck	0.62	0.07
Novartis	0.21	0.05
Pfizer	0.45	0.03
Pharmacia	0.26	0.05
Schering-Plough	0.10	0.01
Wyeth	2.91	0.11



Several things are clear from this data:

- The average market debt/equity ratio for these firms is very small.
- The variability in debt/equity ratios is very large. It does not appear that drug companies appear to be striving for a common debt/equity ratio, whether measured by book or market values.

Can we learn something from this data for pharmaceutical firms? To the author of this book, it appears that there is no evidence that pharmaceuticals are striving for any target debt/equity ratio. If, as we showed in Chapter 17 and section 21.1 above, firm targeting of debt/equity ratios depends on the tax system, then the lack of a clear debt/equity pattern for pharmaceuticals indicates that the tax effects of debt/equity ratios are relatively neutral. In a word: The debt/equity ratios of the pharmaceutical sector are consistent with Merton Miller's hypothesis that $(1-T_D)-(1-T_C)*(1-T_E)=0$, so that there are no net tax benefits to either maximizing or minimizing the corporate debt/equity ratio.

The debt-equity ratio of other industries

How does the pharmaceutical industry compare to retail grocery stores? As the graph below shows, grocery chains appear to have much higher debt/equity ratios than pharmaceutical firms. Having said this, the variation in debt/equity ratios for groceries is enormous. As for drug companies, there appears no evidence of a general trend:

	A	B	C	D	E	F	G	H	I
1	**Debt/Equity ratios for Retail Grocery Companies Source: Yahoo, 20mar02**								
2		**Debt/equity, Book values**	**Debt/equity, Market values**						
3	Ahold (AHO)	2.0100	0.4408						
4	Albertson's Inc. (ABS)	0.9300	0.4189						
5	AMCON Distributing Co. (DIT)	3.2700	3.6333						
6	Arden Group, Inc. (ARDNA)	0.0900	0.0333						
7	BAB, Inc. (BABB.OB)	0.4800	2.8235						
8	B								
9	C								
10	C								
11	D								
12	D								
13	F								
14	F								
15	F								
16	F								
17	G								
18	G								
19	H								
20	I								
21	I								
22	K								
23	M								
24	M								
25	N								
26	P								
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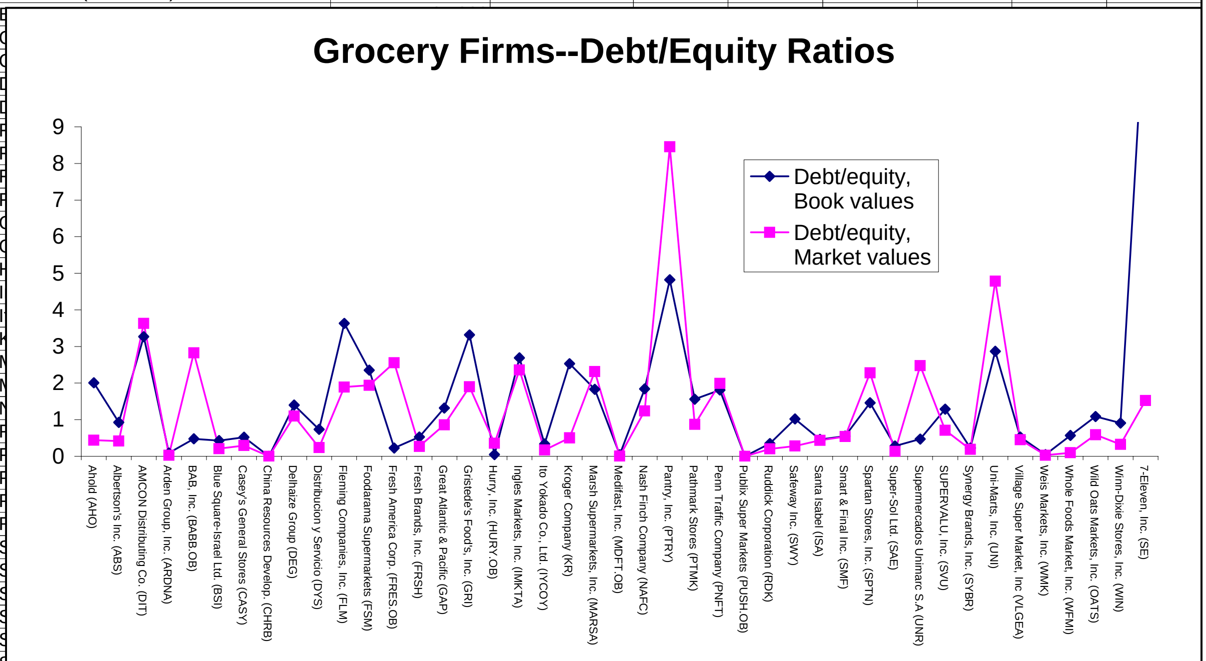
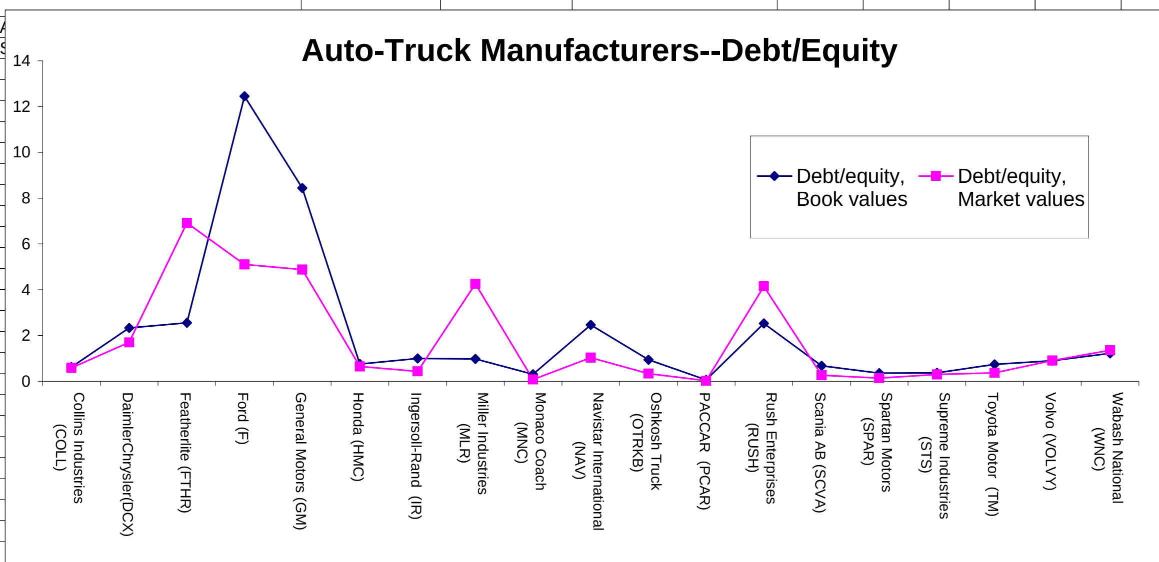
28	P								
29	P								
30	P								
31	S								
32	S								
33	S								
34	S								
35	S								
36	[illegible]	[illegible]	[illegible]						
37	SUPERVALU, Inc. (SVU)	1.2900	0.7127						
38	Synergy Brands, Inc. (SYBR)	0.2300	0.1885						
39	Uni-Marts, Inc. (UNI)	2.8700	4.7833						
40	Village Super Market, Inc (VLGEA)	0.5300	0.4530						
41	Weis Markets, Inc. (WMK)	0.0500	0.0327						
42	Whole Foods Market, Inc. (WFMI)	0.5700	0.1009						
43	Wild Oats Markets, Inc. (OATS)	1.0900	0.5892						
44	Winn-Dixie Stores, Inc. (WIN)	0.9100	0.3273						
45	7-Eleven, Inc. (SE)	12.6100	1.5248						



Here’s similar data for auto manufacturers:

	A	B	C
1	**Debt/Equity ratios for Auto and Truck Manufacturers Source: Yahoo, 20mar02**		
2		**Debt/equity, Book values**	**Debt/equity, Market values**
3	Collins Industries (COLL)	0.6200	0.5905
4	DaimlerChrysler(DCX)	2.3300	1.7007
5	Featherlite (FTHR)	2.5600	6.9189
6	Ford (F)	12.4500	5.1025
7	General Motors (GM)	8.4400	4.8786
8	Honda (HMC)	0.7600	0.6496
9	Ingersoll-Rand (IR)	1.0000	0.4348
10	Miller Industries (MLR)	0.9800	4.2609
11	Monaco Coach (MNC)	0.3100	0.0845
12	Navistar International (NAV)	2.4700	1.0292
13	Oshkosh Truck (OTRKB)	0.9400	0.3345
14	PACCAR (PCAR)	0.0600	0.0241
15	Rush Enterprises (RUSH)	2.5300	4.1475
16	Scania AB (SCVA)	0.6800	0.2698
17	Spartan Motors (SPAR)	0.3600	0.1463
18	Supreme Industries (STS)	0.3700	0.3058
19	Toyota Motor (TM)	0.7400	0.3700
20	Volvo (VOLVY)	0.9000	0.9091
21	Wabash National (WNC)	1.2200	1.3556



In short: As viewed from the data, there does not appear to be a trend in debt/equity ratios, whether measured in book or market values. This is evidence in favor of *tax-neutrality* with respect to debt-equity policy and against theories (like the Modigliani-Miller theory of capital structure with only corporate taxes) which claim that debt is the preferred method of financing.

21.3. Measuring a firm's asset β_{Assets} and WACC, an example

In this section we show how we measure the asset β_{Assets} for Ford Motor Company. We use this β_{Assets} to compute the Ford's WACC using the formula:

$$WACC = r_f + \beta_{Assets} * \left[E(r_M) - r_f \right].$$

Our primary interest in this section, is not the WACC, however. Rather:

- We want to carefully show you how to use public sources of information (in this case Yahoo) to compute a firm's debt beta, debt/equity ratio, and asset beta.
- We want to set the stage for the next section, in which we compute the β_{Assets} for all the firms in the auto and truck industry in the U.S. This enables us to ask whether β_{Assets} is affected by the debt/equity ratio, and to perform our own "homemade" test of the capital structure propositions of the previous chapter. The answer appears to be negative—for this industry we cannot find an effect of debt on β_{Assets}. Our conclusion is that the debt/equity mix does not affect the weighted average cost of capital (WACC).

For the moment we concentrate on the first bullet and compute some numbers for Ford. All the data is derived from Yahoo.

Ford's cost of debt and debt beta β_D

At the end of 2000, Ford reported income expense of $10.902 billion. Combined with the debts on the balance sheets for 2000 and 1999, we can conclude that Ford's interest rate was 6.62%:

	A	B	C
1	**FORD MOTOR COMPANY**		
2		**2000**	**1999**
3	Short-term debt	277,000,000	1,602,000,000
4	Long-term debt	169,503,000,000	158,150,000,000
5	Total debt	169,780,000,000	159,752,000,000
6			
7	Interest expense	10,902,000,000	
8	Implied interest rate	6.62%	<-- =B7/AVERAGE(B5:C5)

Yahoo gives Ford's equity β as 1.07. In order to compute Ford's debt β, we use the following computation:

$$cost\ of\ \ debt = r_D = r_f + \beta_D * \left[E\left(r_M\right) - r_f \right]$$

This is the SML for debt. Since we know that $r_D = 6.62\%$, we can solve for β_D . To do this, we assume that $r_f = 4.80\%$ and that $E\left(r_M\right) - r_f = 5\%$:

$$\beta_D = \frac{r_D - r_f}{E\left(r_M\right) - r_f} = \frac{6.62\% - 4.80\%}{5\%} = 0.3633$$

	A	B	C
10	Risk-free rate	4.80%	
11	Market risk premium	5%	
12	Debt beta	0.3633	<-- =(B8-B10)/B11

Ford's debt/equity ratio

Yahoo gives Ford's debt/equity ratio as 12.45. However, this ratio is in book values, and we want the market value debt/equity ratio. Yahoo also gives Ford's "price/book" ratio as 2.45—by this is meant the ratio of the market price of Ford's shares to their book value. We can now compute the market debt/equity ratio:

$$market\ debt/equity = \frac{book\ debt}{book\,equity} * \frac{1}{\underbrace{\frac{market\ equity}{book\ equity}}_{\substack{\uparrow \\ \text{this is the "market/book"} \\ \text{ratio given in Yahoo}}}}$$

$$= 12.45 * \frac{1}{2.45} = 5.08$$

(Notice that we've assumed that the book value and market value of debt are equal.)

In our spreadsheet:

	A	B	C
14	Debt/Equity, book values	12.45	
15	Price/Book	2.45	
16	Debt/Equity, market values	5.08	<-- =B14/B15

Computing the percentage of debt in the capital structure, $\frac{D}{E+D}$

$\frac{D}{E+D}$ is computed in *market values*. From the previous calculation, we know that the debt/equity ratio in market values is 5.08. We use some algebra to compute $\frac{D}{E+D}$ from the $\frac{D}{E}$:

$$market\ value\ \frac{D}{E+D} = \frac{D}{E+D} * \frac{\frac{1}{E}}{\frac{1}{E}} = \frac{D/E}{1+D/E} = \frac{5.08}{1+5.08} = 0.8356$$

We can now also compute $\frac{E}{E+D} = 1 - \frac{D}{E+D} = 0.1644$

Computing Ford's tax rate T_C

To compute Ford's tax rate, we take its income tax expense and divide it into its pre-tax income:

	A	B	C
21	Income Before Tax	8,234,000,000	
22	Income Tax Expense	2,705,000,000	
23	Tax rate	32.85%	<-- =B22/B21

Computing Ford's asset beta, β_{Assets}

The formula for the asset β is:

$$\beta_{Assets} = \beta_E * \frac{E}{E+D} + \beta_D * (1 - T_C) * \frac{D}{E+D}$$

where

$\beta_E = equity\ beta$

$\beta_D = debt\ beta$

$\frac{E}{E+D} = percent\ of\ equity; \frac{D}{E+D} = percent\ of\ debt$

To do this computation, we use Yahoo's estimate of Ford's equity beta, $\beta_E = 1.07$, and we use our calculation of Ford's $T_C = 32.85\%$. Ford's asset beta is $\beta_{Assets} = 0.3798$:

	A	B	C
1	**FORD MOTOR COMPANY**		
2		**2000**	**1999**
3	Short-term debt	277,000,000	1,602,000,000
4	Long-term debt	169,503,000,000	158,150,000,000
5	Total debt	169,780,000,000	159,752,000,000
6			
7	Interest expense	10,902,000,000	
8	Implied interest rate	6.62%	<-- =B7/AVERAGE(B5:C5)
9			
10	Risk-free rate	4.80%	
11	Market risk premium	5%	
12	Debt beta	0.3633	<-- =(B8-B10)/B11
13			
14	Debt/Equity, book values	12.45	
15	Price/Book	2.45	
16	Debt/Equity, market values	5.08	<-- =B14/B15
17			
18	Debt/Assets, market values	0.8356	<-- =B16/(B16+1)
19	Equity/Assets, market values	0.1644	<-- =1-B18
20			
21	Income Before Tax	8,234,000,000	
22	Income Tax Expense	2,705,000,000	
23	Tax rate	32.85%	<-- =B22/B21
24			
25	Equity beta	1.07	
26	Asset beta	0.3798	<-- =B25*B19+B12*B18*(1-B23)

Address http://biz.yahoo.com/fin/l/f/f_ai.html

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Ford Motor Co Annual Income Statement

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Income Statement Balance Sheet Cash Flow Statement

Period Ending:	**Dec 31, 2000**	**Dec 31, 1999**	**Dec 31, 1998**
Total Revenue	**$170,064,000,000**	**$162,558,000,000**	**$144,607,000,000**
Cost Of Revenue	$126,120,000,000	$119,046,000,000	$104,782,000,000
Gross Profit	$43,944,000,000	$43,512,000,000	$39,825,000,000
Operating Expenses			
Research And Development	N/A	N/A	N/A
Selling General And Administrative Expenses	$14,855,000,000	$14,201,000,000	$12,425,000,000
Non Recurring	N/A	N/A	$38,000,000
Other Operating Expenses	$11,371,000,000	$10,719,000,000	$10,387,000,000
Operating Income	$17,718,000,000	$18,592,000,000	$16,975,000,000
Total Other Income And Expenses Net	$1,418,000,000	$1,510,000,000	$17,286,000,000
Earnings Before Interest And Taxes	$19,136,000,000	$20,102,000,000	$34,261,000,000
Interest Expense	$10,902,000,000	$9,076,000,000	$8,865,000,000
Income Before Tax	$8,234,000,000	$11,026,000,000	$25,396,000,000
Income Tax Expense	$2,705,000,000	$3,670,000,000	$3,176,000,000
Equity Earnings Or Loss Unconsolidated Subsidiary	N/A	N/A	N/A
Minority Interest	($119,000,000)	($119,000,000)	($149,000,000)
Net Income From Continuing Operations	$5,410,000,000	$7,237,000,000	$22,071,000,000
Nonrecurring Events			
Discontinued Operations	($1,943,000,000)	N/A	N/A
Extraordinary Items	N/A	N/A	N/A
Effect Of Accounting Changes	N/A	N/A	N/A
Other Items	N/A	N/A	N/A
Net Income	**$3,467,000,000**	**$7,237,000,000**	**$22,071,000,000**
Preferred Stock And Other Adjustments	N/A	($15,000,000)	N/A
Net Income Applicable To Common Shares	**$3,467,000,000**	**$7,222,000,000**	**$22,071,000,000**

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Ford's income statements, as displayed on Yahoo

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Ford Motor Co Annual Balance Sheet Display Quarterly Data

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Income Statement **Balance Sheet** Cash Flow Statement

Period Ending	**Dec 31, 2000**	**Dec 31, 1999**	**Dec 31, 1998**
Current Assets			
Cash And Cash Equivalents	$4,851,000,000	$6,230,000,000	$4,836,000,000
Short Term Investments	$13,116,000,000	$18,943,000,000	$20,120,000,000
Net Receivables	$136,312,000,000	$9,945,000,000	$139,298,000,000
Inventory	$7,514,000,000	$6,435,000,000	$5,656,000,000
Other Current Assets	$5,318,000,000	$4,126,000,000	$3,405,000,000
Total Current Assets	**$167,111,000,000**	**$45,679,000,000**	**$173,315,000,000**
Long Term Assets			
Long Term Investments	$50,359,000,000	$161,081,000,000	$3,369,000,000
Property Plant And Equipment	$37,508,000,000	$42,317,000,000	$37,320,000,000
Goodwill	N/A	N/A	N/A
Intangible Assets	N/A	N/A	N/A
Accumulated Amortization	N/A	N/A	N/A
Other Assets	$26,101,000,000	$24,336,000,000	$20,366,000,000
Deferred Long Term Asset Charges	$3,342,000,000	$2,816,000,000	$3,175,000,000
Total Assets	**$284,421,000,000**	**$276,229,000,000**	**$237,545,000,000**
Current Liabilities			
Accounts Payable	$48,347,000,000	$39,789,000,000	$35,560,000,000
Short Term And Current Long Term Debt	$277,000,000	$1,602,000,000	$1,191,000,000
Other Current Liabilities	N/A	N/A	$2,447,000,000
Total Current Liabilities	$48,624,000,000	$41,391,000,000	$39,198,000,000
Long Term Debt	$169,503,000,000	$158,150,000,000	$132,532,000,000
Other Liabilities	$37,981,000,000	$40,022,000,000	$36,167,000,000
Deferred Long Term Liability Charges	$9,030,000,000	$8,454,000,000	$6,239,000,000
Minority Interest	N/A	N/A	N/A
Negative Goodwill	N/A	N/A	N/A
Total Liabilities	**$265,138,000,000**	**$248,017,000,000**	**$214,136,000,000**
Stock Holders Equity			
Misc Stocks Options Warrants	N/A	N/A	N/A
Redeemable Preferred Stock	$673,000,000	$675,000,000	N/A
Preferred Stock	N/A	N/A	N/A
Common Stock	$19,000,000	$1,222,000,000	$1,222,000,000
Retained Earnings	$17,884,000,000	$24,606,000,000	$19,659,000,000
Treasury Stock	N/A	($1,417,000,000)	($1,085,000,000)
Capital Surplus	$6,174,000,000	$5,049,000,000	$5,283,000,000
Other Stockholder Equity	($5,467,000,000)	($1,923,000,000)	($1,670,000,000)
Total Stockholder Equity	**$18,610,000,000**	**$27,537,000,000**	**$23,409,000,000**
Net Tangible Assets	**$18,610,000,000**	**$27,537,000,000**	**$23,409,000,000**

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Ford's balance sheets, as displayed on Yahoo

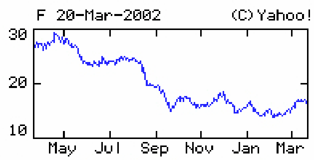
Figure 21.1: The Ford computations in section 21.?? are based on the Yahoo exhibits above for Ford's balance sheets and income statements.

Statistics at a Glance -- NYSE:F — As of 19-Mar-2002

Price and Volume

52-Week Low on 7-Feb-2002	$13.90
Recent Price	$17.18
52-Week High on 18-Apr-2001	$31.42
Beta	1.07
Daily Volume (3-month avg)	11.0M
Daily Volume (10-day avg)	10.3M

Stock Performance



big chart [1d | 5d | 3m | 6m | 1y | 2y | 5y | max]

52-Week Change	-40.4%
52-Week Change relative to S&P500	-41.8%

Share-Related Items

Market Capitalization	$31.1B
Shares Outstanding	1.81B
Float	1.79B

Dividends & Splits

Annual Dividend (indicated)	$0.40
Dividend Yield	2.33%
Last Split: factor 1.748 on 3-Aug-2000	

Per-Share Data

Book Value (mrq*)	$7.00
Earnings (ttm)	-$3.05
Earnings (mrq)	-$2.81
Sales (ttm)	$89.12
Cash (mrq*)	$5.58

Valuation Ratios

Price/Book (mrq*)	2.45
Price/Earnings	N/A
Price/Sales (ttm)	0.19

Income Statements

Sales (ttm)	$162.4B
EBITDA (ttm*)	$12.6B
Income available to common (ttm)	-$5.46B

Profitability

Profit Margin (ttm)	-3.3%
Operating Margin (ttm)	1.4%

Fiscal Year

Fiscal Year Ends	Dec 31
Most recent quarter (fully updated)	30-Sep-2001
Most recent quarter (flash earnings)	31-Dec-2001

Management Effectiveness

Return on Assets (ttm)	-1.91%
Return on Equity (ttm)	-35.37%

Financial Strength

Current Ratio	N/A
Debt/Equity (mrq*)	12.45
Total Cash (mrq)	$0

Short Interest
As of 8-Feb-2002

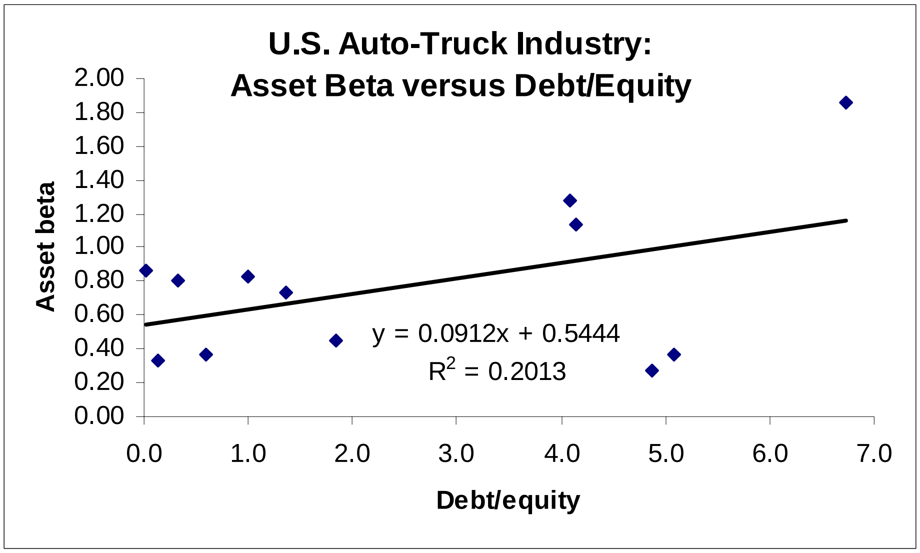
Shares Short	54.7M
Percent of Float	3.1%
Shares Short (Prior Month)	21.1M
Short Ratio	3.13
Daily Volume	17.5M

See Profile Help for a description of each item above; **K** = thousands; **M** = millions; **B** = billions; **mrq** = most-recent quarter; **ttm** = trailing twelve months;
(as of 31-Dec-2001, except **mrq*/ttm*** items as of 30-Sep-2001)

Figure 21.2: Financial profile of Ford, from Yahoo

21.4. Repeating the asset β calculation for an industry

In the previous section we showed how to calculate the asset β_{Asset} for Ford. Suppose we repeat this calculation for all American manufacturers of autos and trucks. The results are displayed on a separate page. Here's the graph which relates the firms' debt/equity ratio and their asset betas:



The graph shows a slight upwards trend:

$$Asset\,beta\ \ for\ autos = 0.5444 + 0.0912 * \frac{Debt}{Equity}, R^2 = 20.13\%$$

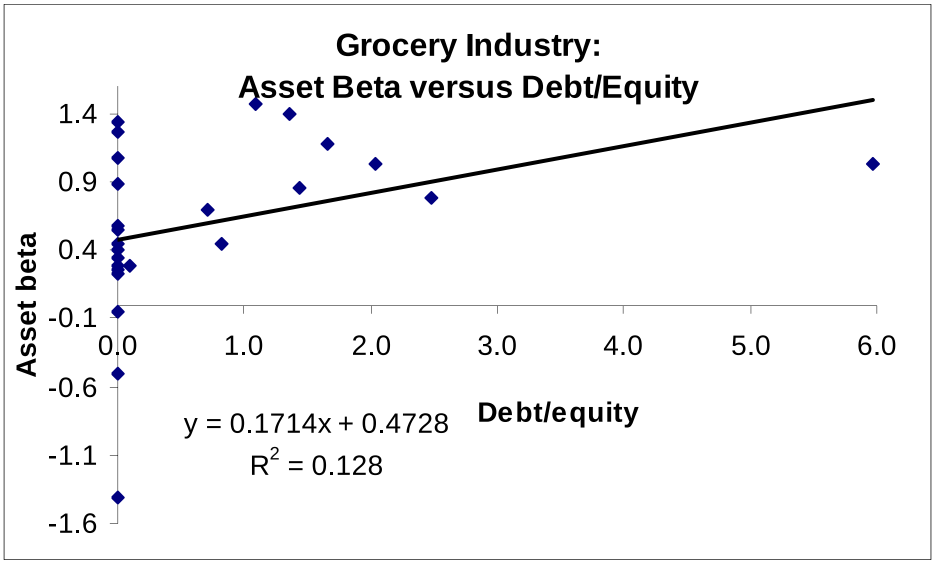
Using this equation, we would conclude that the β for an unlevered auto firm is 0.54, and that increased leverage adds to this β. A more careful analysis (not repeated here, but on the disk with the book) reveals that the positive slope is not statistically significant. Meaning: At least for this small sample, we can conclude that the asset β is not affected by the capital structure. This is Miller's position:

- If the Modigliani-Miller results are representative, then the WACC of will decrease when the amount of debt increases. The effect on the β_{Assets} will be that β_{Assets} should decrease as leverage increases.
- If the Miller results are representative, then the WACC will be unaffected by the amount of debt. The effect on the β_{Assets} will be that β_{Assets} should stay constant as leverage increases.

In the event, β_{Assets} seems to increase slightly with leverage for auto firms. The effect is not large and is statistically insignificant; if it were significant, it would be consistent with a *tax disadvantage* to debt. So—at least for the auto industry, Miller's theory seems to do better at explaining things that the MM theory.

One more industry

The experiment we've performed on the auto industry in the first part of this section is just that—a small experiment to see if we can find any effects of leverage on asset betas. To show that this experiment is not a fluke, we repeat it for the grocery industry:

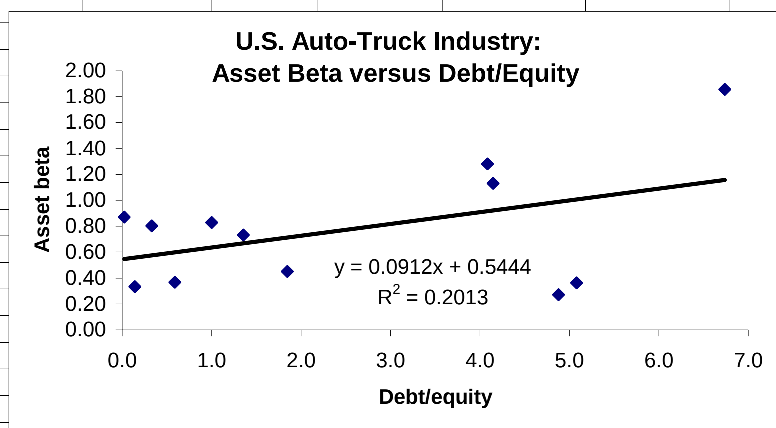


As for the auto industry, there seems to be a slight upward trend in the asset beta as a function of the debt/equity ratio of grocery firms. And, as for the auto industry, this upward slope is not, when we subject it to more statistical scrutiny, significant. We conclude (again) that there is little evidence that leverage affects the asset beta and the WACC.

Several further notes about the grocery industry:

- The average equity β for grocery firms was 0.366 (with a standard deviation of 0.568—meaning that the equity beta was very widely dispersed).
- The average asset β for these firms was 0.594 (with a $\sigma = 0.638$).
- In a period (1999-2000) where the risk-free rate of interest was around 5%, these firms paid average interest rates of 16.75% (with σ = 4.39%). Thus, while shareholders perceived these firms as having fairly low risk, lenders perceived them as having very high risks—the average debt $\beta_D = 2.39$ (with $\sigma = 0.88$).

	A	B	C	D	E	F	G	H	I	J	K
1	**ASSET BETAS FOR AMERICAN TRUCK AND AUTO COMPANIES**										
2		**Equity beta**	**Debt beta**	**Tax rate**	**Market value of equity**	**Debt/Equity book values**	**Price/Book**	**Debt/Equity market values**	**Debt/Assets market values**	**Equity/Assets market values**	**Asset beta**
3	Collins Industries (COLL)	0.150	1.134	35.31%	28.1	0.62	1.05	0.59	0.3713	0.6287	0.3667
4	Featherlite (FTHR)	0.800	2.162	6.87%	7.51	2.56	0.38	6.74	0.8707	0.1293	1.8569
5	Ford (F)	1.070	0.363	37.98%	31.1	12.45	2.45	5.08	0.8356	0.1644	0.3642
6	General Motors (GM)	1.120	0.147	33.40%	34	8.44	1.73	4.88	0.8299	0.1701	0.2715
7	Miller Industries (MLR)	1.560	1.811	33.03%	24.9	0.98	0.24	4.08	0.8033	0.1967	1.2809
8	Navistar International (NAV)	1.490	0.233	29.02%	2.65	2.47	2.47	1.00	0.5000	0.5000	0.8275
9	Oshkosh Truck (OTRKB)	0.930	0.674	37.25%	987.6	0.94	2.82	0.33	0.2500	0.7500	0.8032
10	PACCAR (PCAR)	0.880	0.627	33.57%	5.82	0.06	2.58	0.02	0.0227	0.9773	0.8695
11	Rush Enterprises (RUSH)	0.520	2.133	39.99%	49.4	2.53	0.61	4.15	0.8057	0.1943	1.1325
12	Spartan Motors (SPAR)	0.360	0.199	30.37%	86.6	0.36	2.53	0.14	0.1246	0.8754	0.3324
13	Supreme Industries (STS)	0.390	0.793	39.00%	65.9	2.27	1.23	1.85	0.6486	0.3514	0.4509
14	Wabash National (WNC)	0.910	0.984	39.04%	240.5	1.22	0.9	1.36	0.5755	0.4245	0.7316
15											
16	Average	0.848	0.938	32.90%							0.774
17	Standard deviation	0.437	0.733	0.089							0.473



21.5. Academic evidence

In the previous section we've looked at a specific example—the U.S. auto-truck industry—to try to gauge whether capital structure affects the asset β_{Asset} of these firms. Our conclusion is that, for this industry, they don't: The asset β_{Asset}, and hence the WACC, is not affected by the capital structure.

Recent academic research seems to come to the same conclusion.[3]

- When Eugene Fama and Kenneth French regress firm value on leverage, they conclude that leverage doesn't matter.[4] [graph?]
- John Graham, in a survey published in 2001, concludes that "at the margin the tax costs and tax benefits [of leverage] might be of similar magnitude."[5] To show you how confusing this is, Graham concludes that—using another method—the tax benefit of debt is approximately 9% for the years 1995-1999.[6] This probably represents the costs of bankruptcy.
- Ivo Welch, in a paper written in 2002, finds no evidence whatsoever that firms look for an optimal structure.[7] He finds that firms tend to make few changes in their debt, so that the actual capital structure (i.e., the ratio of debt to the market value of equity) is largely

---

[3] Be warned that this is still controversial. Every finance professor seems to have an opinion on this matter! If you want a good grade in the course, disagree with the book and not with your professor.

[4] "Taxes, Financing Decisions, and Firm Value," *Journal of Finance* 1998, pp. 819-843.

[5] "Taxes and Corporate Finance: A Review," working paper. The quote is from page 25.

[6] Ibid, page 26-27.

[7] Ivo Welch, "Columbus' Egg: The Real Determinants of Capital Structure," Yale School of Management working paper, 2002.

driven by the market prices of the firm's shares. There is little evidence, according to Welch, of any optimizing in the debt decision.

Summing up

The theory of capital structure suggests that the capital structure decision is largely driven by the differential taxation of debt and equity. The empirics of capital structure suggest that it doesn't matter very much in determining the value of the firm.

For practical purposes:

- You can assume that the weighted average cost of capital (WACC) of a firm is invariant to the firm's capital structure.
- This means that the WACC of a firm can be measured by taking the *average WACC* of the firm's industry. It also means that the asset β of a firm's industry is representative of the industry's overall risks and is not a function of the capital structure of the industry.
- The best way to value a firm is to use the WACC to discount the firm's anticipated future free cash flows (recall that these are operating cash flows and do not include interest and other financing). We have illustrated this approach in a number of chapters of this book: Chapter 5, 7, 15.

CHAPTER 22: DIVIDEND POLICY*

This version: February 8, 2004

This chapter is incomplete!

Chapter contents

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* This is a preliminary draft of a chapter of *Principles of Finance with Excel*. (benninga@wharton.upenn.edu).

Overview

The purpose of this chapter is to study the effect of dividend policy on the value of the firm:

- What's a dividend?
- Why might it have an effect on firm value?
- MM dividend irrelevance proposition
- Taxation and dividends: capital gains versus ordinary income
- Empirical evidence?

Finance concepts discussed

- Dividends
- Retained earnings
- Capital gains versus ordinary income

Excel functions used

We use a lot of Excel spreadsheets to put order in things, but truth to tell, this chapter uses hardly any sophisticated Excel concepts. The one function used is **Sum**.

22.1. Dividends

John and Mary both own taxi companies. Operationally the taxi companies are exactly alike: Each company owns the same number of taxis, and has the same income and expenses. Here are the balance sheets for the two companies:

JOHN'S TAXI COMPANY, MARY'S TAXI COMPANY			
Assets		**Liabilities and equity**	
Cash	5,000	Debt	10,000
Taxis	20,000	Equity	
		Stock	5,000
		Accumulated retained earnings	10,000
Total assets	25,000	**Total liabilities and equity**	25,000

John pays himself a dividend

Suppose that John wants some cash and decides to declare a dividend of $3,000. Here's the way his balance sheet looks (Mary's balance sheet is unchanged):

JOHN'S TAXI COMPANY--after dividend			
Assets		**Liabilities and equity**	
Cash	2,000	Debt	10,000
Taxis	20,000	Equity	
		Stock	5,000
		Accumulated retained earnings	7,000
Total assets	22,000	**Total liabilities and equity**	22,000

Notice that there are two changes in John's balance sheet:

- The cash balances decrease from $5,000 to $2,000, reflecting the dividend paid.
- The accumulated retained earnings decrease from $10,000 to $7,000. This is what is meant by the expression that "dividends are paid out of retained earnings." We don't

like this expression, since dividends are paid out of cash; the decrease in retentions simply reflects the matching change made in the balance sheet.

Here are some finance-type questions you could ask about this situation:

The valuation effects of the dividend

Did the dividend paid by John change the value of his taxi business vis-à-vis Mary's business? Obviously not—they both still have the same number of taxis, and Mary has just kept her cash in the business instead of, as John did, pulling it out. A good way to see this is to to write the balance sheets in terms of net debt—subtracting the cash from the debt:

JOHN'S or MARY'S TAXI COMPANY--net debt					**JOHN'S TAXI COMPANY--after dividend**			
Assets		**Liabilities and equity**			**Assets**		**Liabilities and equity**	
		Net debt = Debt - cash	5,000				Net debt = Debt - cash	8,000
Taxis	20,000	Equity			Taxis	20,000	Equity	
		Stock	5,000				Stock	5,000
		Accumulated retained earnings	10,000				Accumulated retained earnings	7,000
Total assets	20,000	**Total liabilities and equity**	20,000		**Total assets**	20,000	**Total liabilities and equity**	20,000

The asset side of the balance sheet is still worth the same, whether or not the dividend has been paid.[1] On the other hand, the liabilities and equity side of the balance sheet is different—John has more debt and less equity than Mary.

Perhaps it's just a capital structure question?

The above balance sheets for the two companies show that—while they are both the same on the asset side, the dividend has changed the capital structure of the companies. So perhaps the dividend question is related to the capital structure problem discussed in Chapters 20 and 21. If so, this suggests

[1] In a more general context we can ***************

- Dividends might matter if capital structure matters: An after-dividend company (like John's) will have a higher debt/equity ratio than a before-dividend company (like Mary's).
- If companies with a higher debt/equity ratio have a higher valuation, then companies should pay dividends.

Now this book takes a definite stand on this question: In the previous chapters we've suggested that the capital structure question is ultimately a question of balancing personal against corporate taxation. We've also suggested that the economic evidence suggests that on balance the taxes are pretty much of a wash, so that capital structure doesn't matter.

Though this argument suggests that dividends do not affect the valuation of a company, there's another tax aspect to this question—the tradeoff between ordinary income taxes and capital gains taxes. We discuss this in the next section.

In the meantime: As long as Mary and John's taxi companies aren't taxed and as long as Mary and John aren't taxed on a personal level, the debt/equity aspects of the dividend decision shouldn't affect the valuation of their companies.

The dividend doesn't affect the enterprise value

Here's another way of thinking about this question: Suppose that both John and Mary are thinking about selling their taxi companies. The "taxi part" of the business is worth $40,000 (this doesn't include the cash balances on the books). John and Mary have slightly different strategies about how to sell the business: John intends to first pay himself a dividend and then sell the business, whereas Mary intends to sell the business first without taking a dividend. Here are the calculations:

Mary sells her taxi company for $40,000					John sells his taxi company for $40,000	
Sale price	40,000				Sale price	40,000
Pay back net debt	5,000				Pay back net debt	8,000
Net to equity	35,000				Net to equity	32,000
Book value of equity	12,000				Book value of equity	12,000
Taxable gain	23,000				Taxable gain	20,000
Taxes on gain (0%)	0				Taxes on gain (0%)	0
Net to Mary from sale	35,000				Net to John from sale	32,000
Add back dividend	0				Add back dividend	3,000
Taxes on dividend (0%)	0				Taxes on dividend (0%)	0
Total	35,000				Total	35,000

Clearly it doesn't matter.[2]

Who cares where the money is as long as it's there?

This is really what it's all about—who cares whether the money is in the taxi company or in the individual bank account of the owner? Of course you can think of many answers to this question which make it appear that it *does* matter:

- Taxes: If the company and its owners pay different tax rates, perhaps dividends are worthwhile (or not—read on).
- Trust: If there are multiple owners of the company, maybe you want the money in *your hands* as opposed to leaving it in the company. Economists call this "agency costs"—an agent being someone you've hired to do your work for you (that is, the manager). The agency cost argument for paying dividends suggests that you and your manager may have different goals; if the manager's goal includes wasting your money, then maybe you should get the money out of his hands by paying a dividend.
-

Miller and Modigliani's "dividend irrelevance" proposition (1961)

[2] Although—to anticipate the next section, the assumption that there are no taxes is critical to this argument.

The enterprise value of the firm—the sum of the values of the operating current assets and fixed assets—is not affected by dividend policy. On the other hand, an increased dividend implies a corresponding decrease in the market value of the firm's equity.

22.2. Taxes!

In the above section we've made two points:

- The value of the "taxi part" of the business—the enterprise value—is not affected by the dividend policy of John and Mary's taxi business
- The proceeds—dividends plus gains from selling the business—to John and Mary are exactly the same, independent of their dividend policy.

Now look at the second point again, and suppose that we introduce taxes. We'll assume that dividends are taxed as "ordinary income" at a rate of 30% and that the gains from selling the business are taxed at a capital gains tax rate of 15%.

We'll start with John, who sells his taxi company for $40,000 right after he's paid himself a $3,000 dividend. As the calculation below shows, John's net from the sale of the company is $31,1000:

	F	G	H	I
1	**JOHN'S TAXI COMPANY--after dividend**			
2				
3	**Assets**		**Liabilities and equity**	
4			Net debt = Debt - cash	8,000
5				
6	Taxis	20,000	Equity	
7			Stock	5,000
8			Accumulated retained earnings	7,000
9				
10	Total assets	20,000		20,000
11				
12	Capital gains tax	15%		
13	Dividend tax	30%		
14				
15	**John sells his taxi company for $40,000**			
16	Sale price	40,000		
17	Pay back net debt	8,000		
18	Net to equity	32,000	<-- =G16-G17	
19	Book value of equity	12,000	<-- =SUM(I7:I8)	
20	Taxable gain	20,000	<-- =G18-G19	
21	Taxes on capital gain (15%)	3,000	<-- =$G$12*G20	
22	Net to John from sale	29,000	<-- =G18-G21	
23				
24	Add back dividend	3,000		
25	Taxes on dividend (30%)	900	<-- =$G$13*G24	
26	Total	31,100	<-- =G22+G24-G25	

Now Mary: She also sells her company, but she hasn't paid herself a dividend. Her net is higher:

	A	B	C	D
1	**MARY'S TAXI COMPANY**			
2				
3	**Assets**		**Liabilities and equity**	
4			Net debt = Debt - cash	5,000
5				
6	Taxis	20,000	Equity	
7			Stock	5,000
8			Accumulated retained earnings	10,000
9				
10	Total assets	20,000	Total liabilities and equity	20,000
11				
12	Capital gains tax	15%		
13	Dividend tax	30%		
14				
15	**Mary sells her taxi company for $40,000**			
16	Sale price	40,000		
17	Pay back net debt	5,000		
18	Net to shareholders (Mary)	35,000	<-- =B16-B17	
19	Book value of equity	15,000	<-- =SUM(D7:D8)	
20	Taxable gain	20,000	<-- =B18-B19	
21	Taxes on capital gain (15%)	3,000	<-- =$B$12*B20	
22	Net to Mary from sale	32,000	<-- =B18-B21	
23				
24	Add back dividend	0		
25	Taxes on dividend (30%)	0	<-- =$B$13*B24	
26	Total	32,000	<-- =B22+B24-B25	

The reason for the difference between John's net of $31,100 and Mary's net of $32,000 is that dividends are taxed. By not paying herself a dividend, Mary has saved herself $900 = 30%*3,000 of taxes on her dividends.[3]

This analysis suggests that *dividends might matter* if there is both a dividend tax and a capital gains tax: In this case you shouldn't pay dividends.

[3] In any case both John and Mary are going to pay the same capital gains taxes. This is because a dividend, paid out of cash, *reduces* the firm's equity and *increases* the firm's net debt. The result, as you can confirm from the examples, is that the capital gain to the firm's shareholders is independent of the dividend.

What if John really needs the money? Solution 1: pay a bonus

Suppose for some reason John really needs the money **now**. Then he should pay himself a bonus, which is a tax-deductible expense for the company:

	A	B	C	D	E	F	G	H	I
1	**JOHN'S TAXI COMPANY--after dividend**					**JOHN'S TAXI COMPANY--after bonus**			
2									
3	**Assets**		**Liabilities and equity**			**Assets**		**Liabilities and equity**	
4	Cash	2,000	Debt	10,000		Cash	3,200	Debt	10,000
5									
6	Taxis	20,000	Equity			Taxis	20,000	Equity	
7			Stock	5,000				Stock	5,000
8			Accumulated retained earnings	7,000				Accumulated retained earnings	8,200
9									
10	**Total assets**	22,000	**Total liabilities and equity**	22,000		**Total assets**	23,200	**Total liabilities and equity**	23,200
11									
12	Corporate tax rate	40%				Corporate tax rate	40%		
13	Capital gains tax	15%				Capital gains tax	15%		
14	Dividend tax	30%				Dividend tax	30%		
15									
16	**John sells his taxi company for $40,000**					**John sells his taxi company for $40,000**			
17	Sale price	40,000				Sale price	40,000		
18	Pay back net debt	8,000	<-- =D4-B4			Pay back net debt	6,800	<-- =I4-G4	
19	Net to equity	32,000	<-- =B17-B18			Net to equity	33,200	<-- =G17-G18	
20	Book value of equity	12,000	<-- =SUM(D7:D8)			Book value of equity	13,200	<-- =SUM(I7:I8)	
21	Taxable gain	20,000	<-- =B19-B20			Taxable gain	20,000	<-- =G19-G20	
22	Taxes on capital gain (15%)	3,000	<-- =$B$13*B21			Taxes on capital gain (15%)	3,000	<-- =$B$13*G21	
23	Net to John from sale	29,000	<-- =B19-B22			Net to John from sale	30,200	<-- =G19-G22	
24									
25	Add back dividend	3,000				Add back dividend	3,000		
26	Taxes on dividend (30%)	900	<-- =$B$14*B25			Taxes on dividend (30%)	900	<-- =$B$14*G25	
27	Total	31,100	<-- =B23+B25-B26			Total	32,300	<-- =G23+G25-G26	

When John pays himself a bonus, it comes out of cash but gets tax deductibility. Here's what happens to the cash balances:

Initial cash balances	$5,000	
After-tax cost of bonus to company	$1,800	The company pays John a $3,000 bonus, which is an expense for tax purposes. At the company's 40% corporate tax rate, the after-tax cost of the bonus is $(1-40\%)*3,000$.
Cash on hand after bonus	$3,200	

This little trick (the tax deductibility of the bonus) is actually more profitable than Mary's not paying a dividend at all (compare John's net of $32,300 to Mary's net of $32,000). However, whether a bonus is better than no bonus depends on the corporate versus the ordinary income tax rate. In the example below the corporate rate is 30%, which is less than John's ordinary income tax rate; he'd be better off by not paying himself a bonus (or a dividend) and selling the company.

	F	G	H	I
1	**JOHN'S TAXI COMPANY--after bonus**			
2				
3	**Assets**		**Liabilities and equity**	
4	Cash	2,900	Debt	10,000
5				
6	Taxis	20,000	Equity	
7			Stock	5,000
8			Accumulated retained earnings	7,900
9				
10	**Total assets**	22,900	**Total liabilities and equity**	22,900
11				
12	Corporate tax rate	30%		
13	Capital gains tax	15%		
14	Ordinary income tax rate	40%		
15				
16	**John sells his taxi company for $40,000**			
17	Sale price	40,000		
18	Pay back net debt	7,100	<-- =I4-G4	
19	Net to equity	32,900	<-- =G17-G18	
20	Book value of equity	12,900	<-- =SUM(I7:I8)	
21	Taxable gain	20,000	<-- =G19-G20	
22	Taxes on capital gain (15%)	3,000	<-- =$B$13*G21	
23	Net to John from sale	29,900	<-- =G19-G22	
24				
25	Add back bonus	3,000		
26	John's taxes on bonus (40%)	1,200	<-- =$G$14*G25	
27	Total	31,700	<-- =G23+G25-G26	

What if John really needs the money? Solution 2: repurchase stock

Maybe John needs the money but can't, for some reason pay himself a bonus. In this case, he should—instead of paying himself a dividend—get the company to repurchase some stock from him. Suppose that John convinces the management of the company (himself!) to buy back $3,000 of stock. Suppose that after this repurchase of equity, John sells the company. Finally, suppose that all of the $3,000 repurchase of stock is taxed to John a capital gain (this is very unlikely—read the note which follows the spreadsheet). In this case, John would still be better off than if he had paid himself a dividend:

	F	G	H	I
1	**JOHN'S TAXI COMPANY--after repurchase**			
2				
3	**Assets**		**Liabilities and equity**	
4	Cash after repurchase	2,000	Debt	10,000
5				
6	Taxis	20,000	Equity	
7			Stock	5,000
8			Accumulated retained earnings	10,000
9			Subtract repurchase of stock	-3,000
10				
11	**Total assets**	22,000	**Total liabilities and equity**	22,000
12				
13	Corporate tax rate	30%		
14	Capital gains tax	15%		
15	Ordinary income tax rate	40%		
16				
17	**John sells his taxi company for $40,000**			
18	Sale price	40,000		
19	Pay back net debt	8,000	<-- =I4-G4	
20	Net to equity	32,000	<-- =G18-G19	
21	Book value of equity	15,000	<-- =SUM(I7:I8)	
22	Taxable gain	17,000	<-- =G20-G21	
23	Taxes on capital gain (15%)	2,550	<-- =$B$14*G22	
24	Net to John from sale	29,450	<-- =G20-G23	
25				
26	Add back repurchase of stock	3,000		
27	John's taxes on repurchase (15%)	450	<-- =$G$14*G26	
28	Total	32,000	<-- =G24+G26-G27	

Some bad tax advice from the author: In order to minimize taxes, John should consult his accountant before repurchasing the stock.[4] It is highly unlikely that the whole repurchase would be taxed as a dividend. It could be structured as a payout of capital (in which case there would be no taxes). The accountant might also be able to value John's *basis* in the stock (what he originally paid for it, plus the accumulated capital gains). Here's an example:

[4] The author of this book barely understands finance and is certainly not a tax accountant. Therefore everything in the next few paragraphs is impressionistic and probably wrong in details (although hopefully right in spirit).

	F	G	H	I
31	**Accountant reasoning?**			
32				
33	**Assets**		**Liabilities and equity**	
34			Net debt	5,000
35	Enterprise value	40,000	Equity, market value	35,000
36	**Total assets**	40,000	**Total liabilities and equity**	40,000
37				
38	Amount spent on repurchase	3,000		
39	As a percent of market value of equity	8.57%	<-- =G38/I35	
40				
41	Book value of equity	15,000		
42	basis = 8.57% of book equity	1,286	<-- =G39*G41	
43				
44	Taxable gain on repurchase	1,714	<-- =G38-G42	
45	Taxes on gain at capital gains tax	257	<-- =G14*G44	
46	Net from repurchase	2,743	<-- =G38-G45	
47				
48				
49	**John sells his taxi company for $40,000**			
50	Sale price	40,000		
51	Pay back net debt	8,000	<-- =I34+G38	
52	Net to equity	32,000	<-- =G50-G51	
53	Book value of equity	13,714	<-- =G41-G42	
54	Taxable gain	18,286	<-- =G52-G53	
55	Taxes on capital gain (274286%)	2,743	<-- =$B$14*G54	
56	Net to John from sale	29,257	<-- =G52-G55	
57				
58	Total: net from sale + net from repurchase	32,000	<-- =G56+G46	

Note: The accountant reckons as follows:

- Before the payout of cash, the company is worth $40,000, which makes the market value of the equity $35,000.
- By paying out $3,000 in cash for stock in the company, John has effectively repurchases 8.57% of the company's equity. Since the book value of the company's equity is $15,000, John has a capital gain of $1,714 (=3,000 - 8.57%*15,000) on the repurchase. This capital gain will be taxed at 15% (=$257), so that John will net $2,743 from the repurchase.

- Now when John sells the company for $40,000, he will first have to pay off its net debt of $8,000 (the repurchase used $3,000 of cash and raised the net debt from $5,000 to $8,000). This leaves him with a market value of equity of $32,000 which has book value of $13,714 (=$15,000 – 8.57%*15,000). This gain also gets taxed at the capital gains tax rate of 15%.
- This leaves John with $32,000.

22.3. Messages

So what are we saying about dividends?

1. In a pure

22.4. Dividends (satisfaction now) versus capital gains (enjoy later)

Up to this point we've established that if you're going to sell your company, dividend taxes make it unwise to first pay yourself a dividend. But what if you're not going to sell the company right away? Should you leave the money in the company, for that golden day when you're going to sell it and benefit from the lowered capital gains taxes? Or should you pay yourself a dividend?

It all depends, of course, on the level of trust you have in the managers of your company. In the case of John and Mary, this is easy—they manage their own companies, and they wouldn't do anything to harm themselves. In this case they should leave the money in the company, where it can earn the same amount (????) as if they paid it out.

FORBES Growth Investor
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Share Repurchases Substitute for Dividends

Forbes Growth Investor, Vahan Janjigian Editor, Vol. 3, No. 9, Page 1, September 2002.

The total return on equities is composed of two components: dividends and capital gains. Since the 1980s, however, the proportion coming from dividends has been shrinking. Furthermore, dividend yields (i.e., dividend per share divided by stock price) and payout ratios (i.e., dividend per share divided by earnings per share) have been falling steadily. Many experienced investors take this as prima facie evidence that stocks remain overvalued despite a tremendous two-year sell-off.	
Value investors in particular believe that steadily rising cash dividends are an indication of financial health. These investors often shun stocks that lack a long history of dividend payments. But others, such as growth investors, believe dividends are not very meaningful.	
A recent article in the Journal of Finance, a leading scholarly publication, provides evidence that the demise of the cash dividend is just an illusion. The authors, Gustavo Grullon and Roni Michaely, argue that focusing only on dividends ignores an increasingly important form of cash payout to stockholders: share repurchases. Cash dividends have been increasing at an annually compounded rate of only 6.3% since 1980. Yet cash spent on share repurchases has been rising at a much more rapid clip of 18.4% compounded annually. Furthermore, cash spent on share repurchases now exceeds that spent on dividends. And total cash paid out (i.e., dividends plus repurchases) as a percentage of earnings has actually been rising during the period studied.	
Our tax code explains much of this behavior. When corporations pay dividends, investors are forced to pay taxes. In fact, dividends are taxed at the ordinary rate. But when corporations initiate share repurchases, investors can avoid taxes altogether by choosing not to sell. Yet if they do sell, they are taxed at the capital gains tax rate, which is much lower than the ordinary tax rate.	
This was the case thirty years ago as well. So why weren't share repurchases as popular then? Grullon and Michaely argue that share repurchases didn't really start growing in popularity until a 1982 regulatory reform, which made it less likely that repurchasing firms would be accused by the SEC of trying to manipulate their stock prices.	

There are a number of lessons to be drawn from this study. First, those who argue that stocks remain overvalued simply because dividend yields or dividend payout ratios are historically low are being shortsighted. They should instead focus on total cash payouts. Second, there should be no doubt that, good or bad, regulatory reforms affect firm behavior. Well-managed firms will do what is best for shareholders. As long as cash dividends are unfavorably taxed, investors will prefer capital gains. And as long as regulators allow it, good corporate boards will deliver what shareholders want.	
Which brings us to a very important point. Dividends are paid from after-tax dollars. Taxing investors again for receiving those dividends imposes a very heavy burden. Regulators should eliminate this double taxation. Dividends should either be treated as a tax-deductible expense for corporations, or tax-exempt income for individuals. *Source*: https://www.forbesnewsletters.com/fagin/index.jhtml?page=sample	

TheStreet.com

Open Book

Why Dividends? There Doesn't Seem to Be a Good Answer

By Don Luskin

Special to TheStreet.com

6/4/01 4:26 PM ET

URL: http://www.thestreet.com/comment/openbook/1449809.html

Remember **Bill Cosby's** act in the 1960s? As a child he used to ask a profound question that adults were too grown up to ask: "Why is there air?"

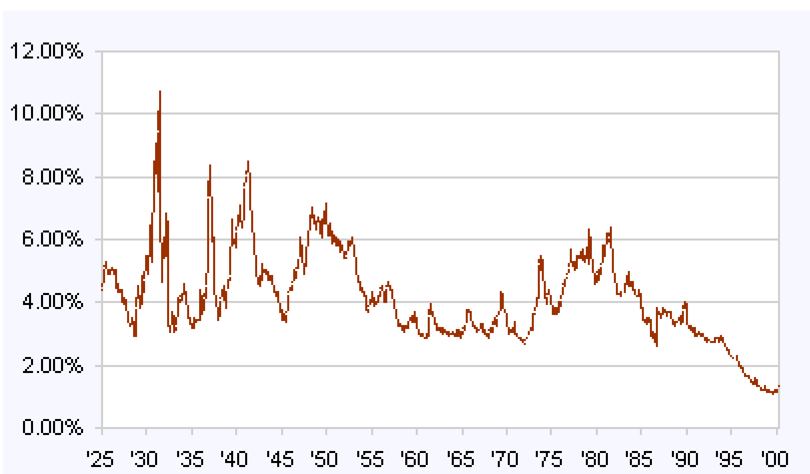
With a child's innocence, let us ask, "Why are there dividends?"

Is this a question we should be too grown up to ask? An awful lot of investment gurus think that dividends are awfully important. For example, I noticed over the weekend that **Jim Jubak**, senior editor at *MSN Money Central*, is saying, "What this stock market needs is a really juicy 5% yield. Instead, the dividend yield on stocks that make up the **Standard & Poor's 500** is down to a paltry 1.23%. Thunderation!"

Jubak is right, in the sense that dividend yields are near historic lows -- even after this year's stiff correction. Take a look at the chart below. Dividend yields today are less than half of what they were at the top of the market in 1929.

Annual Dividend Yield: S&P 500

Source: Global Financial Data



Jubak, and many other observers, would argue that today's low dividend yields are symptomatic of our overvalued markets. Well, maybe markets are overvalued. But I believe dividend yields are so low for another reason: Investors are beginning to ask our childish question: "Why are there dividends?" And they're not finding good answers. So dividends are becoming less and less important.

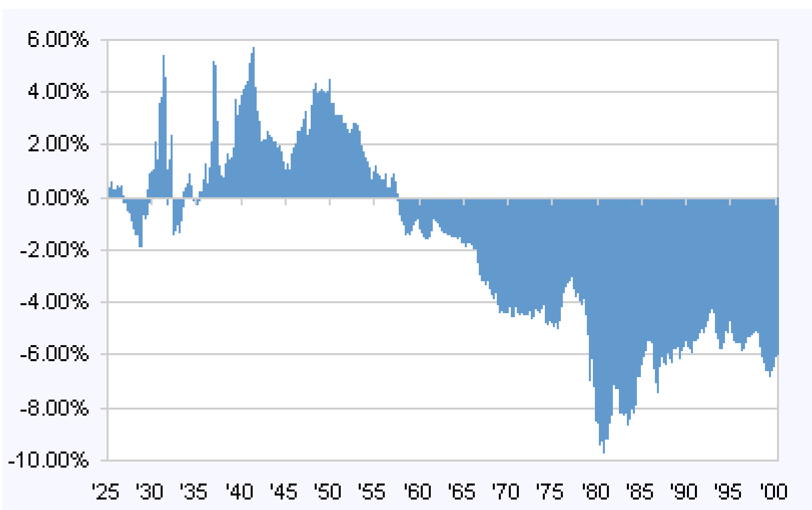
Just think about how silly it all really is. Why would a shareholder want a company to send him his own money back? That's a confession by the investor that he would really rather not invest in that company to begin with. And it's a confession by the company that its investors can invest their money more profitably than the company can.

More and more investors and companies are seeing it this way. But there was a time when everyone felt like Jubak. In those days, dividend yields on stocks had to be high because investors saw high dividends as compensating for the risk of equity ownership. When dividend yields were low, investors reasoned they'd be better off investing in the same company's bonds. After all, why not earn a higher return with less risk?

In fact, for most of the last century, the conventional wisdom was that whenever the dividend yield on the stock market fell below the coupon yield of the bond market, it was time to sell stocks. Take a look at the chart below, which shows the difference between the dividend yield on the S&P 500 and the coupon yield on **Moody's Aaa Corporate Bonds Index**. When stocks yielded less than bonds briefly in 1929, that was one of the greatest stock market sell signals in history.

Yields: Moody's Aaa Corporate Bonds Minus S&P 500

Source: Economagic and Global Financial Data



But then in August 1958, something very strange happened. The dividend yield on the S&P 500 fell below the coupon yield on Moody's Aaa bonds -- and it never came back. For some reason, at that moment, the world decided that dividends weren't so important after all. And dividends have declined in importance ever since.

What happened in 1958 to change investors' preferences so profoundly? Perhaps it happened then because the late 1950s saw the dawn of the age of inflation in America. Bond yields had to rise in relation to equity yields because bonds are completely unprotected against the ravages of inflation.

Or perhaps it was because the 1950s was the dawn of the age of the individual investor, in which pioneers like **Merrill Lynch** brought Wall Street to Main Street. When individuals had poor access to markets, they needed dividends as a low-cost way of getting their money back. As their access improved, they needed dividends less because they could simply pick up the phone and sell shares if they needed cash.

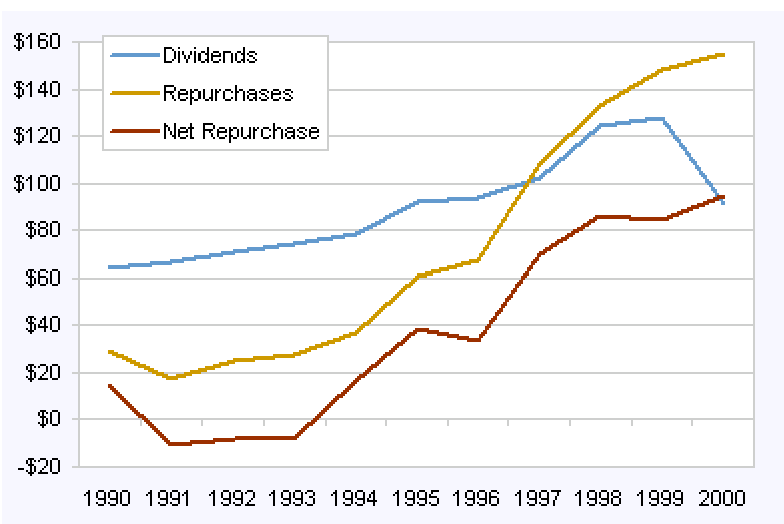
Or perhaps it was because the 1950s was the dawn of the age of financial economics. Academics like Harry Markowitz -- who eventually won the Nobel Prize for this work -- were beginning to codify the realities of investment risk and return. And one of the first lessons was that there was nothing special about dividends as a component of total equity returns.

Twenty-five years later, in the 1980s, another nail was driven into the coffin of dividends. That's when companies started to learn that paying dividends wasn't the most efficient way to return money to shareholders -- buying back their own stock in the open market was smarter. Repurchases allow investors to decide when and how much to cash in. And investors get to decide whether to bear a taxable event -- and if they do, to pay the favorable capital gains tax rate, not the higher ordinary rate they would have paid on dividend income. And investors who don't sell their shares back to the company benefit from reduced earnings dilution.

This logic has been so powerful that, starting in 1997, the total value of share repurchases by S&P 500 companies exceeded the total value of dividends paid. Take a look at the chart below: The spread between repurchases and dividends has gotten wider every year since 1997. And last year, even the value of *net* repurchases (repurchases minus the value of new stock issued) exceeded dividend payouts for the first time.

Dividends and Repurchases (in Billions): S&P 500

Source: Morgan Stanley



This means that to understand the true yield of the equity market -- which has to include both dividends and repurchases -- you would have to at least double the dividend yield quoted by commentators like Jubak.

But while this perhaps blunts Jubak's point about overvaluation, it tends to confirm his broader point that payouts to stock investors are important. Indeed, the mystique of dividend yields is far from dead. Many companies that feel they have lots of good things to do with their money other than pay it out to their investors *still* pay dividends or engage in buybacks. How do they do it? Well, it's simple -- and utterly crazy.

There are many companies that pay dividends that have to borrow money in the debt market in order to do it. If the purpose of a dividend is to return surplus cash to shareholders, then why would any company that had debt pay a dividend? A company in debt has no surplus cash -- by definition. But hundreds and hundreds of companies both borrow and pay dividends at the same time.

For example, in the first quarter **Dow Chemical** (DOW:NYSE) spent $158 million in net interest expenses (accrued interest expense less capitalized interest and debt income) servicing its $10.5 billion in debt. The same company spent $260 million paying dividends of 29 cents a share.

And it gets even nuttier when we start looking at buybacks.

Morgan Stanley Dean Witter's U.S. Equity Strategist Steve Galbraith told me, "Want to know something almost as screwy as borrowing in the bond market to pay dividends? Our wonderful tax and options accounting system has created the following situation -- of the top one-dozen issuers and the top one-dozen buyers back of stock in the S&P, five companies are on both lists! How inefficient and stupid is that?"

So what is there about dividends -- directly, or in the form of repurchases -- that is still so attractive to investors? "Why are there dividends?"

Dividends can't exist simply so that investors can earn income from their investments. Anyone who needs cash from his investment portfolio can have it with a mouse click, just by selling some of his shares. And doing it that way, the investor controls the amount and the timing -- and from which of his stocks he wants the income.

And dividends can't exist as a discipline on company management, forcing them to turn a consistent profit. If they want to pay a dividend, they can simply borrow to do it. And if they want to repurchase shares, they can just issue more later.

Dividends remain one of the great mysteries, like "Why is there air?"

By the way, if you're too young to remember Cos in the 1960s, the answer to the question is: "To blow up basketballs."

Don Luskin is president and CEO of MetaMarkets.com and a portfolio manager of OpenFund, an aggressive growth fund investing in the New Economy. OpenFund strives to be fully invested, expecting to be at least 90% invested under most market conditions. At time of publication, OpenFund was long futures contracts on the S&P 500 and

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CHAPTER 22: INTRODUCTION TO OPTIONS*

this version: September 2002

Chapter contents

Overview

In this chapter we introduce the concept of stock options. We show you the basic definitions and introduce you to the cash flows of options. In addition we show you how option

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strategies—the ability to combine options and stocks in portfolios—can change the payoff patterns available to investors.

Finance concepts discussed

- Call and put options
- Option strategies: protective puts, spreads, butterflies

Excel functions used

- **Max**
- **Min**

22.1. What's an option?

A *call option on a stock* is the right to buy a stock on or before a given date at a pre-determined price. A separate page gives options prices for options on Cisco stock on August 7, 2002; we will use these prices in the examples which follow.

For example, row 24 of the Cisco spreadsheet tells you that on 7 August 2001, a call option on Cisco stock with an exercise price of $20.00 and an exercise date of 21 September 2001 was selling for $1.35:

	A	B	C	D	E	F
7	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
23	Sep01	17.50	2.75	0.90	21 Sep01	45
24	Sep01	20.00	1.35	2.00	21 Sep01	45
25	Sep01	22.50	0.55	3.80	21 Sep01	45

Suppose you purchased this call option on 7 August. Here's the cash flow pattern:

CALL OPTION CASH FLOW PATTERN

7-Aug-01 — 21-Sep-01

Pay: $1.35

In principle, you have the right to buy Cisco stock for $20 on any date before 21 September. In actual fact, you'll only want to exercise the call option at its terminal date (see Section ?? of Chapter 22).

If Cisco's stock price > $20, you'll exercise your call to buy the stock for $20.
Your gain on 21 Sept:
Actual stock price - $20

If Cisco's stock price $\leq$ $20, you will *not* exercise your call option.
Your gain on 21 Sept: 0.

[Separate page]

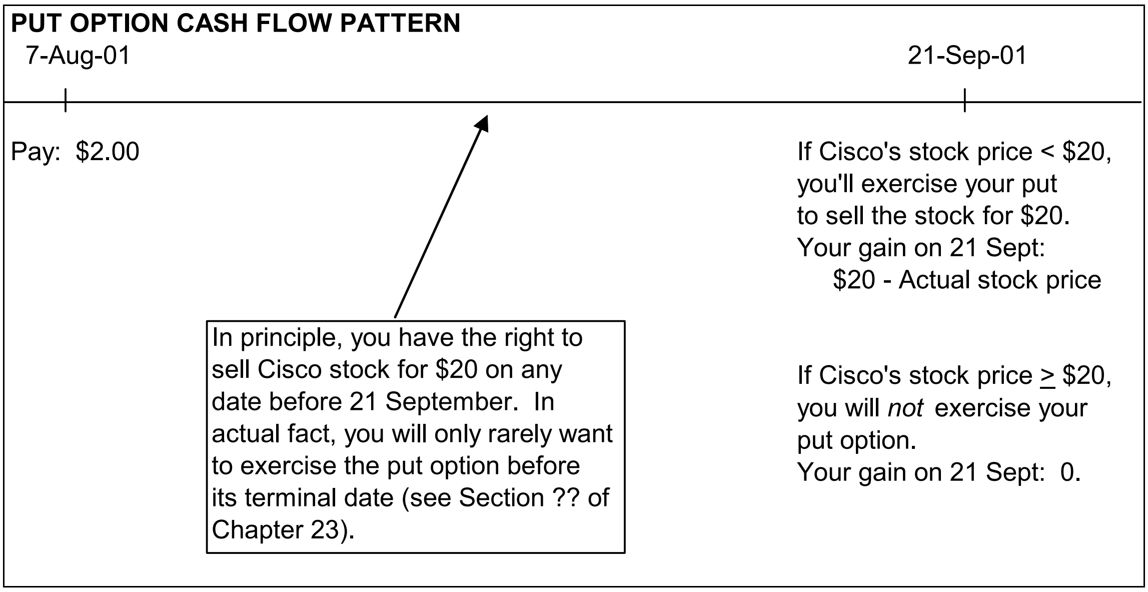
	A	B	C	D	E	F
1	**CISCO OPTIONS, August 7, 2001 CLOSING PRICE ON CHICAGO BOARD OF OPTIONS EXCHANGE**					
2						
3	**August 7, 2001, CSCO closing price**	19.26				
4						
5	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
6	Aug01	7.50	11.90	0.05	17 Aug01	10
7	Aug01	10.00	9.60	0.20	17 Aug01	10
8	Aug01	12.50	6.50	0.10	17 Aug01	10
9	Aug01	15.00	4.20	0.10	17 Aug01	10
10	Aug01	17.50	2.10	0.40	17 Aug01	10
11	Aug01	20.00	0.65	1.45	17 Aug01	10
12	Aug01	22.50	0.15	3.40	17 Aug01	10
13	Aug01	25.00	0.05	5.00	17 Aug01	10
14	Aug01	27.50	0.10	7.50	17 Aug01	10
15	Aug01	30.00	0.10	11.90	17 Aug01	10
16	Aug01	32.50	0.05		17 Aug01	10
17	Aug01	35.00	0.05	16.20	17 Aug01	10
18	Sep01	10.00	9.50		21 Sep01	45
19	Sep01	12.50	6.30	0.15	21 Sep01	45
20	Sep01	15.00	4.50	0.40	21 Sep01	45
21	Sep01	17.50	2.75	0.90	21 Sep01	45
22	Sep01	20.00	1.35	2.00	21 Sep01	45
23	Sep01	22.50	0.55	3.80	21 Sep01	45
24	Sep01	25.00	0.20	5.50	21 Sep01	45
25	Sep01	27.50	0.10		21 Sep01	45
26	Sep01	30.00	0.05		21 Sep01	45
27	Oct01	10.00	10.00	0.10	19 Oct01	73
28	Oct01	12.50	6.90	0.25	19 Oct01	73
29	Oct01	15.00	5.00	0.65	19 Oct01	73
30	Oct01	17.50	3.20	1.40	19 Oct01	73
31	Oct01	20.00	1.80	2.55	19 Oct01	73
32	Oct01	22.50	0.95	4.10	19 Oct01	73
33	Oct01	25.00	0.45	6.00	19 Oct01	73
34	Oct01	27.50	0.20	7.50	19 Oct01	73
35	Oct01	30.00	0.15	10.70	19 Oct01	73
36	Oct01	35.00	0.05	16.30	19 Oct01	73
37	Oct01	40.00	0.05	21.50	19 Oct01	73
38	Oct01	45.00	0.05	29.50	19 Oct01	73
39	Oct01	50.00	0.05	31.12	19 Oct01	73
40	Oct01	55.00	0.10	37.50	19 Oct01	73
41	Oct01	60.00	0.05	36.75	19 Oct01	73
42	Oct01	65.00	0.05		19 Oct01	73

Notes: A blank in the price (for example the Oct01 puts with exercise price 65) indicates that no options were traded. The table includes only some of the traded Cisco options; on this date (7 August 2001), Cisco option with maturities as far out as January 2004 were traded. The Excel notebook for this chapter includes all the option prices for Cisco.

Notice what happens on September 21:

- Suppose the Cisco stock price on September 21 is $35. In this case you get to buy one share of Cisco for $20. Your gain is $35 - $20 = $15.
- If the Cisco stock price on September 21 is $18, you would not exercise your call option to buy a share of Cisco for $20 (why should you? you could buy it on the open market for less). The option expires unexercised, and your gain is $0.

What about the Cisco put option with an exercise price of $20? It was selling, on 7 August 2001, for $2.00. The put option gives you the right to *sell* a share of Cisco on or before the terminal date for its exercise price:



If Cisco's stock price on September 21 is $15, you will exercise your put option and sell a share of Cisco for $20, thus gaining $5.[1] On the other hand if Cisco's share price on September

[1] What if you don't own a share of Cisco on 21 September? No problem: You buy a share on the open market for $15 and use your option to sell it for $20.

21 is \$30, you will not exercise the put option (why sell a share using the option for \$20 when you can sell it on the open market for \$30?).

Option websites

All the data in this chapter was gathered from public sources on the Web. Many of these websites have superb data and also educational features. Here are some websites we especially enjoy.

- The website of the Chicago Board of Options Exchange (CBOE): http://www.cboe.com
- Option metrics: http://www.impliedvol.com/
- Equity analytics: http://www.e-analytics.com/optaaa.htm

European versus American options

Cisco's stock options are *American* stock options—they can be exercised *on or before* the option maturity date *T*. A *European* stock option can be exercised *only on* its maturity date *T*. Clearly an American stock option is worth at least as much as a European stock option.

Two notes about American versus European stock options:

- The terminology has nothing to do with geography. Most traded options, whether in the U.S., Europe, or Asia, are American and not European.
- A remarkable fact about American call options is the following: In many cases an American call option is worth *exactly* the same as an equivalent European call option. This happens if the stock on which the option is written does not pay a dividend before the option expiration date T. Since Cisco stock does not pay dividends, the "American"

feature of Cisco stock options is worthless, and the options on Cisco stock are worth the same as if they are European options. We discuss the reasons for this in Chapter 22.

22.2. Why buy a call option?

There are many conceivable reasons why you might want to buy a call option:

Reason 1: A call option allows you to delay the purchase of a stock: It's 7 August 2001, and you're thinking about buying a share of Cisco for its current market price of $19.26. As an alternative, you can buy a September call option with $X = \$20$. This option will cost you $1.35. Here's your thinking:

- If, on 21 September 2001, Cisco's stock price is > $20.00, you'll exercise the option and purchase the share for $20. If you're careful, you'll realize that there are several "sub-possibilities":
    - Cisco's 21 Sept. stock price = $35. Now you've made out like a bandit: You spent $1.35 for the option, but you bought the stock for $20, saving $15.00. Your net profit is $13.35 ($15.00 - $1.35 cost of the option).
    - If Cisco's 21 Sept. stock price = $21.00, you'll still exercise the option and purchase the stock for $20.00. You've saved $1.00 on the purchase price of the stock , but this time you will have lost a bit of money, since the option cost you $1.35. Your net profit will be -$0.65.
- If on 21 September Cisco's stock is selling for less than $20, you will not exercise your call option. If you still want to purchase the stock, you'll buy it on the open market. In all cases, you will be out only the $1.35 cost of the option.

Reason 2: A call option allows you to make a bet on the stock price going up. This bet is: a) Low cost, b) high upside potential, and c) one-sided

Suppose you buy the Cisco call option above: You spend $1.35 on 7 August 2001 to purchase an option which—on 21 September—gives you the right to purchase Cisco stock for $20. Your purpose is to bet on the price of Cisco stock in September. As you can see in the table below:

- This bet has a low cost: You've put up only $1.35 to make it.
- You will never lose more than the $1.35. This is what we mean when we say that the bet is "one-sided": You can only lose a limited amount of money.
- The bet has very high upside potential: The profits, both in dollars and as a percentage of the money you put up, rise very rapidly when the stock price in September increases over $20.

ANALYZING THE PROFIT FROM A CALL OPTION

Price of Cisco on 21 September	Exercise the option?	Your profit or loss	In percentage
\$15	No—the option gives you the right to buy Cisco for \$20, but the market price is less, so you would *not* exercise the option	-\$1.35	$\frac{Profit/loss}{Option\ cost}=\frac{-1.35}{1.35}=-100\%$
\$20	Yes/No—doesn't matter (you're buying the stock at its market price)	-\$1.35	$\frac{Profit/loss}{Option\ cost}=\frac{-1.35}{1.35}=-100\%$
\$22	Yes—the option lets you buy the stock for \$20, but the market price is \$21. So you should exercise (even though you've lost money—see next column)	$Profit\ on\ exercise - option\ cost$ $=(21-20)-1.35=-0.35$	$\frac{Profit\ on\ exercise - option\ cost}{option\ cost}=$ $\frac{(21-20)-1.35}{1.35}=\frac{-0.35}{1.35}=-26\%$
\$25	Yes	$Profit\ on\ exercise - option\ cost$ $=(25-20)-1.35=3.65$	$\frac{Profit\ on\ exercise - option\ cost}{option\ cost}=$ $\frac{(25-20)-1.35}{1.35}=\frac{3.65}{1.35}=270\%$
\$30	Yes	$Profit\ on\ exercise - option\ cost$ $=(30-20)-1.35=8.65$	$\frac{Profit\ on\ exercise - option\ cost}{option\ cost}=$ $\frac{(30-20)-1.35}{1.35}=\frac{8.65}{1.35}=641\%$

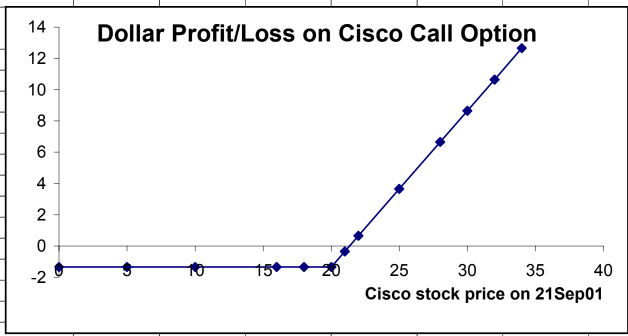
You can summarize all of this in a spreadsheet:

	A	B	C	D
1	**PROFIT FROM BUYING A CISCO CALL**			
2	**Bought for $1.35 on 7Aug01; Exercise price: X=$20**			
3	**Exercise date: 21Sep01**			
4				
5	Call purchase price, 7 Aug	1.35		
6	Call exercise price, X	20		
7				
8	**Market price of Cisco. 21 September 2001**	**Exercise the call?**	**Dollar profit/loss**	**Percentage profit/loss**
9	0	no	-1.35	-100.00%
10	5	no	-1.35	-100.00%
11	10	no	-1.35	-100.00%
12	16	no	-1.35	-100.00%
13	18	no	-1.35	-100.00%
14	20	no	-1.35	-100.00%
15	21	yes	-0.35	-25.93%
16	22	yes	0.65	48.15%
17	25	yes	3.65	270.37%
18	28	yes	6.65	492.59%
19	30	yes	8.65	640.74%
20	32	yes	10.65	788.89%
21	34	yes	12.65	937.04%

=IF(A21>$B$6,"yes","no")

=C21/$B$5

=IF(A21>$B$6,A21-$B$6,0)-$B$5



Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter ???.

22.3. Why buy a put option?

As in the case of the call, there are two primary types of reasons to buy a put:

Reason 1: The put option allows you to delay the decision to sell the stock.

It's 7 August 2001, and you own a share of Cisco stock. You're considering selling the stock; its current market price is $19.26. As an alternative, you can buy a September put option with $X = \$20$. This put option will cost you $2.00. Here's your thinking:

- If, on 21 September 2001, Cisco's stock price is < $20.00, you'll exercise the option and sell the share for $20. As in the case of the call option discussed above, there are several "sub-possibilities":

- Cisco's 21 Sept. stock price = \$5. Now you've made a lot of money: You spent \$2 for the option, but you sold the stock for \$20, which is \$15.00 more than its market price. Your net profit is \$13.00 (\$15.00 - \$2.00 cost of the option).
- If Cisco's 21 Sept. stock price = \$19.00, you'll still exercise the option and sell the stock for \$20.00. Compared to the market price, you've made \$1.00 on the sale of the stock , but this time you will have lost a bit of money, since the option cost you \$2.00. Your net profit will be -\$1.00.

* If on 21 September Cisco's stock is selling for more than \$20, you will not exercise your put option. If you still want to purchase the stock, you'll buy it on the open market. In all cases, you will be out only the \$2.00 cost of the option.

Reason 2: A call option allows you to make a bet on the stock price going down

If you buy a put for \$2.00 and wait until 21 September to exercise, here are your profits:

$$Put\ \ profits = \begin{cases} & Cisco\ \ stock\ \ price, S_T,\ on\ 21Sep01 \le 20 \\ 20.00 - S_T - 2.00 & \text{In this case you exercise the put and} \\ & \text{make } S_T - 20 \text{ minus the cost of the put} \\ & Cisco\ \ stock\ \ price, S_T,\ on\ 21Sep01 > 20 \\ -2.00 & \text{In this case you don't exercise the put;} \\ & \text{your loss is the cost of the put} \end{cases}$$

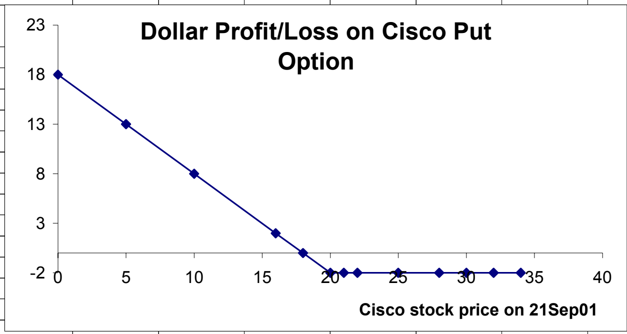
In a spreadsheet:

	A	B	C	D
1	**PROFIT FROM BUYING A CISCO PUT**			
2	**Bought for $2.00 on 7Aug01; Exercise price: X=$20**			
3	**Exercise date: 21Sep01**			
4				
5	Call purchase price, 7 Aug	2		
6	Call exercise price, X	20		
7				
8	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Dollar profit/loss**	**Percentage profit/loss**
9	0	yes	18	900.00%
10	5	yes	13	650.00%
11	10	yes	8	400.00%
12	16	yes	2	100.00%
13	18	yes	0	0.00%
14	20	no	-2	-100.00%
15	21	no	-2	-100.00%
16	22	no	-2	-100.00%
17	25	no	-2	-100.00%
18	28	no	-2	-100.00%
19	30	no	-2	-100.00%
20	32	no	-2	-100.00%
21	34	no	-2	-100.00%

=IF(A21<$B$6,"yes","no")

=C21/$B$5

=IF(A21<$B$6,$B$6-A21,0)-$B$5



Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter ???.

22.4. General properties of option prices

In this section we review some general properties of option prices. We look at the effects of option time to maturity, exercise price, the stock price, interest rates, and risk on option prices. Our discussion is informal and intuitive.

Property 1: Options with more time to maturity are worth more

The longer you have to exercise an option, the more it should be worth. The intuition here is clear: Suppose you have a September call option to buy Cisco stock for $20 and also an October call option to buy Cisco for $20. Since Cisco options are American options, anything the October call gives you all the opportunities associated with the September call—and then some. Thus the October call should be worth more than the September call.[2]

Here's some data for the Cisco options. Notice that the prices of the options increase with maturity:

	A	B	C	D	E	F
7	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
13	Aug01	20.00	0.65	1.45	17 Aug01	10
24	Sep01	20.00	1.35	2.00	21 Sep01	45
33	Oct01	20.00	1.80	2.55	19 Oct01	73
49	Jan02	20.00	2.90	3.40	18 Jan02	164
89	Jan03	20.00	5.40	5.20	17 Jan03	528
108	Jan04	20.00	6.80	5.80	16 Jan04	892

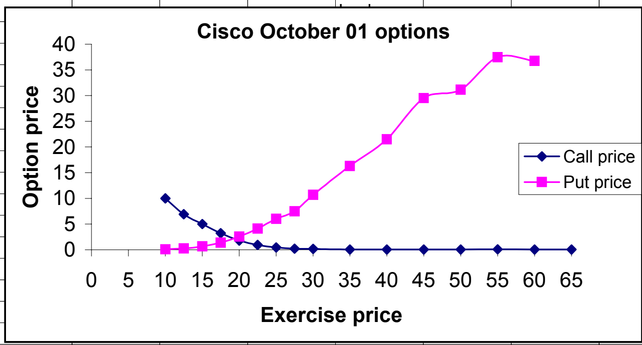
Property 2: Calls with higher exercise prices are worth less; puts with higher exercise prices are worth more

Suppose you had two October calls on Cisco: One call has an exercise price of $20 and the second call has an exercise price of $30. The second call is worth less than the first. Why? Think about calls as *bets* on the stock price: The first call is a bet that the stock price will go over $20, whereas the second call is a bet that the stock price will go over $30. You're always more likely to win the first bet (Cisco will go over $20) than the second bet.

From the table below you can see that Cisco's option prices conform to this property:

[2] The argument in this paragraph seems to depend critically on the calls being American and not European. It holds, however, for European call also—see Chapter 22, Section ???.

	M	N	O	P
7	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**
8	Oct01	10.00	10.00	0.10
9	Oct01	12.50	6.90	0.25
10	Oct01	15.00	5.00	0.65
11	Oct01	17.50	3.20	1.40
12	Oct01	20.00	1.80	2.55
13	Oct01	22.50	0.95	4.10
14	Oct01	25.00	0.45	6.00
15	Oct01	27.50	0.20	7.50
16	Oct01	30.00	0.15	10.70
17	Oct01	35.00	0.05	16.30
18	Oct01	40.00	0.05	21.50
19	Oct01	45.00	0.05	29.50
20	Oct01	50.00	0.05	31.12
21	Oct01	55.00	0.10	37.50
22	Oct01	60.00	0.05	36.75
23	Oct01	65.00	0.05	
24				



Here we've looked at all the options which expire on the same date (October 2001). As you can see: The higher the option exercise price, the lower the call price and the higher the put price. (There are a few exceptions; see paragraph below.)

The logic of this is clear:

- If an October 2001 Cisco call option with exercise price \$10 (the right to buy a share of Cisco for \$10) is worth \$10, then an October 2001 call with exercise price \$12.50 (the right to buy a share of Cisco for \$12.50—more than \$10) is worth less.
- If an October 2001 Cisco put option with exercise price \$10 (the right to sell a share of Cisco in October for \$10) is worth \$0.10, then the right to sell a share of Cisco for \$12.50 should be worth more. And so it is.

The graph and the table show what appear to be a few exceptions to this rule. For example, the Cisco put with $X = \$65$ traded for less than the put with $X = \$60$. If you see this kind of behavior it almost always has to do with the fact that the options in question are infrequently traded. In the example given here, the \$65 and \$60 calls only traded several times during the day in question. The result is that the option prices given in the table refer to options traded on Cisco stock at different times and with different prices. (Notice that one of the options—the October put with exercise price \$65—didn't trade at all.)

When the stock price goes up, call option prices go up and put option prices go down

You can't see this in the original data, but here's an example:

[Example forthcoming?]

The reason for this behavior is obvious, if you think of an option as a bet: Suppose you buy a Cisco $X=20$ October 2001 call option. We can view this option as a bet that Cisco's stock price in October will be above $20. The probability of your winning this bet is higher if Cisco's current stock price is higher, and hence so is the call option's price. Thus, for example, if you're willing to pay $1.80 for the $X=20$ October call when Cisco's current stock price is $19.26, you would be willing to pay more for the same call when Cisco's stock price is $22.

The logic for puts is the same, though the result is opposite: The higher the stock price, the lower the put option price.

When the interest rate goes up, call prices go up and put prices tend to go down

When

[Example forthcoming?]

When the risk of the underlying asset goes up, option prices tend to increase

[Example forthcoming?]

Summary table

Here's a summary table of the effects of basic parameters on option prices:

Variable	Call price?	Put price?
Stock price ↑	↑	↓
Exercise price X ↑	Call price ↓	Put price ↑
Time to maturity ↑	Call price ↑	Put price ↑
Interest rate ↑	Call price ↑	Put price ↓ (usually?)
Stock price volatility ↑	Call price ↑	Put price ↑

22.5. Writing options, shorting stock

Our whole discussion thus far has been from the point of view of the option purchaser. For example in Section 2 we derived the profit pattern from buying a Cisco 20 call for \$1.35 on 7 August 2001 and waiting until the call maturity on 21 September 2001. Similarly in Section 3 we looked at the profit from buying a Cisco 20 put.

There's another side to this story: When you buy a call, someone else sells the call. In the jargon of options markets, the call seller is *writing a call*.

Call buyer: On 7 August 2001 buys, for \$1.35, the *right* to buy one share of Cisco stock for \$20 on or before 21 September 2001.

Call writer: On 7 August 2001 sells, for \$1.35, the *obligation* to sell one share of Cisco stock for \$20—as per demand of the call option buyer—on or before 21 September.

Here's the way the call writer's profit pattern looks:

CALL OPTION CASH FLOW PATTERN--the call writer

7-Aug-01 — 21-Sep-01

Receives: $1.35

In principle, the call buyer has the right to buy Cisco stock for $20 on any date before 21 September. In actual fact, the call buyer will only want to exercise the call option at its terminal date (see next chapter).

If Cisco's stock price > $20 (denote this by $S_T > 20$), the call will be exercised. The call writer has to sell one share of Cisco stock for $20.

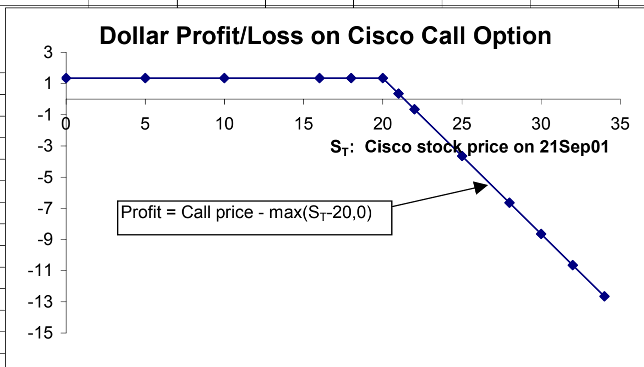
Call writer's loss: S_T - $20

If Cisco's stock price ≤ $20, (denote this by $S_T \le 20$), the call will not be exercised.

Call writer's loss: 0

Here's the profit graph from writing a call option:

	A	B	C
1	**PROFIT FROM WRITING A CISCO CALL**		
2	**Sold for $1.35 on 7Aug01; Exercise price: X=$20**		
3	**Exercise date: 21Sep01**		
4			
5	Call price, 7 August 2001	1.35	
6	Call exercise price, X	20	
7			
8	**S_T: Market price of Cisco, 21 September 2001**	**Will call buyer exercise the call?**	**Dollar profit/loss**
9	0	no	1.35
10	5	no	1.35
11	10	no	1.35
12	16	no	1.35
13	18	no	1.35
14	20	no	1.35
15	21	yes	0.35
16	22	yes	-0.65
17	25	yes	-3.65
18	28	yes	-6.65
19	30	yes	-8.65
20	32	yes	-10.65
21	34	yes	-12.65



=IF(A21>$B$6,"yes","no")

=$B$5-IF(A21>$B$6,A21-$B$6,0)

Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter ???.

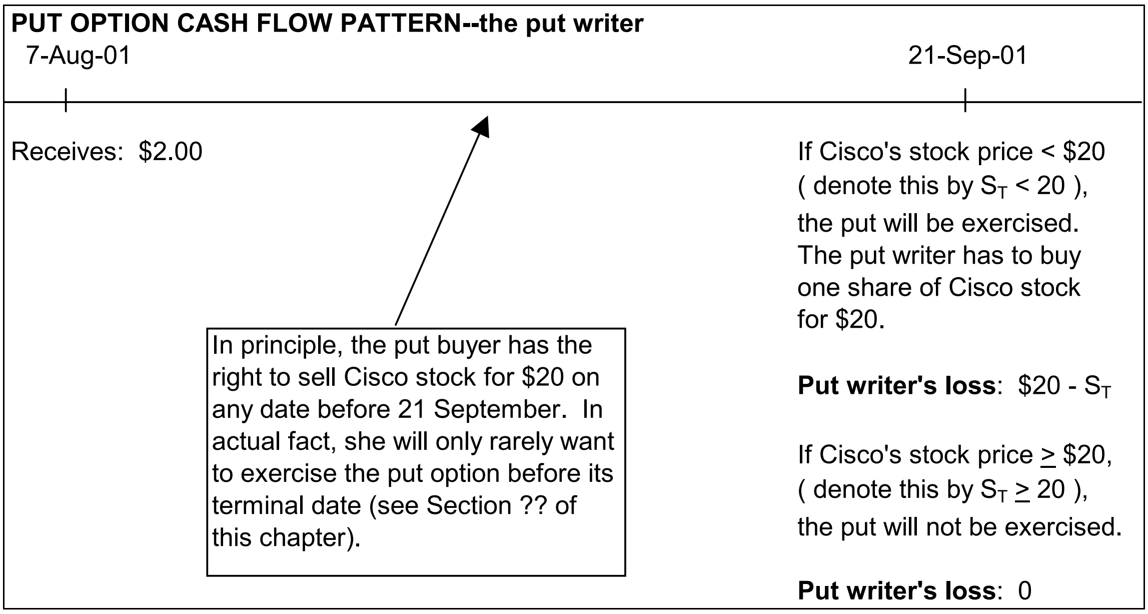
Writing puts

There's a similar story for puts:

Put buyer: On 7 August 2001 buys, for $2.00, the *right* to sell one share of Cisco stock for $20 on or before 21 September 2001.

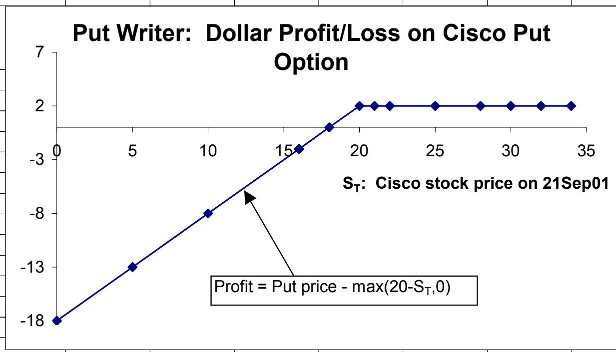
Put writer: On 7 August 2001 sells, for $2.00, the *obligation* to buy one share of Cisco stock for $20—as per demand of the put option buyer—on or before 21 September.

Here's the way the call writer's profit pattern looks:



Here's a graph of the profit pattern from writing a put:

	A	B	C	D	E	F	G	H	I	J	K
1	PROFIT FROM WRITING A CISCO PUT										
2	Sold for $2.00 on 7Aug01; Exercise price: X=$20										
3	Exercise date: 21Sep01										
4											
5	Put price, 7 August 2001	2									
6	Put exercise price, X	20									
7											
8	S_T: Market price of Cisco, 21 September 2001	Will put buyer exercise the put?	Dollar profit/loss to put writer								
9	0	yes	-18								
10	5	yes	-13								
11	10	yes	-8								
12	16	yes	-2								
13	18	yes	0								
14	20	no	2								
15	21	no	2								
16	22	no	2								
17	25	no	2								
18	28	no	2								
19	30	no	2								
20	32	no	2								
21	34	no	2								
22											
23											
24	=IF(A21<$B$6,"yes","no")					Technical note: The data series graphed above					
25						are *non-contiguous* in the spreadsheet (they're					
26						column A and column C). For how to do this: see					
27		=$B$5-IF(A21<$B$6,$B$6-A21,0)				Chapter ???.					
28											



Short-selling a stock

Short-selling a stock ("shorting") is the stock equivalent of writing an option. Here's how shorting a stock compares to buying a stock:

Stock buyer: On 7 August 2001 buys one share of Cisco stock, for $19.26. When you sell the stock—call the date T—you'll get the stock price S_T. Of course you will have also earned any dividends that Cisco will have paid up to and including date T.[3] Ignoring the time value of money, your profit from buying the stock is:

$$S_T + Cisco\ dividends - 19.26$$

Stock shorter: On 7 August 2001 contacts his broker and borrows one share of Cisco stock, which he then sells, thus receiving $19.26. At some future date T, the short-seller of the stock will purchase a share of Cisco on the open market, paying the then-current

[3] Not something you're like to have to worry about: Cisco has never paid a dividend!

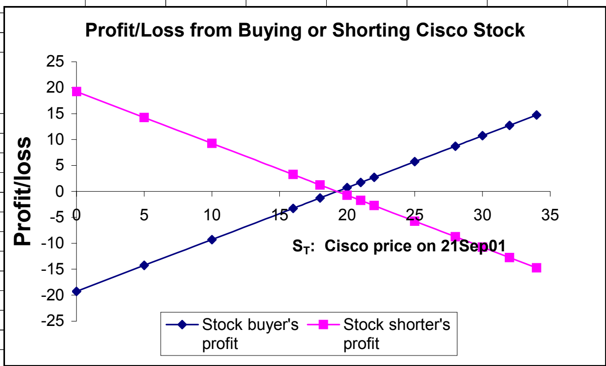
market price S_T. If along the way Cisco has paid any dividends, the short-seller will be obliged to pay these dividends to the person he's borrowed the stock from. His total profit will be:

$$19.26 - \left(S_T + Cisco\ dividends\right).$$

[For more information, see the sidebar from the Motley Fool, one of our favorite websites.]

In the option chapters in this book, we will generally assume that stocks don't pay any dividends between the time you buy them and the time you sell them. This means that the profit from buying or shorting a stock can be represented as follows:

	A	B	C
1	**PROFIT FROM BUYING OR SHORTING CISCO STOCK**		
2	**Market price on 7 August 2001: $19.26**		
3	**Assumption: Position liquidated on 21 September 2001**		
4			
5	Cisco stock price, 7 August 2001	19.26	
6			
7	**S_T: Market price of Cisco, 21 September 2001**	**Stock buyer's profit**	**Stock shorter's profit**
8	0	-19.26	19.26
9	5	-14.26	14.26
10	10	-9.26	9.26
11	16	-3.26	3.26
12	18	-1.26	1.26
13	20	0.74	-0.74
14	21	1.74	-1.74
15	22	2.74	-2.74
16	25	5.74	-5.74
17	28	8.74	-8.74
18	30	10.74	-10.74
19	32	12.74	-12.74
20	34	14.74	-14.74
21			
22		=A20-$B$5	
23			=$B$5-A20



The Motley Fool. Fool.com.

The Fool FAQ : Shorting Stocks

Many times on the Fool boards I've seen references to `selling a stock short' or `taking a short position.' Will someone tell me plainly what shorting is?

An investor who sells stock short borrows shares from a brokerage house and sells them to another buyer. Proceeds from the sale go into the shorter's account. He must buy those shares back (cover) at some point in time and return them to the lender.

Thus, if you sell short 1000 shares of Gardner's Gondolas at $20 a share, your account gets credited with $20,000. If the boats start sinking---since David Gardner, founder and CEO of VENI, knows nothing about their design---and the stock follows suit, tumbling to new lows, then you will start thinking about "covering" your short there for a very nice profit. Here's the record of transactions if the stock falls to $8.

Borrowed and Sold Short 1000 shares at $20: +$20,000

Bought back and returned 1000 shares at $8: -$8,000

Profit: + $12,000

But what happens if as the stock is falling, Tom Gardner, boatsmen extraordinaire, takes over the company at his brother's behest, and the holes and leaks are covered. As the stock begins to takes off, from $14 to $19 to $26 to $37, you finally decide that you'd better swallow hard and close out the transaction. You do so, buying back shares of TOMY (new ticker symbol) at $37.

Here's the record of transaction:

Borrowed and sold short 1000 shares at $20: +$20,000

Bought back and returned 1000 shares at $37: -$37,000

Loss: -$17,000

Ouch. So you see, in the second scenario, when I, your nemesis, took over the company, you lost $17,000...which you'll have to come up with. There's the danger....you have to be able to buy back the shares that you initially borrowed and sold. Whether the price is higher or lower, you're going to need to buy back the shares at some point in time.

To learn more about short selling, try reading the following books: "Tools of the Bear: How Any Investor Can Make Money When Stocks Go Down" - Charles J. Caes; "Financial Shenanigans: How To Detect Accounting Gimmicks & Fraud" - Howard M. Shilit; "When Stocks Crash Nicely: The Finer Art of Short Selling" - Kathry F. Staley; "Selling Short: Risks, Rewards and Strategies for Short Selling Stocks, Options and Futures" - Joseph A. Walker. None of these are perfect in their coverage of short selling but each has its strengths.

Shorting, unlike puts, seems to have an unlimited downside potential, correct? That is, hypothetically, the stock can rise to infinity. Puts, besides the time limit, have a limited downside. Why then, for a short term short, would anyone short instead of purchasing puts?

Theoretically, yes. In reality, no. Because in our number system we count upwards and don't stop, we opine that because numbers go on forever, so can a stock price. But when we think about this objectively, it seems kind of silly, no? Obviously a stock price, which at SOME point reflects actual value in a business, cannot go on to infinity.

Yes, puts do have a limited downside. However, options have an expiration date, which means that they are "time-wasting assets". They also have a "strike price" which means that you need to pick a price and then have the stock below it on expiration date. Finally, you have to pay a premium for an option and if you are not "in the money" more than the premium, by expiration day, you still lose. So, with options, not only do you have to be worried about the direction of the stock, you need to be correct about the magnitude of the move and the time in which it will happen. And even then, even if you successfully manage all 3 of these things, you can still lose money if you don't cover the premium. Not very Foolish. With shorting, you only really need to be concerned about direction. As for limiting liability, you can do that yourself by putting in a buy stop at a price where the loss is "too much" for you.

What is short interest? Does it have anything to do with short attention spans?

Pardon? Short interest? Oh yes! Ahem, short interest is simply the total number of shares of a company that have been sold short. The Fool believes that the best shorts are those with low short interest. They present the maximum chance for price depreciation as few short sales have occurred, driving down the price. Also, low short interest stocks are less susceptible to short squeezes (see below). Short interest figures are available towards the end of each month in financial publications like *Barron's* and the *Investor's Business Daily*.

The significance of short interest is relative. If a company has 100 million shares outstanding and trades 6 million shares a day, a short interest of 3 million shares is probably not significant (depending on how many shares are closely held). But a short interest of 3 million for a company with 10 million shares outstanding trading only 100,000 shares a day is quite high.

I've heard the term 'days to cover' thrown around quite a bit. Does 'days to cover' have anything to do with short interest?

Yes, it does! Days to cover is a function of how many shares of a particular company have been sold short. It is calculated by dividing the number of shares sold short by the average daily trading volume.

Look at Ichabod's Noggins (Nasdaq:HEAD). One million shares of this issue have been sold short (we can find this number, called the short interest, in such publications as Barrons and the IBD). It has an average trading volume of 25,000. The days to cover is 1,000,000/25,000, or 40 days.

When you short a stock, you want the days to cover to be low, say around 7 days or so. This will make the shares less subject to a short squeeze, the nightmare of shorters in which someone starts buying up the shares and driving up the share price. This induces shorters to buy back their shares, which also drives up the price! A short days to cover means the short interest can be eliminated quickly, preventing a short squeeze from working very well.

Also, a lengthy days to cover means that many people have already sold short the stock, making a further decline less likely.

What effect does a large short coverage have (generally) on the stock`s price? Generally, heavy buying increases the price while selling decreases it. Assuming the stocks price has been steady, or climbing, and many shorters attempt to cover their losses, how will this affect the price?

What you are referring to, in investment parlance, is a "short squeeze." When a number of short sellers all try to "cover" their short at the same time, that does indeed drive the stock up.

Our approach when shorting is therefore to avoid in general stocks that already have a fairly hefty amount of existing short sales. We try to set ourselves up so we'll never get squeezed.

I'll point out that short squeezes can be the result of better than expected earnings or some other fundamental aspects of a company's operation. They can also be the result of direct manipulation. That is, profit-seeking individuals with large amounts of cash at their disposal can look on a large short position in a stock as an invitation to start buying, driving up the share prices, thus forcing short-sellers to cover. This in turn drives up the price, and before you know it, the share price has soared!

OK, I understand the potential benefits and risks of shorting, except for one thing. If the stock I've shorted pays a dividend, am I liable for that dividend?

Yes. If you are short as of the ex-dividend date, you are liable to pay the dividend to the person whose shares you have borrowed to make your short sale. I must say, however, that if you are correct in your judgment to sell the issue short, your profits achieved thereby will certainly outweigh the small dollar amount of the dividend payout.

What happens if the stock I've shorted splits?

MF Swagman replies:

Let's say we're speaking of a two-for-one split. In that case, all that happens is that you must cover your short position with twice as many shares as you opened it. If you shorted 100 shares, you must cover with 200. Don't forget, though, that the magnitude of your investment hasn't changed, for while you now have twice as many shares, each one is only worth half as much as before! So, while your original cost basis for the 100 shares may have $36, now, with 200 shares, it is only $18.

This is a very foolish question, I'm sure, but if I sell short I am essentially borrowing the shares from someone else through my broker. Assuming that the lender does NOT need the shares prematurely, what determines how long I can stay short? (pun intended) How long do I have before I am forced to cover my position? Is there any regulation? Is it simply dependent on when/if the broker needs them? Could I possibly stay short for an indefinite period?

As far as I know, there is no pre-determined limit to how long you can keep your short position open. Technically, you could be forced to cover at any time, but typically, having the shares you have borrowed called back is unusual. At least so state all the Schwab representatives of whom I have asked this question.

Source: The Motley Fool website, http://www.fool.com/FoolFAQ/FoolFAQ0033.htm

22.6. Option strategies—more complicated reasons to buy options

"Option strategy" refers to the profits which result from holding a combination of options and shares. In this and the following sections we give a number of examples of such strategies.

Stock + put

We begin with a very simple (but useful) strategy: Suppose we decide, on 7 August 2001, to purchase one share of Cisco stock *and* to purchase a put on the stock with exercise price 20 and expiration date September. The total cost of this strategy is \$21.26: \$19.26 for the share of Cisco and \$2.00 for each put.

Such a strategy effective *insures* your stock returns by guaranteeing that on 17 September 2001 you will have at least \$20 in hand. Your worst-case net profit will be a loss of \$1.65:

Stock price on 17 September	Strategy	Cash in hand	Net profit
Less than \$20	Exercise put option: Give someone else your share of Cisco for \$20.[4]	\$20	\$20 – (\$19.26 + \$2)= -\$1.65
More than \$20	Let the put option expire (don't use it)	Cisco stock price on 17 September, S_T	S_T – (\$19.26 + \$2)= S_T - \$21.26

In a spreadsheet, here's the way this strategy looks:

[4] Not so simple: How does this happen? Bullshit a bit about the exchange mechanism?????

	A	B	C	D	E	F	G
1	**STOCK + PUT: OPTION STRATEGY PROFITS**						
2							
3							
4	Stock price, 7Aug01	19.26					
5	Cost of put option	2	=IF(A9<$B$6,$B$6-A9,0)-$B$5				
6	Put exercise price, X	20					
7					=A9-$B$4		
8	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Profit/loss on the put**	**Profit/loss on the stock**	**Total profit/loss**		
9	0	yes	18	-19.26	-1.26	<-- =C9+D9	
10	5	yes	13	-14.26	-1.26		
11	10	yes	8	-9.26	-1.26		
12	16	yes	2	-3.26	-1.26		
13	18	yes	0	-1.26	-1.26		
14	20	no	-2	0.74	-1.26		
15	21	no	-2	1.74	-0.26		
16	22	no	-2	2.74	0.74		
17	25	no	-2	5.74	3.74		
18	28	no	-2	8.74	6.74		
19	30	no	-2	10.74	8.74		
20	32	no	-2	12.74	10.74		
21	34	no	-2	14.74	12.74		
22							
23							
24							
25							
26							
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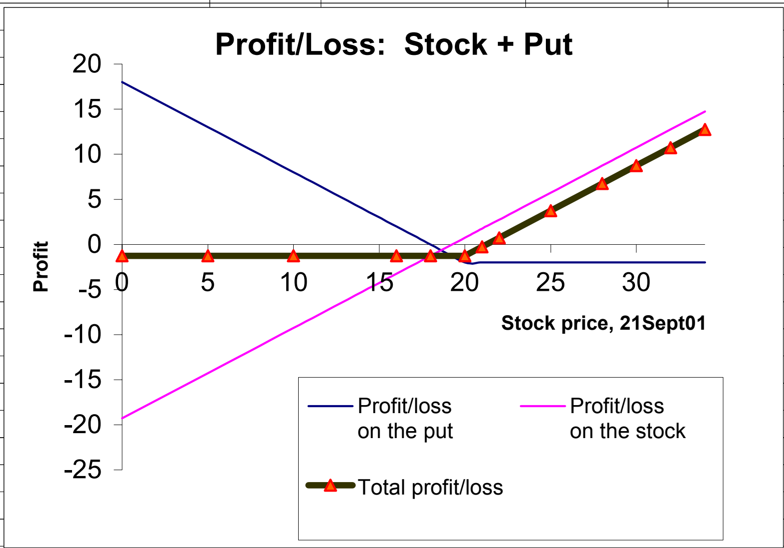


Portfolio insurance strategies—which involve combinations of buying puts and shares together—are popular strategies. They guarantee a minimum return on the investment in the shares (at an extra cost, of course: you have to buy the puts).

ANTICIPATING A BIT—put-call parity

You'll notice that the graph of the stock+put strategy looks a lot like the graph of a call (Section 2). This may lead you to surmise that the payoffs of the combination *stock+put* is somehow equivalent to the payoffs of a *call*. However, this isn't quite true, as you'll see in the next chapter. There we discuss the *put-call parity theorem* and show that—for a put and call written on the same stock and having the same exercise price X:

$$stock + put = call + PV\left(X\right)$$

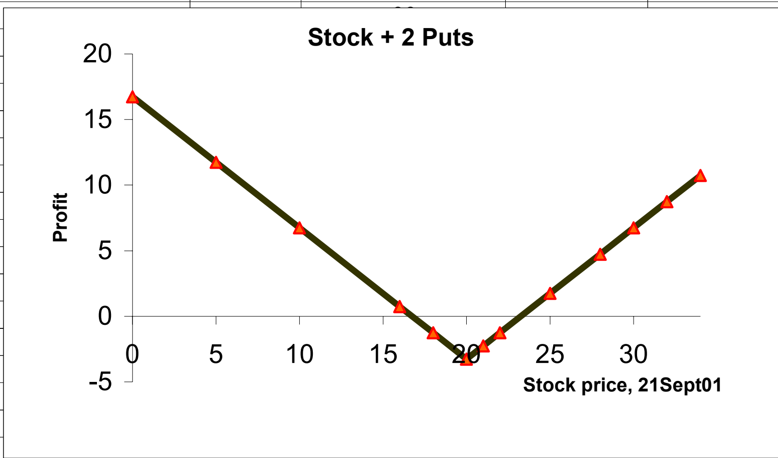
Stock + 2 puts

Suppose you purchased one share of stock and bought 2 puts, each costing \$2 and each having an exercise price of \$20. Here's what your payoff pattern would look like:

Stock price on 17 September	Strategy	Cash in hand, 17 September	Net profit
$S_T \leq \$20$	Exercise both put options. Give someone else your share of Cisco for \$20. Buy an additional share in the market and give it to the put writer for S_T.	2*20 - S_T	2*20 - S_T – (\$19.26 + \$4)= \$16.74 - S_T
More than \$20	Let the put options expire (don't use them)	Cisco stock price on 17 September, S_T	S_T – (\$19.26 + \$4)= S_T - \$21.26

If we make an Excel table, here's what it looks like:

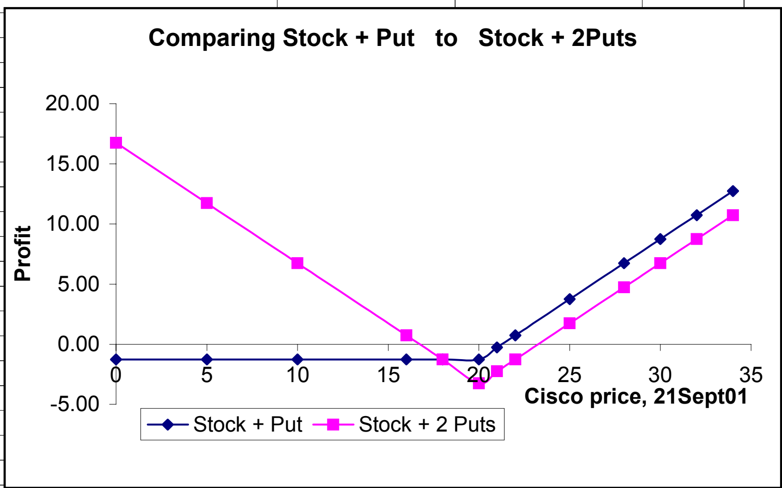
	A	B	C	D	E	F	G
1	**STOCK + 2 PUTS PUT: OPTION STRATEGY PROFITS**						
2							
3	Stock price, 7Aug01	19.26					
4	Cost of put option	2.00		=2*(IF(A8<$B$5,$B$5-A8,0)-$B$4)			
5	Put exercise price, X	20.00					
6					=A8-$B$3		
7	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Profit/loss on the puts**	**Profit/loss on the stock**	**Total profit/loss**		
8	0	yes	36	-19.26	16.74	<-- =C8+D8	
9	5	yes	26	-14.26	11.74		
10	10	yes	16	-9.26	6.74		
11	16	yes	4	-3.26	0.74		
12	18	yes	0	-1.26	-1.26		
13	20	no	-4	0.74	-3.26		
14	21	no	-4	1.74	-2.26		
15	22	no	-4	2.74	-1.26		
16	25	no	-4	5.74	1.74		
17	28	no	-4	8.74	4.74		
18	30	no	-4	10.74	6.74		
19	32	no	-4	12.74	8.74		
20	34	no	-4	14.74	10.74		



Comparing strategies

What's better as a strategy: buying a share of Cisco and buying 1 put, or buying a share of Cisco and buying 2 puts? A little thought will lead you to conclude that—in an efficient market—this is not a sensible question; in an efficient market, where all assets and combinations of assets are properly priced, there's always a tradeoff between assets. Here's a comparison of the graphs:

	A	B	C	D	E	F	G	H
1	**STOCK + PUT COMPARED TO STOCK + 2 PUTS**							
2								
3	Stock price, 7Aug01	19.26						
4	Cost of put option	2.00						
5	Put exercise price, X	20.00						
6								
7	**Market price of Cisco. 21 September 2001**	**Stock + Put**	**Stock + 2 Puts**					
8	0	-1.26	16.74	<-- =2*IF(A8<=$B$5,$B$5-A8,0)+A8-($B$3+2*$B$4)				
9	5	-1.26	11.74					
10	10	-1.26	6.74	=IF(A8<=$B$5,$B$5-A8,0)+A8-($B$3+$B$4)				
11	16	-1.26	0.74					
12	18	-1.26	-1.26					
13	20	-1.26	-3.26					
14	21	-0.26	-2.26					
15	22	0.74	-1.26					
16	25	3.74	1.74					
17	28	6.74	4.74					
18	30	8.74	6.74					
19	32	10.74	8.74					
20	34	12.74	10.74					



The choice between the two strategies involves *tradeoffs* (that's the nature of market efficiency: in an efficient market no asset ever completely dominates another asset).

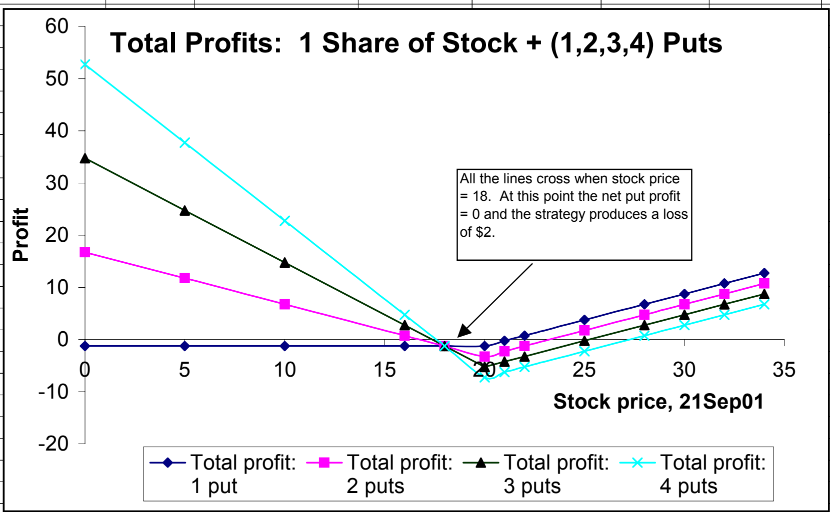
- The stock+put strategy has higher profit when the Cisco September stock price ≥ 20, but it has a negative profit for Cisco $S_T \leq 20$
- The stock + 2 put strategy costs more (you can see this by noting that its payoff when $S_T = 20$ is less than that of the stock + put strategy). On the other hand, it has positive profits both for very low and for high S_T .

Which strategy should you choose? It depends on your prediction of the future: If you think that Cisco is going to make a big move, up or down, then stock + 2 puts is for you. If you think, on the other hand, that Cisco might go up, but you want protection when and if its price goes down (that is, no bets for you), then stock + put is your choice.

Stock + more puts

There's almost nothing to say here, except to show you the graphs:

	A	B	C	D	E	F	G	H	I
1	**STOCK + SEVERAL PUTS PUT: OPTION STRATEGY PROFITS**								
2									
3	Stock price, 7Aug01	19.26							
4	Cost of put option	2							
5	Put exercise price, X	20							
6	Number of puts purchased	2							
7									
8	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Profit/loss on single put**	**Profit/loss on the stock**	**Total profit: 1 put**	**Total profit: 2 puts**	**Total profit: 3 puts**	**Total profit: 4 puts**	
9	0	yes	18	-19.26	-1.26	16.74	34.74	52.74	
10	5	yes	13	-14.26	-1.26	11.74	24.74	37.74	
11	10	yes	8	-9.26	-1.26	6.74	14.74	22.74	
12	16	yes	2	-3.26	-1.26	0.74	2.74	4.74	
13	18	yes	0	-1.26	-1.26	-1.26	-1.26	-1.26	
14	20	no	-2	0.74	-1.26	-3.26	-5.26	-7.26	
15	21	no	-2	1.74	-0.26	-2.26	-4.26	-6.26	
16	22	no	-2	2.74	0.74	-1.26	-3.26	-5.26	
17	25	no	-2	5.74	3.74	1.74	-0.26	-2.26	
18	28	no	-2	8.74	6.74	4.74	2.74	0.74	
19	30	no	-2	10.74	8.74	6.74	4.74	2.74	
20	32	no	-2	12.74	10.74	8.74	6.74	4.74	
21	34	no	-2	14.74	12.74	10.74	8.74	6.74	



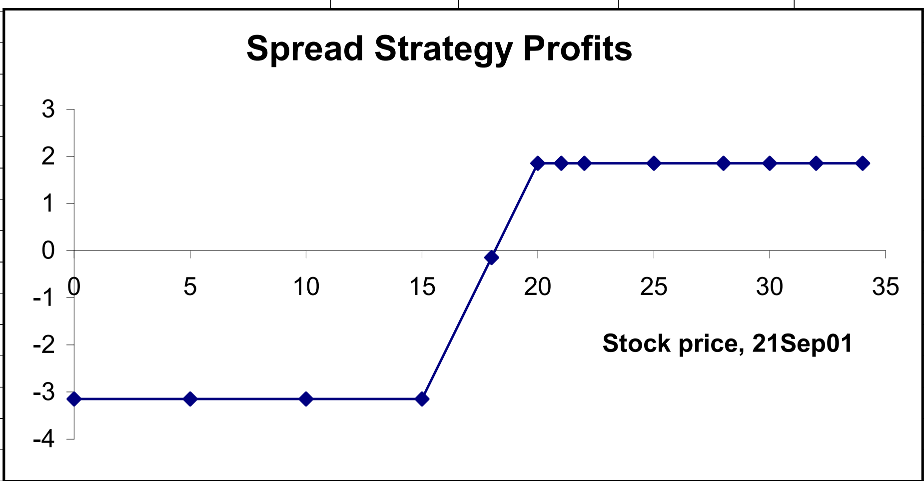
22.7. Spread

A spread strategy involves buying one option on a stock and writing another option. In the example below on August 7, 2001:

- We buy one X=15 September call on Cisco. This option costs $4.50.
- We write one X=20 September call on Cisco. This option costs 1.35; since we're writing the option, this is income on August 7.

In the spreadsheet below we examine this strategy's payoffs and graph them:

	A	B	C	D	E	F
1	**SPREAD: BUY ONE OPTION, SELL ANOTHER**					
2						
3	Cost of September, X=15 call	4.5				
4	Number of X=15 calls purchased	1				
5						
6	Cost of Sept. X=20 call	1.35				
7	Number of X=20 calls purchased	-1				
8						
9	**Market price of Cisco. 21 September 2001**	**Exercise X=15 call?**	**Profit/loss on X=15 call**	**Exercise X=20 call?**	**Profit/loss on X=20 call**	**Total profit**
10	0	no	-4.5	no	1.35	-3.15
11	5	no	-4.5	no	1.35	-3.15
12	10	no	-4.5	no	1.35	-3.15
13	15	no	-4.5	no	1.35	-3.15
14	18	yes	-1.5	no	1.35	-0.15
15	20	yes	0.5	no	1.35	1.85
16	21	yes	1.5	yes	0.35	1.85
17	22	yes	2.5	yes	-0.65	1.85
18	25	yes	5.5	yes	-3.65	1.85
19	28	yes	8.5	yes	-6.65	1.85
20	30	yes	10.5	yes	-8.65	1.85
21	32	yes	12.5	yes	-10.65	1.85
22	34	yes	14.5	yes	-12.65	1.85



There's another way to think about the strategy profits: On September 21, 2001 (the option expiration date) we will have:

$$\underbrace{-4.50+\underbrace{Max\left[S_{CSCO,21Sep01}-15,0\right]}_{\substack{\uparrow\\ \text{This is the option payoff on 21Sep01}\\ \text{from buying a call with X=15}}}}_{\substack{\uparrow\\ \text{This is the profit from buying}\\ \text{the X=15 option}}}\underbrace{+1.35\underbrace{-Max\left[S_{CSCO,21Sep01}-20,0\right]}_{\substack{\uparrow\\ \text{Writing an option means}\\ \text{taking a loss if Cisco's stock}\\ \text{price is > 20}}}}_{\substack{\uparrow\\ \text{This is the profit from writing the X=20 option}}}$$

$$=-3.15+\begin{cases} 0 & S_{CSCO,21Sep01}<15 \\ S_{CSCO,21Sep01}-15 & 15\le S_{CSCO,21Sep01}\le 20 \\ 5 & S_{CSCO,21Sep01}>20 \end{cases}$$

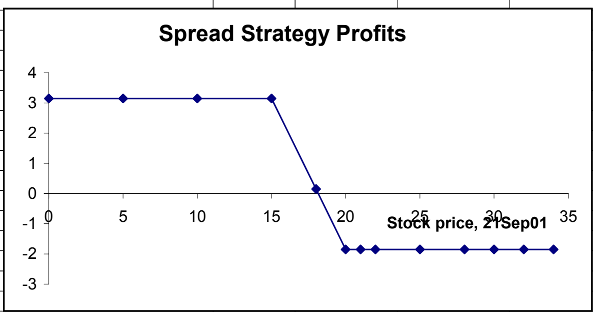
In column G we've put this equation as an Excel formula. Notice the two **If** functions, one within the other:

	A	B	C	D	E	F	G	H
1	**SPREAD: BUY ONE OPTION, SELL ANOTHER**							
2								
3	Cost of September, X=15 call	4.5						
4	Number of X=15 calls purchased	1						
5								
6	Cost of Sept. X=20 call	1.35						
7	Number of X=20 calls purchased	-1						
8								
9	**Market price of Cisco. 21 September 2001**	**Exercise X=15 call?**	**Profit/loss on X=15 call**	**Exercise X=20 call?**	**Profit/loss on X=20 call**	**Total profit**	**Equation**	
10	0	no	-4.5	no	1.35	-3.15	-3.15	<-- =-3.15+IF(A10<15,0,IF(A10>20,5,A10-15))
11	5	no	-4.5	no	1.35	-3.15	-3.15	<-- =-3.15+IF(A11<15,0,IF(A11>20,5,A11-15))
12	10	no	-4.5	no	1.35	-3.15	-3.15	
13	15	no	-4.5	no	1.35	-3.15	-3.15	
14	18	yes	-1.5	no	1.35	-0.15	-0.15	
15	20	yes	0.5	no	1.35	1.85	1.85	
16	21	yes	1.5	yes	0.35	1.85	1.85	
17	22	yes	2.5	yes	-0.65	1.85	1.85	
18	25	yes	5.5	yes	-3.65	1.85	1.85	
19	28	yes	8.5	yes	-6.65	1.85	1.85	
20	30	yes	10.5	yes	-8.65	1.85	1.85	
21	32	yes	12.5	yes	-10.65	1.85	1.85	
22	34	yes	14.5	yes	-12.65	1.85	1.85	

Why buy a spread? In this case the spread is a not-too-risky bet on the stock price going up. If it goes up, you profit (moderately); if the stock price goes down, your loss is limited to \$3.15. This kind of a spread is called a *bull spread*—you're bullish on the stock (meaning that you think the stock price will go up).

Here's a *bear spread*: In this case we write the $X = 15$ call and buy the $X=20$ call.

	A	B	C	D	E	F	G	H
1	**BULL SPREAD: A MODERATE BET ON STOCK DECLINE**							
2								
3	Cost of September, X=15 call	4.5						
4	Number of X=15 calls purchased	-1						
5								
6	Cost of Sept. X=20 call	1.35						
7	Number of X=20 calls purchased	1						
8								
9	**Market price of Cisco. 21 September 2001**	**Exercise X=15 call?**	**Profit/loss on X=15 call**	**Exercise X=20 call?**	**Profit/loss on X=20 call**	**Total profit**	**Equation**	
10	0	no	4.5	no	-1.35	3.15	3.15	<-- =3.15-IF(A10<15,0,IF(A10>20,5,A10-15))
11	5	no	4.5	no	-1.35	3.15	3.15	<-- =3.15-IF(A11<15,0,IF(A11>20,5,A11-15))
12	10	no	4.5	no	-1.35	3.15	3.15	
13	15	no	4.5	no	-1.35	3.15	3.15	
14	18	yes	1.5	no	-1.35	0.15	0.15	
15	20	yes	-0.5	no	-1.35	-1.85	-1.85	
16	21	yes	-1.5	yes	-0.35	-1.85	-1.85	
17	22	yes	-2.5	yes	0.65	-1.85	-1.85	
18	25	yes	-5.5	yes	3.65	-1.85	-1.85	
19	28	yes	-8.5	yes	6.65	-1.85	-1.85	
20	30	yes	-10.5	yes	8.65	-1.85	-1.85	
21	32	yes	-12.5	yes	10.65	-1.85	-1.85	
22	34	yes	-14.5	yes	12.65	-1.85	-1.85	



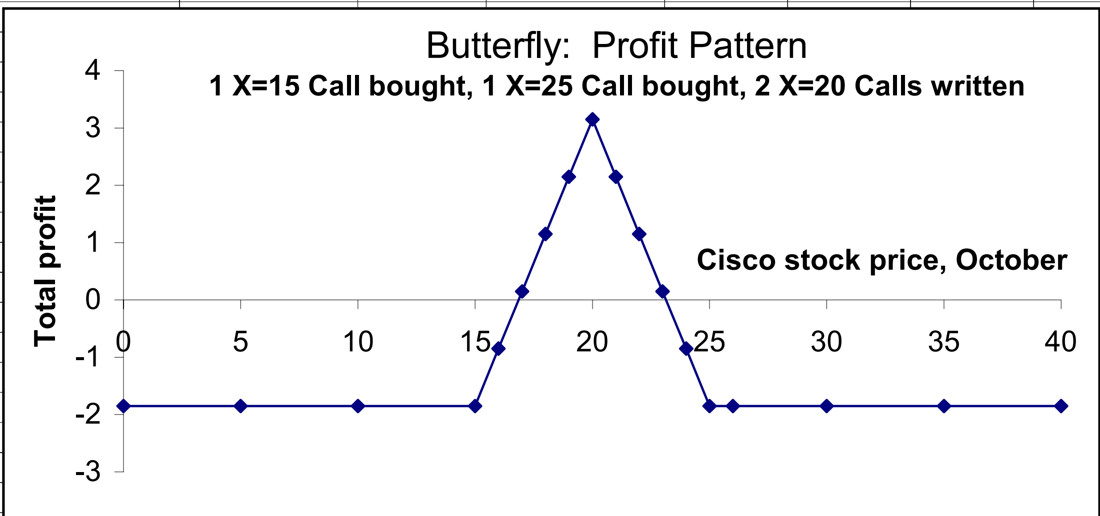
22.8. Butterfly

A *butterfly* is a combination of 3 options. In the butterfly illustrated below:

- We buy one Cisco October $X=15$ call for \$5
- We write two Cisco October $X=20$ calls for \$1.80 each
- We buy one Cisco October $X=25$ call for \$0.45

Here's the resulting profit pattern:

	A	B	C	D	E	F	G	H
1	**GRAPHING THE PROFIT FROM A BUTTERFLY IN CISCO OPTIONS**							
2	**Strategy: Buy 1 October 15 Call, Write 2 October 20 Calls, Buy 1 October 25 Call**							
3								
4	**Call prices**							
5		**X**	**Price**					
6		15	5.00					
7		20	1.80					
8		25	0.45					
9								
10	**Payoff and profits**					=MAX(A12-15,0)-$C$6		
11	**October Cisco stock price**	**Payoff on October X=15 call**	**Payoff on October X=20 call**	**Payoff on October X=25 call**	**Total profit**	=-2*(MAX(A12-20,0)-$C$7)		
12	0	-5	3.6	-0.45	-1.85			
13	5	-5	3.6	-0.45	-1.85			
14	10	-5	3.6	-0.45	-1.85			
15	15	-5	3.6	-0.45	-1.85	=MAX(A12-25,0)-$C$8		
16	16	-4	3.6	-0.45	-0.85			
17	17	-3	3.6	-0.45	0.15			
18	18	-2	3.6	-0.45	1.15			
19	19	-1	3.6	-0.45	2.15			
20	20	0	3.6	-0.45	3.15			
21	21	1	1.6	-0.45	2.15			
22	22	2	-0.4	-0.45	1.15			
23	23	3	-2.4	-0.45	0.15			
24	24	4	-4.4	-0.45	-0.85			
25	25	5	-6.4	-0.45	-1.85			
26	26	6	-8.4	0.55	-1.85			
27	30	10	-16.4	4.55	-1.85			
28	35	15	-26.4	9.55	-1.85			
29	40	20	-36.4	14.55	-1.85			



Why buy a butterfly? Looking at the graph you can see that it's a bet on the stock price not moving very much. If Cisco's October stock price is close to $20, we'll make money from our butterfly. If it deviates (up or down) by a lot, we'll lose money, but only moderately.

In Section ??? of the Chapter 22 we'll return to butterflies and use them to derive a remarkable fact about option prices.

Summary

In this section we've looked at the basics of option markets. We've discussed definitions (calls, puts, American versus European options) and profit patterns of both individual options and combinations of options.

EXERCISES

Put butterflies and put convexity

CHAPTER 23: INTRODUCTION TO OPTIONS*

this version: November 20, 2004

Chapter contents

Overview

The financial assets we have discussed so far in this book are stocks (Chapter 000) and bonds (Chapter 000). In Chapters 23-26 we discuss another kind of financial asset—options. As you will see in these chapters, an option is different in many respects from a stock or a bond:

- The value of an option is derived from the value of another asset, usually a stock. For this reason options are sometimes called *derivative assets*.
- The buyer of an option buys upside gains but has only limited downside losses.
- Options are more complicated than bonds or stocks. In order to understand options we will have to introduce you to some new terminology and some new ways of thinking about financial assets.

A simple example of an option

In order to give some meaning to these somewhat mysterious statements, we start with a simple example.[1] It is 1 January 2006, and the price of an ounce of gold is $400. You have a very strong hunch that the price of gold will be $500 in three months. Your hunches have never failed you, so this must be a sure-fire way to make some money. Taking your total savings of $400, you go to your local jewelry mart to buy some gold. But with your paltry savings you can buy only one ounce of gold, and you can make a maximum of only $100—only a 25% return on your initial investment.

However, the jeweler has another offer for you: For $50, he is willing to sell you a contract which gives you the right to buy one ounce of gold in three months for $400. You

[1] Even this simple example is non-trivial. Options are like that!

realize that this contract—a *call option on gold*—gives you the opportunity of making much more money that actually buying gold.

Here's your calculation:

- Using your \$400 savings, you can buy eight call options.
- In three months you can use the call options to buy eight ounces of gold for \$400 per ounce. If your hunch is correct, the gold price in three months will be \$500 per ounce, so that you can make \$100 per ounce of gold purchased.
- Your total profit using the gold call options will be \$800—a profit of 200% on your initial investment. This compares to the profit of 25% you will make if you use your \$400 savings to buy one ounce of physical gold.

Downside: Suppose your hunch is wrong, and the price of gold in three months is \$300. Now compare the profits of buying one ounce of physical gold to the profits of buying eight call options:

- If you bought one ounce of physical gold, you would have lost 25% of your initial \$400 investment.
- If you bought eight options and the price of gold on 31 March 2006 is \$300 per ounce, the *options will be worthless*. In this case you would have lost 100% of your initial \$400 investment.

CALL OPTION ON ONE OUNCE OF GOLD
Price on 1 January 2006: $50

If presented at Asheville Jewelry Mart on or before 31 March 2006, this piece of paper gives you the right to buy one ounce of gold for $400. After 31 March 2006, this piece of paper is worthless.

The owner of this piece of paper can sell it to anyone else at any time.

Figure 23.1. The gold call option certificates sold by the Asheville Jewelry Mart.

Peacemount stock options—an example

Our gold example should convince you that options are an interesting way to make money. In this subsection we give an example of a stock option. Stock options give their holders the right to either buy or sell a stock in the future for a predetermined price. Stock options come in two flavors: A *call option* on a stock allows you to make money if the stock price goes up without losing too much if the stock price goes down. A *put option* on a stock allows you to make money if the stock's price goes down without losing too much if the stock's price goes up.

Take a look at Figure 23.2, which shows a call option (the right to buy a share of stock) on one share of a fictional company called Peacemount. On 26 November 2003 it would cost you $3 to buy this option. Having bought, you then have the right for the next three months to buy a share of Peacemount stock for $36.

Why buy this option? By spending $3 now, you lock in $36 as the maximum price Peacemount stock will cost you in the next three months. If the price of the stock goes up in the next three months, this will save you a lot of money. If, for example, Peacemount stock is selling on 26 February 2004 for $50, then by using your call option, you can buy the stock for $36. You will have a profit of $11 (buying the stock for $36 instead of $50 saves you $14; from this amount you have to deduct the $3 cost of the option). If, on the other hand, Peacemount stock declines below $36, then you will not exercise the option but you will only lose your $3 investment. In option market jargon: *The call option offers upside gains but only limited downside losses.*

There's another reason to buy the option: You might be able to sell it at some time during the next three months and make a profit. Suppose that in one week the price of Peacemount stock is $45. Then the price of the call option should be at least $9, since the owner of the option could immediately make a profit of $9 by exercising it.[2] Notice that in this example the price of the stock increases by 25% (from $36 to $45), whereas the price of the option increases by at least 300%. This makes the option a very interesting speculation. In option market jargon: *The call option's market price is very sensitive to the price of the underlying asset (in our case: the price of Peacemount stock).*

In addition to call options, this chapter also discusses *put options*. Whereas a call option is the right to buy a share of stock in the future, a put option is the right to sell a share of stock.

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[2] Whoever holds the option can purchase a share of Peacemount for $36. The stock price is now $45, so the immediate realizable profit is $9.

An example is given in Figure 23.3: For $2.50 you could, on 26 November 2003, buy the right to sell one share of Peacemount stock for $36 during the next three months.

Why might you be interested in buying this put option? One reason is that, for holders of Peacemount stock, the put option places a *floor on your losses*. Suppose you own a share of Peacemount stock. On 26 November 2003 shares of Peacemount are selling for $35.50. If you buy the put option today for $2.50, you guarantee yourself that at any point during the next three months you will realize at least $33.50 from your stock.

To see this, suppose that on 26 February 2004 the price of Peacemount is $20. Instead of selling your share on the open market, you will use ("exercise") the put option to sell the share for $36. Accounting for the cost of the option, your net receipts will be $33.50 ($36 for the share minus the $2.50 cost of the put option).

CALL OPTION ON PEACEMOUNT STOCK

Price on 26 November 2003: $3

If presented at the Asheville Stock Exchange on or before 26 February 2004, this piece of paper gives you the right to buy one share of Peacemount stock for $36. After 26 February 2004 this piece of paper is worthless.

The holder of this piece of paper can sell it to someone else at any time.

Some additional information:

- On 26 November 2003, shares of Peacemount stock sold for $35.50.
- Peacemount's stock price has experienced considerable variations during the past three months:

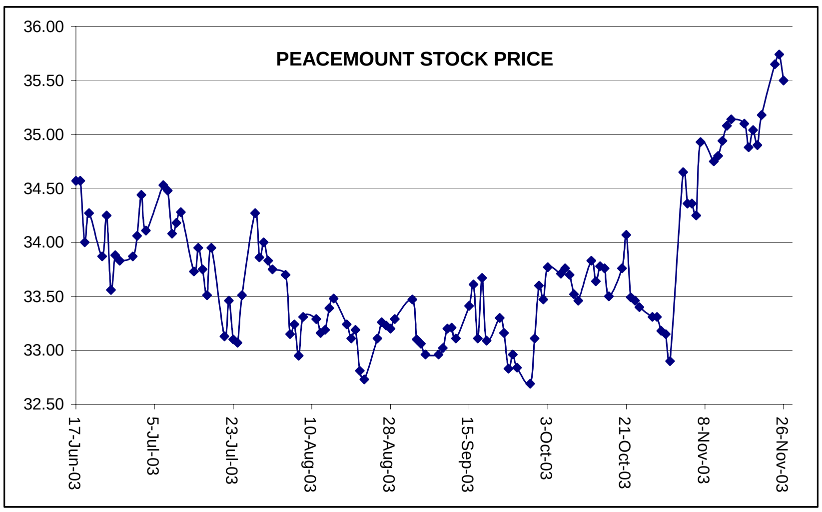


Figure 23.2. A call option on Peacemount stock. The option gives the holder the right to buy a share of Peacemount on or before 26 February 2004 for $36. The market price of the option on 26 November 2003 is $3.

PUT OPTION ON PEACEMOUNT STOCK

Price on 26 November 2003: $2.50

If presented at the Asheville Stock Exchange on or before 26 February 2004, this piece of paper gives you the right to sell one share of Peacemount stock for $36. After 26 February 2004 this piece of paper is worthless.

The holder of this piece of paper can sell it to someone else at any time.

Figure 23.3. A (hypothetical) put option on Peacemount stock. The option gives the holder the right to sell a share of Peacemount on or before 26 February 2004 for $36. The market price of the option on 26 November 2003 is $2.50.

What's next?

In this chapter shows you basic option definitions and introduces you to option cash flows. In addition we show you how option strategies—the ability to combine options and stocks in portfolios—can change the payoff patterns available to investors. When you finish this chapter, you will understand why stock options are *really interesting* securities, and why you might want to invest in them.

Finance concepts discussed

- Call and put options
- Option strategies: protective puts, spreads, butterflies

Excel functions used

- **Max**
- **Min**

BASIC OPTION TERMINOLOGY AND SYMBOLS

Name	Definition	Symbol
Call option	The right to buy a stock or other asset at a pre-determined price on or before some future date.	C
Put option	The right to sell a stock or other asset at a pre-determined price on or before some future date.	P
Exercise price	The pre-determined price of the option—the price at which the stock/asset can be purchased in the future. Also called the *strike price*.	X
Exercise date	The last date on which the option can be exercised. Past this date the option is worthless.	T
Underlying asset	The stock or other asset which can be purchased with an option (in our previous examples: Gold or one share of Peacemount stock).	S S_0: stock price today S_T: the stock price on the exercise date T

Figure 23.4: Option pricing involves a lot of terminology. Here are some very basic terms.

23.1. What's an option?

A *call option on a stock* is the right to buy a stock on or before a given date at a pre-determined price. Figure 23.6 gives options prices for options on Cisco stock on August 7, 2002; we will use these prices in the examples which follow.

Cisco call options

For example, row 21 of the Cisco spreadsheet tells you that on 7 August 2001, a call option on Cisco stock with an exercise price of $20.00 and an exercise date of 21 September 2001 was selling for $1.35:

	A	B	C	D	E	F
4	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
20	Sep01	17.50	2.75	0.90	21 Sep01	45
21	Sep01	20.00	1.35	2.00	21 Sep01	45
22	Sep01	22.50	0.55	3.80	21 Sep01	45

Suppose you purchased this call option on 7 August. Figure 23.5 shows the option's cash flow pattern:

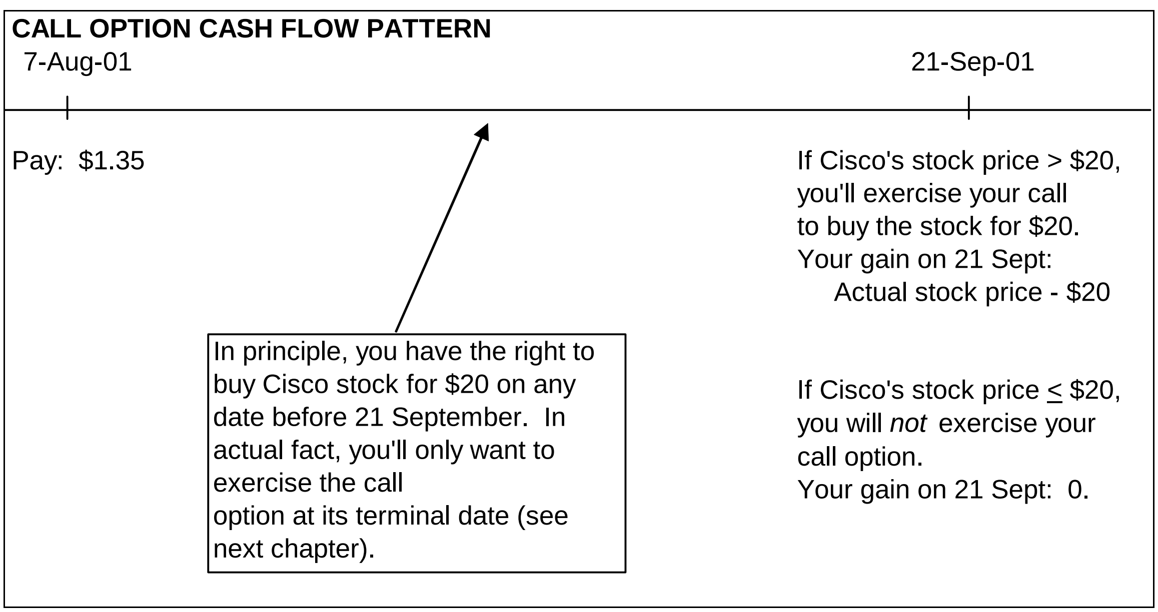


Figure 23.5: The cash flows from buying a Cisco call option on 7 August 2001 for $1.35 and possibly exercising it on 21 September 2001. The option has exercise price *X = $20*.

	A	B	C	D	E	F
1	**CISCO OPTIONS, August 7, 2001 CLOSING PRICE ON CHICAGO BOARD OF OPTIONS EXCHANGE**					
2	**August 7, 2001, CSCO closing price**	19.26				
3						
4	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
5	Aug01	7.50	11.90	0.05	17 Aug01	10
6	Aug01	10.00	9.60	0.20	17 Aug01	10
7	Aug01	12.50	6.50	0.10	17 Aug01	10
8	Aug01	15.00	4.20	0.10	17 Aug01	10
9	Aug01	17.50	2.10	0.40	17 Aug01	10
10	Aug01	20.00	0.65	1.45	17 Aug01	10
11	Aug01	22.50	0.15	3.40	17 Aug01	10
12	Aug01	25.00	0.05	5.00	17 Aug01	10
13	Aug01	27.50	0.10	7.50	17 Aug01	10
14	Aug01	30.00	0.10	11.90	17 Aug01	10
15	Aug01	32.50	0.05		17 Aug01	10
16	Aug01	35.00	0.05	16.20	17 Aug01	10
17	Sep01	10.00	9.50		21 Sep01	45
18	Sep01	12.50	6.30	0.15	21 Sep01	45
19	Sep01	15.00	4.50	0.40	21 Sep01	45
20	Sep01	17.50	2.75	0.90	21 Sep01	45
21	Sep01	20.00	1.35	2.00	21 Sep01	45
22	Sep01	22.50	0.55	3.80	21 Sep01	45
23	Sep01	25.00	0.20	5.50	21 Sep01	45
24	Sep01	27.50	0.10		21 Sep01	45
25	Sep01	30.00	0.05		21 Sep01	45
26	Oct01	10.00	10.00	0.10	19 Oct01	73
27	Oct01	12.50	6.90	0.25	19 Oct01	73
28	Oct01	15.00	5.00	0.65	19 Oct01	73
29	Oct01	17.50	3.20	1.40	19 Oct01	73
30	Oct01	20.00	1.80	2.55	19 Oct01	73
31	Oct01	22.50	0.95	4.10	19 Oct01	73
32	Oct01	25.00	0.45	6.00	19 Oct01	73
33	Oct01	27.50	0.20	7.50	19 Oct01	73
34	Oct01	30.00	0.15	10.70	19 Oct01	73
35	Oct01	35.00	0.05	16.30	19 Oct01	73
36	Oct01	40.00	0.05	21.50	19 Oct01	73
37	Oct01	45.00	0.05	29.50	19 Oct01	73
38	Oct01	50.00	0.05	31.12	19 Oct01	73
39	Oct01	55.00	0.10	37.50	19 Oct01	73
40	Oct01	60.00	0.05	36.75	19 Oct01	73
41	Oct01	65.00	0.05		19 Oct01	73
42	Jan02	10.00	9.50	0.30	18 Jan02	164
43	Jan02	12.50	8.20	0.60	18 Jan02	164
44	Jan02	15.00	5.70	1.20	18 Jan02	164
45	Jan02	17.50	4.10	2.00	18 Jan02	164
46	Jan02	20.00	2.90	3.40	18 Jan02	164
47	Jan02	22.50	1.85	4.90	18 Jan02	164
48	Jan02	25.00	1.20	7.00	18 Jan02	164
49	Jan02	26.25	0.95	7.50	18 Jan02	164
50	Jan02	27.50	0.80	9.80	18 Jan02	164
51	Jan02	30.00	0.45	11.30	18 Jan02	164
52	Jan02	32.50	0.45	13.10	18 Jan02	164
53	Jan02	35.00	0.15	15.00	18 Jan02	164
54	Jan02	37.50	0.20	20.10	18 Jan02	164
55	Jan02	40.00	0.10	19.30	18 Jan02	164
56	Jan02	42.50	0.10	25.90	18 Jan02	164
57	Jan02	45.00	0.10	27.00	18 Jan02	164
58	Jan02	47.50	0.15	34.12	18 Jan02	164
59	Jan02	50.00	0.05	30.10	18 Jan02	164
60	Jan02	52.50	0.10	34.50	18 Jan02	164
61	Jan02	55.00	0.05	37.00	18 Jan02	164
62	Jan02	57.50	0.05	39.70	18 Jan02	164
63	Jan02	60.00	0.10	40.10	18 Jan02	164
64	Jan02	62.50	0.05	43.00	18 Jan02	164
65	Jan02	65.00	0.05	45.80	18 Jan02	164
66	Jan02	67.50	0.05	31.37	18 Jan02	164
67	Jan02	70.00	0.05	50.80	18 Jan02	164
68	Jan02	72.50	0.05	53.50	18 Jan02	164
69	Jan02	75.00	0.05	58.50	18 Jan02	164
70	Jan02	77.50	0.05	59.70	18 Jan02	164
71	Jan02	80.00	0.05	62.20	18 Jan02	164
72	Jan02	82.50	0.19	30.00	18 Jan02	164
73	Jan02	85.00	0.05	46.00	18 Jan02	164
74	Jan02	87.50	0.06	52.50	18 Jan02	164
75	Jan02	90.00	0.05	56.50	18 Jan02	164
76	Jan02	95.00	0.12	65.37	18 Jan02	164
77	Jan02	100.00	0.05	70.87	18 Jan02	164
78	Jan02	105.00	0.05	48.00	18 Jan02	164
79	Jan02	110.00	0.06	84.12	18 Jan02	164
80	Jan02	115.00	0.31	63.00	18 Jan02	164
81	Jan02	120.00	0.05	97.90	18 Jan02	164
82	Jan03	10.00	10.60	0.95	17 Jan03	528
83	Jan03	12.50	9.50	1.60	17 Jan03	528
84	Jan03	15.00	7.70	2.60	17 Jan03	528
85	Jan03	17.50	6.50	3.80	17 Jan03	528
86	Jan03	20.00	5.40	5.20	17 Jan03	528
87	Jan03	25.00	3.70	7.80	17 Jan03	528
88	Jan03	30.00	2.30	11.60	17 Jan03	528
89	Jan03	35.00	1.75	15.90	17 Jan03	528
90	Jan03	40.00	1.10	20.90	17 Jan03	528
91	Jan03	45.00	0.95	25.50	17 Jan03	528
92	Jan03	50.00	0.65	32.20	17 Jan03	528
93	Jan03	55.00	0.50	37.20	17 Jan03	528
94	Jan03	60.00	0.40	40.20	17 Jan03	528
95	Jan03	65.00	0.30	48.40	17 Jan03	528
96	Jan03	70.00	0.25	49.00	17 Jan03	528
97	Jan03	75.00	0.15	48.62	17 Jan03	528
98	Jan03	80.00	0.15	42.87	17 Jan03	528
99	Jan03	85.00	0.20	29.75	17 Jan03	528
100	Jan03	90.00	0.40	56.50	17 Jan03	528
101	Jan03	95.00	0.25	56.75	17 Jan03	528
102	Jan03	100.00	0.10	83.10	17 Jan03	528
103	Jan04	10.00	11.90	1.30	16 Jan04	892
104	Jan04	15.00	9.00	3.20	16 Jan04	892
105	Jan04	20.00	6.80	5.80	16 Jan04	892
106	Jan04	25.00	5.50	8.50	16 Jan04	892
107	Jan04	30.00	4.00	11.90	16 Jan04	892

Figure 23.6: Cisco stock option prices on 7 August 2001. A blank in the price (for example the Oct01 puts with exercise price 65) indicates that no options were traded. On 7 August 2001 Cisco option with maturities as far out as January 2004 were traded.

Now let's see what happens on September 21:

- Suppose the Cisco stock price on September 21 is \$35. In this case you get to buy one share of Cisco for \$20. Your gain is \$35 - \$20 = \$15.
- If the Cisco stock price on September 21 is \$18, you would not exercise your call option to buy a share of Cisco for \$20 (why should you? you could buy it on the open market for less). The option expires unexercised, and your gain is \$0.

Cisco put options

What about the Cisco put option with an exercise price of \$20? It was selling, on 7 August 2001, for \$2.00. The put option gives you the right to *sell* a share of Cisco on or before the terminal date for its exercise price. The put option's cash flow pattern is shown in Figure 23.7.

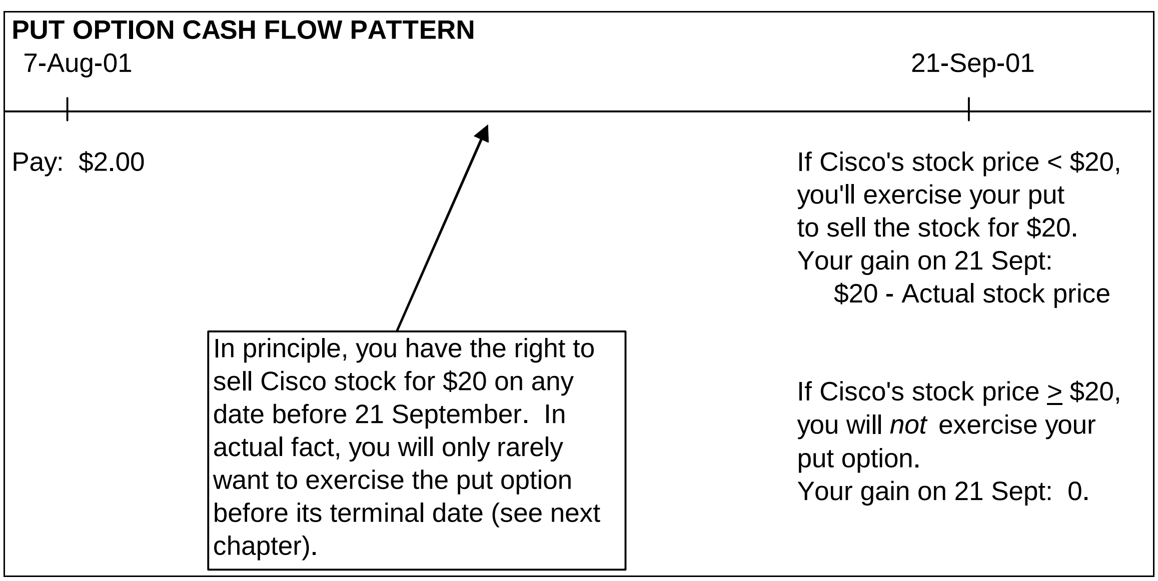


Figure 23.7: The cash flows from buying a Cisco put option on 7 August 2001 for \$2 and possibly exercising it on 21 September 2001. The option has exercise price $X = \$20$.

If Cisco's stock price on September 21 is $15, you will exercise your put option and sell a share of Cisco for $20, thus gaining $5.[3] On the other hand if Cisco's share price on September 21 is $30, you will not exercise the put option (why sell a share using the option for $20 when you can sell it on the open market for $30?).

Option websites

All the data in this chapter was gathered from public sources on the Web. Many of these websites have superb data and also educational features. Here are some websites we especially enjoy.

- The website of the Chicago Board of Options Exchange (CBOE): http://www.cboe.com
- Option metrics: http://www.impliedvol.com/
- Equity analytics: http://www.e-analytics.com/optaaa.htm

European versus American options

Cisco's stock options are *American* stock options—they can be exercised *on or before* the option maturity date *T*. A *European* stock option can be exercised *only on* its maturity date *T*. Clearly an American stock option is worth at least as much as a European stock option.

Two notes about American versus European stock options:

[3] What if you don't own a share of Cisco on 21 September? No problem: You buy a share on the open market for $15 and use your option to sell it for $20.

- The terminology has nothing to do with geography. Most traded options, whether in the U.S., Europe, or Asia, are American and not European.
- A remarkable fact about American call options is the following: In many cases an American call option is worth *exactly* the same as an equivalent European call option. This happens if the stock on which the option is written does not pay a dividend before the option expiration date T. Since Cisco stock does not pay dividends, the "American" feature of Cisco stock call options is worthless, and the call options on Cisco stock are worth the same as if they are European options. We discuss the reasons for this in Chapter 24.

In the money, out of the money, at the money

A call option is said to be "in the money" if the current stock price is larger than the option's exercise price. Look at the Cisco October calls in the spreadsheet below. The call with the exercise price of $12.50 (currently selling for $6.90) has an exercise price *less* than Cisco's current stock price of $19.26. Thus this call is *in the money*—the stock price is greater than the call's exercise price.

	A	B	C	D	E
1	**CISCO OCTOBER OPTIONS In or out of the money?**				
2	**August 7, 2001, CSCO closing price**	19.26			
3					
4	**Expiration date**	**Exercise price, X**	**Call price**	**In or out of the money?**	
20	Oct01	12.50	6.90	in the money	<-- =IF($B$2>B20,"in the money","out of the money")
21	Oct01	15.00	5.00	in the money	<-- =IF($B$2>B21,"in the money","out of the money")
22	Oct01	17.50	3.20	in the money	<-- =IF($B$2>B22,"in the money","out of the money")
23	Oct01	20.00	1.80	out of the money	<-- =IF($B$2>B23,"in the money","out of the money")
24	Oct01	22.50	0.95	out of the money	<-- =IF($B$2>B24,"in the money","out of the money")
25	Oct01	25.00	0.45	out of the money	
26	Oct01	27.50	0.20	out of the money	
27	Oct01	30.00	0.15	out of the money	
28	Oct01	35.00	0.05	out of the money	
29	Oct01	40.00	0.05	out of the money	
30	Oct01	45.00	0.05	out of the money	
31	Oct01	50.00	0.05	out of the money	
32	Oct01	55.00	0.10	out of the money	
33	Oct01	60.00	0.05	out of the money	

The call with exercise price $50 (selling for $0.05) is *out of the money*—its exercise price is more than Cisco's current share price.

If the call's exercise price is equal to the current stock price, it is termed an *at-the-money* call. The call with an exercise price of $20 is almost at the money, and option traders would refer to it loosely as the at-the-money call.

A put is said to be *in the money* if the put's exercise price is greater than the current stock price. In the table below, showing Cisco's October put options, the $50 call (currently selling for $29.50) is in the money and the $12.50 put (selling for $0.10) is *out of the money*. There is no actual at-the-money put, but traders would refer to the $20 exercise put (selling for $1.40) as the *at-the-money* put.

	A	B	C	D	E
35		Exercise price, X	Put price	In or out of the money?	
36	Oct01	12.50	0.10	out of the money	<-- =IF(B36>$B$2,"in the money","out of the money")
37	Oct01	15.00	0.25	out of the money	<-- =IF(B37>$B$2,"in the money","out of the money")
38	Oct01	17.50	0.65	out of the money	<-- =IF(B38>$B$2,"in the money","out of the money")
39	Oct01	20.00	1.40	in the money	<-- =IF(B39>$B$2,"in the money","out of the money")
40	Oct01	22.50	2.55	in the money	<-- =IF(B40>$B$2,"in the money","out of the money")
41	Oct01	25.00	4.10	in the money	
42	Oct01	27.50	6.00	in the money	
43	Oct01	30.00	7.50	in the money	
44	Oct01	35.00	10.70	in the money	
45	Oct01	40.00	16.30	in the money	
46	Oct01	45.00	21.50	in the money	
47	Oct01	50.00	29.50	in the money	
48	Oct01	55.00	31.12	in the money	
49	Oct01	60.00	37.50	in the money	

MORE OPTION TERMINOLOGY

Terminology	Definition
European option	The option is exercisable *only on* the exercise date *T*.
American option	The option is exercisable *on or before* the exercise date *T*. Most options traded on exchanges are American options. Although in principle an American option should be worth more than a European option, in many cases this is not true (see Chapter 24).
At-the-money option	An option whose exercise price *X* is equal to the underlying stock's current stock price S_0. "In-the-money" is often loosely used to describe an option whose exercise price *X* is approximately equal to the current stock price S_0.
In-the-money option	An option from which money can be made by immediate exercise. A call option is in the money if the current stock price S_0 is greater than the option's exercise price *X*. A put option is in the money if the current stock price S_0 is less than the option's exercise price *X*.
Out-of-the-money option	An option from which no money can be made by immediate exercise. A call option is out of the money if $X > S_0$. A put option is out of the money if $S_0 >$ X.

Figure 23.8: Option pricing involves a lot of terminology. Here are some very basic terms.

23.2. Why buy a call option?

Here are two simple reasons why you might want to buy a call option.

Reason 1: A call option allows you to delay the purchase of a stock: It's 7 August 2001, and you're thinking about buying a share of Cisco for its current market price of \$19.26. As an alternative, you can buy a September call option with $X = \$20$. This option will cost you \$1.35. Here's your thinking:

- If, on 21 September 2001, Cisco's stock price is > \$20.00, you'll exercise the option and purchase the share for \$20. If you're careful, you'll realize that there are several "sub-possibilities":
    - Cisco's 21 Sept. stock price = \$35. Now you've made out like a bandit: You spent \$1.35 for the option, but you bought the stock for \$20, saving \$15.00. Your net profit is \$13.35 (\$15.00 - \$1.35 cost of the option).
    - If Cisco's 21 Sept. stock price = \$21.00, you'll still exercise the option and purchase the stock for \$20.00. You've saved \$1.00 on the purchase price of the stock, but this time you will have lost a bit of money, since the option cost you \$1.35. Your net profit will be -\$0.65.
- If on 21 September Cisco's stock is selling for less than \$20, you will not exercise your call option. If you still want to purchase the stock, you'll buy it on the open market. In all cases, you will be out only the \$1.35 cost of the option.

Reason 2: A call option allows you to make a bet on the stock price going up. This bet is: a) low cost, b) high upside potential, and c) one-sided

Suppose you buy the Cisco call option above: You spend $1.35 on 7 August 2001 to purchase an option which—on 21 September—gives you the right to purchase Cisco stock for $20. Your purpose is to bet on the price of Cisco stock in September. As you can see in the Figure 23.8:

- This bet has a low cost: You've put up only $1.35 to make it.
- You will never lose more than the $1.35. This is what we mean when we say that the bet is "one-sided": You can only lose a limited amount of money.
- The bet has very high upside potential: The profits, both in dollars and as a percentage of the money you put up, rise very rapidly when the stock price in September increases over $20.

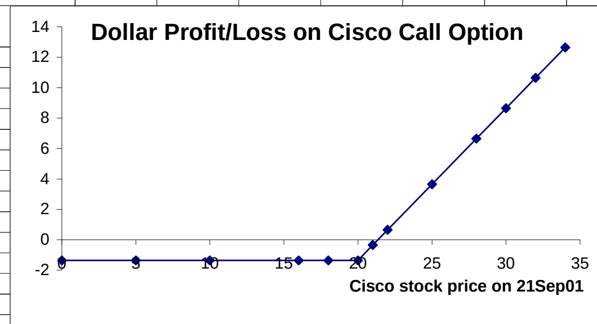
ANALYZING THE PROFIT FROM A CALL OPTION

Price of Cisco on 21 September	Exercise the option?	Your profit or loss	In percentage
\$15	No—the option gives you the right to buy Cisco for \$20, but the market price is less, so you would *not* exercise the option	-\$1.35	$\frac{Profit/loss}{Option\ cost}=\frac{-1.35}{1.35}=-100\%$
\$20	Yes/No—doesn't matter (you're buying the stock at its market price)	-\$1.35	$\frac{Profit/loss}{Option\ cost}=\frac{-1.35}{1.35}=-100\%$
\$21	Yes—the option lets you buy the stock for \$20, but the market price is \$21. So you should exercise (even though you've lost money—see next column)	$Profit\ on\ exercise - option\ cost$ $=(21-20)-1.35=-0.35$	$\frac{Profit\ on\ exercise - option\ cost}{option\ cost}=$ $\frac{(21-20)-1.35}{1.35}=\frac{-0.35}{1.35}=-26\%$
\$25	Yes	$Profit\ on\ exercise - option\ cost$ $=(25-20)-1.35=3.65$	$\frac{Profit\ on\ exercise - option\ cost}{option\ cost}=$ $\frac{(25-20)-1.35}{1.35}=\frac{3.65}{1.35}=270\%$
\$30	Yes	$Profit\ on\ exercise - option\ cost$ $=(30-20)-1.35=8.65$	$\frac{Profit\ on\ exercise - option\ cost}{option\ cost}=$ $\frac{(30-20)-1.35}{1.35}=\frac{8.65}{1.35}=641\%$

Figure 23.8: Analyzing the profit from a call option. If the stock price is down on September 21, your loss is limited to \$1.35. However, if the price goes above \$20, your percentage gains from the option are very large. The call option is a "one-sided" bet on the stock price going up—if the stock price goes up, you make money; if the stock price goes down, you lose a limited amount of money.

You can summarize all of this in a spreadsheet:

	A	B	C	D
1	**PROFIT FROM BUYING A CISCO CALL Bought for $1.35 on 7Aug01; Exercise price: X=$20 Exercise date: 21Sep01**			
2	Call purchase price, 7 August 2001	1.35		
3	Call exercise price, X	20		
4				
5	**Market price of Cisco. 21 September 2001**	**Exercise the call?**	**Dollar profit/loss**	**Percentage profit/loss**
6	0	no	-1.35	-100.00%
7	5	no	-1.35	-100.00%
8	10	no	-1.35	-100.00%
9	16	no	-1.35	-100.00%
10	18	no	-1.35	-100.00%
11	20	no	-1.35	-100.00%
12	21	yes	-0.35	-25.93%
13	22	yes	0.65	48.15%
14	25	yes	3.65	270.37%
15	28	yes	6.65	492.59%
16	30	yes	8.65	640.74%
17	32	yes	10.65	788.89%
18	34	yes	12.65	937.04%



=IF(A18>$B$3,"yes","no")

=C18/$B$2

=IF(A18>$B$3,A18-$B$3,0)-$B$2

Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter 26.

23.3. Why buy a put option?

As in the case of the call, there are two simple reasons to buy a put:

Reason 1: The put option allows you to delay the decision to sell the stock.

It's 7 August 2001, and you own a share of Cisco stock. You're considering selling the stock; its current market price is $19.26. As an alternative, you can buy a September put option with *X = $20*. This put option will cost you $2.00. Here's your thinking:

- If, on 21 September 2001, Cisco's stock price is < $20.00, you'll exercise the option and sell the share for $20. As in the case of the call option discussed above, there are several "sub-possibilities":

- Cisco's 21 Sept. stock price = \$5. Now you've made a lot of money: You spent \$2 for the option, but you sold the stock for \$20, which is \$15.00 more than its market price. Your net profit is \$13.00 (\$15.00 - \$2.00 cost of the option).
- If Cisco's 21 Sept. stock price = \$19.00, you'll still exercise the option and sell the stock for \$20.00. Compared to the market price, you've made \$1.00 on the sale of the stock, but this time you will have lost a bit of money, since the option cost you \$2.00. Your net profit will be -\$1.00.

- If on 21 September Cisco's stock is selling for more than \$20, you will not exercise your put option. If you still want to sell the stock, you'll sell it on the open market. In all cases, you will be out only the \$2.00 cost of the option.

Reason 2: A put option allows you to make a bet on the stock price going down

If you buy a put for \$2.00 and wait until 21 September to exercise, here are your profits:

$$Put\ \ profits = \begin{cases} & Cisco\ stock\ \ price, S_T,\ on\ 21Sep01 \le 20 \\ 20.00 - S_T - 2.00 & \text{In this case you exercise the put and} \\ & \text{make } S_T - 20 \text{ minus the cost of the put} \\ & Cisco\ stock\ \ price, S_T,\ on\ 21Sep01 > 20 \\ -2.00 & \text{In this case you don't exercise the put;} \\ & \text{your loss is the cost of the put} \end{cases}$$

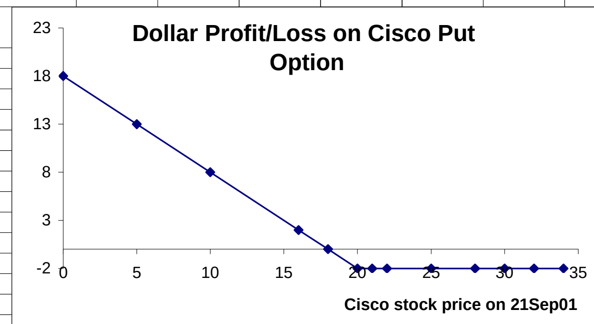
In a spreadsheet:

	A	B	C	D
1	**PROFIT FROM BUYING A CISCO PUT Bought for $2.00 on 7Aug01; Exercise price: X=$20 Exercise date: 21Sep01**			
2	Call purchase price, 7 August 2001	2		
3	Call exercise price, X	20		
4				
5	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Dollar profit/loss**	**Percentage profit/loss**
6	0	yes	18	900.00%
7	5	yes	13	650.00%
8	10	yes	8	400.00%
9	16	yes	2	100.00%
10	18	yes	0	0.00%
11	20	no	-2	-100.00%
12	21	no	-2	-100.00%
13	22	no	-2	-100.00%
14	25	no	-2	-100.00%
15	28	no	-2	-100.00%
16	30	no	-2	-100.00%
17	32	no	-2	-100.00%
18	34	no	-2	-100.00%

=IF(A18<$B$3,"yes","no")

=C18/$B$2

=IF(A18<$B$3,$B$3-A18,0)-$B$2



Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter 26.

23.4. General properties of option prices

In this section we review three general properties of option prices. We look at the effects of option time to maturity, exercise price, the stock price, interest rates, and risk on option prices. Our discussion is informal and intuitive.

Property 1: Options with more time to maturity are worth more

The longer you have to exercise an option, the more it should be worth. The intuition here is clear: Suppose you have a September call option to buy Cisco stock for $20 and also an October call option to buy Cisco for $20. Since Cisco options are American options, anything the October call gives you all the opportunities associated with the September call—and then some. Thus the October call should be worth more than the September call.[4]

Here's some data for the Cisco options. Notice that the prices of the options increase with maturity:

	A	B	C	D
1	**CISCO OPTIONS: THE EFFECT OF EXPIRATION DATE ON OPTION PRICE**			
2				
3	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**
4	Aug01	20.00	0.65	1.45
5	Sep01	20.00	1.35	2.00
6	Oct01	20.00	1.80	2.55
7	Jan02	20.00	2.90	3.40
8	Jan03	20.00	5.40	5.20
9	Jan04	20.00	6.80	5.80
10				
11				
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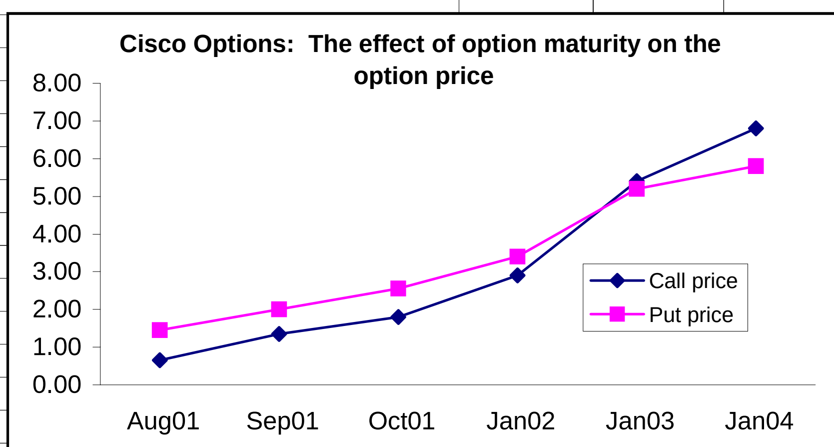
24				



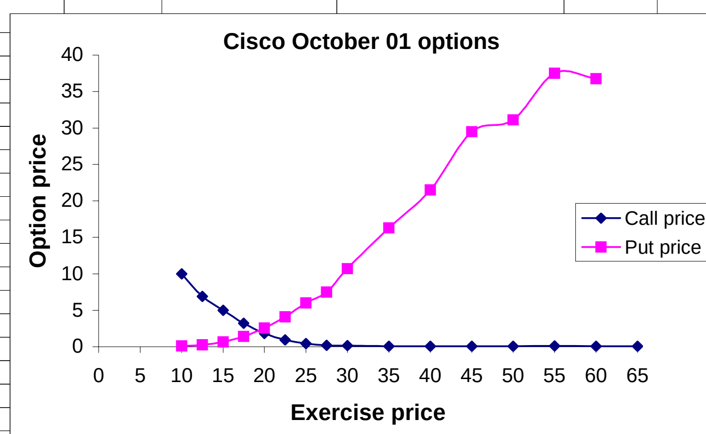
[4] The argument in this paragraph seems to depend critically on the calls being American and not European. It holds, however, for European call also—see Chapter 21, Section ???.

Property 2: Calls with higher exercise prices are worth less; puts with higher exercise prices are worth more

Suppose you had two October calls on Cisco: One call has an exercise price of $20 and the second call has an exercise price of $30. The second call is worth less than the first. Why? Think about calls as *bets* on the stock price: The first call is a bet that the stock price will go over $20, whereas the second call is a bet that the stock price will go over $30. You're always more likely to win the first bet (Cisco will go over $20) than the second bet.

From the table below you can see that Cisco's option prices conform to this property:

	A	B	C	D
1	**CISCO OCTOBER OPTIONS: THE EFFECT OF THE EXERCISE PRICE ON THE OPTION PRICE**			
2	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**
3	Oct01	10.00	10.00	0.10
4	Oct01	12.50	6.90	0.25
5	Oct01	15.00	5.00	0.65
6	Oct01	17.50	3.20	1.40
7	Oct01	20.00	1.80	2.55
8	Oct01	22.50	0.95	4.10
9	Oct01	25.00	0.45	6.00
10	Oct01	27.50	0.20	7.50
11	Oct01	30.00	0.15	10.70
12	Oct01	35.00	0.05	16.30
13	Oct01	40.00	0.05	21.50
14	Oct01	45.00	0.05	29.50
15	Oct01	50.00	0.05	31.12
16	Oct01	55.00	0.10	37.50
17	Oct01	60.00	0.05	36.75
18	Oct01	65.00	0.05	
19				
20				



Here we've looked at all the options which expire on the same date (October 2001). As you can see: The higher the option exercise price, the lower the call price and the higher the put price. (There are a few exceptions; see paragraph below.)

The logic of this is clear:

- If an October 2001 Cisco call option with exercise price $10 (the right to buy a share of Cisco for $10) is worth $10, then an October 2001 call with exercise price $12.50 (the right to buy a share of Cisco for $12.50—more than $10) is worth less.

- If an October 2001 Cisco put option with exercise price $10 (the right to sell a share of Cisco in October for $10) is worth $0.10, then the right to sell a share of Cisco for $12.50 should be worth more. And so it is.

The graph and the table show what appear to be a few exceptions to this rule. For example, the Cisco put with $X = \$60$ traded for less than the put with $X = \$55$. If you see this kind of behavior it almost always has to do with the fact that the options in question are infrequently traded. In the example given here, the $65 and $60 calls only traded several times during the day in question. The result is that the option prices given in the table refer to options traded on Cisco stock at different times and with different prices. (Notice that one of the options—the October put with exercise price $65—didn't trade at all.)

Property 3: When the stock price goes up, call option prices go up and put option prices go down

The reason for this behavior is obvious, if you think of an option as a bet: Suppose you buy a Cisco $X=20$ October 2001 call option. We can view this option as a bet that Cisco's stock price in October will be above $20. The probability of your winning this bet is higher if Cisco's current stock price is higher, and hence so is the call option's price. Thus, for example, if you're willing to pay $1.80 for the $X=20$ October call when Cisco's current stock price is $19.26, you would be willing to pay more for the same call when Cisco's stock price is $22.

The logic for puts is the same, though the result is opposite: The higher the stock price, the lower the put option price.

23.5. Writing options, shorting stock

Our discussion thus far has been from the point of view of the option purchaser. For example in Section 23.2 we derived the profit pattern from buying a Cisco 20 call for $1.35 on 7 August 2001 and waiting until the call maturity on 21 September 2001. Similarly in Section 23.3 we looked at the profit from buying a Cisco 20 put.

There's another side to this story: When you buy a call, someone else sells the call. In the jargon of options markets, the call seller is *writing a call*.

Call buyer: On 7 August 2001 buys, for $1.35, the *right* to buy one share of Cisco stock for $20 on or before 21 September 2001.

Call writer: On 7 August 2001 sells, for $1.35, the *obligation* to sell one share of Cisco stock for $20—as per demand of the call option buyer—on or before 21 September.

Here's the way the call writer's profit pattern looks:

CALL OPTION CASH FLOW PATTERN--the call writer

7-Aug-01

Receives: $1.35

In principle, the call buyer has the right to buy Cisco stock for $20 on any date before 21 September. In actual fact, the call buyer will only want to exercise the call option at its terminal date (see next chapter).

21-Sep-01

If Cisco's stock price > $20 (denote this by $S_T > 20$), the call will be exercised. The call writer has to sell one share of Cisco stock for $20.

Call writer's loss: S_T - $20

If Cisco's stock price $\leq$ $20, (denote this by $S_T \leq 20$), the call will not be exercised.

Call writer's loss: 0

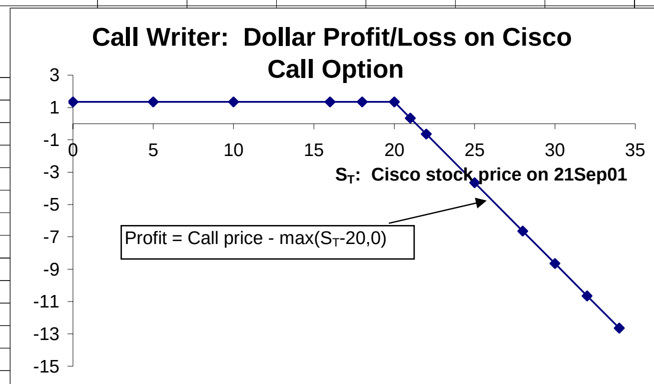
Figure 23.9. The cash flows from writing a Cisco call option on 7 August 2001 for $1.35 and possibly having it exercised against the writer on 21 September 2001. The option has exercise price *X = $20*.

Here's the profit graph from writing a call option:

	A	B	C
1	**PROFIT FROM WRITING A CISCO CALL Bought for $1.35 on 7Aug01; Exercise price: X=$20 Exercise date: 21Sep01**		
2	Call price, 7 August 2001	1.35	
3	Call exercise price, X	20	
4			
5	**S_T: Market price of Cisco, 21 September 2001**	**Will call buyer exercise the call?**	**Dollar profit/loss**
6	0	no	1.35
7	5	no	1.35
8	10	no	1.35
9	16	no	1.35
10	18	no	1.35
11	20	no	1.35
12	21	yes	0.35
13	22	yes	-0.65
14	25	yes	-3.65
15	28	yes	-6.65
16	30	yes	-8.65
17	32	yes	-10.65
18	34	yes	-12.65

=IF(A18>$B$3,"yes","no")

=$B$2-IF(A18>$B$3,A18-$B$3,0)



Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter 26.

Writing puts

There's a similar story for puts:

Put buyer: On 7 August 2001 buys, for $2.00, the *right* to sell one share of Cisco stock for $20 on or before 21 September 2001.

Put writer: On 7 August 2001 sells, for $2.00, the *obligation* to buy one share of Cisco stock for $20—as per demand of the put option buyer—on or before 21 September.

Here's the way the call writer's profit pattern looks:

PUT OPTION CASH FLOW PATTERN--the put writer

7-Aug-01 | 21-Sep-01

Receives: $2.00

In principle, the put buyer has the right to sell Cisco stock for $20 on any date before 21 September. In actual fact, she will only rarely want to exercise the put option before its terminal date (see Chapter 21).

If Cisco's stock price < $20 (denote this by $S_T < 20$), the put will be exercised. The put writer has to buy one share of Cisco stock for $20.

Put writer's loss: $\$20 - S_T$

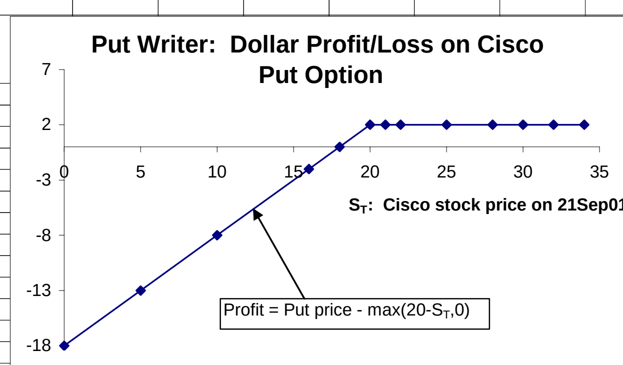
If Cisco's stock price $\geq$ $20, (denote this by $S_T \geq 20$), the put will not be exercised.

Put writer's loss: 0

Figure 23.10. The cash flows from writing a Cisco put option on 7 August 2001 for $2.00 and possibly having it exercised against the writer on 21 September 2001. The option has exercise price *X = $20*.

Here's a graph of the profit pattern from writing a put:

	A	B	C
1	**PROFIT FROM WRITING A CISCO PUT Bought for $2.00 on 7Aug01; Exercise price: X=$20 Exercise date: 21Sep01**		
2	Put price, 7 August 2001	2	
3	Put exercise price, X	20	
4			
5	**S_T: Market price of Cisco, 21 September 2001**	**Will put buyer exercise the put?**	**Dollar profit/loss to put writer**
6	0	yes	-18
7	5	yes	-13
8	10	yes	-8
9	16	yes	-2
10	18	yes	0
11	20	no	2
12	21	no	2
13	22	no	2
14	25	no	2
15	28	no	2
16	30	no	2
17	32	no	2
18	34	no	2
19			
20			
21	=IF(A18<$B$3,"yes","no")		
22			
23		=$B$2-IF(A17<$B$3,$B$3-A18,0)	
24			



Technical note: The data series graphed above are *non-contiguous* in the spreadsheet (they're column A and column C). For how to do this: see Chapter 26.

Short-selling a stock

Short-selling a stock ("shorting") is the stock equivalent of writing an option. Here's how shorting a stock compares to buying a stock:

Stock buyer: On 7 August 2001 buys one share of Cisco stock, for $19.26. When you sell the stock—call the date *T*—you'll get the stock price S_T. Of course you will have also earned any dividends that Cisco will have paid up to and including date *T*.[5] Ignoring the time value of money, your profit from buying the stock is:

$$S_T + Cisco\ dividends - 19.26$$

Stock shorter: On 7 August 2001 contacts his broker and borrows one share of Cisco stock, which he then sells, thus receiving $19.26. At some future date *T*, the short-seller of the stock will purchase a share of Cisco on the open market, paying the then-current market price S_T. If along the way Cisco has paid any dividends, the short-seller will be obliged to pay these dividends to the person he's borrowed the stock from. His total profit will be:

$$19.26 - \left(S_T + Cisco\ dividends\right).$$

[For more information, see the sidebar from the Motley Fool, one of our favorite websites.]

In the option chapters in this book, we will generally assume that stocks don't pay any dividends between the time you buy them and the time you sell them. This means that the profit from buying or shorting a stock can be represented as follows:

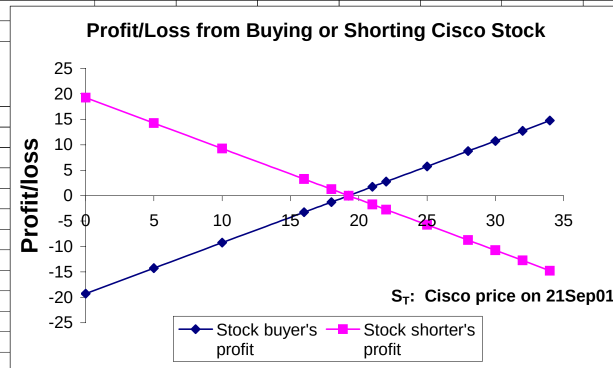
[5] Not something you're like to have to worry about: Cisco has never paid a dividend!

PROFIT FROM BUYING OR SHORTING CISCO STOCK
Market price on 7 August 2001: $19.26
Assumption: Position liquidated on 21 September 2001

	A	B	C
2	Cisco stock price, 7 August 2001	19.26	
3			
4	**S_T: Market price of Cisco, 21 September 2001**	**Stock buyer's profit**	**Stock shorter's profit**
5	0.00	-19.26	19.26
6	5.00	-14.26	14.26
7	10.00	-9.26	9.26
8	16.00	-3.26	3.26
9	18.00	-1.26	1.26
10	19.26	0	0
11	21.00	1.74	-1.74
12	22.00	2.74	-2.74
13	25.00	5.74	-5.74
14	28.00	8.74	-8.74
15	30.00	10.74	-10.74
16	32.00	12.74	-12.74
17	34.00	14.74	-14.74

=A17-$B$2

=$B$2-A17



Terminology Note

A *long* position in a stock involves buying the stock on a particular date and possibly selling the stock on a later date. When you have a long position in a stock, you can choose to hold on to the stock forever (in this case you will collect the dividends that the stock pays).

A *short* position in a stock involves selling borrowed stock on a particular date and buying the stock on a later date in order to give the shares back to the stock lender. Buying the stock in order to return the shares to the lender is called *closing out the short position*. When you have a short position in a stock you must close out the position at some future date.

The Fool FAQ : Shorting Stocks

Many times on the Fool boards I've seen references to `selling a stock short' or `taking a short position.' Will someone tell me plainly what shorting is?

An investor who sells stock short borrows shares from a brokerage house and sells them to another buyer. Proceeds from the sale go into the shorter's account. He must buy those shares back (cover) at some point in time and return them to the lender.

Thus, if you sell short 1000 shares of Gardner's Gondolas at $20 a share, your account gets credited with $20,000. If the boats start sinking---since David Gardner, founder and CEO of VENI, knows nothing about their design---and the stock follows suit, tumbling to new lows, then you will start thinking about "covering" your short there for a very nice profit. Here's the record of transactions if the stock falls to $8.

Borrowed and Sold Short 1000 shares at $20: +$20,000

Bought back and returned 1000 shares at $8: -$8,000

Profit: + $12,000

But what happens if as the stock is falling, Tom Gardner, boatsmen extraordinaire, takes over the company at his brother's behest, and the holes and leaks are covered. As the stock begins to takes off, from $14 to $19 to $26 to $37, you finally decide that you'd better swallow hard and close out the transaction. You do so, buying back shares of TOMY (new ticker symbol) at $37.

Here's the record of transaction:

Borrowed and sold short 1000 shares at $20: +$20,000

Bought back and returned 1000 shares at $37: -$37,000

Loss: -$17,000

Ouch. So you see, in the second scenario, when I, your nemesis, took over the company, you lost $17,000...which you'll have to come up with. There's the danger....you have to be able to buy back the shares that you initially borrowed and sold. Whether the price is higher or lower, you're going to need to buy back the shares at some point in time.

To learn more about short selling, try reading the following books: "Tools of the Bear: How Any Investor Can Make Money When Stocks Go Down" - Charles J. Caes; "Financial Shenanigans: How To Detect Accounting Gimmicks & Fraud" - Howard M. Shilit; "When Stocks Crash Nicely: The Finer Art of Short Selling" - Kathry F. Staley; "Selling Short: Risks, Rewards and Strategies for Short Selling Stocks, Options and Futures" - Joseph A. Walker. None of these are perfect in their coverage of short selling but each has its strengths.

Shorting, unlike puts, seems to have an unlimited downside potential, correct? That is, hypothetically, the stock can rise to infinity. Puts, besides the time limit, have a limited downside. Why then, for a short term short, would anyone short instead of purchasing puts?

Theoretically, yes. In reality, no. Because in our number system we count upwards and don't stop, we opine that because numbers go on forever, so can a stock price. But when we think about this objectively, it seems kind of silly, no? Obviously a stock price, which at SOME point reflects actual value in a business, cannot go on to infinity.

Yes, puts do have a limited downside. However, options have an expiration date, which means that they are "time-wasting assets". They also have a "strike price" which means that you need to pick a price and then have the stock below it on expiration date. Finally, you have to pay a premium for an option and if you are not "in the money" more than the premium, by expiration day, you still lose. So, with options, not only do you have to be worried about the direction of the stock, you need to be correct about the magnitude of the move and the time in which it will happen. And even then, even if you successfully manage all 3 of these things, you can still lose money if you don't cover the premium. Not very Foolish. With shorting, you only really need to be concerned about direction. As for limiting liability, you can do that yourself by putting in a buy stop at a price where the loss is "too much" for you.

What is short interest? Does it have anything to do with short attention spans?

Pardon? Short interest? Oh yes! Ahem, short interest is simply the total number of shares of a company that have been sold short. The Fool believes that the best shorts are those with low short interest. They present the maximum chance for price depreciation as few short sales have occurred, driving down the price. Also, low short interest stocks are less susceptible to short squeezes (see below). Short interest figures are available towards the end of each month in financial publications like *Barron's* and the *Investor's Business Daily*.

The significance of short interest is relative. If a company has 100 million shares outstanding and trades 6 million shares a day, a short interest of 3 million shares is probably not significant (depending on how many shares are closely held). But a short interest of 3 million for a company with 10 million shares outstanding trading only 100,000 shares a day is quite high.

I've heard the term 'days to cover' thrown around quite a bit. Does 'days to cover' have anything to do with short interest?

Yes, it does! Days to cover is a function of how many shares of a particular company have been sold short. It is calculated by dividing the number of shares sold short by the average daily trading volume.

Look at Ichabod's Noggins
http://quote.yahoo.com/quotes?SYMBOLS=HEAD&detailed=t(Nasdaq:HEAD). One million shares of this issue have been sold short (we can find this number, called the short interest, in such publications as Barrons and the IBD). It has an average trading volume of 25,000. The days to cover is 1,000,000/25,000, or 40 days.

When you short a stock, you want the days to cover to be low, say around 7 days or so. This will make the shares less subject to a short squeeze, the nightmare of shorters in which someone starts buying up the shares and driving up the share price. This induces shorters to buy back their shares, which also drives up the price! A short days to cover means the short interest can be eliminated quickly, preventing a short squeeze from working very well.

Also, a lengthy days to cover means that many people have already sold short the stock, making a further decline less likely.

What effect does a large short coverage have (generally) on the stock`s price? Generally, heavy buying increases the price while selling decreases it. Assuming the stocks price has been steady, or climbing, and many shorters attempt to cover their losses, how will this affect the price?

What you are referring to, in investment parlance, is a "short squeeze." When a number of short sellers all try to "cover" their short at the same time, that does indeed drive the stock up.

Our approach when shorting is therefore to avoid in general stocks that already have a fairly hefty amount of existing short sales. We try to set ourselves up so we'll never get squeezed.

I'll point out that short squeezes can be the result of better than expected earnings or some other fundamental aspects of a company's operation. They can also be the result of direct manipulation. That is, profit-seeking individuals with large amounts of cash at their disposal can look on a large short position in a stock as an invitation to start buying, driving up the share prices, thus forcing short-sellers to cover. This in turn drives up the price, and before you know it, the share price has soared!

OK, I understand the potential benefits and risks of shorting, except for one thing. If the stock I've shorted pays a dividend, am I liable for that dividend?

Yes. If you are short as of the ex-dividend date, you are liable to pay the dividend to the person whose shares you have borrowed to make your short sale. I must say, however, that if you are correct in your judgment to sell the issue short, your profits achieved thereby will certainly outweigh the small dollar amount of the dividend payout.

What happens if the stock I've shorted splits?

MF Swagman replies:

Let's say we're speaking of a two-for-one split. In that case, all that happens is that you must cover your short position with twice as many shares as you opened it. If you shorted 100 shares, you must cover with 200. Don't forget, though, that the magnitude of your investment hasn't changed, for while you now have twice as many shares, each one is only worth half as much as before! So, while your original cost basis for the 100 shares may have $36, now, with 200 shares, it is only $18.

This is a very foolish question, I'm sure, but if I sell short I am essentially borrowing the shares from someone else through my broker. Assuming that the lender does NOT need the shares prematurely, what determines how long I can stay short? (pun intended) How long do I have before I am forced to cover my position? Is there any regulation? Is it simply dependent on when/if the broker needs them? Could I possibly stay short for an indefinite period?

As far as I know, there is no pre-determined limit to how long you can keep your short position open. Technically, you could be forced to cover at any time, but typically, having the shares you have borrowed called back is unusual. At least so state all the Schwab representatives of whom I have asked this question.

Source: The Motley Fool website, http://www.fool.com/FoolFAQ/FoolFAQ0033.htm

23.6. Option strategies—more complicated reasons to buy options

In the previous section we studied the profit and loss from buying and selling calls, puts, and shares. In this and the following two sections we look at the profit involved in more complicated option strategies. “Option strategy” refers to the profits which result from holding a combination of options and shares.

A simple option strategy: buy a stock and buy a put

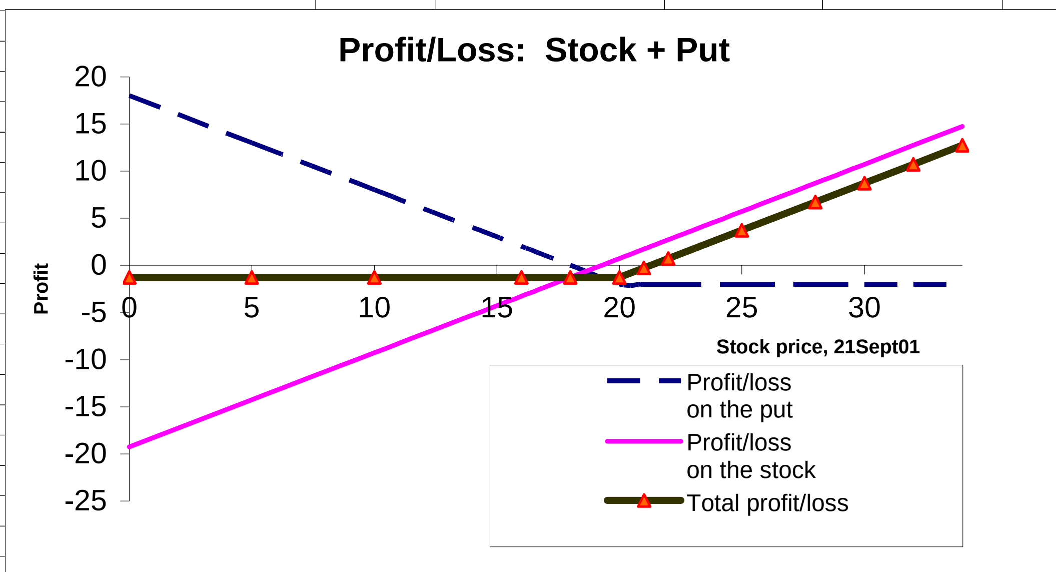
We begin with a very simple (but useful) strategy: Suppose we decide, on 7 August 2001, to purchase one share of Cisco stock *and* to purchase a put on the stock with exercise price 20 and expiration date September. The total cost of this strategy is $21.26: $19.26 for the share of Cisco and $2.00 for each put.

Such a strategy effective *insures* your stock returns by guaranteeing that on 17 September 2001 you will have at least $20 in hand. Your worst-case net profit will be a loss of $1.65:

Stock price on 17 September	Strategy	Cash in hand	Net profit
Less than $20	Exercise put option and sell your share of Cisco for $20.	$20	$20 – ($19.26 + $2)= -$1.65
More than $20	Let the put option expire (don’t use it)	Cisco stock price on 17 September, S_T	S_T – ($19.26 + $2)= S_T - $21.26

In a spreadsheet, here’s the way this strategy looks:

	A	B	C	D	E	F
1	**STOCK + PUT: OPTION STRATEGY PROFITS**					
2	Stock price, 7Aug01	19.26				
3	Cost of put option	2				
4	Put exercise price, X	20	=IF(A7<$B$4,$B$4-A7,0)-$B$3			
5					=A7-$B$2	
6	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Profit/loss on the put**	**Profit/loss on the stock**	**Total profit/loss**	
7	0	yes	18	-19.26	-1.26	<-- =C7+D7
8	5	yes	13	-14.26	-1.26	
9	10	yes	8	-9.26	-1.26	
10	16	yes	2	-3.26	-1.26	
11	18	yes	0	-1.26	-1.26	
12	20	no	-2	0.74	-1.26	
13	21	no	-2	1.74	-0.26	
14	22	no	-2	2.74	0.74	
15	25	no	-2	5.74	3.74	
16	28	no	-2	8.74	6.74	
17	30	no	-2	10.74	8.74	
18	32	no	-2	12.74	10.74	
19	34	no	-2	14.74	12.74	



Buying a stock or a portfolio *and* buying a put on the stock or portfolio is often called a *portfolio insurance* strategy. Portfolio insurance strategies are very popular among investors. They guarantee a minimum return on the investment in the shares (at an extra cost, of course: you have to buy the puts).

ANTICIPATING A BIT—put-call parity

You'll notice that the graph of the stock+put strategy looks a lot like the graph of a call (Section 23.2). This may lead you to surmise that the payoffs of the combination *stock+put* is somehow equivalent to the payoffs of a *call*. However, this isn't quite true, as you'll see in the next chapter. There we discuss the *put-call parity theorem* and show that—for a put and call written on the same stock and having the same exercise price *X*:

$$stock + put = call + PV(X)$$

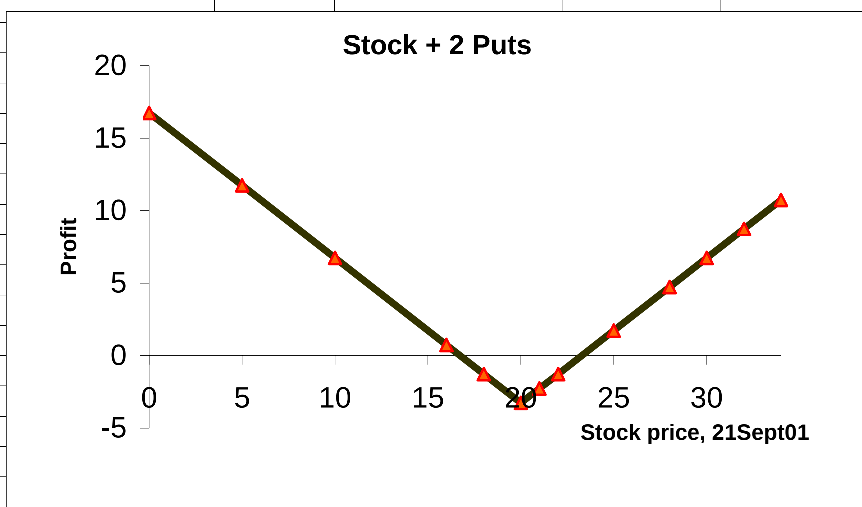
A more complicated strategy: Stock + 2 puts

Suppose you purchased one share of stock and bought 2 puts, each costing $2 and each having an exercise price of $20. Here's what your payoff pattern would look like:

Stock price on 17 September	Strategy	Cash in hand, 17 September	Net profit
$S_T \leq \$20$	Exercise both put options. Give someone else your share of Cisco for $20. Buy an additional share in the market and give it to the put writer for S_T.	2*20 - S_T	2*20 - S_T – ($19.26 + $4)= $16.74 - S_T
More than $20	Let the put options expire (don't use them)	Cisco stock price on 17 September, S_T	S_T – ($19.26 + $4)= S_T - $23.26

If we make an Excel table, here's what it looks like:

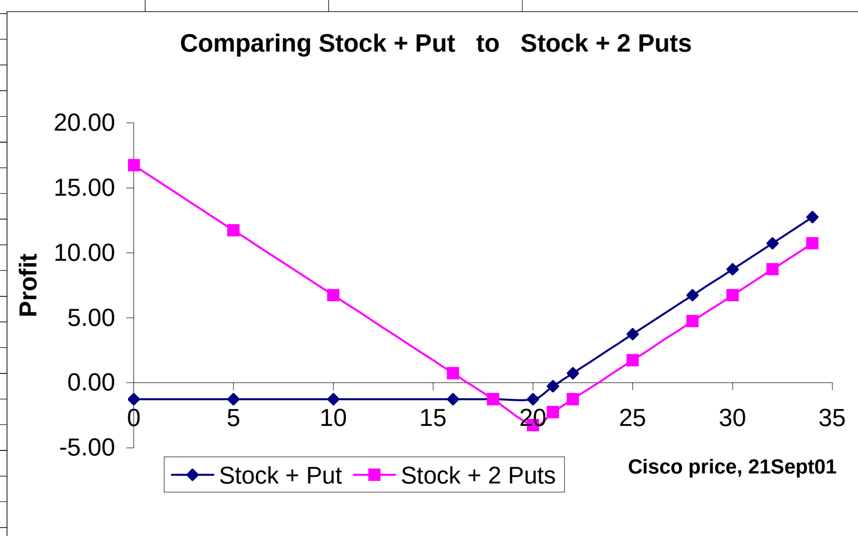
	A	B	C	D	E	F
1	**STOCK + 2 PUTS PUT: OPTION STRATEGY PROFITS**					
2	Stock price, 7Aug01	19.26				
3	Cost of put option	2.00		=2*(IF(A7<$B$4,$B$4-A7,0)-$B$3)		
4	Put exercise price, X	20.00				
5					=A7-$B$2	
6	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Profit/loss on the puts**	**Profit/loss on the stock**	**Total profit/loss**	
7	0	yes	36	-19.26	16.74	<-- =C7+D7
8	5	yes	26	-14.26	11.74	
9	10	yes	16	-9.26	6.74	
10	16	yes	4	-3.26	0.74	
11	18	yes	0	-1.26	-1.26	
12	20	no	-4	0.74	-3.26	
13	21	no	-4	1.74	-2.26	
14	22	no	-4	2.74	-1.26	
15	25	no	-4	5.74	1.74	
16	28	no	-4	8.74	4.74	
17	30	no	-4	10.74	6.74	
18	32	no	-4	12.74	8.74	
19	34	no	-4	14.74	10.74	



Comparing strategies

What's better as a strategy: buying a share of Cisco and buying 1 put, or buying a share of Cisco and buying 2 puts? Look at the graphs of the two strategies:

	A	B	C	D
1	**STOCK + PUT COMPARED TO STOCK + 2 PUTS**			
2	Stock price, 7Aug01	19.26		
3	Cost of put option	2.00		
4	Put exercise price, X	20.00		
5				
6	**Market price of Cisco. 21 September 2001**	**Stock + Put**	**Stock + 2 Puts**	
7	0	-1.26	16.74	<-- =2*IF(A7<=$B$4,$B$4-A7,0)+A7-($B$2+2*$B$3)
8	5	-1.26	11.74	
9	10	-1.26	6.74	=IF(A7<=$B$4,$B$4-A7,0)+A7-($B$2+$B$3)
10	16	-1.26	0.74	
11	18	-1.26	-1.26	
12	20	-1.26	-3.26	
13	21	-0.26	-2.26	
14	22	0.74	-1.26	
15	25	3.74	1.74	
16	28	6.74	4.74	
17	30	8.74	6.74	
18	32	10.74	8.74	
19	34	12.74	10.74	



The choice between the two strategies involves *tradeoffs* (that's the nature of market efficiency: in an efficient market no asset ever completely dominates another asset).

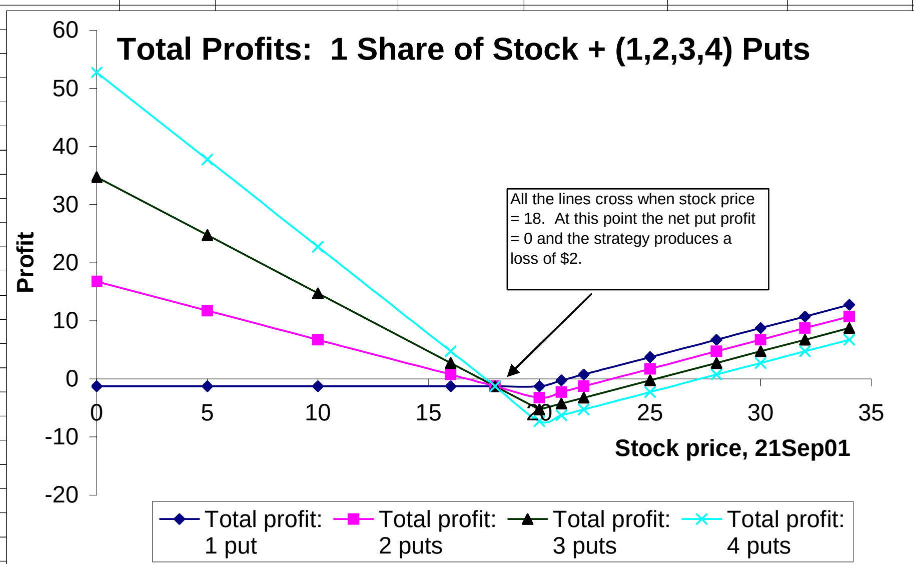
- The stock+put strategy has higher profit when the Cisco September stock price $\geq$ 20, but it has a negative profit for Cisco $S_T \leq 20$
- The stock + 2 put strategy costs more (you can see this by noting that its payoff when $S_T = 20$ is less than that of the stock + put strategy). On the other hand, it has positive profits both for very low and for high S_T .

Which strategy should you choose? It depends on your prediction of the future: If you think that Cisco is going to make a big move, up or down, then stock + 2 puts is for you, since this strategy makes profits on "big moves" of the stock price (whether up or down). If you think, on the other hand, that Cisco might go up, but you want protection when and if its price goes down (that is, no bets for you), then stock + put is your choice.

Another strategy: One share of stock + 1, 2, 3, or 4 puts

There's almost nothing to say here, except to show you the graphs:

	A	B	C	D	E	F	G	H
1	**STOCK + SEVERAL PUTS PUT: OPTION STRATEGY PROFITS**							
2	Stock price, 7Aug01	19.26						
3	Cost of put option	2						
4	Put exercise price, X	20						
5	Number of puts purchased	2						
6								
7	**Market price of Cisco. 21 September 2001**	**Exercise the put**	**Profit/loss on single put**	**Profit/loss on the stock**	**Total profit: 1 put**	**Total profit: 2 puts**	**Total profit: 3 puts**	**Total profit: 4 puts**
8	0	yes	18	-19.26	-1.26	16.74	34.74	52.74
9	5	yes	13	-14.26	-1.26	11.74	24.74	37.74
10	10	yes	8	-9.26	-1.26	6.74	14.74	22.74
11	16	yes	2	-3.26	-1.26	0.74	2.74	4.74
12	18	yes	0	-1.26	-1.26	-1.26	-1.26	-1.26
13	20	no	-2	0.74	-1.26	-3.26	-5.26	-7.26
14	21	no	-2	1.74	-0.26	-2.26	-4.26	-6.26
15	22	no	-2	2.74	0.74	-1.26	-3.26	-5.26
16	25	no	-2	5.74	3.74	1.74	-0.26	-2.26
17	28	no	-2	8.74	6.74	4.74	2.74	0.74
18	30	no	-2	10.74	8.74	6.74	4.74	2.74
19	32	no	-2	12.74	10.74	8.74	6.74	4.74
20	34	no	-2	14.74	12.74	10.74	8.74	6.74



23.7. Another option strategy: Spread

A spread strategy involves buying one option on a stock and writing another option. In the example below on August 7, 2001:

- We buy one X=15 September call on Cisco. This option costs $4.50.

- We write one X=20 September call on Cisco. This option costs 1.35; since we're writing the option, this is income on August 7.

In the spreadsheet below we examine this strategy's payoffs and graph them:

	A	B	C	D	E	F	G
1	**BULL SPREAD: A MODERATE BET ON STOCK PRICE INCREASE**						
2	Cost of September, X=15 call	4.5					
3	Number of X=15 calls purchased	1			=$B$6*(MAX(A9-20,0)-$B$5)		
4			=$B$3*(MAX(A9-15,0)-$B$2)				
5	Cost of Sept. X=20 call	1.35					
6	Number of X=20 calls purchased	-1				=C9+E9	
7							
8	**Market price of Cisco. 21 September 2001**	**Exercise X=15 call?**	**Profit/loss on X=15 call**	**Exercise X=20 call?**	**Profit/loss on X=20 call**	**Total profit**	
9	0	no	-4.50	no	1.35	-3.15	
10	5	no	-4.50	no	1.35	-3.15	
11	10	no	-4.50	no	1.35	-3.15	
12	15	no	-4.50	no	1.35	-3.15	
13	18	yes	-1.50	no	1.35	-0.15	
14	20	yes	0.50	no	1.35	1.85	
15	21	yes	1.50	yes	0.35	1.85	
16	22	yes	2.50	yes	-0.65	1.85	
17	25	yes	5.50	yes	-3.65	1.85	
18	28	yes	8.50	yes	-6.65	1.85	
19	30	yes	10.50	yes	-8.65	1.85	
20	32	yes	12.50	yes	-10.65	1.85	
21	34	yes	14.50	yes	-12.65	1.85	
22							
23							
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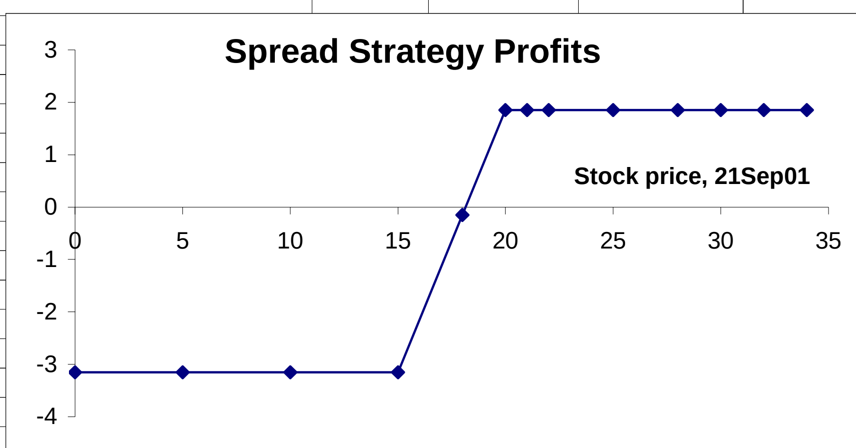


There's another way to think about the strategy profits: On September 21, 2001 (the option expiration date) we will have:

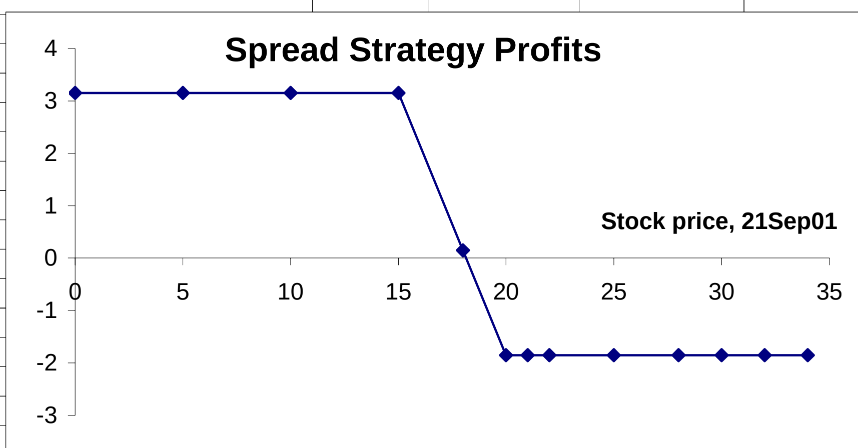
$$\underbrace{-4.50+\underbrace{Max\left[S_{CSCO,21Sep01}-15,0\right]}_{\substack{\uparrow \\ \text{This is the option payoff on 21Sep01} \\ \text{from buying a call with X=15}}}}_{\substack{\uparrow \\ \text{This is the profit from buying} \\ \text{the X=15 option}}}\underbrace{+1.35\underbrace{-Max\left[S_{CSCO,21Sep01}-20,0\right]}_{\substack{\uparrow \\ \text{Writing an option means} \\ \text{taking a loss if Cisco's stock} \\ \text{price is > 20}}}}_{\substack{\uparrow \\ \text{This is the profit from writing the X=20 option}}}$$

$$=-3.15+\begin{cases} 0 & S_{CSCO,21Sep01}<15 \\ S_{CSCO,21Sep01}-15 & 15\le S_{CSCO,21Sep01}\le 20 \\ 5 & S_{CSCO,21Sep01}>20 \end{cases}$$

In this case the spread is a not-too-risky bet on the stock price going up. If it goes up, you profit (moderately); if the stock price goes down, your loss is limited to \$3.15. This kind of a spread is called a *bull spread*—you're bullish on the stock (meaning that you think the stock price will go up).

Here's a *bear spread*: In this case we write the *X = 15* call and buy the *X=20* call. As you can see from the graph below, the bear spread is a bet that the stock price will decline.

	A	B	C	D	E	F	G
1	**BEAR SPREAD: A MODERATE BET ON STOCK PRICE DECLINE**						
2	Cost of September, X=15 call	4.5					
3	Number of X=15 calls purchased	-1			=$B$6*(MAX(A9-20,0)-$B$5)		
4			=$B$3*(MAX(A9-15,0)-$B$2)				
5	Cost of Sept. X=20 call	1.35					
6	Number of X=20 calls purchased	1				=C9+E9	
7							
8	**Market price of Cisco. 21 September 2001**	**Exercise X=15 call?**	**Profit/loss on X=15 call**	**Exercise X=20 call?**	**Profit/loss on X=20 call**	**Total profit**	
9	0	no	4.50	no	-1.35	3.15	
10	5	no	4.50	no	-1.35	3.15	
11	10	no	4.50	no	-1.35	3.15	
12	15	no	4.50	no	-1.35	3.15	
13	18	yes	1.50	no	-1.35	0.15	
14	20	yes	-0.50	no	-1.35	-1.85	
15	21	yes	-1.50	yes	-0.35	-1.85	
16	22	yes	-2.50	yes	0.65	-1.85	
17	25	yes	-5.50	yes	3.65	-1.85	
18	28	yes	-8.50	yes	6.65	-1.85	
19	30	yes	-10.50	yes	8.65	-1.85	
20	32	yes	-12.50	yes	10.65	-1.85	
21	34	yes	-14.50	yes	12.65	-1.85	



23.8. The butterfly option strategy

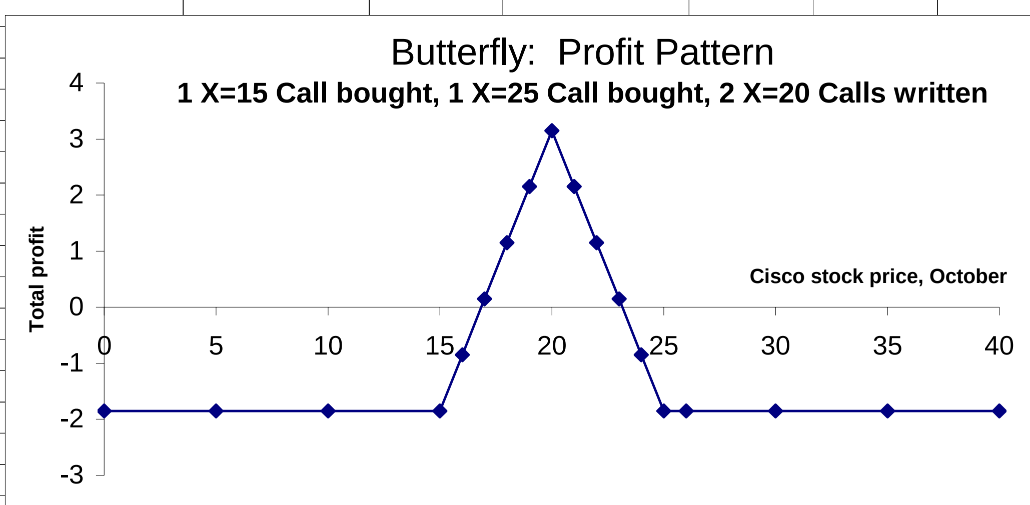
The last option strategy we consider in this chapter is a *butterfly*, the combination of three options. In the butterfly illustrated below:

- We buy one Cisco, October $X=15$, call for $5

- We write two Cisco, October $X=20$, calls for \$1.80 each
- We buy one Cisco, October $X=25$, call for \$0.45

Here's the resulting profit pattern:

	A	B	C	D	E	F	G	H
1	**GRAPHING THE PROFIT FROM A BUTTERFLY IN CISCO OPTIONS Strategy: Buy 1 October 15 Call, Write 2 October 20 Calls, Buy 1 October 25 Call**							
2	**Call prices**							
3		**X**	**Price**					
4		15	5.00					
5		20	1.80					
6		25	0.45					
7								
8	**Payoff and profits**					=MAX(A10-15,0)-$C$4		
9	**October Cisco stock price**	**Payoff on October X=15 call**	**Payoff on October X=20 call**	**Payoff on October X=25 call**	**Total profit**	=-2*(MAX(A10-20,0)-$C$5)		
10	0	-5	3.6	-0.45	-1.85			
11	5	-5	3.6	-0.45	-1.85			
12	10	-5	3.6	-0.45	-1.85			
13	15	-5	3.6	-0.45	-1.85	=MAX(A10-25,0)-$C$6		
14	16	-4	3.6	-0.45	-0.85			
15	17	-3	3.6	-0.45	0.15			
16	18	-2	3.6	-0.45	1.15			
17	19	-1	3.6	-0.45	2.15			
18	20	0	3.6	-0.45	3.15			
19	21	1	1.6	-0.45	2.15			
20	22	2	-0.4	-0.45	1.15			
21	23	3	-2.4	-0.45	0.15			
22	24	4	-4.4	-0.45	-0.85			
23	25	5	-6.4	-0.45	-1.85			
24	26	6	-8.4	0.55	-1.85			
25	30	10	-16.4	4.55	-1.85			
26	35	15	-26.4	9.55	-1.85			
27	40	20	-36.4	14.55	-1.85			



Why buy a butterfly? Looking at the graph you can see that it's a bet on the stock price not moving very much. If Cisco's October stock price is close to $20, we'll make money from our butterfly. If it deviates (up or down) by a lot, we'll lose money, but only moderately.

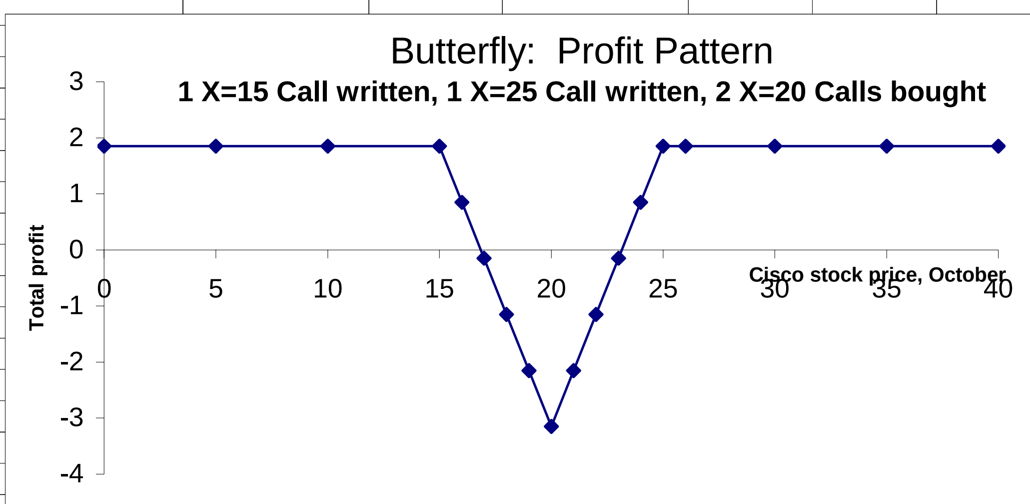
Of course, if we reverse the option positions in the butterfly, we'll get a bet on the stock price moving a lot (big movements either up or down will lead to profits, small movements in the stock price will lead to losses):

	A	B	C	D	E	F	G	H
1	**THE OPPOSITE BUTTERFLY--A BET ON LARGE STOCK PRICE MOVEMENTS Strategy: Write 1 October 15 Call, Buy 2 October 20 Calls, Write 1 October 25 Call**							
2	**Call prices**							
3		**X**	**Price**					
4		15	5.00					
5		20	1.80					
6		25	0.45					
7								
8	**Payoff and profits**							
9	**October Cisco stock price**	**Payoff on October X=15 call**	**Payoff on October X=20 call**	**Payoff on October X=25 call**	**Total profit**			
10	0	5.00	-3.6	0.45	1.85			
11	5	5.00	-3.6	0.45	1.85			
12	10	5.00	-3.6	0.45	1.85			
13	15	5.00	-3.6	0.45	1.85			
14	16	4.00	-3.6	0.45	0.85			
15	17	3.00	-3.6	0.45	-0.15			
16	18	2.00	-3.6	0.45	-1.15			
17	19	1.00	-3.6	0.45	-2.15			
18	20	0.00	-3.6	0.45	-3.15			
19	21	-1.00	-1.6	0.45	-2.15			
20	22	-2.00	0.4	0.45	-1.15			
21	23	-3.00	2.4	0.45	-0.15			
22	24	-4.00	4.4	0.45	0.85			
23	25	-5.00	6.4	0.45	1.85			
24	26	-6.00	8.4	-0.55	1.85			
25	30	-10.00	16.4	-4.55	1.85			
26	35	-15.00	26.4	-9.55	1.85			
27	40	-20.00	36.4	-14.55	1.85			

=$C$4-MAX(A10-15,0)

=2*(MAX(A10-20,0)-$C$5)

=$C$6-MAX(A10-25,0)



Summary

Stock options are securities which make it possible to bet on an increase in the stock price (calls) or a decrease in the price (puts). In this chapter we've looked at the basics of option markets. We've discussed definitions (calls, puts, American versus European options) and profit patterns of both individual options and combinations of options.

In the next chapter we discuss some facts about stock option prices.

Exercises

Note: Templates for many of these problems are on the CD-ROM which comes with this book.

1. On 2 September 2004 Kellogg stock closed at $41.78. For $2.60 you can buy a call option on Kellogg with an exercise price of $40. The option expires 17 December 2004.

1.a. What right does this call option give you?

1.b. Suppose you buy the call option and hold it until the expiration date. If the price of Kellogg on 17 December 2004, is $52, will you exercise the option? What will be your profit?

1.c. If the price of Kellogg on 17 December 2004, is $38, will you exercise the option? What will be your profit?

2. It is mid-July 2008. Intel stock is currently trading at $30, and you think that the price of the stock will go down by 22 October 2008. For $3 you can buy a put on Intel stock that expires in October and that has an exercise price of $25.

2.a. What right does this put option give you?

2.b. What happens if the stock does not go below $25 by the time your option expires?

2.c. Suppose you buy the option and hold it until the expiration date. If the price of Intel on 22 October 2008 is $20, what will be your profit from the option? What if the price is $38?

3. It is 18 July 2006 and you've just bought one call option on ForeverYours stock. The option cost you $6, expires on 18 September 2004, and has an exercise price of $20.

3.a. Complete the following Excel table.

3.b. Make a graph which shows the Profit (column C) on the *y*-axis and the Stock Price on 18Sep04 (column A) on the *x*-axis.

	A	B	C
2	**ForeverYours stock price, 18sep04--S_T**	**Exercise the call option?**	**Profit**
3	0		
4	5		
5	10		
6	15		
7	20		
8	25		
9	30		
10	35		
11	40		
12	45		
13	50		

4. It is 31 December 2007 and you've just bought one put option on ItStinks stock. The option cost you $3, expires on 13 March 2008, and has an exercise price of $35.

4.a. Complete the following Excel table.

4.b. Make a graph which shows the Profit (column C) on the *y*-axis and the Stock Price on 13Mar08 (column A) on the *x*-axis.

	A	B	C
2	**ItStinks stock price, 13mar08--S_T**	**Exercise the put option?**	**Profit**
3	0		
4	5		
5	10		
6	15		
7	20		
8	25		
9	30		
10	35		
11	40		
12	45		
13	50		

5.

5.a. On 1 September 2004 Ford's stock price was $13.90 per share. Call options on Ford expiring on 17 September 2004 with an exercise price of $12.50 sold for $1.50. Should a call with an exercise price of $12.50 expiring on the 21 January 2005 sell for more that $1.50? Explain.

5.b. Look at the following table. Is there an option which is clearly mispriced?

Ford Call Options Expiring 21jan05	
Exercise price X	Call option price
$ 2.50	$ 12.80
$ 5.00	$ 9.00
$ 7.50	$ 7.90
$ 10.00	$ 5.20
$ 12.50	$ 3.00
$ 15.00	$ 3.30
$ 17.50	$ 0.60
$ 20.00	$ 0.15
$ 22.50	$ 0.10
$ 25.00	$ 0.05

6.

6.a. On 1 September 2004 GM's stock price was $41.21 per share. Put options on GM expiring on 17 September 2004 with an exercise price of $40 sold for $1.80. Should a call with an exercise price of $40 expiring on the 21 January 2005 sell for more that $1.80? Explain.

6.b. Look at the following table. Is there an option which is clearly mispriced?

GM Put Options Expiring 21jan05	
Exercise price X	Put option price
$ 5.00	$ 37.00
$ 10.00	$ 33.60
$ 15.00	$ 29.20
$ 20.00	$ 24.20
$ 30.00	$ 14.30
$ 35.00	$ 9.70
$ 40.00	$ 6.00
$ 45.00	$ 2.70
$ 50.00	$ 0.95
$ 55.00	$ 0.35
$ 60.00	$ 0.09
$ 65.00	$ 0.05
$ 70.00	$ 0.25

7. The price of IBM stock on 1 June 2004 was $88.00.

7.a. If you purchased the stock on 1 June and sold it on 1 September 2004 for $84.05, what would have been your profit?

7.b. If you shorted IBM stock on 1 June and closed out your short position on 1 September 2004, what would have been your profit?

8. It is 15 December 2006, and John is considering buying 100 shares of GoodLuck stock (the stock price is currently $40 per share). On the same date Mary is considering shorting 100

shares of GoodLuck stock. If both John and Mary intend to close out their positions on 1 April 2007, fill in the following table and graph their profits.

	A	B	C
1	GoodLuck stock price, 15dec06	$ 50.00	
2			
3	**GoodLuck stock price on 1apr07**	**John's profit from buying 100 shares**	**Mary's profit from shorting 100 shares**
4	$0.00		
5	$10.00		
6	$20.00		
7	$30.00		
8	$40.00		
9	$50.00		
10	$60.00		
11	$70.00		
12	$80.00		
13	$90.00		
14	$100.00		

9. You've decided to add 100 shares of ABC Corp. to your portfolio. ABC stock is currently trading at $50 a share. As an alternative to buying the shares now, you're considering buying 1000 call options on ABC. Each option has an exercise price of $50 and expires in 3 months. The options cost $5 each.

9a. Compare the two strategies by filling in the following table and graphing the percentage profits of each strategy against the stock price S_T in three months.

9.b. Which strategy is riskier?

	A	B	C	D	E
1	Investment today in buying 100 shares	$ 5,000			
2	Investment today in buying 1,000 call options	$ 5,000			
3					
4	**ABC stock price in 3 months, S_T**	**Dollar profit from buying 100 shares**	**Dollar profit from buying 100 call options now**	**Percentage profit from buying 100 shares**	**Percentage profit from buying 1,000 call options now**
5	0				
6	10				
7	20				
8	30				
9	40				
10	50				
11	60				
12	70				
13	80				
14	90				
15	100				

10. On 14 February 2002 Microsoft (MSFT) stock is trading at $48.30 per share. The price of a call option on MSFT expiring March 2003 is $1.45 for options with X = $47.50 and $0.35 for options with X = $50.

10.a. You think that shares of MSFT will rise in price in the immediate future, and you want to speculate in the stock. Compare (graphically) the following two alternatives: purchasing 1,000 MSFT options with an exercise price of $47.5 versus purchasing 1,000 MSFT options with a strike of $50.

10.b. Compare the two strategies. Which is preferable?

Use the following template:

	A	B	C
1	**Exercise price**	**Call price**	
2	47.5	1.45	
3	50	0.35	
4			
5	Investment		
6	A: 1000 options, X=47.5	1,450	<-- =B2*1000
7	B: 1000 options, X=50	350	<-- =B3*1000
8			
9	**Stock price in 3 months, S_T**	**Percentage profit on strategy A**	**Percentage profit on strategy B**
10	0.0		
11	10.0		
12	20.0		
13	30.0		
14	40.0		
15	42.5		
16	45.0		
17	47.5		
18	50.0		
19	55.0		
20	60.0		
21	70.0		
22	80.0		
23	90.0		
24	100.0		

11. On 1 September 2004 McDonald's (MCD) stock is trading at \$27.19 per share. The price of a put option on MCD expiring 17 September 2004 is \$0.10 for options with X = \$25.00 and \$0.70 for options with X = \$27.50.

10.a. You think that shares of MCD will fall in price in the immediate future, and you want to speculate on the stock. Compare (graphically) the following two alternatives: purchasing 1,000 MCD options with an exercise price of \$25 versus purchasing 1,000 MCD put options with a strike of \$27.50.

10.b. Compare the two strategies. Which is preferable?

Use the following template:

Exercise price	Put price	
25	0.1	
27.5	0.7	
Investment		
A: 1000 options, X=25	100	<-- =B3*1000
B: 1000 options, X=27.50	700	<-- =B4*1000
Stock price in 3 months, S_T	**Percentage profit on strategy A**	**Percentage profit on strategy B**
0.0		
15.0		
20.0		
22.0		
24.0		
25.0		
26.0		
26.5		
27.0		
27.5		
28.0		
30.0		
32.5		
35.0		
40.0		

12. A put option written on ENERGY-R-US Corporation's stock is selling for $2.50. The option has an exercise price of $20 and 6 months to expiration. The current market price for a share of ENERGY-R-US is $26. Determine the profit from a strategy of buying the stock and buying the put; graph these profits. Use the following template.

	A	B	C	D
1	Energy-R-Us, stock	26.00		
2	Put price, X = 20	2.50		
3				
4	**Stock price, S_T in 6 months**	**Profit from put**	**Profit from stock**	**Total profit**
5	0			
6	5			
7	10			
8	15			
9	20			
10	25			
11	30			
12	35			
13	40			
14	45			
15	50			

13. Using the data from the previous problem, compare the following three strategies:

- Purchase one share of stock and one put on the stock.
- Purchase one share of stock and two puts on the stock.
- Purchase of one share of stock and three puts on the stock.

14. Using the data and the template below: Suppose you bought a GM call with exercise (strike) price X = 50 and wrote a GM call with exercise price X = 45 Graph the profit of this strategy at option expiration. Why might this be an attractive strategy?

	A	B	C	D
1	**Call Prices**			
2	**X**	**Price**		
3	45	4.10		
4	50	1.65		
5				
6	**GM stock price, S_T at option expiration**	**Profit on X=45 call (written)**	**Profit on X=50 call (bought)**	**Total profit**
7	20			
8	25			
9	30			
10	35			
11	40			
12	45			
13	50			
14	55			
15	60			
16	65			

15. Using the data from the previous problem, compute and graph the profit from a strategy in which you buy a GM call with exercise price X = 45 and write a calll with exercise price X = 50. Explain why this strategy might be attractive.

16. The following options are traded on WOW Corporation's stock. State which property of option prices is violated and show that you can design a strategy to profit from this mispricing.[6]

Option	Exercise Price	Expiration Date	Price
Call	40	1-Jan-04	$13.50
Call	40	1-Jul-04	$12.95

[6] Such a strategy is called an *arbitrage strategy*. Such strategies are discussed at length in the next chapter.

17. The following options are traded on Smow Corporation's stock. State which property of option prices is violated and show that you can design a strategy to profit from this mispricing.

Option	Exercise Price	Expiration Date	Price
Put	50	1-Mar-04	$4.25
Put	60	1-Mar-04	$4.00

18. David wants to buy a call option written on RAIDER Corp. stock. Patrick is willing to sell David a call option on RAIDER Corp. stock with an exercise price of $50 for $8.20. The option will mature in exactly one year. The current market price for RAIDER Corp. stock is $50.

18.a. Determine and graph the payoffs of both David and Patrick's respective positions.

18.b. For what stock price S_T is the profit of both David and Patrick zero?

Call price	8.20	
RAIDER stock price at option expiration, S_T	**Patrick's profit**	**David's profit**
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		

19. Portfolio insurance describes a position in which an investor buys put options to insure that the value of his portfolio does not fall below a certain point. Suppose Jerry has a portfolio that consists of 100 shares of RTY stock. The current market price of RTY is $35 per share. The following options are also being traded on RTY stock:

Expiration date	Exercise price	Call price	Put price
1-Jun-04	20	$ 18.00	$ 0.10
1-Jun-04	25	$ 11.35	$ 0.45
1-Jun-04	35	$ 3.50	$ 3.20
1-Jun-04	40	$ 0.75	$ 5.65

19.a. What options must Jerry buy if he wants to insure that the value of his portfolio will not drop below $2000?

19.b. How much will this cost?

20. A covered call position entails entering into a long position in stock and writing a call option with a high strike price. The purpose of such a position is to finance a portion of the stock purchase from the sale of the call option.

Sam thinks that STF Corp. stock, currently priced at $80/share, will go up in price by about $15 in the next 6 months. He would like to buy 10,000 shares of STF today and cash in on his bullish sentiment. In order to cut the initial costs of his purchase, he would like to enter into a covered call position. The following options are being traded on STF Corp:

Expiration date	Exercise price	Call price
1-Aug-04	$ 70	$ 18.95
1-Aug-04	$ 80	$ 7.65
1-Aug-04	$ 90	$ 2.70
1-Aug-04	$ 100	$ 0.50

Suppose Sam writes 10,000 of the $90 calls. Show Sam's profit. Use the following template:

STF stock price	$ 80		
Number of shares purchased	10,000		
Stock price of STF in 6 months, S_T	**Profit from stock position**	**Profit from option position, 10,000 options with X = $90**	**Profit from covered call strategy**
50			
60			
70			
80			
90			
100			
110			
120			

21. Referring to the facts in the previous problem:

21.a. Compare the profits from a covered call strategy using the $90 calls with one using the $100 calls.

21.b. Which of the two covered call strategies would you recommend?

22. Given the three calls below, design a butterfly strategy which pays off if the stock does not make a major move from its current value of $60. Graph the strategy profits. Use the following template.

Call prices				
	X	**Price**		
	50	22.00		
	60	15.00		
	70	10.00		
Payoff and profits				
Stock price at option expiration, S_T	**Payoff on X = 50 call**	**Payoff on X = 60 call**	**Payoff on X = 70 call**	**Total profit**
30.0				
35.0				
40.0				
45.0				
50.0				
52.5				
55.0				
57.5				
60.0				
62.5				
65.0				
67.5				
70.0				
72.5				
75.0				
80.0				
85.0				
90.0				

23. Given the data from the previous problem, design a butterfly strategy which pays off if the stock price makes a large move from its current price of $60. Graph the strategy profits.

CHAPTER 23: OPTION PRICING FACTS*

This version: September 22, 2002

Chapter contents

Overview

In this chapter we discuss some *facts* about option pricing. Our emphasis is on a set of propositions knows as *arbitrage restrictions* on option prices. These restrictions specify relations between the prices of puts and calls and the prices of either the stock underlying the options or a risk-free asset.

Notation: Throughout the chapter we use the following notation

S_0 = price of stock at time 0 (today)
S_T = price of stock on option exercise date T
X = option exercise price
r = interest rate
C = call option price at time 0 (today); sometimes we also write this as C_0
P = put option price at time 0 (today); sometimes we write this as P_0
C_t = call option price at time t
P_t = put option price at time t

Dividends: We assume that the stock on which the options are written does not pay dividends before the option maturity date. This is not an overly-restrictive assumption: Stocks which pay dividends tend to do so at regular intervals (quarterly, semi-annually, or annually). Holders of options on these stocks are thus reasonably sure when the stocks will pay a dividends. There are thus long periods of time when market participants can be assured that a stock will not pay a dividend.

For example: General Motors pays a regular quarterly dividend in February, May, August, and November. An investor who purchases an option on GM in March with an April maturity knows that in the intervening period no dividends will be paid on the stock.

Many other stocks have never paid a dividend and investors in these stocks' options can be reasonably assured that the dividend pricing restriction imposed in this chapter is not restrictive. Stocks which fall into this category include many of the high-tech stocks whose options tend to attract the most investor interest.

Finance concepts discussed in this chapter

- Option pricing restrictions
- No early exercise of calls
- Put-call parity
- Early exercise of American puts
- Option price convexity

Excel functions used

- Max
- Sum

23.1. Fact 1: Call price of an option > $Max\left[S_0 - PV(X), 0\right]$

It's 15 August 2001, and you're considering buying a call option on Microsoft. Currently the MSFT share itself is selling for S_0 = *$63*; you want to buy a call on MSFT with an exercise price $X = 60$ and with time to maturity $T = 1$ *year*. Furthermore, we'll suppose that the option is an *American call option*, and can be exercised at any time on or before T.

We will examine Fact 1 in two stages. We start with a "dumb fact," something that is obvious once we say it, and then proceed to demonstrate Fact 1 for you.

Dumb fact: $Call\ \ price \geq Max\left[S_0 - X, 0\right]$.

Now it's probably clear to you that the Microsoft option should be selling for *at least $3*. To see this, suppose that the option is selling for $2. We'll devise an *arbitrage strategy*—a strategy which will make us money risklessly:

Action taken today	**Cash flow (negative numbers indicate costs)**
Buy the option	-$2
Immediately exercise the option, buying the stock	-$60
Immediately sell the stock on the open market	+$63
Arbitrage profit	**+$1**

So the "dumb fact"—that an American call option should sell for more than the difference between the stock price and the exercise price—is pretty obvious.

Smart fact: $Call\ \ price > Max\left[S_0 - PV\left(X\right),0\right]$.[1]

This is a lot less obvious than the previous fact. It's also a lot more powerful. The "dumb fact" above says that the option should sell for at least $3. As the spreadsheet below shows, the "smart fact" says much more; if, for example, the interest rate is 10%, then the smart fact says that the option should sell for at least $8.45.

[1] How smart? Robert Merton, who first established this and lots of other facts about options, subsequently won the Nobel Prize for economics, in part for his work on option pricing.

	A	B	C
1	**FACT 1: Lower bound on call price**		
2			
3	Microsoft stock price, 15 August 2001, S_0	63	
4	Option exercise price, X	60	
5	Option exercise time, T (in years)	1	
6	Interest rate, r	10%	
7			
8	Lower bound on call price		
9	Dumb fact, call price > Max[S_0 - X,0]	3	<-- =MAX(B3-B4,0)
10	Fact 1: call price > Max[S_0 - PV(X),0]	8.45	<-- =MAX(B3-B4/(1+B6)^B5)

To prove the "smart fact," let's assume that you can buy the call for \$5. We'll show that there exists an *arbitrage strategy*, and we will therefore conclude that the option price is too low.

Definition: An arbitrage strategy is a combination of assets—usually short or long positions in the stock, calls and puts on the stock, and a risk-free security—which produces positive cash flows at all points in time. If you can design an arbitrage strategy for a given set of asset prices (as we do below), it shows that at least one of the prices is *wrong*.

Here's the strategy. At time 0 (today), we will:

At time 0 (today):

- Short one share of the stock
- Invest in a riskless security paying off the call's exercise price at time *T*.
- Buy a call on the option.

At time T:

- Purchase the stock on the open market at the time-*T* price, in order to close the short position

- Collect from our investment in the riskless security
- Exercise the option if this is profitable

Here's an example, which assumes that the stock price at time 0 is 63 and that the interest rate is 10%:

	A	B	C
14	**Arbitrage proof**		
15			
16	Call price at time 0 (today)	5	
17			
18	**Actions at time 0 (today)**		
19	Short the stock	63	<-- =B3
20	Buy a bond which pays of X at time T	-54.55	<-- =-B4/(1+B6)^B5
21	Buy a call	-5	<-- =-B16
22	**Total cash flow at time 0**	3.45	<-- =SUM(B19:B21)
23			
24	**Cash flow at time T**		
25	S_T, stock price at time T	33	
26			
27	Repay the shorted stock	-33	<-- =-B25
28	Collect money from the bond	60	<-- =B4
29	Exercise the call?	0	<-- =MAX(B25-B4,0)
30	Total	27	<-- =SUM(B27:B29)

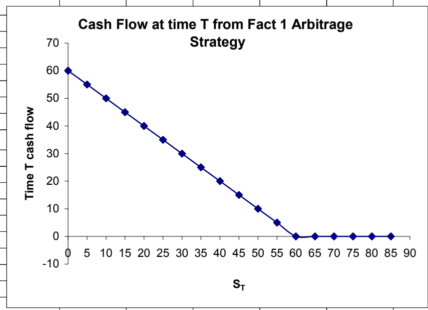
In cells B25:B30 we calculate the cash flow at time $T=1$ from the strategy. In the example above, Microsoft stock at T is selling for $33. In this case, we would have a positive time T cash flow of $27.

In the example below, we assume that Microsoft stock at T is 90. In this case you exercise the call (giving you a positive cash flow of $30), but the total payoff from the strategy is now $0:

	A	B	C
24	**Cash flow at time T**		
25	S_T, stock price at time T	90	
26			
27	Repay the shorted stock	-90	<-- =-B25
28	Collect money from the bond	60	<-- =B4
29	Exercise the call?	30	<-- =MAX(B25-B4,0)
30	Total	0	<-- =SUM(B27:B29)

If we build a data table (see Chapter ???) for the time-T cash flow from the strategy, we see that the strategy always has a positive cash flow:

	A	B	C	D	E	F	G
13					**Data table: Cash**		
14	**Arbitrage proof**				**Flow from strategy**		
15							
16	Call price at time 0 (today)	10			S_T		<-- Data table header is hidden
17					0	60	
18	**Actions at time 0 (today)**				5	55	
19	Short the stock	63	<-- =B3		10	50	
20	Buy a bond which pays of X at time T	-54.55	<-- =-B4/(1+B6)^B5		15	45	
21	Buy a call	-10	<-- =-B16		20	40	
22	**Total cash flow at time 0**	-1.55	<-- =SUM(B19:B21)		25	35	
23					30	30	
24	**Cash flow at time T**				35	25	
25	S_T, stock price at time T	33			40	20	
26					45	15	
27	Repay the shorted stock	-33	<-- =-B25		50	10	
28	Collect money from the bond	60	<-- =B4		55	5	
29	Exercise the call?	0	<-- =MAX(B25-B4,0)		60	0	
30	Total	27	<-- =SUM(B27:B29)		65	0	
31					70	0	
32					75	0	
33					80	0	
34					85	0	
35							
36							



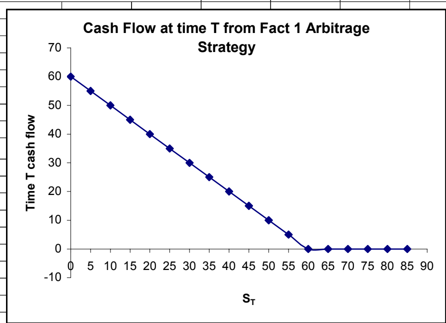
So: We've designed an *arbitrage*:

- At time 0, the cash flow is \$3.45 > 0
- At time T, the cash flow is either positive (if the stock price $S_T < 60$) or zero.

You can't lose from this strategy!! In a rational world this means that something is wrong with the asset prices. In this case, it's clear what's wrong—the call price is too low.

To see this, consider the case where the call price is \$10:

	A	B	C	D	E	F	G	H
14	**Arbitrage proof**					Flow from strategy		
15								
16	Call price at time 0 (today)	10				S_T		<-- Data table header is hidden
17						0	60	
18	**Actions at time 0 (today)**					5	55	
19	Short the stock	63	<-- =B3			10	50	
20	Buy a bond which pays of X at time T	-54.54545	<-- =-B4/(1+B6)^B5			15	45	
21	Buy a call	-10	<-- =-B16			20	40	
22	**Total cash flow at time 0**	-1.545455	<-- =SUM(B19:B21)			25	35	
23						30	30	
24	**Cash flow at time T**					35	25	
25	S_T, stock price at time T	90				40	20	
26						45	15	
27	Repay the shorted stock	-90	<-- =-B25			50	10	
28	Collect money from the bond	60	<-- =B4			55	5	
29	Exercise the call?	30	<-- =MAX(B25-B4,0)			60	0	
30	Total	0	<-- =SUM(B27:B29)			65	0	
31						70	0	
32						75	0	
33						80	0	
34						85	0	
35								
36								



The cash flows at T (shown in the graph) don't change, but the initial cash flow (cell B22) is now negative. This makes more sense: If the call price is > 8.45, then you have to invest money today in order to have a non-negative cash flow in the future.

We've proved our first option pricing fact: $Call\ \ price > Max\left[S_0 - PV\left(X\right),0\right]$.

23.2. Fact 2: It's never worthwhile to exercise a call early.[2]

Suppose that on 15 August 2001 you bought a Microsoft call option for \$12 (note that this price does not violate Fact 1's price restriction). Furthermore, suppose that the option expires one year from today, on 15 August 2002.

Now suppose that after 8 months (approximately 2/3 of a year), you want to get rid of the option. To make the problem interesting, we'll assume that the price of Microsoft has risen to \$80. You have two possibilities:

- You could exercise the option. In this case you would collect \$20 = $Max\left[S_t - X,0\right] = Max\left[80-60,0\right]$.
- You could also *sell* the option on the open market. Of course, we don't know what the option's price would be, but Fact 1 tells us that in no case will the price be less than

$$Max\left[S_t - PV\left(X\right),0\right] = Max\left[S_t - \frac{X}{\left(1+r\right)^{1-t}},0\right]$$
$$= Max\left[80 - \frac{60}{\left(1+10\%\right)^{1-2/3}},0\right] = 21.876$$

What should you do? Clearly you should *sell* rather than *exercise* the call.

[2] When the call is written on a non-dividend-paying stock.

	A	B	C	D	E	F	G	H	I
1	**FACT 2: No early exercise of calls**								
2	Assumption: Stock pays no dividends between *t = 0* and *T*								
3									
4	Microsoft stock price, 15 August 2001, S_0	63							
5	Option exercise price, X	60							
6	Option exercise time, T (in years)	1							
7	Interest rate, r	10%							
8	Call price at time 0	12							
9									
10	t=0				t=0.6		1		
11									
12	Buy option for $12.00			Consider selling the option					
13				or exercising it.					
14									
15				Stock price, S_t	80				
16									
17				Payoff from option exercise	20	<-- =MAX(E15-B5,0)			
18				Minimum value of option					
19				according to Fact 1	22.24439	<-- =MAX(E15-B5/(1+$B$7)^(1-0.6),0)			
20									
21				Exercise option or sell it?	sell	<-- =IF(E19>E17,"sell","exercise")			

23.3. Fact 3: Put-call parity $Put_0 = Call_0 + PV\left(X\right) - S_0$

Put-call parity states that the put price is determined by the call price, the stock price, and the risk-free rate of interest.[3] Here's an example: Suppose that we're considering a one-year put option on the Microsoft stock we've been discussing throughout this chapter. What should be the put price on Microsoft—where we assume that the put has the same exercise price $X=60$ and the same time to maturity $T=1$?

	A	B	C
1	**FACT 3: Put-Call Parity**		
2			
3	Microsoft stock price, 15 August 2001, S_0	63	
4	Option exercise price, X	60	
5	Option exercise time, T (in years)	1	
6	Interest rate, r	10%	
7			
8	Call price	15	
9	Put price by put-call parity	6.55	<-- =B8+B4/(1+B6)^B5-B3

Here's a proof of the important fact about option pricing. We assume that t

[3] Again: Recall that the assumption is that the stock pays no dividends before the option maturity date *T*.

	A	B	C
11	**Arbitrage proof of put-call parity**		
12			
13	Put price today (t=0)	3	
14			
15	**Actions at time 0 (today)**		
16	Buy stock	-63	<-- =-B3
17	Buy put	-3	<-- =-B13
18	Write call	15	
19	Take a loan of PV(X) at risk-free interest	54.55	<-- =B4/(1+B6)^B5
20	**Total cash flow at time 0**	3.55	<-- =SUM(B16:B19)
21			
22	**Cash flow at time T**		
23	S_T, stock price at time T	90	
24			
25	Sell stock	90	<-- =B23
26	Exercise the put?	0	<-- =MAX(B4-B23,0)
27	Cash flow from call	-30	<-- =-MAX(B23-B4,0)
28	Repay loan	-60	<-- =-B4
29	Total	0	<-- =SUM(B25:B28)

In the example above we assumed that at time 0 the put was priced at $3. We then designed an arbitrage strategy:

At time 0 (today):

- Buy one share of Microsoft stock for $63
- Buy one put with exercise price $X = \$60$ for $3
- Write one call with $X = \$60$, collecting (today) $15
- Take a loan of $54.55; the loan has a one-year maturity (like the options). At the current interest rate of 10% you will have to pay off $60 in one year.

At time T we close out all our positions

- Sell our share of Microsoft at the prevailing market price S_T
- Exercise the put, if this is profitable
- Have the call exercised against us, if this is profitable for the call buyer
- Repay the loan

Our example above shows that the cash flow at $T=1$ will be zero if S_T = \$90. The cash flow will also be zero if $S_T = \$35$:

	A	B	C
22	**Cash flow at time T**		
23	S_T, stock price at time T	35	
24			
25	Sell stock	35	<-- =B23
26	Exercise the put?	25	<-- =MAX(B4-B23,0)
27	Cash flow from call	0	<-- =-MAX(B23-B4,0)
28	Repay loan	-60	<-- =-B4
29	Total	0	<-- =SUM(B25:B28)

As you can see, no matter what the Microsoft stock price in one year, the cash flow at $T=1$ from this strategy will be zero. However, the strategy has a positive initial cash flow of \$3.55. Clearly this is an arbitrage!

Symbolically, the future cash flow is given by:

$$\underbrace{S_T}_{\text{Stock value}} + \underbrace{Max\left[X - S_T, 0\right]}_{\text{Put payoff}} \underbrace{- Max\left[S_T - X, 0\right]}_{\substack{\text{Cash flow to call} \\ \text{writer at } T=1}} \underbrace{-X}_{\text{Loan repayment}}$$

$$= \begin{cases} S_T + X - S_T - X & \text{if } S_T < X \\ S_T - \left(S_T - X\right) - X & \text{if } S_T \geq X \end{cases}$$

$$= 0$$

A little thought will reveal that—given the stock price $S_0 = 60$, the interest rate $r=10\%$, the exercise price $X = 60$ of both the put and the call, and the call option price of \$15—the put option price must be \$6.55 to prevent arbitrage.

23.4. Fact 4: Bound on an American put option price: $P > Max\left[X - S_0, 0\right]$

Suppose you're contemplating buying an American put on Microsoft stock. The stock's price today is $S_0 = 63$ and the option exercise price is $X=70$. Clearly the option should sell for at least $7. If not, you could easily devise an arbitrage, as illustrated in the spreadsheet below:

	A	B	C
1	**FACT 4: Lower bound on *American* put price**		
2			
3	Microsoft stock price, 15 August 2001, S_0	63	
4	Option exercise price, X	70	
5	Option exercise time, T (in years)	1	
6			
7	Fact 4: Lower bound of American put: Max[X - S_0, 0]	7	<-- =MAX(B4-B3,0)
8			
9	**Arbitrage**		
10	American put option price	3	
11	Buy option	-3	
12	Buy stock now	-63	
13	Exercise put option immediately: deliver stock and get *X*	70	
14	Immediate profit	4	<-- =SUM(B11:B13)

If the American put option is mispriced (that is, its price is less than $7), you can make money by; buying the option, buying the stock, and exercising the option immediately. This arbitrage profit will not exist if the option's price is greater than $7.

23.5. Fact 5: Bounds on European put option prices $P > Max\left[PV(X) - S_0, 0\right]$

Fact 5 is the "put parallel" for Fact 1 about calls.[4]

[4] There's a crucial difference in the parallel between Facts 1 and 5: Fact 1 applies to *all* calls, whether European or American. Fact 5 applies only to European puts. Of course in both cases, the assumption is that the stock pays no dividends before option maturity.

	A	B	C
1	**FACT 5: Lower bound on *European* put price**		
2			
3	Microsoft stock price, 15 August 2001, S_0	63	
4	Option exercise price, X	70	
5	Option exercise time, T (in years)	1	
6	Interest rate, r	10%	
7			
8	Lower bound on call price		
9	Lower bound of American put: Max[X - S_0, 0]	7	<-- =MAX(B4-B3,0)
10	Fact 4: put price > Max[PV(X) - S_0,0]	0.6364	<-- =MAX(B4/(1+B6)^B5-B3,0)

This fact says that the price of a European put can actually be much lower than the price of an American put. Look at the example above, in which we look at the price of a put option on Microsoft stock with $T = 1$ and $X = 70$. If our put was an American put, then it couldn't sell for less than $7. On the other hand, a *European put*, which cannot be exercised until date *T*, can sell for anything more than $0.6364.

Meaning: An arbitrage with European puts will exist only when the put price goes *below* $0.6364. Here's an example of an arbitrage when the put price is $0.5:

	A	B	C
14	**Arbitrage proof**		
15	Put price at time 0 (today)	0.5	
16			
17	**Actions at time 0 (today)**		
18	Buy the stock	-63	<-- =-B3
19	Borrow PV(X)	63.64	<-- =B4/(1+B6)^B5
20	Buy a put	-0.5	<-- =-B15
21	**Total cash flow at time 0**	0.14	<-- =SUM(B18:B20)
22			
23	**Cash flow at time T**		
24	S_T, stock price at time T	50	
25			
26	Sell the stock	50	<-- =B24
27	Repay the loan	-70	<-- =-B4
28	Put cash flow	20	<-- =MAX(B4-B24,0)
29	Total	0	<-- =SUM(B26:B28)

23.6. Fact 6: You might find it optimal to early-exercise an American put on a non-dividend paying stock

Recall that you'll *never* find it optimal to early-exercise an American *call* on a non-dividend paying stock. But this is not necessarily true for a put option. Here's an example:

Suppose that you're currently holding an option on PFE stock (a fictional company). You bought the option some time ago, when PFE stock's price was still healthy. However, at the current date, the stock has taken a plunge and is selling for $1 per share. Your American put option has an exercise price of $X = 100$ and expires in one year. The interest rate is 10%. If you exercise the option now, you'll have a net payoff of $99 ($100 minus the current value of the stock of $1), which—if you invest it in bonds with an interest rate of 10%--will be $99*1.10=$108.90 in one year. This is more than anyone would have if they waited for a year until exercise.

Therefore any rational holder of an American put option will choose to early exercise the option if the current stock price is very low.

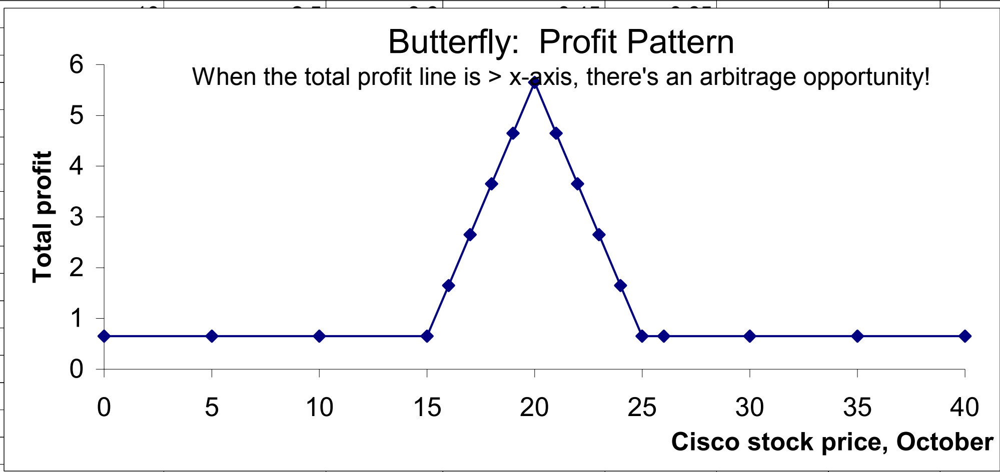
23.7. Fact 7: Option prices are convex

To see the meaning of this somewhat opaque statement, we return to the Cisco example from Chapter 21. Consider the three call options indicated below. Recall that a *butterfly* strategy consists of buying one low-priced and one high-priced call and selling two medium-priced calls.

	A	B	C	D	E	F
1	**CISCO OPTIONS, August 7, 2001**					
2	**CLOSING PRICE ON CHICAGO**					
3	**BOARD OF OPTIONS EXCHANGE**					
4						
5	August 7, 2001, CSCO closing price	19.26				
6						
7	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
8	Aug01	7.50	11.90	0.05	17 Aug01	10
9	Aug01	10.00	9.60	0.20	17 Aug01	10
10	Aug01	12.50	6.50	0.10	17 Aug01	10
11	Aug01	15.00	4.20	0.10	17 Aug01	10
12	Aug01	17.50	2.10	0.40	17 Aug01	10
13	Aug01	20.00	0.65	1.45	17 Aug01	10
14	Aug01	22.50	0.15	3.40	17 Aug01	10
15	Aug01	25.00	0.05	5.00	17 Aug01	10
16	Aug01	27.50	0.10	7.50	17 Aug01	10
17	Aug01	30.00	0.10	11.90	17 Aug01	10
18	Aug01	32.50	0.05		17 Aug01	10
19	Aug01	35.00	0.05	16.20	17 Sep01	41
20	Sep01	10.00	9.50		21 Sep01	45
21	Sep01	12.50	6.30	0.15	21 Sep01	45
22	Sep01	15.00	4.50	0.40	21 Sep01	45
23	Sep01	17.50	2.75	0.90	21 Sep01	45
24	Sep01	20.00	1.35	2.00	21 Sep01	45
25	Sep01	22.50	0.55	3.80	21 Sep01	45
26	Sep01	25.00	0.20	5.50	21 Sep01	45
27	Sep01	27.50	0.10		21 Sep01	45

Suppose that the call option prices for Cisco were different from those actually seen in the market. In the example below, we show how our butterfly would have looked had the $X = 15$ call been priced at $2.50 instead of $5.00:

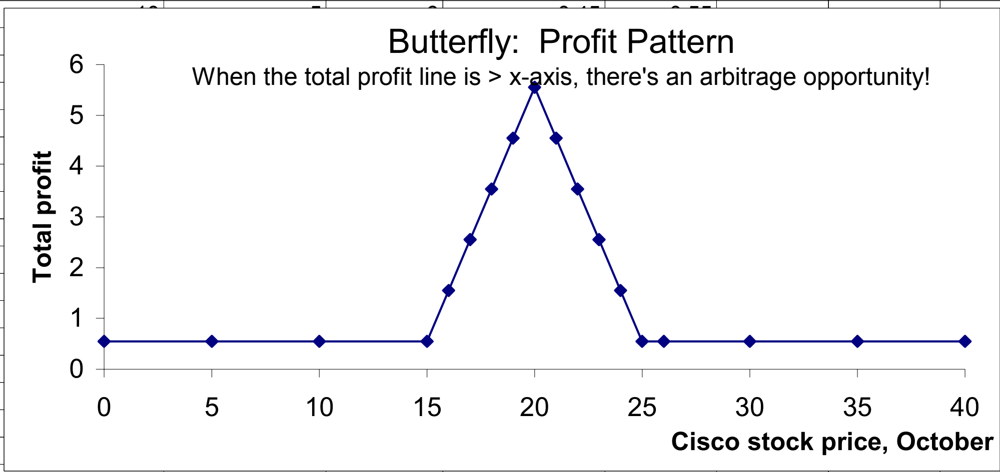
	A	B	C	D	E	F	G	H	I
1	**WHEN DOES A BUTTERFLY INDICATE AN ARBITRAGE OPPORTUNITY?**								
2	**Strategy: Buy 1 October 15 Call, Write 2 October 20 Calls, Buy 1 October 25 Call**								
3									
4	**Call prices**								
5		**X**	**Price**						
6		15	2.50						
7		20	1.80						
8		25	0.45						
9									
10	**Payoff and profits**								
11	**October Cisco stock price**	**Payoff on October X=15 call**	**Payoff on October X=20 call**	**Payoff on October X=25 call**	**Total profit**				
12	0	-2.5	3.6	-0.45	0.65				
13	5	-2.5	3.6	-0.45	0.65				
14									
15–30									
31									
32									



Notice that the total profit graph is *completely above the x-axis*. This means that—no matter what the stock price in October, you will make a profit. This is clearly not logical—something is wrong with these prices!

You get the same thing if you assume that the $X = 20$ call option is priced at \$3.00 instead of \$1.80:

	A	B	C	D	E	F	G	H	I
1	**WHEN DOES A BUTTERFLY INDICATE AN ARBITRAGE OPPORTUNITY?**								
2	**Strategy: Buy 1 October 15 Call, Write 2 October 20 Calls, Buy 1 October 25 Call**								
3									
4	**Call prices**								
5		**X**	**Price**						
6		15	5.00						
7		20	3.00						
8		25	0.45						
9									
10	**Payoff and profits**								
11	**October Cisco stock price**	**Payoff on October X=15 call**	**Payoff on October X=20 call**	**Payoff on October X=25 call**	**Total profit**				
12	0	-5	6	-0.45	0.55				
13	5	-5	6	-0.45	0.55				

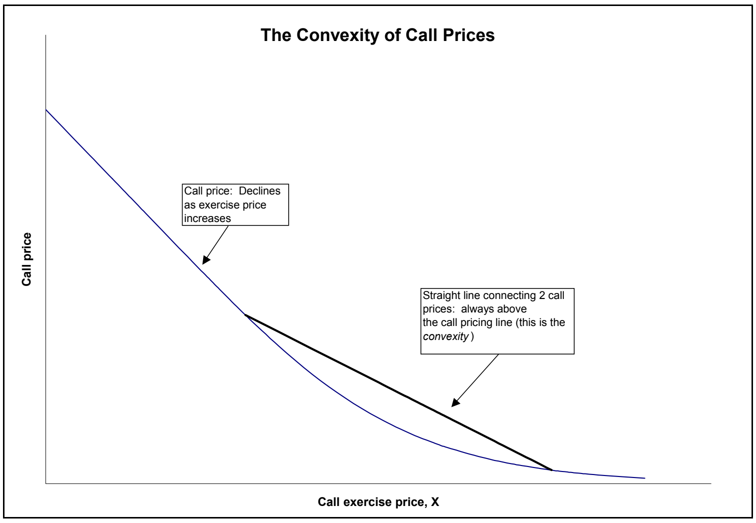


What's wrong?

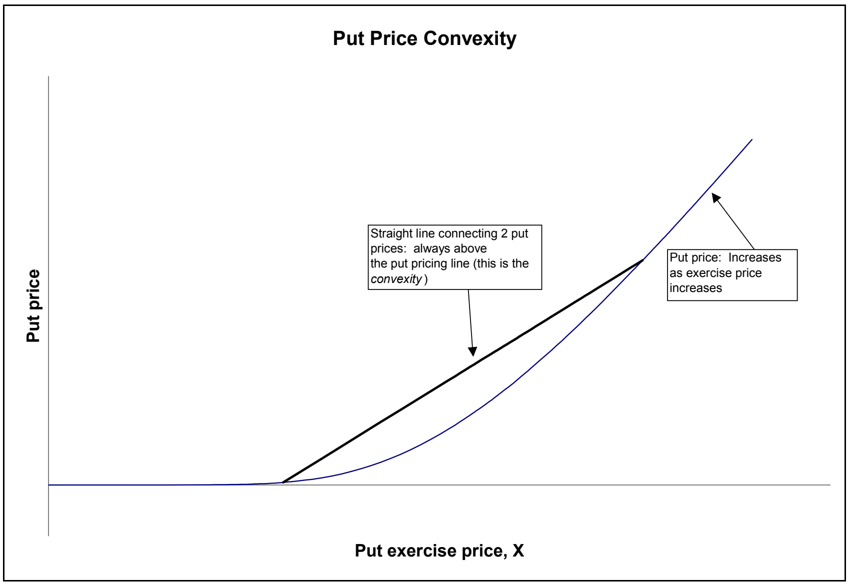
Playing around a bit with the numbers will convince you that a *condition necessary for the butterfly graph to straddle the x-axis* is:

$$Call\ \ price\left(X=20\right)<\frac{Call\ \ price\left(X=15\right)+Call\ \ price\left(X=25\right)}{2}$$

This condition—in the jargon of the options markets referred to as the *convexity property of call prices*—says that for 3 "equally spaced" calls, the middle call price must be less than the average of the two extreme call prices. Another way of saying this is that the line connecting two call prices always lies *above* the graph of the call prices:



Put prices are also convex. We leave put butterflies as an exercise and let you prove this on your own. Here's the way put prices look:



Summary

In this chapter we have derived restrictions on option prices which stem from their being related to other securities in the market. These arbitrage restrictions help us bound option prices (that is, establish minimum prices for put and call options) as well as establish relations between the prices of various options and the underlying security (as in the case of the put-call parity theorem).

In this chapter we have dealt with 7 such option pricing restrictions, but there are many more which deal with cases involving dividends and transactions costs. Understanding the seven restrictions discussed in this chapter will help you understand not only the pricing of options (we will have more to say on this topic in the next chapter), but it will also help you understand the way option traders think—they are constantly busy trying to figure out how to arbitrage option prices.

CHAPTER 24: OPTION PRICING FACTS*

This version: November 20, 2004

Chapter contents

Overview

In Chapter 23 we discussed basic option concepts: definitions of a call and a put, the reasons why you might want to buy or sell an option, and the profits resulting from various options strategies. In this chapter we discuss some basic facts about option pricing. Our emphasis is on a set of propositions known as *arbitrage restrictions* on option prices. These restrictions specify relations between the prices of puts and calls and the prices of either the stock underlying the options or a risk-free asset.

By understanding the option pricing restrictions in this chapter, you can often easily judge whether an option is mispriced. Here's an example: Suppose you're considering buying a call option on Microsoft stock, which is currently selling for $63 a share. Suppose the option expires in one year and has exercise price X = *$60*. The interest rate is 10%. The option is priced at $7. Is it a good buy or not? Our first option pricing fact (Section 24.1) will enable you to say that the option is *underpriced* and that it is definitely a good buy.

Notation: Throughout the chapter we use the following notation

S_0 = price of stock at time 0 (today)
S_T = price of stock on option exercise date T
X = option exercise price
r = interest rate
C = call option price at time 0 (today);
sometimes we also write this as C_0 and occasionally as $Call_0$
P = put option price at time 0 (today);
sometimes we write this as P_0 and occasionally as Put_0
C_t = call option price at time t
P_t = put option price at time t

Dividends: Throughout the chapter we assume that the stock on which the options are written does not pay dividends before the option maturity date.[1] This is not an overly-restrictive assumption: Stocks which pay dividends tend to do so at regular intervals (quarterly, semi-annually, or annually). Holders of options on these stocks are thus reasonably sure when the stocks will pay a dividends. There are thus long periods of time when market participants can be assured that a stock will not pay a dividend.

For example: General Motors pays a regular quarterly dividend in February, May, August, and November. An investor who purchases an option on GM in March with an April maturity knows that in the intervening period no dividends will be paid on the stock.

Many other stocks have never paid a dividend and investors in these stocks' options can be reasonably assured that the dividend pricing restriction imposed in this chapter is not restrictive. Stocks which fall into this category include many of the high-tech stocks whose options tend to attract the most investor interest.

Finance concepts discussed in this chapter

- Option pricing restrictions
- No early exercise of calls
- Put-call parity
- Early exercise of American puts
- Option price convexity

[1] The one exception is Section 24.6, where we briefly discuss the effect of dividends.

Excel functions used

- **Max**
- **Sum**
- **If**

24.1. Fact 1: Call price of an option > $Max\left[S_0 - PresentValue\left(X\right),0\right]$

It's 15 August 2001, and you're considering buying a call option on Microsoft. Currently the MSFT share itself is selling for S_0 = \$63; you want to buy a call on MSFT with an exercise price X = \$60 and with time to maturity T = *1 year*. Furthermore, we'll suppose that the option is an *American call option*, and can be exercised at any time on or before T.

We will examine Fact 1 in two stages. We start with a "dumb fact," something that is obvious once we say it, and then proceed to demonstrate Fact 1 for you.

Dumb fact: $Call\ \ price \geq Max\left[S_0 - X, 0\right]$**.**

Now it's probably clear to you that the Microsoft option should be selling for *at least* $\$3 = S_0 - X = \$63 - \$60$. To see this, suppose that the option is selling for \$2. We'll devise an *arbitrage strategy*—a strategy which will make us money risklessly:

Arbitrage strategy to profit from call price $C = \$2$ when stock price is $S_0 = \$63$ and $X = \$60$	
Action taken today	**Cash flow (negative numbers indicate costs)**
Buy the option	-\$2
Immediately exercise the option, buying the stock	-\$60
Immediately sell the stock on the open market	+\$63
Arbitrage profit	**+\$1**

So the "dumb fact"—that an American call option should sell for more than the difference between the stock price and the exercise price—is pretty obvious.

> Definition: Arbitrage Strategy
>
> An arbitrage strategy is a combination of assets—usually short or long positions in the stock, calls and puts on the stock, and a risk-free security—which produces positive cash flows at all points in time. If you can design an arbitrage strategy for a given set of asset prices (as we do below), it shows that at least one of the prices is *wrong*.

Smart fact: $Call\ \ price > Max\left[S_0 - PV\left(X\right), 0\right]$.

This is a lot less obvious than the previous fact. It's also a lot more powerful.[2] The "dumb fact" above says that the option should sell for at least \$3. As the spreadsheet below shows, the "smart fact" says much more; for example, if the interest rate is 10%, then the smart fact says that the option should sell for at least \$8.45.

[2] How smart? Robert Merton, who first established this and lots of other facts about options, subsequently won the Nobel Prize for economics, in part for his work on option pricing.

	A	B	C
1	**FACT 1: Lower bound on call price**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	60	
4	Option exercise time, T (in years)	1	
5	Interest rate, r	10%	
6			
7	Lower bound on call price		
8	Dumb fact, call price > Max[S_0 - X,0]	3	<-- =MAX(B2-B3,0)
9	Fact 1: call price > Max[S_0 - PV(X),0]	8.45	<-- =MAX(B2-B3/(1+B5)^B4,0)

To prove the "smart fact," let's assume that you can buy the call for \$5. We'll show that there exists an *arbitrage strategy*, and we will therefore conclude that the option price is too low.

The arbitrage strategy involves a set of actions at time 0 (today) and at time T (the option expiration date):

At time 0 (today):

- Short one share of the stock
- Invest in a riskless security paying off the call's exercise price at time *T*.
- Buy a call on the option.

At time T:

- Purchase the stock on the open market at the time-*T* price, in order to close the short position
- Collect from our investment in the riskless security
- Exercise the option if this is profitable

Here's an example, which assumes that the stock price at time 0 is 63 and that the interest rate is 10%:

	A	B	C
1	**ARBITRAGE PROOF OF FACT 1**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	60	
4	Option exercise time, T (in years)	1	
5	Interest rate, r	10%	
6			
7	**Call price at time 0 (today)**	**5**	Below examine if this price <-- violates the arbitrage restriction
8			
9	**ARBITRAGE STRATEGY**		
10	**Actions at time 0 (today)**		
11	Short the stock	63	<-- =B2
12	Buy a bond which pays of X at time T	-54.55	<-- =-B3/(1+B5)^B4
13	Buy a call	-5	<-- =-B7
14	**Total cash flow at time 0**	3.45	<-- =SUM(B11:B13)
15			
16	**Cash flow at time T**		
17	S_T, stock price at time T	33	
18			
19	Repay the shorted stock	-33	<-- =-B17
20	Collect money from the bond	60	<-- =B3
21	Exercise the call?	0	<-- =MAX(B17-B3,0)
22	**Total cash flow at time T**	27	<-- =SUM(B19:B21)

In cells B19:B22 we calculate the cash flow at time *T=1* from the strategy. In the example above, Microsoft stock at *T* is selling for S_T = $33. In this case, we would have a positive time *T* cash flow of $27.

In the example below, we assume that Microsoft stock at *T* is S_T = $90. In this case you exercise the call (giving you a positive cash flow of $30), but the total payoff from the strategy is now $0:

	A	B	C
1	**ARBITRAGE PROOF OF FACT 1**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	60	
4	Option exercise time, T (in years)	1	
5	Interest rate, r	10%	
6			
7	**Call price at time 0 (today)**	**5**	Below examine if this price <-- violates the arbitrage restriction
8			
9	**ARBITRAGE STRATEGY**		
10	**Actions at time 0 (today)**		
11	Short the stock	63	<-- =B2
12	Buy a bond which pays of X at time T	-54.55	<-- =-B3/(1+B5)^B4
13	Buy a call	-5	<-- =-B7
14	**Total cash flow at time 0**	3.45	<-- =SUM(B11:B13)
15			
16	**Cash flow at time T**		
17	S_T, stock price at time T	90	
18			
19	Repay the shorted stock	-90	<-- =-B17
20	Collect money from the bond	60	<-- =B3
21	Exercise the call?	30	<-- =MAX(B17-B3,0)
22	**Total cash flow at time T**	0	<-- =SUM(B19:B21)

By changing the stock price S_T , you can see that our strategy always produces no worse than a zero cash flow at time *T*. This makes it an *arbitrage strategy*:

- At time 0, the cash flow is $3.45 > 0
- At time *T*, the cash flow is either positive (if the stock price $S_T < 60$) or zero.

You can't lose from this strategy!! In a rational world this means that something is wrong with the asset prices. In this case, it's clear what's wrong—the call price is too low.

To see this, consider the case where the call price is $10. As you can see below (cell B14), this means that the initial cash flow from the arbitrage strategy is negative. If the stock price at time *T* is less than $60, say S_T = $55, then you will make a future profit (cell B22 below), but this profit is no longer an arbitrage profit (recall that arbitrage occurs when you can *never* lose money—with a $10 call price, you start off with an initial negative cash flow):

	A	B	C
7	**Call price at time 0 (today)**	**10**	Below examine if this price <-- violates the arbitrage restriction
8			
9	**ARBITRAGE STRATEGY**		
10	**Actions at time 0 (today)**		
11	Short the stock	63	<-- =B2
12	Buy a bond which pays of X at time T	-54.55	<-- =-B3/(1+B5)^B4
13	Buy a call	-10	<-- =-B7
14	**Total cash flow at time 0**	-1.55	<-- =SUM(B11:B13)
15			
16	**Cash flow at time T**		
17	S_T, stock price at time T	55	
18			
19	Repay the shorted stock	-55	<-- =-B17
20	Collect money from the bond	60	<-- =B3
21	Exercise the call?	0	<-- =MAX(B17-B3,0)
22	**Total cash flow at time T**	5	<-- =SUM(B19:B21)

The cash flow at *T* (cell B22) is zero, but the initial cash flow (cell B14) is now negative. This makes more sense:

Negative initial cash flows in this arbitrage strategy start when the call price is > 8.45. If this is true, then you have to invest money today in order to have a non-negative cash flow in the future.

We've proved our first option pricing fact: $Call\ \ price > Max\left[S_0 - PV\left(X\right), 0\right]$.

24.2. Fact 2: It's never worthwhile to exercise a call early.[3]

Suppose that on 15 August 2001 you bought a Microsoft call option for $12 (note that this price does not violate Fact 1's price restriction). Furthermore, suppose that the option expires one year from today, on 15 August 2002.

[3] When the call is written on a non-dividend-paying stock.

Now suppose that after 8 months (approximately 2/3 of a year), you want to get rid of the option. To make the problem interesting, we'll assume that the price of Microsoft has risen to S_t = \$80. You have two possibilities:

- You could exercise the option. In this case you would collect \$20 = $Max\left[S_t - X, 0\right] = Max\left[80 - 60, 0\right]$.
- You could also *sell* the option on the open market. Of course, we don't know what the option's price would be, but Fact 1 tells us that in no case will the price be less than

$$Max\left[S_t - PV\left(X\right), 0\right] = Max\left[S_t - \frac{X}{\left(1+r\right)^{1/3}}, 0\right]$$
$$= Max\left[80 - \frac{60}{\left(1+10\%\right)^{1/3}}, 0\right] = 21.876$$

The present value $\frac{X}{\left(1+r\right)^{1/3}}$ expresses the fact that there is 1/3 of a year left before the option's exercise.

What should you do? Clearly you should *sell* rather than *exercise* the call.

	A	B	C	D	E	F
1	**FACT 2: No early exercise of calls**					
2	Microsoft stock price, 15 August 2001, S_0	63				
3	Option exercise price, X	60				
4	Option exercise time, T (in years)	1				
5	Interest rate, r	10%				
6	Call price at time 0	12				
7						
8	**Time line**					
9	**t=0**			**t=2/3**		**T=1**
10						
11	Buy option for \$12.00		Consider selling the option			
12			or exercising it.			
13						
14			Stock price, S_t	80.00		
15						
16			Payoff from option exercise	20.00	<-- =MAX(D14-B3,0)	
17			Minimum value of option			
18			according to Fact 1	21.8762	<-- =MAX(D14-B3/(1+$B$5)^(1-2/3),0)	
19						
20			Exercise option or sell it?	sell	<-- =IF(D18>=D16,"sell","exercise")	

24.3. Fact 3: Put-call parity $Put_0 = Call_0 + PV(X) - S_0$

Put-call parity states that the put price is determined by the call price, the stock price, and the risk-free rate of interest.[4] Here's an example: Suppose that we're considering a one-year put option on the Microsoft stock we've been discussing throughout this chapter. Recall that Microsoft stock is currently selling for S_0 = \$63. What should be the put price on Microsoft—where we assume that the put has the same exercise price X = \$60 and the same time to maturity T = 1?

	A	B	C
1	**FACT 3: Put-Call Parity**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	60	
4	Option exercise time, T (in years)	1	
5	Interest rate, r	10%	
6			
7	Call price	15.00	
8	Put price by put-call parity	6.55	<-- =B7+B3/(1+B5)^B4-B2

Another interpretation of put-call parity is that the put price plus the stock price always equals the call price plus the present value of the exercise price:

$$Put_0 + S_0 = Call_0 + PV(X).$$

This means that given any three of the following four variables—$Put_0, S_0, Call_0, X$—the fourth variable is determined.

[4] Again: Recall that the assumption is that the stock pays no dividends before the option maturity date T.

An arbitrage proof of put-call parity (can be skipped on first reading)

We can prove put-call parity by using arbitrage, as specified in the spreadsheet below. We assume that the stock price is S_0 = \$63, the exercise price is X = \$60, the time to exercise is T = 1 year, the interest rate is r = 10%, and the call price is $Call_0$ = \$15. Given these facts, put-call parity says that the put price should be P_0 = \$6.55 (cell B8).

In cell B11 we suppose that the put price is \$3, different from its put-call parity value; we then show that this makes an arbitrage profitable.

	A	B	C
1	**Arbitrage Proof of Put-Call Parity**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	60	
4	Option exercise time, T (in years)	1	
5	Interest rate, r	10%	
6			
7	Call price	15	
8	Put price by put-call parity	6.55	<-- =B7+B3/(1+B5)^B4-B2
9			
10	**Arbitrage proof of put-call parity**		
11	Put price today (t=0)	3	If this price differs from the price in cell B8, we will show that there is a profitable arbitrage strategy.
12			
13	**Actions at time 0 (today)**		
14	Buy stock	-63	<-- =-B2
15	Buy put	-3	<-- =-B11
16	Write call	15	
17	Take a loan of PV(X) at risk-free interest	54.55	<-- =B3/(1+B5)^B4
18	**Total cash flow at time 0**	3.55	<-- =SUM(B14:B17)
19			
20	**Cash flow at time T**		
21	S_T, stock price at time T	90	
22			
23	Sell stock	90	<-- =B21
24	Exercise the put?	0	<-- =MAX(B3-B21,0)
25	Cash flow from call	-30	<-- =-MAX(B21-B3,0)
26	Repay loan	-60	<-- =-B3
27	Total	0	<-- =SUM(B23:B26)

Here's the arbitrage strategy we designed:

At time 0 (today):

- Buy one share of Microsoft stock for \$63
- Buy one put with exercise price $X = \$60$ for \$3
- Write one call with $X = \$60$, collecting (today) $C_0 = \$15$
- Take a loan of \$54.55; the loan has a one-year maturity (like the options). At the current interest rate of 10% you will have to pay off \$60 in one year.

At time T we close out all our positions

- Sell our share of Microsoft at the prevailing market price S_T
- Exercise the put, if this is profitable
- Have the call exercised against us, if this is profitable for the call buyer
- Repay the loan

Our example above shows that the cash flow at $T=1$ will be zero if S_T = \$90. The cash flow will also be zero if $S_T = \$35$:

	A	B	C
20	**Cash flow at time T**		
21	S_T, stock price at time T	35	
22			
23	Sell stock	35	<-- =B21
24	Exercise the put?	25	<-- =MAX(B3-B21,0)
25	Cash flow from call	0	<-- =-MAX(B21-B3,0)
26	Repay loan	-60	<-- =-B3
27	Total	0	<-- =SUM(B23:B26)

As you can see, no matter what the Microsoft stock price in one year, the cash flow at $T=1$ from this strategy will be zero. However, the strategy has a positive initial cash flow of \$3.55. Clearly this is an arbitrage!

Symbolically, the future cash flow is given by:

$$\underbrace{S_T}_{\text{Stock value}} + \underbrace{Max\left[X - S_T, 0\right]}_{\text{Put payoff}} \underbrace{-Max\left[S_T - X, 0\right]}_{\substack{\text{Cash flow to call} \\ \text{writer at } T=1}} \underbrace{-X}_{\text{Loan repayment}}$$

$$= \begin{cases} S_T + X - S_T - X & if \;\; S_T < X \\ S_T - \left(S_T - X\right) - X & if \;\; S_T \ge X \end{cases}$$

$$= 0$$

A little thought will reveal that—given the stock price $S_0 = 60$, the interest rate $r=10\%$, the exercise price $X = 60$ of both the put and the call, and the call option price of \$15—the put option price must be \$6.55 to prevent arbitrage. This follows from the put-price parity relation:

$$Put_0 = Call_0 + PV\left(X\right) - S_0$$

$$= 15 + \frac{60}{1.10} - 63 = 6.55$$

24.4. Fact 4: Bound on an American put option price: $P_0 > Max\left[X - S_0, 0\right]$

Suppose you're contemplating buying an American put on Microsoft stock. The stock's price today is S_0 = \$63 and the option exercise price is X = \$70. Clearly the option should sell for at least \$7. If not, you could easily devise an arbitrage, as illustrated in the spreadsheet below:

	A	B	C
1	**FACT 4: Lower bound on *American* put price**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	70	
4	Option exercise time, T (in years)	1	
5			
6	Fact 4: Lower bound of American put: $P_0 > Max[X - S_0, 0]$	7	<-- =MAX(B3-B2,0)
7			
8	**Arbitrage strategy**		
9	American put option price	3	
10	Buy option	-3	
11	Buy stock now	-63	
12	Exercise put option immediately: deliver stock and get *X*	70	
13	Immediate profit	4	<-- =SUM(B10:B12)

If the American put option is mispriced (that is, its price is less than $7), you can make money by; buying the option, buying the stock, and exercising the option immediately. This arbitrage profit will not exist if the option's price is greater than $7.

24.5. Fact 5: Bounds on European put option prices $P_0 > Max\left[PV(X) - S_0, 0\right]$

Fact 5 is the "put parallel" for Fact 1 about calls.[5]

	A	B	C
1	**FACT 5: Lower bound on *European* put price**		
2	Microsoft stock price, 15 August 2001, S_0	63	
3	Option exercise price, X	70	
4	Option exercise time, T (in years)	1	
5	Interest rate, r	10%	
6			
7	Lower bound on call price		
8	Lower bound of American put: P_0 > Max[X - S_0, 0]	7	<-- =MAX(B3-B2,0)
9	Fact 5: P_0 > Max[PV(X) - S_0,0]	0.6364	<-- =MAX(B3/(1+B5)^B4-B2,0)

[5] There's a crucial difference in the parallel between Facts 1 and 5: Fact 1 applies to *all* calls, whether European or American. Fact 5 applies only to European puts. Of course in both cases, the assumption is that the stock pays no dividends before option maturity.

American versus European Puts

Fact 5 says that the price of a European put can actually be much lower than the price of an American put. Consider the example above, in which we look at the price of a put option on Microsoft stock with $T = 1$ and $X = 70$. If our put was an American put, then it couldn't sell for less than \$7. On the other hand, a *European put*, which cannot be exercised until date *T*, can sell for anything more than \$0.6364.

24.6. Fact 6: You might find it optimal to early-exercise an American put on a non-dividend paying stock

Recall that you'll *never* find it optimal to early-exercise an American *call* on a non-dividend paying stock. But this is not necessarily true for a put option. Here's an example:

Suppose that you're currently holding an option on PFE stock. You bought the option some time ago, when PFE stock's price was still healthy. However, at the current date, the stock has taken a plunge and is selling for \$1 per share. Your American put option has an exercise price of X = \$100 and expires in one year. The interest rate is 10%. If you exercise the option now, you'll have a net payoff of \$99 (\$100 minus the current value of the stock of \$1), which—if you invest it in bonds with an interest rate of 10%--will be \$99*1.10=\$108.90 in one year. This is more than anyone would have if they waited for a year until exercise.

Therefore any rational holder of an American put option will choose to early exercise the option if the current stock price is very low.

24.7. Fact 7: Option prices are convex (somewhat advanced)

Suppose we have three calls, each with a different exercise price but with the same time to exercise *T*, written on the same stock. Suppose that the exercise price of the first call is *X* = \$15, the exercise price of the second call is *X* = \$20, and the exercise price of the third call is *X* = \$25. Call price convexity says that for 3 such "equally spaced" calls, the middle call price must be less than the average of the two extreme call prices. In an equation:

$$Call\ \ price\left(X=20\right)<\frac{Call\ \ price\left(X=15\right)+Call\ \ price\left(X=25\right)}{2}$$

To see the meaning of convexity, we return to the Cisco example from Chapter 23. Consider the three call options in rows 18, 20, and 22 of the next spreadsheet. The convexity relation says that:

$$Call\ \ price\left(X=20\right)<\frac{Call\ \ price\left(X=15\right)+Call\ \ price\left(X=25\right)}{2}=\frac{4.50+0.20}{2}=2.35$$

Since the Cisco call with *X* = \$20 is selling for \$1.35 (cell C20), it fulfills the convexity relation.

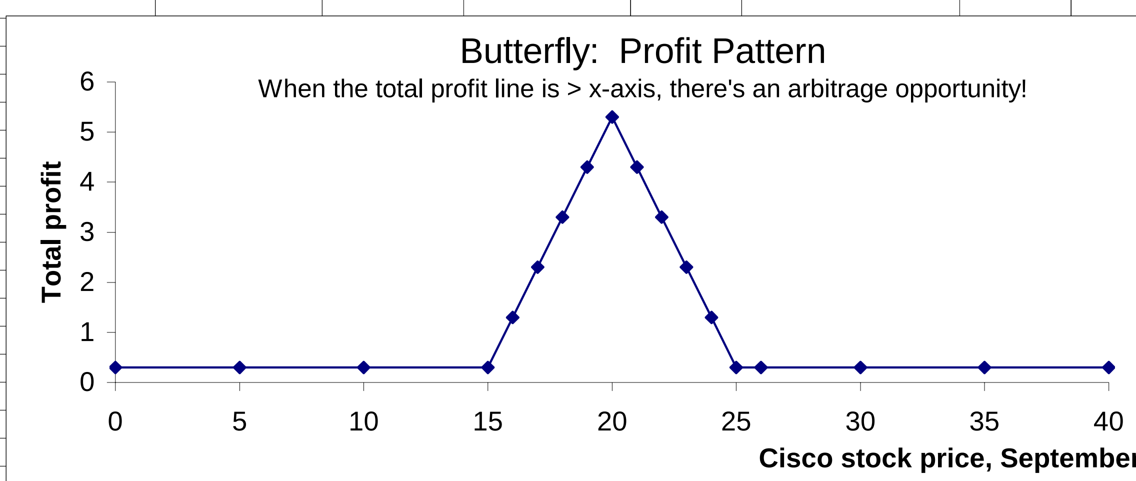
	A	B	C	D	E	F
1	**CISCO OPTIONS, August 7, 2001 CLOSING PRICE ON CHICAGO BOARD OF OPTIONS EXCHANGE**					
2	August 7, 2001, CSCO closing price	19.26				
3	**Stated expiration date**	**Exercise price, X**	**Call price**	**Put price**	**Actual expiration date**	**Days to maturity**
4	Aug01	7.50	11.90	0.05	17 Aug01	10
5	Aug01	10.00	9.60	0.20	17 Aug01	10
6	Aug01	12.50	6.50	0.10	17 Aug01	10
7	Aug01	15.00	4.20	0.10	17 Aug01	10
8	Aug01	17.50	2.10	0.40	17 Aug01	10
9	Aug01	20.00	0.65	1.45	17 Aug01	10
10	Aug01	22.50	0.15	3.40	17 Aug01	10
11	Aug01	25.00	0.05	5.00	17 Aug01	10
12	Aug01	27.50	0.10	7.50	17 Aug01	10
13	Aug01	30.00	0.10	11.90	17 Aug01	10
14	Aug01	32.50	0.05		17 Aug01	10
15	Aug01	35.00	0.05	16.20	17 Sep01	41
16	Sep01	10.00	9.50		21 Sep01	45
17	Sep01	12.50	6.30	0.15	21 Sep01	45
18	Sep01	15.00	4.50	0.40	21 Sep01	45
19	Sep01	17.50	2.75	0.90	21 Sep01	45
20	Sep01	20.00	1.35	2.00	21 Sep01	45
21	Sep01	22.50	0.55	3.80	21 Sep01	45
22	Sep01	25.00	0.20	5.50	21 Sep01	45

Why do call prices have to be convex?

In this subsection we use a butterfly strategy (Chapter 23, page000) to show you why call prices always have to be convex. Recall that a *butterfly* strategy consists of buying one low-priced and one high-priced call and selling two medium-priced calls.

Suppose that the call option prices for Cisco were different from those actually seen in the market. In the example below, we show how our butterfly would have looked had the X = \$20 call been priced at \$2.50 instead of \$1.35:

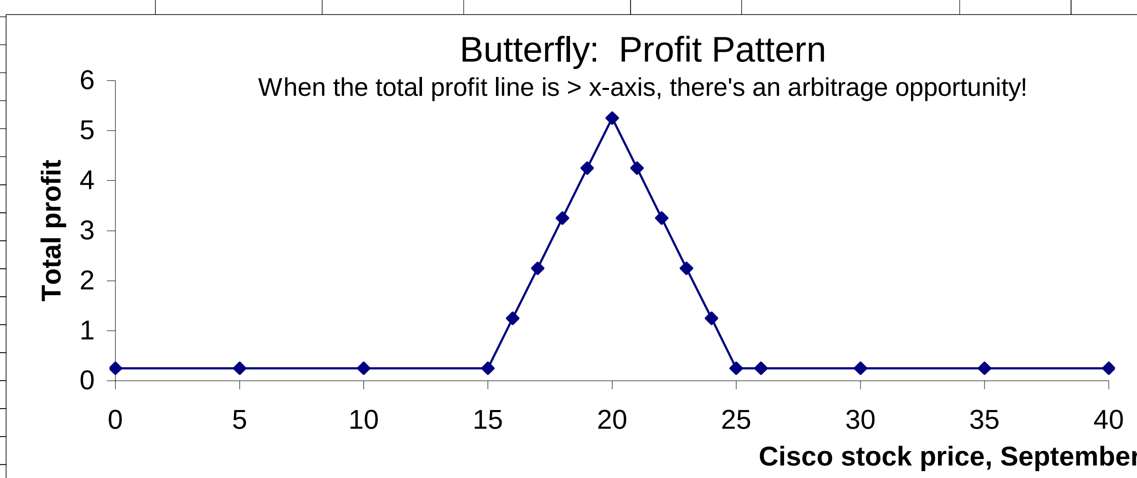
	A	B	C	D	E	F	G	H
1	**WHEN DOES A BUTTERFLY INDICATE AN ARBITRAGE OPPORTUNITY? Strategy: Buy 1 September X=15 Call, Write 2 September X=20 Calls, Buy 1 September X=25 Call**							
2	**Call prices**							
3	**X**	**Price**						
4	15	4.50						
5	20	2.50	<-- The actual price is $1.35. To illustrate arbitrage, we assume $2.50					
6	25	0.20						
7								
8	**Butterfly payoff and profits**					=MAX(A10-15,0)-$B$4		
9	**September Cisco stock price**	**Payoff on September X=15 call**	**Payoff on September X=20 call**	**Payoff on September X=25 call**	**Total profit**	=-2*(MAX(A10-20,0)-$B$5)		
10	0	-4.5	5	-0.2	0.3	<-- =D10+C10+B10		
11	5	-4.5	5	-0.2	0.3			
12	10	-4.5	5	-0.2	0.3	=MAX(A10-25,0)-$B$6		
13	15	-4.5	5	-0.2	0.3			
14	16	-3.5	5	-0.2	1.3			
15	17	-2.5	5	-0.2	2.3			
16	18	-1.5	5	-0.2	3.3			
17	19	-0.5	5	-0.2	4.3			
18	20	0.5	5	-0.2	5.3			
19	21	1.5	3	-0.2	4.3			
20	22	2.5	1	-0.2	3.3			
21	23	3.5	-1	-0.2	2.3			
22	24	4.5	-3	-0.2	1.3			
23	25	5.5	-5	-0.2	0.3			
24	26	6.5	-7	0.8	0.3			
25	30	10.5	-15	4.8	0.3			
26	35	15.5	-25	9.8	0.3			
27	40	20.5	-35	14.8	0.3			



Notice that the total profit graph is *completely above the x-axis*. This means that—no matter what the stock price in September, you will make a profit. This is clearly not logical—something is wrong with these prices!

You get the same thing if you assume that the X = \$15 call option is priced at \$2.25 instead of \$4.50:

	A	B	C	D	E	F	G	H
1	**WHEN DOES A BUTTERFLY INDICATE AN ARBITRAGE OPPORTUNITY? Strategy: Buy 1 September X=15 Call, Write 2 September X=20 Calls, Buy 1 September X=25 Call**							
2	**Call prices**							
3	**X**	**Price**						
4	15	2.25	<-- The actual price is \$4.50. To illustrate arbitrage, we assume \$2.25					
5	20	1.35						
6	25	0.20						
7								
8	**Butterfly payoff and profits**							
9	**September Cisco stock price**	**Payoff on September X=15 call**	**Payoff on September X=20 call**	**Payoff on September X=25 call**	**Total profit**			
10	0	-2.25	2.7	-0.2	0.25	<-- =D10+C10+B10		
11	5	-2.25	2.7	-0.2	0.25			
12	10	-2.25	2.7	-0.2	0.25			
13	15	-2.25	2.7	-0.2	0.25			
14	16	-1.25	2.7	-0.2	1.25			
15	17	-0.25	2.7	-0.2	2.25			
16	18	0.75	2.7	-0.2	3.25			
17	19	1.75	2.7	-0.2	4.25			
18	20	2.75	2.7	-0.2	5.25			
19	21	3.75	0.7	-0.2	4.25			
20	22	4.75	-1.3	-0.2	3.25			
21	23	5.75	-3.3	-0.2	2.25			
22	24	6.75	-5.3	-0.2	1.25			
23	25	7.75	-7.3	-0.2	0.25			
24	26	8.75	-9.3	0.8	0.25			
25	30	12.75	-17.3	4.8	0.25			
26	35	17.75	-27.3	9.8	0.25			
27	40	22.75	-37.3	14.8	0.25			
28								



What's wrong?

Playing around a bit with the numbers will convince you that a *condition necessary for the butterfly graph to straddle the x-axis* is:

$$Call\ \ price\left(X_{Middle}\right) < \frac{Call\ \ price\left(X_{Low}\right) + Call\ \ price\left(X_{High}\right)}{2},$$

where $X_{Low}, X_{Middle}, X_{High}$ are three equally-spaced exercise prices.

This condition—in the jargon of the options markets referred to as the *convexity property of call prices*—says that for 3 "equally spaced" calls, the middle call price must be less than the average of the two extreme call prices. Another way of saying this is that the line connecting two call prices always lies *above* the graph of the call prices:

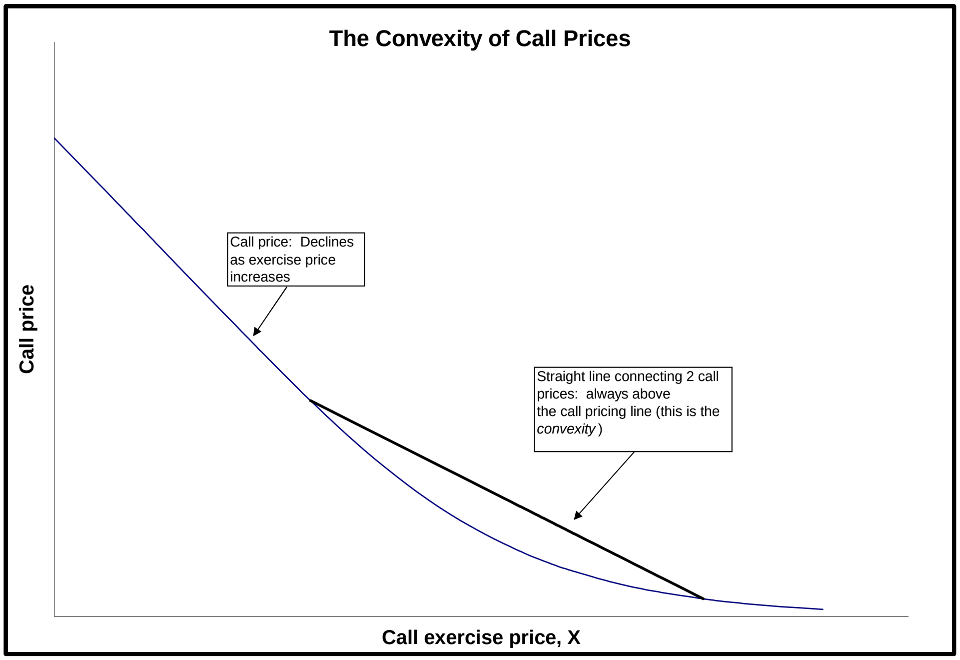


Figure 24.1: The curved line illustrates the actual call prices for various exercise prices. Call price convexity means that the line connecting two call prices is always above the actual call pricing curve.

Put prices are also convex.

$$Put\ \ price\left(X_{Middle}\right) < \frac{Put\ \ price\left(X_{Low}\right) + Put\ \ price\left(X_{High}\right)}{2}$$

We leave put butterflies as an exercise and let you prove this on your own. Here's the way put prices look:

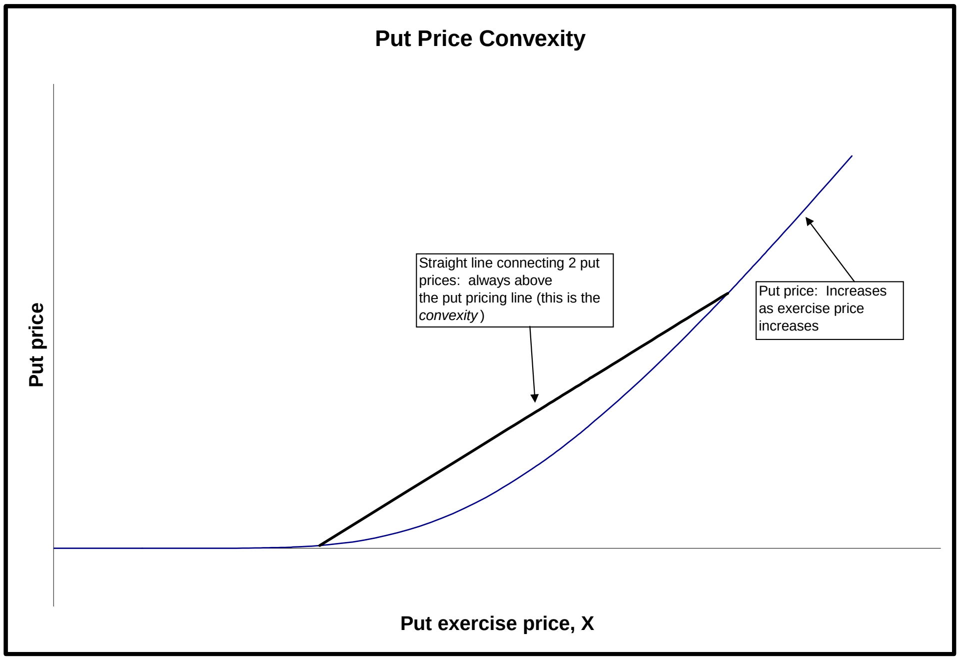


Figure 24.2: The curved line illustrates the actual put prices for various exercise prices. Put price convexity means that the line connecting two put prices is always above the actual put pricing curve.

Summary

In this chapter we have derived restrictions on option prices which stem from their being related to other securities in the market. These arbitrage restrictions help us bound option prices (that is, establish minimum prices for put and call options) as well as establish relations between the prices of various options and the underlying security (as in the case of the put-call parity theorem).

In this chapter we have dealt with seven such option pricing restrictions, but there are many more which deal with cases involving dividends and transactions costs. Understanding the seven restrictions discussed in this chapter will help you understand not only the pricing of options (we will have more to say on this topic in the next chapter), but it will also help you understand the way option traders think—they are constantly busy trying to figure out how to arbitrage option prices.

Exercises

1. You want to buy one American call option contract on Dell Computer Corporation, expiring in six months, with a strike price of $25. The current stock price is at $24.80. Can the option price be lower than $0.60? Assume that the interest rate is 8%.

2. Assume that you can buy the above call for $0.50 (which is less than the theoretical minimum). How can you exploit the mispricing to make a riskless gain?

3. Your generous uncle gives you 10,000 units of the above option as a birthday gift. The stock price has risen to $28. Will you exercise the option early or rather sell it? Explain.

4. Cash dividends affect option prices through their effect on the underlying stock price. Because the stock price is expected to drop by the amount of the dividend on the ex-dividend date, high cash dividends imply lower call premiums. Suppose you own a call option with a strike price of 90 that expires in one week. The stock is currently trading at $100 and is expected to pay a $2.00 dividend tomorrow. The call option has a value of $10. What are you going to do: hold the option or exercise the option early?

5. The Fashion Corporation has stock outstanding that is currently selling for $83 per share. Both a put and call with a strike price of $80 and an expiration of 6 months are trading. The put option premium is $2.50, and the risk-free rate is 5 percent. If put-call parity holds, what is the call option premium?

6. The current market price of a two-month European put option on a non-dividend-paying stock with strike price of $50 is $4. The stock price is $47 and the risk-free interest rate is 6%.

6.a. If a two-month call option with the same strike price is currently selling for $1, what opportunities are there for an arbitrageur? How can she exploit arbitrage?

6.b. Would the above market prices still provide an arbitrage opportunity if the stock would be $46.8/per share in 1 month?

7. In general, what is the problem of using Wall Street Journal prices to search for violations of the put-call parity relationship?

8. Recall that a Butterfly spread is an options strategy built on four trades at one expiration date and three different strike prices. For call options, one option each at the high and low strike prices are bought, and two options at the middle strike price are sold. A Butterfly spread for ABC stock is created as follows: Sell 1 ABC Jun $180 Call for $20 and buy 2 ABC Jun $200 Call each at $10 and sell 1 ABC Jun $220 Call for $5 (Net premium receive $20-2*$10+$5=5$). For put options, the trades are reversed: Sell 1 ABC Jun $180 Put, buy 2 ABC Jun $200 Put and sell 1 ABC Jun $220 Put. Use put-call parity to show that the cost of a butterfly spread created from the European calls is identical to the cost of the butterfly spread created from European puts.

9. (Challenge). You have the following information, 25 calendar days before the March 2004 option expiration day:

Strike	Put/Call	Price

1025	Call	$19.8
1025	Put	$14.5
1040	Call	$12.5
1040	Put	$22.17

In the absence of arbitrage, what does the annualized riskfree rate have to be?

10. A European put and a call option both expire in a year and have the same exercise price of $20. The options are currently traded at the same market price of $3. Assume that the annual interest rate is 8%. What is the current stock price? In general: If a European put and a call have the same price and expire at the same time, what can you say about the relationships between the stock price and the exercise price. $S_0>X$? $S_0<X$? $S_0=X$?

11. You consider buying an American put option on Dell Computer Corporation, expiring in six months, with a strike price of $25. The current stock price is at $18. What is the minimum price that you are willing to pay? If you can buy the above put for $5 (which is less than the theoretical minimum), how can you exploit the mispricing to make a riskless gain?

12. ABC is a non-dividend paying stock. Suppose that S=$17, X=$20, r=5% per annum.

    12.a. Can a **European** put option that expires in 6 month trade at $2.50? Note that a European put option may sometimes be worth less than its intrinsic value.

    12.b. Consider a situation where the European put option is traded at $2.4. Show how you can gain from arbitrage.

13. Suppose that you are currently holding an American put option on National Australia Bank that has an exercise price of $45. The option expires in 6 months. The share price is currently traded at $23.00.

13.a. Consider a situation where the American put option is traded at $21. Show how you can gain from arbitrage.

13.b. What is your net payoff if

i. you exercise the put option today (assume that you invest your proceeds in bonds with an interest rate of 8%).

ii. you hold the option until its expiration date.

14. You need to weigh the benefits of early exercise of a put option you hold that expires in 6 month, X=$50, r=20%, with profit you may be giving up by selling the stock today instead of later. Assume:

14.a. S=$20

14.b. S=$3

In which case you are sure better off exercising the option?

15. Refer to the call options prices given in exercise 8 above. Show that the convexity property of call prices holds.

16. A butterfly spread is created using the following put options: The investor buys a put option with a strike price of 55 and pays $15, buys a put with a strike price of 65 and pays $5 and sells two put with an intermediate strike price of $60.

16.a. What is the upper bound for the X=$60 put price, according to the convexity property?

16.b. Assume that the X=$60 put price is $12. Draw the profit pattern at maturity for the butterfly using Excel (let the stock prices at maturity range between $40 and $80). Does the chart indicate an arbitrage opportunity?

17. At the expiration date the put call parity $Put_0(X) = Call_0(X) + PV(X) - S_0$ has the following form**:** $Put_T(X) = Call_T(X) + X - S_T$ or $S_T = Call_T(X) - Put_T(X) + X$**.** Verify this equation using Excel: Let S_T range from $20 to $100 and the exercise price X=$60. The option value at expiry are: Put(X)=Max(S_T -X,0), Call(X)=Max(X- S_T,0).

18. Cisco (CSCO) stock sells for $25. The CSCO April 24 call sells for $3 3/8 and the CSCO April 24 put sells for $1 3/4. The call, put and a Treasury Bill all mature in 4 months. Today's price for a Treasury bill which pays off $100 in four months $94.92. Assume that Cisco does not pay dividends in this period. Use the put-call parity relation to find the arbitrage profit today, if exists.

CHAPTER 24: OPTION PRICING—THE BLACK-SCHOLES FORMULA*

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Chapter contents

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it.

Overview

In the two previous chapters on option pricing, we've discussed some facts about options, but we haven't discussed *how to determine the price of an option*. This is the subject of this chapter.

In this chapter we discuss the Black-Scholes formula. This is the most important option pricing formula—it's in wide use in option markets. Everybody "knows" this formula, in the sense that even non finance people (lawyers, accountants, judges, bankers . . .) know that options are priced using Black-Scholes; they may not know how to apply it, and they certainly wouldn't know why the formula is correct, but they know that it's used.

In our discussion of the Black-Scholes model, we'll make no attempt whatsoever to give a theoretical background to the model. It's hopeless, unless you know a lot more math than 99% of all beginning finance students will ever know.[1]

The next chapter discusses the other major technique for solving option prices, the *binomial option pricing model*. This model gives some insights into how to price an option, and it's also used widely (though not as widely as the Black-Scholes equation). This approach is somewhat idiosyncratic, since most books discuss the binomial model—which, in a theoretical sense, underlies the Black-Scholes formula—first and then discuss Black-Scholes. However, since we have no intention of making the theoretical connection between the binomial model and Black-Scholes, we've chosen to reverse the order and deal with the more important model first.

[1] A bitter truth, perhaps. But get this—your professor probably can't prove the Black-Scholes equation either (don't ask him, he'll be embarrassed). On the other hand—you know how to drive a car but may not know how an internal combustion engine works, you know how to use a computer but can't make a central processing unit chip, ...

What does "pricing an option" mean?

Suppose we're discussing a call option on Microsoft stock which is sold on 8 February 2002: On this date, Microsoft's stock price is \$60.65. We will look at options on this stock which have an exercise price of $X = \$60$, and which expire on July 19, 2002. Here's what you've learned so far:

- From Chapter 21, we know what the terminology means.
- Chapter 21 also tells us what the payoff *pattern* and *profit pattern* of the call option looks like—by itself and in combination with other assets
- Chapter 22 tells us some pricing *restrictions* on the call option. A simple restriction ("Fact 1" from Chapter 21) says that $Call > Max\left[S_0 - PV\left(X\right), 0\right]$. A more sophisticated restriction ("Fact 3," put-call parity) says that—once we know the price of Microsoft stock, the call price, and the interest rate—the put price is determined.

All of these facts are –by themselves—interesting. However, they don't tell us what the *price* of the call option should be. This is the subject of this chapter—the Black-Scholes formula gives us the an answer—a good one, as you'll see—to how to price the option.

Finance concepts in this chapter

- Black-Scholes formula
- Put-call parity
- Stock price volatility
- Implied volatility

Excel functions used

- Exp
- Ln
- Stdevp
- Varp
- Data Table

24.1. The Black-Scholes Model

In a famous paper published in 1973, Fisher Black and Myron Scholes proved a formula for pricing European call and put options on non-dividend-paying stocks. Their model is probably the most famous model of modern finance. The Black-Scholes model uses the following formula to price calls on the stock:

$$C = SN(d_1) - Xe^{-rT}N(d_2),$$
$$where$$
$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Here C denotes the price of a call, S is the current price of the underlying stock, X is the exercise price of the call, T is the call's time to exercise, r is the interest rate, and σ is the standard deviation of the logarithm of the stock's return. *N( )* denotes a value of the standard normal distribution. It is assumed that the stock will pay no dividends before date T.

The spreadsheet below prices an option on a stock whose current price is $S=100$. The option's exercise price is $X=90$ and its time to maturity is $T=0.5$ (one-half year). The interest rate is $r=4\%$, and sigma (σ, the stock's volatility—a measure of the stock's riskiness; more about this later) is $\sigma = 35\%$.

	A	B	C
1	**The Black-Scholes Option-Pricing Formula**		
2			
3	S	100	Current stock price
4	X	90	Exercise price
5	T	0.50000	Time to maturity of option (in years)
6	r	4.00%	Risk-free rate of interest
7	Sigma	35%	Stock volatility
8			
9	d_1	0.6303	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))
10	d_2	0.3828	<-- d_1-sigma*SQRT(T)
11			
12	N(d_1)	0.7357	<-- Uses formula NormSDist(d_1)
13	N(d_2)	0.6491	<-- Uses formula NormSDist(d_2)
14			
15	Call price	16.32	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)
16	Put price	4.53	<-- call price - S + X*Exp(-r*T): by Put-Call parity
17		4.53	<-- X*exp(-r*T)*N(-d_2) - S*N(-d_1): direct formula

By the put-call parity theorem (see Chapter 22), a put with the same exercise date T and exercise price X written on the same stock will have price $P = C - S + Xe^{-rT}$. We've used this formula in cell B16. Cell B17 includes another version of put pricing—a direct formula which follows from the Black-Scholes formula.

24.2. What do the Black-Scholes parameters mean? How to calculate them?

The Black-Scholes option pricing model depends on 5 parameters:

- S, the current price of the stock. By this we always mean the stock price on the date we're calculating the option price.
- X, the exercise price of the option (this is also called the "strike price").

- *T*, the time to the option's expiration. In the Black-Scholes formula, this is always given in *annual terms*—meaning: an option with 3 months to expiration has *T = 0.25*, an option with 51 days until expiration has $T = \frac{51}{365} = 0.1397$).

- *r*, the risk-free interest rate. This is also given in annual terms. Meaning: If the interest rate is 6% per year and if an option has *T = 0.25*, then we write *r=6%* in the Black-Scholes formula. The Black-Scholes formula assumes that there is only one risk-free rate, whereas in reality there are many rates. In actual calculations we usually use the Treasury bill rate for a maturity which is closest to the option maturity.

Here's an example: The table below (from Yahoo) shows the annualized U.S. Treasury bill rates on 8 February 2002. If we were valuing an option with *T = 0.125* (i.e., with maturity of 1 month), we would take *r* to be the 3-month Treasury bill rate (1.66%). If we were valuing an option with maturity *T= 1*, then we would take some rate intermediate between the 6-month yield of 1.76% and the 2-year yield of 2.99%.[2]

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- Economic Calendar
- Education
- Glossary
- Message Boards
- News
- Screener

U.S. Treasury Bonds

Maturity	Yield	Yesterday	Last Week	Last Month
3 Month	1.66	1.67	1.71	1.63
6 Month	1.76	1.76	1.81	1.71
2 Year	2.99	4.91	3.07	3.00
5 Year	4.26	4.19	4.29	4.32
10 Year	4.95	4.89	4.98	5.05
30 Year	5.43	5.34	5.39	5.49

[2] Two notes: It turns out that the Black-Scholes price is not that sensitive to variations in interest rates (see exercise ???). 2) The Yahoo table is very minimal! In actual practice we would try to find the rate on a zero-coupon bond of maturity similar to *T*.

- σ *("sigma")* is a measure of the riskiness of the stock. This is not simple to calculate (we discuss this in Sections 3 and 4 below). But before we discuss this, here are some facts to help you get your bearings:
    - If the stock is riskless, then $\sigma = 0\%$. A stock is riskless if its future price is completely predictable.
    - An "average" U.S. stock has σ of between 10% and 25%
    - A risky stock may have a σ of as much as 80% or 100 %.

  A remarkable fact is that the *Black-Scholes option price depends only on the sigma of the stock and <u>not on the stock's expected return</u>*.

24.3. Computing σ from stock prices

There are two main ways to compute the sigma: We can either calculate the sigma by looking at the series of past stock prices. Alternatively, we can calculate the *implied sigma* by looking at options prices; this calculation is often called the *implied volatility*. This section describes the first method, and the next section describes how to compute the implied volatility.

Below we show the annual prices for Microsoft for the decade from 1991 - 2001. Column C shows the *continuously compounded return* for the prices: $r_t^{continuous} = \ln\left(\frac{P_t}{P_{t-1}}\right)$. As you can see, the σ computed from this date is $\sigma = 36.90\%$:

	A	B	C	D
1	**MICROSOFT STOCK PRICES--ANNUAL DATA**			
2				
3	**Date**	**Closing stock price**	**Return**	
4	31-Dec-90	2.7257		
5	31-Dec-91	5.0104	60.88%	<-- =LN(B5/B4)
6	31-Dec-92	5.4062	7.60%	
7	31-Dec-93	5.3203	-1.60%	
8	31-Dec-94	7.4219	33.29%	
9	31-Dec-95	11.5625	44.33%	
10	31-Dec-96	25.5000	79.09%	
11	31-Dec-97	37.2969	38.02%	
12	31-Dec-98	87.5000	85.27%	
13	31-Dec-99	97.8750	11.21%	
14	31-Dec-00	61.0625	-47.18%	
15	31-Dec-01	66.2500	8.15%	
16	Average return		29.01%	<-- =AVERAGE(C5:C15)
17	Return variance		13.61%	<-- =VARP(C5:C15)
18	Return standard deviation		36.90%	<-- =STDEVP(C5:C15)

In the world of option pricing it is not usual to compute σ from annual data. Most traders prefer daily, weekly, or monthly data. The use of non-annual data requires some adjustment to the calculations. We show these adjustments in the example below, where we calculate Microsoft's σ from monthly data; a discussion of what we did follows the spreadsheet:

	A	B	C	D
1	**MICROSOFT STOCK PRICES MONTHLY DATA FOR 2001**			
2				
3	**Date**	**Close**		
4	29-Dec-00	43.3750		
5	31-Jan-01	61.0630	34.20%	<-- =LN(B5/B4)
6	28-Feb-01	59.0000	-3.44%	<-- =LN(B6/B5)
7	30-Mar-01	54.6880	-7.59%	<-- =LN(B7/B6)
8	30-Apr-01	67.7500	21.42%	
9	31-May-01	69.1800	2.09%	
10	29-Jun-01	73.0000	5.37%	
11	31-Jul-01	66.1900	-9.79%	
12	31-Aug-01	57.0500	-14.86%	
13	28-Sep-01	51.1700	-10.88%	
14	31-Oct-01	58.1500	12.79%	
15	30-Nov-01	64.2100	9.91%	
16	31-Dec-01	66.2500	3.13%	
17				
18	**Monthly return statistics**			
19	Average return		3.53%	<-- =AVERAGE(C5:C16)
20	Return variance		1.91%	<-- =VARP(C5:C16)
21	Return standard deviation		13.81%	<-- =STDEVP(C5:C16)
22				
23	**Annualized return statistics**			
24	Average return		42.36%	<-- =12*C19
25	Return variance		22.88%	<-- =12*C20
26	Return standard deviation		47.84%	<-- =SQRT(C25)

The standard deviation required for the Black-Scholes formula is 47.84%--the annualized standard deviation. Notice that since

$$\begin{aligned} \textit{annual variance} &= 12 * \textit{monthly variance} \\ \textit{annual standard deviation} &= \sqrt{12 * \textit{monthly variance}} \\ &= \sqrt{12} * \textit{monthly standard deviation} \end{aligned}$$

In general, if we're calculating from non-annual data:

$$\begin{aligned} &\sigma, \textit{ annual standard deviation} \\ &\qquad \sqrt{12} * \textit{monthly standard deviation} \\ &\qquad \sqrt{52} * \textit{weekly standard deviation} \\ &\qquad \sqrt{260} * \textit{daily standard deviation} \end{aligned}$$

(The last calculation may be a bit confusing—since there are 52 weeks per year and 5 business days per week, many traders assume that there are 260 business days per year. However, others use 250 and 365.)

Continuous versus discrete returns—a reminder

The Black-Scholes formula uses *continuously compounded* returns, whereas in most of this book we use *discretely compounded returns*. We discussed the difference between these two concepts in Chapter 2. Suppose you have an investment which is worth P_t at time t and worth P_{t+1} one period later. There are two ways to define the return on the investment. The *discrete return* is $r_t^{discrete} = \frac{P_{t+1}}{P_t} - 1$, and the *continuously compounded return* is $r_t^{continuous} = \ln\left(\frac{P_{t+1}}{P_t}\right)$. The example below shows the difference:

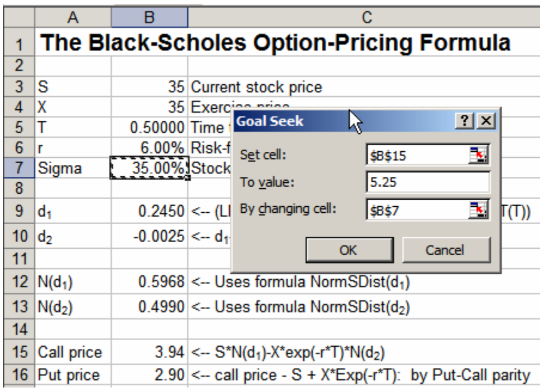
	A	B	C
1	**DISCRETE VERSUS CONTINUOUS RETURNS**		
2			
3	**Computing the returns from prices**		
4	P_t	100	
5	P_{t+1}	120	
6			
7	Discrete return	20.00%	<-- =B5/B4-1
8	Continosly-compounded return	18.23%	<-- =LN(B5/B4)
9			
10	**Computing the future price from the returns**		
11	Annual return, *r*	12%	
12	Period over which you get the return (in years)	0.25	
13			
14	Initial investment P_t	100	
15	Future value P_{t+1}		
16	If *r* is the annual *discrete* return	102.8737	<-- =B14*(1+B11)^B12
17	If *r* is the annual *continuous* return	103.0455	<-- =B14*EXP(B11*B12)

24.4. Calculating the implied volatility from option prices

When we calculate the implied volatility from option prices, we use the Black-Scholes formula to *find the σ which gives a specific options price*. Suppose, for example, that a share of ABC Corp. is currently selling for $35, and that a 6-month at-the-money call option on ABC Corp. is selling for $12. Suppose the interest rate is 6%. The spreadsheet below shows that σ must be greater than 35% (since the call prices increases with σ, and since σ = 35% gives a call price of $3.94, we'll have to make σ larger to get a call price of $5.25):

	A	B	C	D	E	F	G
1	**The Black-Scholes Option-Pricing Formula**						
2							
3	S	35	Current stock price				
4	X	35	Exercise price				
5	T	0.50000	Time to maturity of option (in years)				
6	r	6.00%	Risk-free rate of interest				
7	Sigma	35.00%	Stock volatility				
8							
9	d_1	0.2450	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))				
10	d_2	-0.0025	<-- d_1-sigma*SQRT(T)				
11							
12	$N(d_1)$	0.5968	<-- Uses formula NormSDist(d_1)				
13	$N(d_2)$	0.4990	<-- Uses formula NormSDist(d_2)				
14							
15	Call price	3.94	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)				

Using **Goal Seek**, we can compute the σ which gives the market price; it turns out to be σ = 48.71%. Here's the **Goal Seek** dialog box:



And here's the final result:

	A	B	C	D	E	F	G
1	**The Black-Scholes Option-Pricing Formula**						
2							
3	S	35	Current stock price				
4	X	35	Exercise price				
5	T	0.50000	Time to maturity of option (in years)				
6	r	6.00%	Risk-free rate of interest				
7	Sigma	48.71%	Stock volatility				
8							
9	d_1	0.2593	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))				
10	d_2	-0.0851	<-- d_1-sigma*SQRT(T)				
11							
12	$N(d_1)$	0.6023	<-- Uses formula NormSDist(d_1)				
13	$N(d_2)$	0.4661	<-- Uses formula NormSDist(d_2)				
14							
15	Call price	5.25	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)				

What's used in practice—implied σ or σ from historical prices?

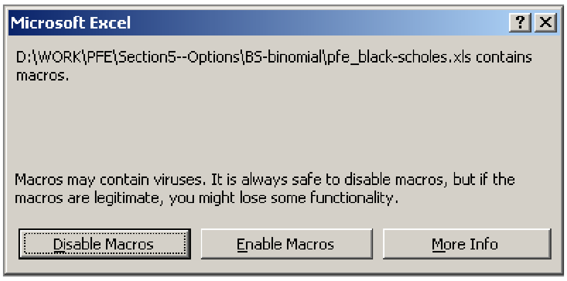
The answer is a bit of both. Smart traders compare the implied volatility with the historical volatility and try to form estimates of what the stock volatility actually is. There are whole websites devoted to this subject, and lots of proprietary software. Our own favorite (and, as of the writing of this book, still free) website is Option Metrics (http://www.impliedvol.com/).

24.5. An Excel Black-Scholes function

The spreadsheet which comes with this chapter includes two Excel functions which compute the Black-Scholes call and put prices. These functions are not part of the original Excel package; they have been defined by the author. Here's an example of how to use them:

	A	B	C	D
1	**BLACK-SCHOLES OPTION FUNCTIONS**			
2	The functions in this spreadsheet--**Calloption** and **Putoption**--were			
3	defined by the author; they are part of this spreadsheet.			
4				
5	S	100	Current stock price	
6	X	90	Exercise price	
7	T	0.50000	Time to maturity of option (in years)	
8	r	4.00%	Risk-free rate of interest	
9	Sigma	35%	Stock volatility	
10				
11	Call price	16.32	<-- =calloption(B5,B6,B7,B8,B9)	
12	Put price	4.53	<-- =putoption(B5,B6,B7,B8,B9)	

The function **Calloption(stock price, exercise price, time to maturity, interest, sigma)** is a defined macro which is attached to the spreadsheet.[3] When you first open the spreadsheet Excel will display the following message, which asks if you really want to open this macro. In this case the correct answer is **Enable macros**.



An implied volatility function

The spreadsheet also comes with two functions which compute the implied volatility for a call and a put option. The function **CallVolatility(stock price, exercise price, option maturity, interest rate, target)** calculates the σ which gives the Black-Scholes price given the

[3] As you can see in the spreadsheet, **putoption** has the same syntax.

other parameters. The spreadsheet also includes a function called **PutVolatility** which computes the implied volatility for a put option.[4] Both functions are illustrated below:

	A	B	C
1	**TWO IMPLIED VOLATILITY FUNCTIONS**		
2			
3	**Using CallVolatility to compute the implied volatility for a call**		
4	S	35	Current stock price
5	X	35	Exercise price
6	T	0.50000	Time to maturity of option (in years)
7	r	6.00%	Risk-free rate of interest
8	Target	5.25	<-- This is the current call price we want to match
9			
10	Implied volatility	48.71%	<-- =CallVolatility(B4,B5,B6,B7,B8)
11			
12	**Using PutVolatility to compute the implied volatility for a call**		
13	S	35	Current stock price
14	X	35	Exercise price
15	T	1.00000	Time to maturity of option (in years)
16	r	6.00%	Risk-free rate of interest
17	Target	3.44	<-- This is the current put price we want to match
18			
19	Implied volatility	32.49%	<-- =putVolatility(B13,B14,B15,B16,B17)

24.6. Doing sensitivity analysis

We can use Excel to do a lot of Black-Scholes sensitivity analysis. In this section we give two examples, leaving other examples for the chapter exercises.

Example 1: The sensitivity of the Black-Scholes call price to the stock price

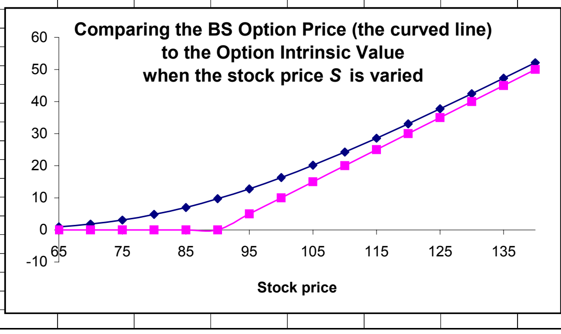
[4] In the spirit of this chapter, we do not explain how these functions work. For details see my book *Financial Modeling*.

For example, the following **Data|Table** (see Chapter ???) gives—as the stock price *S* varies—the Black-Scholes value of the call compared to its intrinsic value (i.e., *max(S-X,0)*) . Note that we have not shown the header of the data table.

	A	B	C	D	E
1	**BLACK-SCHOLES OPTION FUNCTIONS**				
2	The functions in this spreadsheet--**Calloption** and **Putoption**--were				
3	defined by the author; they are part of this spreadsheet.				
4					
5	S	100	Current stock price		
6	X	90	Exercise price		
7	T	0.50000	Time to maturity of option (in years)		
8	r	4.00%	Risk-free rate of interest		
9	Sigma	35%	Stock volatility		
10					
11	Call price	16.32	<-- =calloption(B5,B6,B7,B8,B9)		
12	Put price	4.53	<-- =putoption(B5,B6,B7,B8,B9)		
13					
14					
15			Stock price	BS call price	Option intrinsic value
16				16.32	10
17			65	0.974881	0
18			70	1.823552	0
19			75	3.084287	0
20			80	4.808631	0
21			85	7.015908	0
22			90	9.695162	0
23			95	12.81164	5
24			100	16.31546	10
25			105	20.14963	15
26			110	24.25671	20
27			115	28.58313	25
28			120	33.08179	30
29			125	37.713	35
30			130	42.4445	40
31			135	47.25076	45
32			140	52.11206	50
33					
34					
35					

This cell is part of the data table header; it contains the formula **=B11.**

This cell is part of the data table header. It contains a formula **=MAX(B5-B6,0)** which the option's intrinsic value.



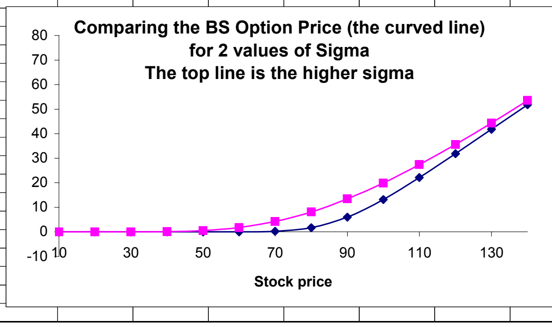
Example 2: The sensitivity of the Black-Scholes price to different estimates of σ

Here's the sensitivity analysis of the Black-Scholes price to the σ.

	A	B	C	D	E
1	**BLACK-SCHOLES SENSITIVITY ON SIGMA**				
2					
3	S	100	Current stock price		
4	X	90	Exercise price		
5	T	0.50000	Time to maturity of option (in years)		
6	r	4.00%	Risk-free rate of interest		
7					
8					
9					
10					
11					
12					
13			**Stock price**	**BS call price**	**Option intrinsic value**
14				13.15	19.90771512
15			10	0.00	0.00
16			20	0.00	0.00
17			30	0.00	0.01
18			40	0.00	0.09
19			50	0.00	0.53
20			60	0.01	1.78
21			70	0.24	4.25
22			80	1.72	8.14
23			90	5.96	13.41
24			100	13.15	19.91
25			110	22.14	27.38
26			120	31.86	35.60
27			130	41.80	44.37
28			140	51.78	53.53
29			150	61.78	62.96
30			160	71.78	72.57
31					
32					
33					

This cell is part of the data table header; it contains the formula **=calloption(B5,B6,B7,B8,20%).**

This cell is part of the data table header. It contains a formula **=calloption(B5,B6,B7,B8,50%)**.



24.7. Does the Black-Scholes model work? Applying it to Microsoft options

To examine whether and how well the Black-Scholes model works, we do two experiments in this section. First we compare the Black-Scholes option prices for a set of put and call options on Microsoft stock to the actual market prices. Then we compare the implied volatilities for the same options.

Our conclusion: Black and Scholes works "pretty well." That's a big complement for a financial model!

Comparing actual market prices to Black-Scholes prices

The experiment we run here looks at options on Microsoft stock.

- On 8 February 2002 we look at the call and put options on Microsoft stock which expire in July 2002.

- We calculate the Black-Scholes price of these options and compare it to the actual market price.

As you will see, our conclusion is that the Black-Scholes model works pretty well.

We get our data from Yahoo, which allows us to look up the stock price of Microsoft on 8 February 2002 and also look up the prices of Microsoft options.

Symbol	Last Trade		Change		Volume
MSFT	Feb 8	**60.65**	+0.85	+1.42%	30,642,600

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Profile, Reports, Research, SEC Filings, Upgrades, **more...**

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The closing stock price of Microsoft stock on 8 February 2002 was $60.65. The stock was up 1.42% from the previous day's close, and the total volume of stock traded was 30,642,600 shares.

We now look at the closing prices of options on Microsoft stock which expire in July 2002. Clicking on options in the above box leads us to the option prices:

Expires After: Fri 19-Jul-02 — Options Center | Analyzer NEW! | Most Actives | Symbology | Calendar

Options: Feb-02 | Mar-02 | Apr-02 | **Jul-02** | Jan-03 | Jan-04 — Highlighted options are in-the-money

Calls							Strike Price	Puts						
Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int		Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int
MQFGE.X	**35.10**	0.00	35.70	36.10	0	166	**25**	MQFSE.X	**0.05**	0.00	0.00	0.15	0	70
MQFGF.X	**30.20**	0.00	30.80	31.20	0	99	**30**	MQFSF.X	**0.15**	0.00	0.00	0.15	0	171
MQFGG.X	**25.40**	0.00	25.90	26.30	0	53	**35**	MQFSG.X	**0.30**	0.00	0.15	0.30	0	1,644
MQFGH.X	**20.80**	0.00	21.20	21.60	0	161	**40**	MQFSH.X	**0.55**	0.00	0.40	0.55	0	451
MQFGI.X	**15.70**	-0.70	16.80	17.20	11	829	**45**	MQFSI.X	**1.00**	0.00	0.90	1.05	23	15,930
MSQGJ.X	**12.30**	0.00	12.70	13.10	3	1,086	**50**	MSQSJ.X	**2.00**	0.00	1.75	1.95	1,040	19,919
MSQGK.X	**8.70**	0.00	9.10	9.40	0	400	**55**	MSQSK.X	**3.30**	0.00	3.00	3.30	67	8,163
MSQGL.X	**5.60**	-0.20	6.00	6.30	81	2,444	**60**	MSQSL.X	**5.40**	0.00	5.00	5.30	39	14,609
MSQGM.X	**3.80**	+0.20	3.70	4.00	88	9,474	**65**	MSQSM.X	**8.30**	+0.10	7.60	7.90	32	8,441
MSQGN.X	**2.15**	+0.05	2.10	2.40	1,125	18,565	**70**	MSQSN.X	**12.30**	+0.70	11.00	11.40	15	9,112
MSQGO.X	**1.10**	0.00	1.10	1.30	86	19,073	**75**	MSQSO.X	**15.70**	0.00	14.90	15.30	0	692
MSQGP.X	**0.60**	0.00	0.50	0.65	35	14,770	**80**	MSQSP.X	**20.30**	0.00	19.40	19.80	0	757
MSQGQ.X	**0.35**	+0.10	0.20	0.35	23	8,636	**85**	MSQSQ.X	**25.10**	0.00	24.20	24.60	0	385
MSQGR.X	**0.20**	0.00	0.10	0.25	0	2,684	**90**	MSQSR.X	**30.10**	0.00	29.20	29.60	0	657
MSQGS.X	**0.15**	0.00	0.05	0.15	0	3,203	**95**	MSQSS.X	**35.10**	0.00	34.20	34.60	0	286
MSQGT.X	**0.10**	0.00	0.00	0.15	0	698	**100**	MSQST.X	**40.10**	0.00	39.20	39.60	0	6
MSQGA.X	**0.15**	0.00	0.00	0.15	0	306	**105**	MSQSA.X	**45.10**	0.00	44.20	44.60	0	10

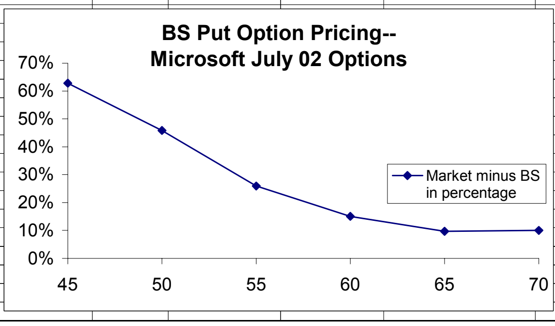
Look carefully at the above box:

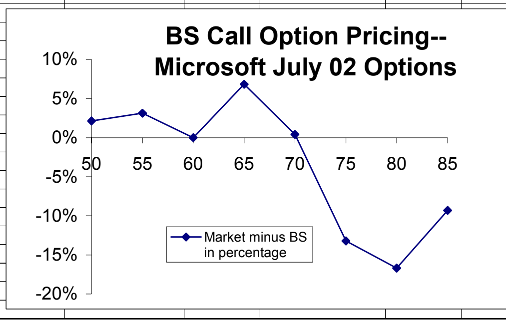
- Not all the options were traded on 8 February. For example—there was no "volume" (and hence no trading) of either calls or puts with exercise price ("strike price") of 25.
- Significant amounts of call options traded on 8 February were only for exercise prices *X=60, 65, 70, 75, 80, 85*. Significant amounts of put options traded were only for exercise prices *X = 45, 50, 55, 60, 65, 70*.
- The price of the "last trade" is in bold face black. But where there is no volume for this day, the price refers to a previous day's trading.

In the spreadsheet below we look at the Microsoft July options which actually traded on 8 February and compare the Black-Scholes price to the actual market price. We use the 6-month Treasury bill rate of 1.7% as our risk-free rate.[5]

[5] We computed MSFT's volatility by using **Goal Seek** to find σ such that the difference between the market price and the Black-Scholes price of the at-the-money call is zero.

	A	B	C	D	E	F	G	H
1	**MICROSOFT OPTIONS: Comparing BS to actual prices**							
2	This spreadsheet computes the Black-Scholes value of the Microsoft							
3	July 02 options on 8 February 2002 and compares							
4	the prices to the actual market prices. As you can							
5	see, the Black-Scholes formula works pretty well!							
6						**Computing the time to maturity**		
7	S	60.65	Microsoft stock, closing price 8 Feb 02			Current date	8-Feb-02	
8	T	0.35890	Time to maturity of option (in years)			Expiration date	19-Jun-02	
9	r	1.70%	Risk-free rate of interest			Time (days)	131	<-- =G8-G7
10	Sigma	35.38%	Stock volatility			Time (% of year)	0.3589	<-- =G9/365
11								
12								
13		**Exercise price**	**BS call price**	**Actual call market price**	**Market minus BS in dollars**	**Market minus BS in percentage**		
14		50	12.04	12.30	0.26	2.14%	<-- =(D14-C14)/D14	
15		55	8.43	8.70	0.27	3.11%	<-- =(D15-C15)/D15	
16		60	5.60	5.60	0.00	0.00%		
17		65	3.54	3.80	0.26	6.82%		
18		70	2.14	2.15	0.01	0.40%		
19		75	1.25	1.10	-0.15	-13.20%		
20		80	0.70	0.60	-0.10	-16.69%		
21		85	0.38	0.35	-0.03	-9.30%		
22								
23								
24								
25		**Exercise price**	**BS put price**	**Actual put market price**	**Market minus BS in dollars**	**Market minus BS in percentage**		
26		45	0.37	1.00	0.63	62.83%	<-- =(D26-C26)/D26	
27		50	1.08	2.00	0.92	45.88%		
28		55	2.44	3.30	0.86	25.92%		
29		60	4.59	5.40	0.81	15.09%		
30		65	7.50	8.30	0.80	9.69%		
31		70	11.07	12.30	1.23	10.04%		





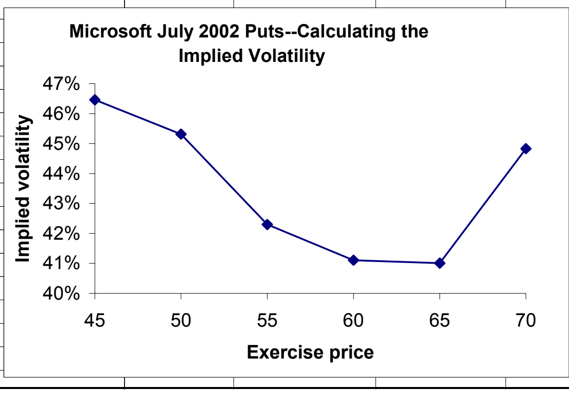
The pattern of the prices is interesting:

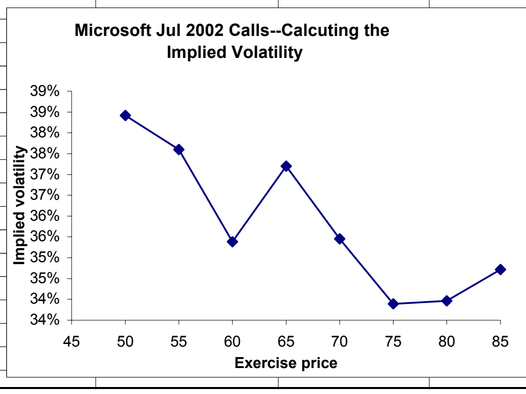
- In fact the BS model does a remarkably good job of pricing the calls.
- There appears to be a much bigger bias in the put prices. Investors appear to price *low exercise* puts at more than the Black-Scholes price. This phenomenon is often seen in markets—it apparently stems from investor demand for puts as insurance. Having said this, however, the market prices and the Black-Scholes prices show a remarkable convergence.

Does the Black-Scholes model work? Looking at implied volatilities

This is our second experiment. We take the Microsoft data above calculate the implied volatility for each option (using the functions **CallVolatility** and **PutVolatility** discussed in Section ????). Here's our spreadsheet:

	A	B	C	D	E	F	G	H
1	**MICROSOFT OPTIONS: Computing the implied volatilities**							
2	This spreadsheet computes the implied volatility of the Microsoft							
3	July 02 options on 8 February 2002 and compares							
4	The average volatility of the calls appears to be lower							
5	than the average implied volatility of the puts							
6						**Computing the time to maturity**		
7	S	60.65	Microsoft stock, closing price 8 Feb 02			Current date	8-Feb-02	
8	T	0.35890	Time to maturity of option (in years)			Expiration date	19-Jun-02	
9	r	1.70%	Risk-free rate of interest			Time (days)	131	<-- =G8-G7
10						Time (% of year)	0.3589	<-- =G9/365
11								
12								
13		**Exercise price**	**Actual call market price**	**Implied volatility**				
14		50	12.30	38.42%	<-- =CallVolatility($B$7,B14,$B$8,$B$9,C14)			
15		55	8.70	37.60%	<-- =CallVolatility($B$7,B15,$B$8,$B$9,C15)			
16		60	5.60	35.38%				
17		65	3.80	37.20%				
18		70	2.15	35.45%				
19		75	1.10	33.89%				
20		80	0.60	33.96%				
21		85	0.35	34.72%				
22								
23								
24								
25		**Exercise price**	**Actual put market price**	**Implied volatility**				
26		45	1.00	46.46%	<-- =putVolatility($B$7,B26,$B$8,$B$9,C26)			
27		50	2.00	45.32%	<-- =putVolatility($B$7,B27,$B$8,$B$9,C27)			
28		55	3.30	42.30%	<-- =putVolatility($B$7,B28,$B$8,$B$9,C28)			
29		60	5.40	41.10%				
30		65	8.30	41.01%				
31		70	12.30	44.83%				
32								





The results are both encouraging and discouraging:

- The implied volatilities for the calls are pretty close together, as are the implied volatilities for the puts. This is good news.

- On the other hand the implied volatilities for the puts are uniformly larger than the implied volatilities for the calls. This is strange, since in the Black-Scholes formulation, the implied volatility refers to the volatility of the stock's return and hence has nothing to do with whether we're discussing a put or a call option.
- On the third hand,[6] the actual difference between the implied volatilities for the calls and the puts is not that great (only about 6%).

This is not the place to summarize the vast finance literature on implied volatilities. For our purposes, the Black-Scholes model works pretty well. That's enough!

Summary

This chapter has given you a quick and hopefully practical insight into how to use the Black-Scholes model. Of all the financial models developed in the past 50 years, this model works best. It is remarkably good at pricing options and is widely used. It is also easy to use, provided you don't get too hung up on the details of where the formula comes from (in this chapter we've left these hang-ups behind us, and concentrated exclusively on implementational details).

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[6] Harry Truman is reported to have gotten so sick of hearing economists say "On the hand, ... But on the other hand, ... " that he asked his chief of staff to get him a "one-handed economist." History does not record if he succeeded. The economist in this section's bullets has at least 3 hands. Harry Truman would not have liked him.

Exercises

1. Use the Black-Scholes model to price the following:

   - A call option on a stock whose current price is 50, with exercise price $X = 50$, $T = 0.5$, $r = 10\%$, $\sigma = 25\%$.
   - A put option with the same parameters.

2. Use the data from exercise 1 and **Data|Table** to produce graphs that show:

   - The sensitivity of the Black-Scholes call price to changes in the initial stock price S.
   - The sensitivity of the Black-Scholes put price to changes in σ.
   - The sensitivity of the Black-Scholes call price to changes in the time to maturity T.
   - The sensitivity of the Black-Scholes call price to changes in the interest rate r.
   - The sensitivity of the put price to changes in the exercise price X.

3. Produce a graph comparing a call's *intrinsic value* (defined as $Max(S\text{-}X,0)$) and its Black-Scholes price. From this graph you should be able to deduce that it is never optimal to exercise early a call priced by the Black-Scholes.

4. Produce a graph comparing a put's intrinsic value (= $Max(X\text{-}S,0)$) and its Black-Scholes price. From this graph you should be able to deduce that it is may be optimal to exercise early a put priced by the Black-Scholes formula.

6. Use the Excel **Solver** to find the stock price for which there is the maximum difference between the Black-Scholes call option price and the option's intrinsic value. Use the following values: $S = 45, X = 45, T = 1, \sigma = 40\%, r = 8\%$.

Repeat the MSFT exercise in the text for the March 2002 options:

Expires After: Fri 15-Mar-02 Options Center | Analyzer NEW! | Most Actives | Symbology | Calendar

Options: Feb-02 | **Mar-02** | Apr-02 | Jul-02 | Jan-03 | Jan-04 Highlighted options are in-the-money

Calls							Strike Price	Puts						
Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int		Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int
MQFCF.X	**29.30**	-0.70	30.50	30.90	15	11	**30**	MQFOF.X	**0.05**	0.00	0.00	0.10	15	10
MQFCG.X	**25.00**	0.00	25.50	25.90	0	143	**35**	MQFOG.X	**0.00**	0.00	0.00	0.10	0	0
MQFCH.X	**20.00**	0.00	20.50	20.90	0	0	**40**	MQFOH.X	**0.00**	0.00	0.00	0.10	0	20
MQFCI.X	**15.10**	0.00	15.60	16.00	0	455	**45**	MQFOI.X	**0.05**	0.00	0.05	0.15	0	16
MSQCJ.X	**9.80**	-0.60	10.80	11.20	26	1,325	**50**	MSQOJ.X	**0.50**	+0.20	0.20	0.35	60	776
MSQCK.X	**5.50**	-0.60	6.40	6.70	2	449	**55**	MSQOK.X	**0.95**	-0.15	0.80	0.95	311	4,845
MSQCL.X	**3.00**	+0.25	3.00	3.20	300	2,138	**60**	MSQOL.X	**2.55**	-0.35	2.25	2.45	188	20,754
MSQCM.X	**0.95**	+0.05	0.90	1.00	2,155	19,806	**65**	MSQOM.X	**5.70**	-0.20	5.10	5.40	171	8,376
MSQCN.X	**0.20**	-0.10	0.15	0.30	114	11,777	**70**	MSQON.X	**10.00**	-0.20	9.40	9.70	20	1,014
MSQCO.X	**0.15**	0.00	0.00	0.10	0	6,643	**75**	MSQOO.X	**15.10**	0.00	14.20	14.60	0	594
MSQCP.X	**0.00**	0.00	0.00	0.15	0	1,209	**80**	MSQOP.X	**20.10**	0.00	19.20	19.60	0	2

9. Note that you can use the Black-Scholes formula to calculate the call option premium as a percentage of the exercise price in terms of S/X:

$$C = SN(d_1) - Xe^{-rT}N(d_2) \Rightarrow \frac{C}{X} = \frac{S}{X}N(d_1) - e^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Implement this in a spreadsheet.

10. Note that you can also calculate the Black-Scholes put option premium as a percentage of the exercise price in terms of S/X:

$$P = -SN\left(-d_1\right) + Xe^{-rT}N\left(-d_2\right) \Rightarrow \frac{P}{X} = e^{-rT}N\left(-d_2\right) - \frac{S}{X}N\left(-d_1\right)$$

where

$$d_1 = \frac{\ln(S/X) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Implement this in a spreadsheet. Find the ratio of S/X for which C/X and P/X cross when $T = 0.5$, $\sigma = 25\%$, $r = 10\%$. (You can use a graph or you can use Excel's Solver.) Note that this crossing point is affected by the interest rate and the option maturity, but not by σ.

CHAPTER 25: OPTION PRICING—THE BLACK-SCHOLES FORMULA*

Current version: July 18, 2004

Chapter contents

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* This is a preliminary draft of a chapter of *Principles of Finance with Excel*. (benninga@wharton.upenn.edu).

Overview

In the two previous chapters on option pricing, we've discussed some facts about options, but we haven't discussed *how to determine the price of an option*. In this chapter we show how to price options using the Black-Scholes formula. The Black-Scholes formula is the most important option pricing formula. The formula is in wide use in options markets. It has also achieved a certain degree of notoriety, in the sense that even non finance people (lawyers, accountants, judges, bankers . . .) know that options are priced using Black-Scholes. They may not know how to apply it, and they certainly wouldn't know why the formula is correct, but they know that it is used to price options.

In our discussion of the Black-Scholes model, we'll make no attempt whatsoever to give a theoretical background to the model. It's hopeless, unless you know a lot more math than 99% of all beginning finance students will ever know.[1]

The next chapter discusses the other major model for pricing options, the *binomial option pricing model*. The binomial model gives some insights into how to price an option, and it's also used widely (though not as widely as the Black-Scholes equation). Most books discuss the binomial model—which, in a theoretical sense, underlies the Black-Scholes formula—first and then discuss Black-Scholes. However, since we have no intention of making the theoretical connection between the binomial model and Black-Scholes, we've chosen to reverse the order and deal with the more important model first.

[1] A bitter truth, perhaps. But get this—your professor probably can't prove the Black-Scholes equation either (don't ask him, he'll be embarrassed). On the other hand—you know how to drive a car but may not know how an internal combustion engine works, you know how to use a computer but can't make a central processing unit chip, ...

What does "pricing an option" mean?

Suppose we're discussing a call option on Microsoft stock which is sold on 8 February 2002. On this date, Microsoft's stock price is S_0 = \$60.65. Suppose that the call option has an exercise price X = *\$60* and expires on July 19, 2002. Here's what you've learned so far:

- From Chapter 22, you know the basic option terminology. You know what an exercise price X is, you know the difference between a call and a put, etc. Using Excel you can also compute the time T to option expiration:

$$T = \frac{\begin{array}{c} Days\ between\ 8\ Feb\ 2002 \\ and\ 19\ July\ 2002 \end{array}}{Number\ of\ \ days\ in\ 2002} = \frac{161}{365} = 0.4411 \ .$$

- From Chapter 22, you also know what the *payoff pattern* and *profit pattern* of the call option looks like—by itself and in combination with other assets
- From Chapter 23, you know that there are some pricing *restrictions* on the call option. A simple restriction ("Fact 1" from Chapter 23) says that $Call > Max\left[S_0 - PV\left(X\right), 0\right]$. A more sophisticated restriction ("Fact 3," put-call parity) says that—once we know the price of Microsoft stock, the call price, and the interest rate—the put price is determined.

All of these facts are –by themselves—interesting. However, they don't tell us what the *price* of the call option should be. This is the subject of this chapter—the Black-Scholes formula tells us how what the market price of the option should be.

Finance concepts in this chapter

- Black-Scholes formula
- Put-call parity

- Stock price volatility
- Implied volatility
- Real options

Excel functions used

- **Exp**
- **Ln**
- **Stdevp**
- **Varp**
- Data Table

25.1. The Black-Scholes Model

In 1973, Fisher Black and Myron Scholes proved a formula for pricing European call and put options on non-dividend-paying stocks. Their model is probably the most famous model of modern finance.[2] The Black-Scholes model uses the following formula to price calls on the stock:

[2] The 1997 Nobel Prize for Economics was awarded to Myron Scholes and Robert Merton for their role in developing the option pricing formula. Fisher Black, who died in 1995, would have undoubtedly shared in the prize had he still been alive.

$$C = S_0 N(d_1) - Xe^{-rT} N(d_2),$$
$$\text{where}$$
$$d_1 = \frac{\ln(S_0 / X) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Don't let this formula frighten you! We're going to show you how to use Excel to implement the Black-Scholes formula, and you won't really have to understand the mechanics or the math (look back at footnote 1 for an explanation). However, if you want some explanations: C denotes the price of a call, S_0 is the current price of the underlying stock, X is the exercise price of the call, T is the call's time to exercise, r is the interest rate, and σ is the standard deviation of the stock's return. *N( )* denotes a value of the standard normal distribution. It is assumed that the stock will pay no dividends before date T.

The spreadsheet below prices an option on a stock whose current price is $S_0 = 100$. The option's exercise price is $X = 90$ and its time to maturity is $T = 0.5$ (one-half year). The interest rate is $r=4\%$, and sigma (σ, the stock's volatility—a measure of the stock's riskiness; more about this later) is $\sigma = 35\%$.

	A	B	C
1	**The Black-Scholes Option-Pricing Formula**		
2	S	100	Current stock price
3	X	90	Exercise price
4	T	0.50000	Time to maturity of option (in years)
5	r	4.00%	Risk-free rate of interest
6	Sigma	35%	Stock volatility
7			
8	d_1	0.6303	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))
9	d_2	0.3828	<-- d_1-sigma*SQRT(T)
10			
11	N(d_1)	0.7357	<-- Uses formula NormSDist(d_1)
12	N(d_2)	0.6491	<-- Uses formula NormSDist(d_2)
13			
14	Call price	16.32	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)
15	Put price	4.53	<-- call price - S + X*Exp(-r*T): by Put-Call parity
16		4.53	<-- X*exp(-r*T)*N(-d_2) - S*N(-d_1): direct formula

By the put-call parity theorem (see Chapter 23, page000), a put with the same exercise date T and exercise price X written on the same stock will have price $P = C - S_0 + Xe^{-rT}$. We've used this formula in cell B15. Cell B16 includes another version of put pricing—a direct formula which follows from the Black-Scholes formula.

25.2. What do the Black-Scholes parameters mean? How to calculate them?

The Black-Scholes option pricing model depends on 5 parameters:

- S_0, the *current price of the stock*. By this we always mean the stock price on the date we're calculating the option price.
- X, the *exercise price of the option* (this is also called the *strike price*).
- T, the *time to the option's expiration* (sometimes called the *option maturity*). In the Black-Scholes formula, T is always given in *annual terms*—meaning: an option with 3 months to expiration has $T = 0.25$, an option with 51 days until expiration has $T = \frac{51}{365} = 0.1397$).
- r, the *risk-free interest rate*. This is also given in annual terms. Meaning: If the interest rate is 6% per year and if an option has $T = 0.25$, then we write $r=6\%$ in the Black-Scholes formula. We usually use the Treasury bill rate for a maturity which is closest to the option maturity.
- σ *("sigma")* is a measure of the *riskiness of the stock*. Sigma is an important variable in determining the option price, and it is not a simple concept to explain. We discuss it at

length in Sections 25.3 and 25.4. However, here are some facts to help you get your bearings on sigma:

- If the stock is riskless, then $\sigma = 0\%$. A stock is riskless if its future price is completely predictable.
- An "average" U.S. stock has σ of between 10% and 25%
- A risky stock may have a σ of as much as 80% or 100 %.

25.3. Historical volatility: Computing σ from stock prices

There are two main ways to compute the sigma: We can either calculate the sigma by looking at the series of past stock prices. This computation is sometimes called the *historical sigma* or the *historical volatility*. Alternatively, we can calculate the *implied sigma* by looking at options prices; this calculation is often called the *implied volatility*. This section describes the computation of the historical volatility, and the next section describes how to compute the implied volatility.

Below we show the annual prices for Microsoft for the decade from 1991 - 2001. Column C shows the *continuously compounded return* for the prices: $r_t^{continuous} = \ln\left(\frac{P_t}{P_{t-1}}\right)$. Sigma is the standard deviation of these annual returns (cell C18). As you can see, the σ computed from these prices is $\sigma = 36.90\%$:

	A	B	C	D
1	**MICROSOFT STOCK PRICES--ANNUAL DATA**			
2	**Date**	**Closing stock price**	**Return**	
3	31-Dec-90	2.7257		
4	31-Dec-91	5.0104	60.88%	<-- =LN(B4/B3)
5	31-Dec-92	5.4062	7.60%	
6	31-Dec-93	5.3203	-1.60%	
7	31-Dec-94	7.4219	33.29%	
8	31-Dec-95	11.5625	44.33%	
9	31-Dec-96	25.5000	79.09%	
10	31-Dec-97	37.2969	38.02%	
11	31-Dec-98	87.5000	85.27%	
12	31-Dec-99	97.8750	11.21%	
13	31-Dec-00	61.0625	-47.18%	
14	31-Dec-01	66.2500	8.15%	
15				
16	Average return		29.01%	<-- =AVERAGE(C4:C14)
17	Return variance		13.61%	<-- =VARP(C4:C14)
18	Return standard deviation		36.90%	<-- =STDEVP(C4:C14)

In the world of option pricing it is not usual to compute σ from annual data. Most traders prefer daily, weekly, or monthly data. The use of non-annual data requires some adjustment to the calculations. We show these adjustments in the example below, where we calculate Microsoft's σ from monthly data; a discussion of what we did follows the spreadsheet:

	A	B	C	D
1	**MICROSOFT STOCK PRICES MONTHLY DATA FOR 2001**			
2	**Date**	**Close**		
3	29-Dec-00	43.3750		
4	31-Jan-01	61.0630	34.20%	<-- =LN(B4/B3)
5	28-Feb-01	59.0000	-3.44%	<-- =LN(B5/B4)
6	30-Mar-01	54.6880	-7.59%	<-- =LN(B6/B5)
7	30-Apr-01	67.7500	21.42%	
8	31-May-01	69.1800	2.09%	
9	29-Jun-01	73.0000	5.37%	
10	31-Jul-01	66.1900	-9.79%	
11	31-Aug-01	57.0500	-14.86%	
12	28-Sep-01	51.1700	-10.88%	
13	31-Oct-01	58.1500	12.79%	
14	30-Nov-01	64.2100	9.91%	
15	31-Dec-01	66.2500	3.13%	
16				
17	**Monthly return statistics**			
18	Average return		3.53%	<-- =AVERAGE(C4:C15)
19	Return variance		1.91%	<-- =VARP(C4:C15)
20	Return standard deviation		13.81%	<-- =STDEVP(C4:C15)
21				
22	**Annualized return statistics**			
23	Average return		42.36%	<-- =12*C18
24	Return variance		22.88%	<-- =12*C19
25	Return standard deviation		47.84%	<-- =SQRT(C24)

The standard deviation of the monthly returns is 13.81% (cell C20). The annualized standard deviation required for the Black-Scholes formula is 47.84% (cell C25). Notice that since

$$\begin{aligned} annual\ variance &= 12 * monthly\ variance \\ annual\ standard\ deviation &= \sqrt{12 * monthly\ variance} \\ &= \sqrt{12} * monthly\ standard\ deviation \end{aligned}$$

In general, if we're calculating from non-annual data:

$$\begin{aligned} &\sigma,\ annual\ standard\ deviation = \\ &\quad \sqrt{12} * monthly\ standard\ deviation \\ &\quad \sqrt{52} * weekly\ standard\ deviation \\ &\quad \sqrt{260} * daily\ standard\ deviation \end{aligned}$$

(The use of 260 in calculating the annualized σ from weekly data may be a bit confusing: Since there are 52 weeks per year and 5 business days per week, many traders assume that there are 260 business days per year. However, others use 250 and 365.)

Continuous versus discrete returns—a reminder

The Black-Scholes formula uses *continuously compounded* returns, whereas in most of this book we use *discretely compounded returns*. We discussed the difference between these two concepts in Chapter 6. Suppose you have an investment which is worth P_t at time t and worth P_{t+1} one period later. There are two ways to define the return on the investment. The *discrete return* is $r_t^{discrete} = \frac{P_{t+1}}{P_t} - 1$, and the *continuously compounded return* is $r_t^{continuous} = \ln\left(\frac{P_{t+1}}{P_t}\right)$. The example below shows the difference:

	A	B	C
1	**DISCRETE VERSUS CONTINUOUS RETURNS**		
2	**Computing the returns from prices**		
3	P_t	100	
4	P_{t+1}	120	
5			
6	Discrete return	20.00%	<-- =B4/B3-1
7	Continously-compounded return	18.23%	<-- =LN(B4/B3)

25.4. Implied volatility: Calculating σ from option prices

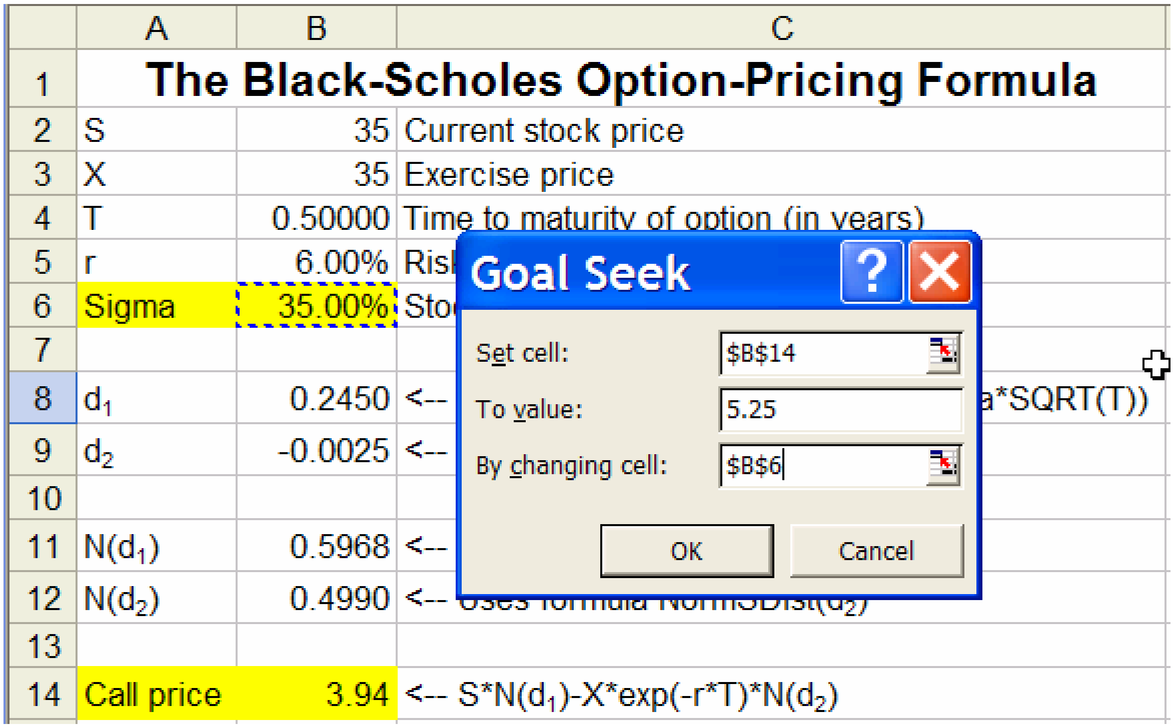
In the previous section we computed the annualized standard deviation of returns σ from historical stock prices. In this section we compute σ from option prices.

When we calculate the implied volatility from option prices, we use the Black-Scholes formula to *find the σ which gives a specific options price*. Suppose, for example, that a share of

ABC Corp. is currently selling for \$35, and that a 6-month at-the-money call option on ABC Corp. is selling for \$12. Suppose the interest rate is 6%. The spreadsheet below shows that σ must be greater than 35% (since the call prices increases with σ, and since $\sigma = 35\%$ gives a call price of \$3.94, we'll have to make σ larger to get a call price of \$5.25):

	A	B	C
1	**The Black-Scholes Option-Pricing Formula**		
2	S	35	Current stock price
3	X	35	Exercise price
4	T	0.50000	Time to maturity of option (in years)
5	r	6.00%	Risk-free rate of interest
6	Sigma	35.00%	Stock volatility
7			
8	d_1	0.2450	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))
9	d_2	-0.0025	<-- d_1-sigma*SQRT(T)
10			
11	N(d_1)	0.5968	<-- Uses formula NormSDist(d_1)
12	N(d_2)	0.4990	<-- Uses formula NormSDist(d_2)
13			
14	Call price	3.94	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)

Using **Goal Seek**, we can compute the σ which gives the market price; it turns out to be $\sigma = 48.71\%$. Here's the **Goal Seek** dialog box:



And here's the final result:

	A	B	C
1	**The Black-Scholes Option-Pricing Formula**		
2	S	35	Current stock price
3	X	35	Exercise price
4	T	0.50000	Time to maturity of option (in years)
5	r	6.00%	Risk-free rate of interest
6	Sigma	48.71%	Stock volatility
7			
8	d_1	0.2593	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))
9	d_2	-0.0851	<-- d_1-sigma*SQRT(T)
10			
11	N(d_1)	0.6023	<-- Uses formula NormSDist(d_1)
12	N(d_2)	0.4661	<-- Uses formula NormSDist(d_2)
13			
14	Call price	5.25	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)

What's used in practice—implied σ or σ from historical prices?

The answer is a bit of both. Smart traders compare the implied volatility with the historical volatility and try to form estimates of what the stock volatility actually is. There are whole websites devoted to this subject, and lots of proprietary software. Our own favorite (and, as of the writing of this book, still free) website is Option Metrics (http://www.impliedvol.com/).

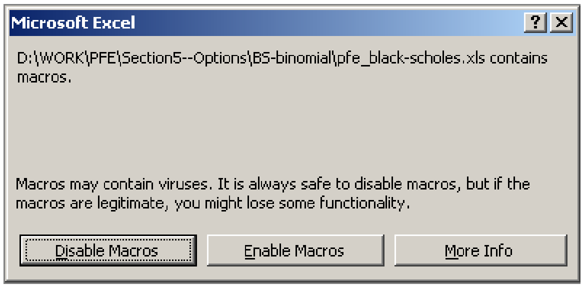
25.5. An Excel Black-Scholes function

The spreadsheet **pfe_chap25.xls** which accompanies this chapter includes two Excel functions which compute the Black-Scholes call and put prices. These functions are not part of the original Excel package; they have been defined by the author. Here's an example of how to use them:

	A	B	C
1	**BLACK-SCHOLES OPTION FUNCTIONS**		
2	The functions in this spreadsheet--**Calloption** and **Putoption**--were defined by the author.		
3	S	100	Current stock price
4	X	90	Exercise price
5	T	0.50000	Time to maturity of option (in years)
6	r	4.00%	Risk-free rate of interest
7	Sigma	35%	Stock volatility
8			
9	Call price	16.32	<-- =calloption(B3,B4,B5,B6,B7)
10	Put price	4.53	<-- =putoption(B3,B4,B5,B6,B7)

The function **Calloption(stock price, exercise price, time to maturity, interest, sigma)** is a defined macro which is attached to the spreadsheet.[3] When you first open the spreadsheet Excel will display the following message, which asks if you really want to open this macro. In this case the correct answer is **Enable macros**.

[3] As you can see in the spreadsheet, **putoption** has the same format for the variables.



An implied volatility function

The spreadsheet also comes with two functions which compute the implied volatility for a call and a put option. The function **CallVolatility(stock price, exercise price, option maturity, interest rate, target)** calculates the σ which gives the Black-Scholes price given the other parameters. The spreadsheet also includes a function called **PutVolatility** which computes the implied volatility for a put option.[4] Both functions are illustrated below:

	A	B	C
1	**TWO IMPLIED VOLATILITY FUNCTIONS**		
2	**Using CallVolatility to compute the implied volatility for a call**		
3	S	35	Current stock price
4	X	35	Exercise price
5	T	0.50000	Time to maturity of option (in years)
6	r	6.00%	Risk-free rate of interest
7	Target	5.25	<-- This is the current call price we want to match
8			
9	Implied volatility	48.71%	<-- =CallVolatility(B3,B4,B5,B6,B7)
10			
11	**Using PutVolatility to compute the implied volatility for a call**		
12	S	35	Current stock price
13	X	35	Exercise price
14	T	1.00000	Time to maturity of option (in years)
15	r	6.00%	Risk-free rate of interest
16	Target	3.44	<-- This is the current put price we want to match
17			
18	Implied volatility	32.49%	<-- =putVolatility(B12,B13,B14,B15,B16)

[4] In the spirit of this chapter, we do not explain how these functions work. For details see my book *Financial Modeling*.

25.6. Doing sensitivity analysis on the Black-Scholes formula

We can use Excel to do a lot of Black-Scholes sensitivity analysis. In this section we give two examples, leaving other examples for the chapter exercises.

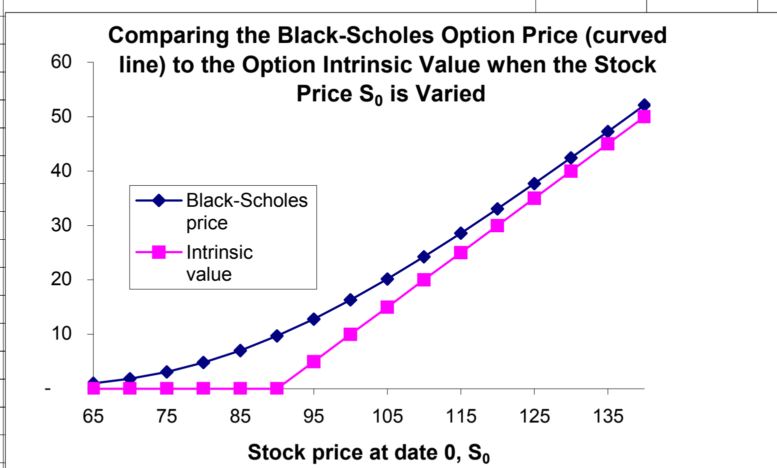
Example 1: The sensitivity of the Black-Scholes call price to the stock price S_0

The following **Data|Table** (see Chapter 000) shows the sensitivity of the Black-Scholes call value to the current stock price S_0. It compares the Black-Scholes call value to the call's intrinsic value $max(S_0 - X,0)$.

	A	B	C	D	E	F	G
1	**Black-Scholes Price Sensitivity to S_0**						
2	S_0	100	Current stock price				
3	X	90	Exercise price				
4	T	0.50000	Time to maturity of option (in years)				
5	r	4.00%	Risk-free rate of interest				
6	Sigma	35%	Stock volatility				
7							
8	Call price	16.3155	<-- =calloption(B2,B3,B4,B5,B6)				
9	Put price	4.5333	<-- =putoption(B2,B3,B4,B5,B6)				
10							
11							
12			Stock price at time 0, S_0	**Black-Scholes price**	**Intrinsic value**		
13				16.32	10.00		
14			65	0.97	0.00		
15			70	1.82	0.00		
16			75	3.08	0.00		
17			80	4.81	0.00		
18			85	7.02	0.00		
19			90	9.70	0.00		
20			95	12.81	5.00		
21			100	16.32	10.00		
22			105	20.15	15.00		
23			110	24.26	20.00		
24			115	28.58	25.00		
25			120	33.08	30.00		
26			125	37.71	35.00		
27			130	42.44	40.00		
28			135	47.25	45.00		
29			140	52.11	50.00		

This cell is part of the data table header. It contains the formula **=B8.**

This cell is part of the data table header. It contains the formula **=Max(B2-B3,0)**; this is the option's *intrinsic value*.



The option's intrinsic value $Max(S_0 - X, 0)$ shows what it would be worth if exercised immediately. The option's Black-Scholes price shows what the option would be worth on the open market. Notice that the Black-Scholes price for the call option is always greater than the intrinsic value—it is not worthwhile early-exercising the call option.

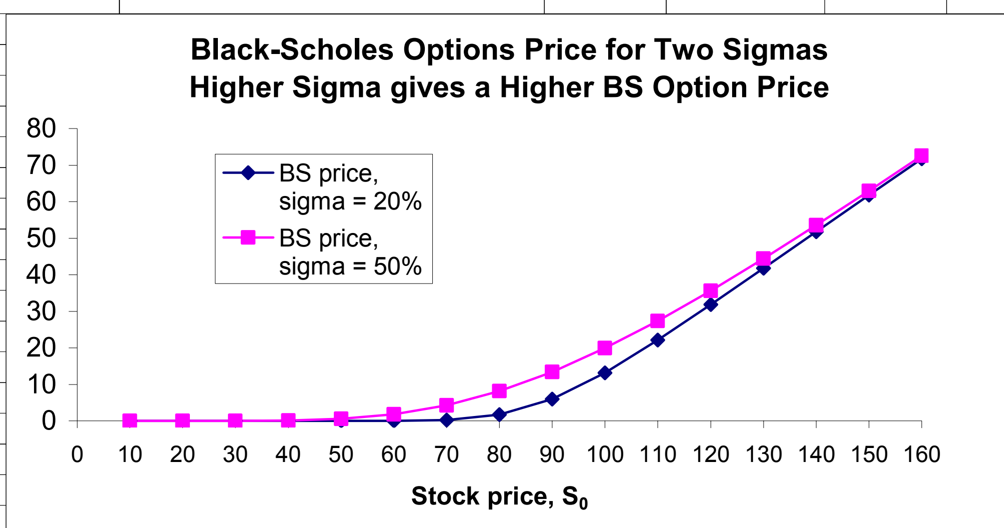
Example 2: The sensitivity of the Black-Scholes price to different estimates of σ

Here's the sensitivity analysis of the Black-Scholes price to the σ.

	A	B	C	D	E	F	G
1			**BLACK-SCHOLES SENSITIVITY ON SIGMA**				
2	S_0	100	Current stock price				
3	X	90	Exercise price				
4	T	0.50000	Time to maturity of option (in years)				
5	r	4.00%	Risk-free rate of interest				
6							
7			**Stock price**	**BS price, sigma = 20%**	**BS price, sigma = 50%**		
8				13.15	19.91		
9			10	0.00	0.00		
10			20	0.00	0.00		
11			30	0.00	0.01		
12			40	0.00	0.09		
13			50	0.00	0.53		
14			60	0.01	1.78		
15			70	0.24	4.25		
16			80	1.72	8.14		
17			90	5.96	13.41		
18			100	13.15	19.91		
19			110	22.14	27.38		
20			120	31.86	35.60		
21			130	41.80	44.37		
22			140	51.78	53.53		
23			150	61.78	62.96		
24			160	71.78	72.57		

This cell is part of the data table header. It contains the formula **=calloption(B2,B3,B4,B5,50%)**.

This cell is part of the data table header; it contains the formula **=calloption(B2,B3,B4,B5,20%).**



The higher the stock's sigma σ, the higher the Black-Scholes option price.

25.7. Does the Black-Scholes model work? Applying it to Microsoft options

In this section we do two experiments to examine whether and how well the Black-Scholes model works. First we compare the Black-Scholes option prices for a set of put and call options on Microsoft stock to the actual market prices. Then we compare the implied volatilities for the same options.

Our conclusion: Black and Scholes works pretty well. That's a big complement for a financial model!

Comparing actual market prices to Black-Scholes prices

The experiment we run here looks at options on Microsoft stock.

- On 8 February 2002 we look at the call and put options on Microsoft stock which expire on 19 July 2002.
- We calculate the Black-Scholes price of these options and compare it to the actual market price.

We get our data from Yahoo, which allows us to look up the stock price of Microsoft on 8 February 2002 and also look up the prices of Microsoft options.

Symbol	Last Trade		Change		Volume
MSFT	Feb 8	**60.65**	+0.85	+1.42%	30,642,600
Chart, Financials, Historical Prices, Insider, Messages, News, Options Profile, Reports, Research, SEC Filings, Upgrades, **more...**					
Get your tax refund fast. Use TurboTax® on Yahoo! Finance					

The closing stock price of Microsoft stock on 8 February 2002 was $60.65. The stock was up 1.42% from the previous day's close, and the total volume of stock traded was 30,642,600 shares.

We now look at the closing prices of options on Microsoft stock which expire in July 2002. Clicking on options in the above box leads us to the option prices:

Expires After: Fri 19-Jul-02 — Options Center | Analyzer NEW! | Most Actives | Symbology | Calendar

Options: Feb-02 | Mar-02 | Apr-02 | **Jul-02** | Jan-03 | Jan-04 — Highlighted options are in-the-money

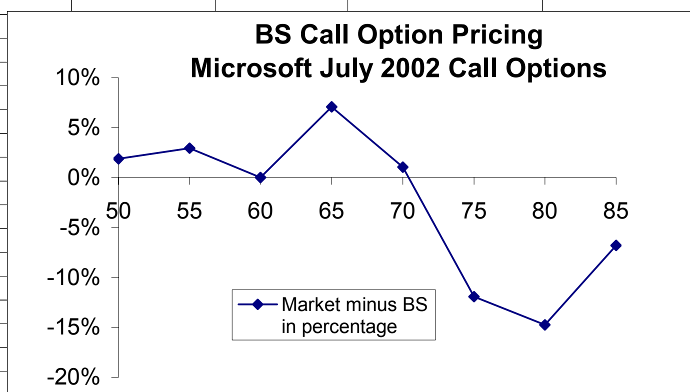
Calls							Strike Price	Puts						
Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int		Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int
MQFGE.X	**35.10**	0.00	35.70	36.10	0	166	**25**	MQFSE.X	**0.05**	0.00	0.00	0.15	0	70
MQFGF.X	**30.20**	0.00	30.80	31.20	0	99	**30**	MQFSF.X	**0.15**	0.00	0.00	0.15	0	171
MQFGG.X	**25.40**	0.00	25.90	26.30	0	53	**35**	MQFSG.X	**0.30**	0.00	0.15	0.30	0	1,644
MQFGH.X	**20.80**	0.00	21.20	21.60	0	161	**40**	MQFSH.X	**0.55**	0.00	0.40	0.55	0	451
MQFGI.X	**15.70**	-0.70	16.80	17.20	11	829	**45**	MQFSI.X	**1.00**	0.00	0.90	1.05	23	15,930
MSQGJ.X	**12.30**	0.00	12.70	13.10	3	1,086	**50**	MSQSJ.X	**2.00**	0.00	1.75	1.95	1,040	19,919
MSQGK.X	**8.70**	0.00	9.10	9.40	0	400	**55**	MSQSK.X	**3.30**	0.00	3.00	3.30	67	8,163
MSQGL.X	**5.60**	-0.20	6.00	6.30	81	2,444	**60**	MSQSL.X	**5.40**	0.00	5.00	5.30	39	14,609
MSQGM.X	**3.80**	+0.20	3.70	4.00	88	9,474	**65**	MSQSM.X	**8.30**	+0.10	7.60	7.90	32	8,441
MSQGN.X	**2.15**	+0.05	2.10	2.40	1,125	18,565	**70**	MSQSN.X	**12.30**	+0.70	11.00	11.40	15	9,112
MSQGO.X	**1.10**	0.00	1.10	1.30	86	19,073	**75**	MSQSO.X	**15.70**	0.00	14.90	15.30	0	692
MSQGP.X	**0.60**	0.00	0.50	0.65	35	14,770	**80**	MSQSP.X	**20.30**	0.00	19.40	19.80	0	757
MSQGQ.X	**0.35**	+0.10	0.20	0.35	23	8,636	**85**	MSQSQ.X	**25.10**	0.00	24.20	24.60	0	385
MSQGR.X	**0.20**	0.00	0.10	0.25	0	2,684	**90**	MSQSR.X	**30.10**	0.00	29.20	29.60	0	657
MSQGS.X	**0.15**	0.00	0.05	0.15	0	3,203	**95**	MSQSS.X	**35.10**	0.00	34.20	34.60	0	286
MSQGT.X	**0.10**	0.00	0.00	0.15	0	698	**100**	MSQST.X	**40.10**	0.00	39.20	39.60	0	6
MSQGA.X	**0.15**	0.00	0.00	0.15	0	306	**105**	MSQSA.X	**45.10**	0.00	44.20	44.60	0	10

Look carefully at the above box:

- Not all the options were traded on 8 February. For example—there was no "volume" (and hence no trading) of either calls or puts with exercise price ("strike price") of 25.
- Significant amounts of call options traded on 8 February were only for exercise prices $X=60, 65, 70, 75, 80, 85$. Significant amounts of put options traded were only for exercise prices $X = 45, 50, 55, 60, 65, 70$.
- The price of the "last trade" is in bold face black. But where there is no volume for this day, the price refers to a previous day's price.

In the spreadsheet below we look at the Microsoft July call options which actually traded on 8 February and compare the Black-Scholes price to the actual market price. We use the 6-month Treasury bill rate of 1.7% as our risk-free rate.

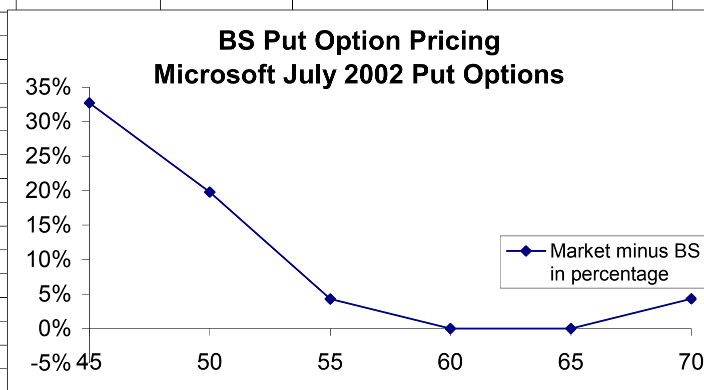
	A	B	C	D	E	F	G	H
1	**MICROSOFT CALL OPTIONS: Comparing BS to actual prices**							
2	This spreadsheet computes the Black-Scholes value of the Microsoft July 2002 options on 8 February 2002 and compares the prices to the actual market prices. As you can see, the Black-Scholes formula works pretty well!							
3								
4						**Computing the time to maturity T**		
5	S_0	60.65	Microsoft stock, closing price 8 Feb 02			Current date	8-Feb-02	
6	T	0.44110	Time to maturity of option (in years)			Expiration date	19-Jul-02	
7	r	1.70%	Risk-free rate of interest			Time (days)	161	<-- =G6-G5
8	Sigma	31.66%	<-- =CallVolatility(B5,60,B6,B7,D13)			Time (% of year)	0.4411	<-- =G7/365
9								
10		**Exercise price**	**BS call price**	**Actual call market price**	**Market minus BS in dollars**	**Market minus BS in percentage**		
11		50	12.07	12.30	0.23	1.89%	<-- =(D11-C11)/D11	
12		55	8.44	8.70	0.26	2.94%	<-- =(D12-C12)/D12	
13		60	5.60	5.60	0.00	0.00%		
14		65	3.53	3.80	0.27	7.08%		
15		70	2.13	2.15	0.02	1.05%		
16		75	1.23	1.10	-0.13	-11.93%		
17		80	0.69	0.60	-0.09	-14.74%		
18		85	0.37	0.35	-0.02	-6.80%		



We first use the function **CallVolatility** to compute the implied volatility of an at-the-money call (cell B8). We then use this volatility to price all the Microsoft calls using the Black-Scholes formula (cells C11:C18). Columns E and F compare the Black-Scholes prices to the actual market prices in cells D11:D18. The Black-Scholes model does a very good job of pricing the calls.

Below we repeat this exercise for Microsoft July puts.

	A	B	C	D	E	F	G	H
1	**MICROSOFT PUT OPTIONS: Comparing BS to actual prices**							
2	This spreadsheet computes the Black-Scholes value of the Microsoft July 2002 options on 8 February 2002 and compares the prices to the actual market prices. As you can see, the Black-Scholes formula works pretty well!							
3								
4						**Computing the time to maturity T**		
5	S_0	60.65	Microsoft stock, closing price 8 Feb 02			Current date	8-Feb-02	
6	T	0.44110	Time to maturity of option (in years)			Expiration date	19-Jul-02	
7	r	1.70%	Risk-free rate of interest			Time (days)	161	<-- =G6-G5
8	Sigma	37.35%	<-- =putVolatility(B5,60,B6,B7,D14)			Time (% of year)	0.4411	<-- =G7/365
9								
10		**Exercise price**	**BS put price**	**Actual put market price**	**Market minus BS in dollars**	**Market minus BS in percentage**		
11		45	0.67	1.00	0.33	32.72%	<-- =(D11-C11)/D11	
12		50	1.60	2.00	0.40	19.79%		
13		55	3.16	3.30	0.14	4.27%		
14		60	5.40	5.40	0.00	0.00%		
15		65	8.30	8.30	0.00	0.00%		
16		70	11.77	12.30	0.53	4.31%		

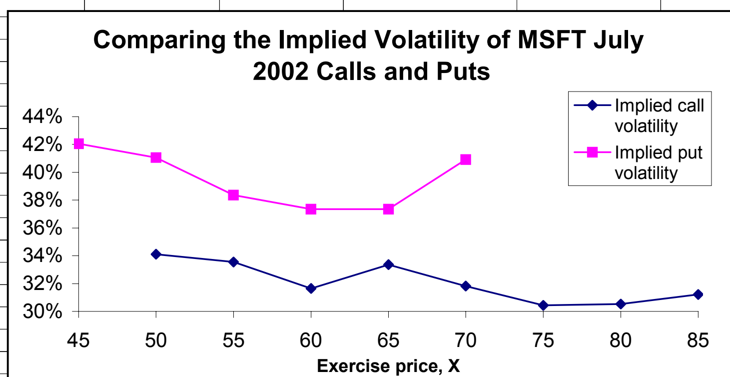


The Black-Scholes model works for both puts and calls. The one problematic feature of the pricing is that the puts are priced at a higher implied volatility than the calls: The implied volatility of the at-the-money calls is 31.66% versus an implied volatility for at-the-money puts of 37.35%.

Does the Black-Scholes model work? Looking at implied volatilities

This is our second experiment. We take the Microsoft data above to calculate the implied volatility for each option (using the functions **CallVolatility** and **PutVolatility** discussed in Section ????). Here's our spreadsheet:

	A	B	C	D	E	F	G	H
1	**MICROSOFT OPTIONS: Computing the implied volatilities**							
2	This spreadsheet computes the implied volatility of the Microsoft July 2002 options on 8 February 2002. The average implied volatility of the calls is lower than the average implied volatility of the puts.							
3								
4						**Computing the time to maturity**		
5	S_0	60.65	Microsoft stock, closing price 8 Feb 02			Current date	8-Feb-02	
6	T	0.44110	Time to maturity of option (in years)			Expiration date	19-Jul-02	
7	r	1.70%	Risk-free rate of interest			Time (days)	161	<-- =G6-G5
8						Time (% of year)	0.4411	<-- =G7/365
9								
10		**Exercise price**	**Actual call market price**	**Implied call volatility**	**Actual put market price**	**Implied put volatility**		
11		45			1.00	42.05%	<-- =putVolatility($B$5,B11,$B$6,$B$7,E11)	
12		50	12.30	34.11%	2.00	41.05%		
13		55	8.70	33.56%	3.30	38.37%		
14		60	5.60	31.66%	5.40	37.35%		
15		65	3.80	33.36%	8.30	37.36%		
16		70	2.15	31.82%	12.30	40.92%		
17		75	1.10	30.44%				
18		80	0.60	30.52%	=CallVolatility($B$5,B12,$B$6,$B$7,C12)			
19		85	0.35	31.22%				



The results are both encouraging and discouraging:

- The implied volatilities for the calls are pretty close together, as are the implied volatilities for the puts. This is good news.
- On the other hand the implied volatilities for the puts are uniformly larger than the implied volatilities for the calls. This is strange, since in the Black-Scholes formulation, the implied volatility refers to the volatility of the stock's return and hence has nothing to do with whether we're discussing a put or a call option.

- On the third hand,[5] the actual difference between the implied volatilities for the calls and the puts is not that great (only about 6%).

This is not the place to summarize the vast finance literature on implied volatilities. For our purposes, the Black-Scholes model works pretty well. That's enough!

25.8. Real options (advanced topic)

Thus far in this chapter we have discussed the use of the Black-Scholes model to price call or put options on shares. Such options are sometimes termed *financial options* because the option is written on a stock, which is a financial asset. A growing field in finance discusses *real options*. A real option is an option which becomes available as the result of an investment opportunity. Here are some examples of real options:

- Caulk Shipping is considering the purchase of a license to operate a ferry service from Philadelphia to Camden. The license requires the company to operate one boat on the ferry line, but allows Caulk Shipping the possibility of operating as many as ten ferry boats on the line. This possibility—the *option to expand* the ferry service—should be taken into account when Caulk Shipping evaluates the economics of buying the license.
- Jones Oil is considering the purchase a plot which is known to contain a large quantity of oil. Tom Shale, the company's financial analyst has computed the NPV of the lease—he

[5] Harry Truman is reported to have gotten so sick of hearing economists say "On the hand, ... But on the other hand, ... " that he asked his chief of staff to get him a "one-handed economist." History does not record if he succeeded. The economist in this section's bullets has at least 3 hands. Harry Truman would not have liked him.

assumes that once the oil drilling equipment is in place, the company will pump the oil out of the ground at the maximum feasible rate. However, Tom also realizes that the financial analysis of the plot purchase should include an important *real option*: If the future oil price is low, Jones Oil can stop pumping the oil and wait until the price gets higher. This *option to delay* has obvious value.

- Merrill Widgets is considering the purchase of six new widget machines to replace machines which are currently in place. The new machines employ an innovative production technology and are much more sophisticated than the old machines. Simona Mba, the company's financial analyst, has determined that the NPV of replacing a single machine is negative, and thus recommends against the replacement. Roberta Merrill, the company's owner, has a slightly different logic: She wants to purchase one widget machine in order to learn about the machine's possibilities; after a year she will then decide whether to buy the remaining five widget machines. The purchase of a single new widget machine gives Merrill Widgets the *option to learn*. The company's financial analysis should value this option. Below we return to this case and show how to value the option to learn.

A simple example of the option to learn

In the rest of this section we will show how the Black-Scholes model can be used to value Merrill Widget's option to learn. Recall that the company is considering replacing each of its existing six widget machines with a new machines. The new machines cost $1,000 each and have a five-year life. Simona Mba, the company's financial analyst, has estimated the expected per-machine cash flows; these flows are defined as the incremental cash flow of replacing a single old machine by a new machine and include the after-tax savings from introducing new machines, the tax shield on incremental depreciation from replacing an old by a new machine, and the sale of the old machine. It is important to emphasize that management does not know the exact realization of these annual cash flows, but knows only their expected values. The expected cash flows for the new machine are given below.

	A	B	C	D	E	F	G
3	Year	0	1	2	3	4	5
4	CF of single machine	-1000	220	300	400	200	150

Simona estimates the risk-adjusted cost of capital for the project as 12%. Using the expected cash flows and a cost of capital of 12% for the project; Simona has concluded that the replacement of a single old machine by a new machine is unprofitable, since the NPV is negative:

$$-1000+\frac{220}{1.12}+\frac{300}{(1.12)^2}+\frac{400}{(1.12)^3}+\frac{200}{(1.12)^4}+\frac{150}{(1.12)^5}=-67.48$$

Now comes the (real options) twist. Roberta Merrill, the company's owner, says: "I want to try one of the new machines for a year and learn the true realization of its cash flows. At the end of the year, if the experiment is successful, I want to replace five other similar machines on the line with the new machines. If I do not try one of the new machines, I will never know their true cash flows."

Does this change our previously negative conclusion about replacing a single machine? The answer is "yes." To see this, we now realize that what we have is a package:

- Replacing a single machine today. This has a NPV of –67.48.
- The *option* of replacing 5 more machines in one year. We can view each such option as a call option on an asset which has current value of:

$$S = \frac{220}{1.12} + \frac{300}{(1.12)^2} + \frac{400}{(1.12)^3} + \frac{200}{(1.12)^4} + \frac{150}{(1.12)^5} = 932.52$$

and an exercise price $X = 1,000$. Of course these call options can be exercised only if we purchase the first machine now; in effect the real options model will be pricing the learning costs.

Let's suppose that the Black-Scholes option pricing model can price this call option. We further suppose that the risk-free rate is 6% and the standard deviation of the cash flows is $\sigma =$ 40%. The next figure shows that the value of the each of the options to acquire one machine in one year is \$143.98. It now follows that the value of the whole project is \$652.39 (cell B12):

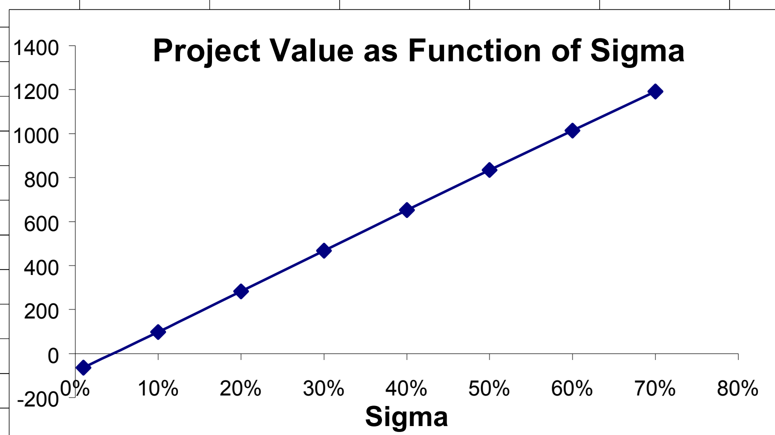
$$\begin{aligned} \textit{Project value} &= \textit{NPV of first machine} + \textit{5 options to acquire} \\ &= -67.48 + 5*143.98 = 652.39 \end{aligned}$$

	A	B	C	D	E	F	G
1	**THE OPTION TO EXPAND**						
2							
3	Year	0	1	2	3	4	5
4	CF of single machine	-1000	220	300	400	200	150
5							
6	Discount rate for machine cash flows	12%					
7	Riskless discount rate	6%					
8	NPV of single machine	-67.48					
9							
10	Number of machines bought next year	5					
11	Option value of single machine purchased in one more year	143.98	<-- =B24				
12	NPV of total project	652.39	<-- =B8+B10*B11				
13							
14	**Black-Scholes Option Pricing Formula**						
15	S	932.52	PV of machine CFs				
16	X	1000.00	Exercise price = Machine cost				
17	r	6.00%	Risk-free rate of interest				
18	T	1	Time to maturity of option (in years)				
19	Sigma	40%	<-- Volatility				
20	d_1	0.1753	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))				
21	d_2	-0.2247	<-- d_1 - sigma*SQRT(T)				
22	N(d_1)	0.5696	<--- Uses formula NormSDist(d_1)				
23	N(d_2)	0.4111	<--- Uses formula NormSDist(d_2)				
24	Option value = BS call price	143.98	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)				

Thus, buying one machine today, and in the process acquiring the option to purchase five more machines in one year is a worthwhile project.

One critical element here is the volatility. The lower the volatility (i.e., the lower the uncertainty), the less worthwhile this project is. By building a data table we can examine the relation between the standard deviation σ and the project value:

	B	C	D	E	F	G	H	I	J
27	**Data Table**								
28	σ	652.39	<-- Table header: =B12						
29	1%	-63.48							
30	10%	97.16							
31	20%	283.09							
32	30%	468.40							
33	40%	652.39							
34	50%	834.59							
35	60%	1014.54							
36	70%	1191.81							
37									
38									
39									
40									
41									
42									



The value of the project as a whole comes from our uncertainty about the actual cash flows one year from now. The less is this uncertainty (measured by σ), the less valuable the project. In this particular example a very low uncertainty ($\sigma \geq 4.75\%$) with respect the machine cash flow returns is sufficient to justify its purchase.[6]

---

[6] Estimating the σ for real option cash flows is problematic, since there is little market data (as there is for stocks) to guide us. Many authors use estimates in the range of 30% - 50% for the standard deviation of real option returns; this is somewhat higher than the average standard deviation of U.S. market returns for equity, which are in the range of 15% - 30%. To explore this issue, consult one of the three leading books in the area: Lenos Trigeorgis, *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, MIT Press, 1996; Martha Amram and Nalin Kulik, *Real Options*, Harvard Business School, 1998; Tom Copeland and Vladimir Antikarov, *Real Options: A Practitioner's Guide*, Texere, 2001.

Real options: where do we go from here?

Real options are increasingly used in finance to value corporate investments. The example of Merrill Widgets given above is only a small example of the use of the real options technique. For deeper discussions, we suggest you consult one of the books mentioned in footnote 000.

Summary

This chapter has given you a quick and hopefully practical insight into how to use the Black-Scholes model. The Black-Scholes model is remarkably good at pricing options and is widely used. It is also easy to use, provided you don't get too hung up on the details of where the formula comes from (in this chapter we've left these hang-ups behind us, and concentrated exclusively on implementational details).

Exercises

1. Use the Black-Scholes model to price the following:

   - A call option on a stock whose current price is 50, with exercise price $X = 50$, $T = 0.5$, $r = 10\%$, $\sigma = 25\%$.
   - A put option with the same parameters.

2. A call option on a stock is priced at \$5.35. The option has an exercise price of $X = \$40$. The current stock price $S_0 = \$33$, the option's time to maturity is 6 months, and the interest rate $r = 6\%$. Use the Black-Scholes model to determine the implied volatility, the σ used to price the option. (Excel hint: use **Goal Seek**.)

3. A put option on a stock is priced at \$5. The option has an exercise price of $X = \$25$. The stock's current price is $S_0 = \$25$, the option's time to maturity is 1 year, and the interest rate is $r = 5\%$. Use the Black-Scholes model to determine the option's implied volatility, the σ used to price the option. (Excel hint: use **Solver**)

4. A call option with ½ year to maturity is written on a stock whose current price is \$40. The option's exercise price is \$38, the interest rate is 4%, and the stock's volatility is 30%.

   4.a. Find the call option price using the Black-Scholes model.

   4.b. Make a table showing the option's price for volatilities ranging from 10%, 20%, …, 60%. (Excel hint: by far the easiest way to do this is to use **Data Table**, explained in Chapter 28.)

5. A put option with ½ year to maturity is written on a stock whose current price is \$40. The option's exercise price is \$38, the interest rate is 4%, and the stock's volatility is 30%.

5.a. Find the put option price using the Black-Scholes model.

5.b. Make a table showing the option's price for maturities ranging from $T = 0.2, 0.4, \ldots, 2.0$. (Excel hint: by far the easiest way to do this is to use **Data Table**, explained in Chapter 28.)

6. Use the data from exercise 1 and **Data|Table** to produce graphs that show:

- The sensitivity of the Black-Scholes call price to changes in the initial stock price S_0.
- The sensitivity of the Black-Scholes put price to changes in σ.
- The sensitivity of the Black-Scholes call price to changes in the time to maturity T.
- The sensitivity of the Black-Scholes call price to changes in the interest rate r.
- The sensitivity of the put price to changes in the exercise price X.

7. Produce a graph comparing a call's *intrinsic value* (defined as $Max(S_0 - X,0)$) and its Black-Scholes price. From this graph you should be able to deduce that it is never optimal to exercise early a call priced by the Black-Scholes.

8. Produce a graph comparing a put's intrinsic value (= $Max(X-S_0,0)$) and its Black-Scholes price. From this graph you should be able to deduce that it is may be optimal to exercise early a put priced by the Black-Scholes formula.

9. Use the Excel **Solver** to find the stock price for which there is the maximum difference between the Black-Scholes call option price and the option's intrinsic value. Use the following values: $S_0 = 45$, $X = 45$, $T = 1$, $\sigma = 40\%$, $r = 8\%$.

10. Repeat the MSFT exercise in the text for the March 2002 options:

Expires After: Fri 15-Mar-02 — Options Center | Analyzer NEW! | Most Actives | Symbology | Calendar

Options: Feb-02 | **Mar-02** | Apr-02 | Jul-02 | Jan-03 | Jan-04 — Highlighted options are in-the-money

Calls							Strike Price	Puts						
Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int		Symbol	Last Trade	Chg	Bid	Ask	Vol	Open Int
MQFCF.X	**29.30**	-0.70	30.50	30.90	15	11	**30**	MQFOF.X	**0.05**	0.00	0.00	0.10	15	10
MQFCG.X	**25.00**	0.00	25.50	25.90	0	143	**35**	MQFOG.X	**0.00**	0.00	0.00	0.10	0	0
MQFCH.X	**20.00**	0.00	20.50	20.90	0	0	**40**	MQFOH.X	**0.00**	0.00	0.00	0.10	0	20
MQFCI.X	**15.10**	0.00	15.60	16.00	0	455	**45**	MQFOI.X	**0.05**	0.00	0.05	0.15	0	16
MSQCJ.X	**9.80**	-0.60	10.80	11.20	26	1,325	**50**	MSQOJ.X	**0.50**	+0.20	0.20	0.35	60	776
MSQCK.X	**5.50**	-0.60	6.40	6.70	2	449	**55**	MSQOK.X	**0.95**	-0.15	0.80	0.95	311	4,845
MSQCL.X	**3.00**	+0.25	3.00	3.20	300	2,138	**60**	MSQOL.X	**2.55**	-0.35	2.25	2.45	188	20,754
MSQCM.X	**0.95**	+0.05	0.90	1.00	2,155	19,806	**65**	MSQOM.X	**5.70**	-0.20	5.10	5.40	171	8,376
MSQCN.X	**0.20**	-0.10	0.15	0.30	114	11,777	**70**	MSQON.X	**10.00**	-0.20	9.40	9.70	20	1,014
MSQCO.X	**0.15**	0.00	0.00	0.10	0	6,643	**75**	MSQOO.X	**15.10**	0.00	14.20	14.60	0	594
MSQCP.X	**0.00**	0.00	0.00	0.15	0	1,209	**80**	MSQOP.X	**20.10**	0.00	19.20	19.60	0	2

Note that you can use the Black-Scholes formula to calculate the call option premium as a percentage of the exercise price in terms of S_0/X:

$$C = S_0 N(d_1) - Xe^{-rT} N(d_2) \Rightarrow \frac{C}{X} = \frac{S_0}{X} N(d_1) - e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/X) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Implement this in a spreadsheet.

Note that you can also calculate the Black-Scholes put option premium as a percentage of the exercise price in terms of S_0/X:

$$P = -S_0 N(-d_1) + Xe^{-rT} N(-d_2) \Rightarrow \frac{P}{X} = e^{-rT} N(-d_2) - \frac{S_0}{X} N(-d_1)$$

where

$$d_1 = \frac{\ln(S_0 / X) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Implement this in a spreadsheet. Find the ratio of S_0/X for which C/X and P/X cross when $T = 0.5$, $\sigma = 25\%$, $r = 10\%$. (You can use a graph or you can use Excel's Solver.) Note that this crossing point is affected by the interest rate and the option maturity, but not by σ.

11. As Shown in Chapter 21 the call option value is always greater than its immediate exercise value (S_0 -K) for S_0 >K. However, the value of the European put is sometimes less that its intrinsic value (K- S_0) for S_0 < K. Use the put option pricing model to find such an example.

12. The probability that a European call option on the stock will be exercised is N(d_2) (same expression as in Black-Scholes option pricing formula). What is the probability that a European call option on a stock with an exercise price of $40 and a maturity date in six months will be exercised? The current stock price is at $38, the interest rate is at 5%, stock return volatility is at 25%.

13. A stock price is currently $50 and the risk-free interest rate is 5%. Use the Black-Scholes model to translate the following table of European call options on the stock into a table of implied volatilities, assuming no dividends (Excel hint: use **Solver**)

Are the option prices consistent with Black-Scholes?

Exercise Price($)/Maturity (months)

EXERCISE PRICE($)/MATURITY (MONTHS)	**3**	**6**	**9**
45	7	8.3	10.5
50	3.7	5.2	7.5
55	1.6	2 .9	5 .1

14. A put option with 1 year to maturity is written on a stock. The current underlying stock price is \$20. The option's exercise price is \$18, the interest rate is 3.74%, and the stock's volatility is 32.7%. The price of a call option written on the same stock with the same exercise price and time to maturity is \$4.3 Use Black and Scholes model to determine: Does put-call parity hold?

15. The stock price of ABC-Corp is currently $S_0 =$ \$50. What is the price of a European call option which expires in 2 months and which has a exercise price of \$60? Assume the yearly interest rate is 5.5%, and the *monthly* volatility of the stock prices is 7.8%.

16. The price of a share of ABC-Corp stock is currently $S_0 =$ \$55. Assume that the yearly interest 2%, and that the stock's volatility is 0.4.

16.a. Determine the prices of European call and put options with a exercise price of \$55, and expiration in three months.

16.a. Verify put-call parity.

17. A one month European call option is currently selling for \$3.90. The exercise price of the option is \$40, and the current stock price is $S_0 =$ \$43. The monthly interest rate is 0.5% and the

monthly volatility of the stock return is at 7%. Does this price present an opportunity for arbitrage, according to B&S?

18. Consider an option trading on a stock with a year to maturity. The implied volatility of the option at the opening is 25% and at closing 22%. Assume that the stock prices hasn't changed, what do you conclude about the price; has it increased or decreased?

19. If the volatility of a stock is 30% and assuming 250 trading days a year, what is the standard deviation of the return in one trading day?

CHAPTER 27: INTRODUCTION TO EXCEL*

this version: March 2003

Chapter contents

Introduction

This chapter introduces you to Excel and shows you how to do the most important initial operations. Excel is not difficult to learn to use, provided you're willing make many mistakes along the way, and you take an occasional look at the online Help (press function key F1).

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* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it. All the material is copyright and the rights belong to the author.

Contents of this chapter

- Turning Excel on
- Saving, creating a new directory
- Copying—relative versus absolute
- Formatting numbers
- Making a graph
- Fiddling with the default settings for Excel
- Using a few functions
- Printing

27.1. Getting started

Ok, you've started your computer and pressed on the Excel icon on your desktop (or maybe it's not on your desktop—maybe you got there through the button ...). You're facing a blank spreadsheet, and you want to play. Let's write a spreadsheet describing how $1000 deposited in the bank at 15% will grow over time:

	A	B	C
1	**COMPOUND INTEREST**		
2	Year	Bank balance	
3	0	1000	
4	1		
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		

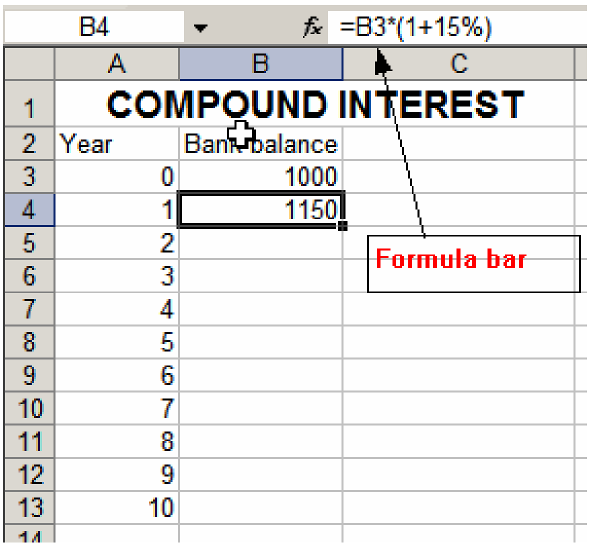
After you've finished typing in the above, put the cursor in cell B4. We're going to make a formula that describes how much money will be in the bank at the end of year 1. When you're in cell B4, type in the following formula and then hit [Enter] (no spaces, please!):

=B3*(1+15%)

Here's what the spreadsheet should look like:

	A	B	C
1	COMPOUND INTEREST		
2	Year	Bank balance	
3	0	1000	
4	1	1150	
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		

If you put the cursor on cell B4 and look at the *formula bar* (next to the f_x symbol), you'll see what you've written in the spreadsheet:

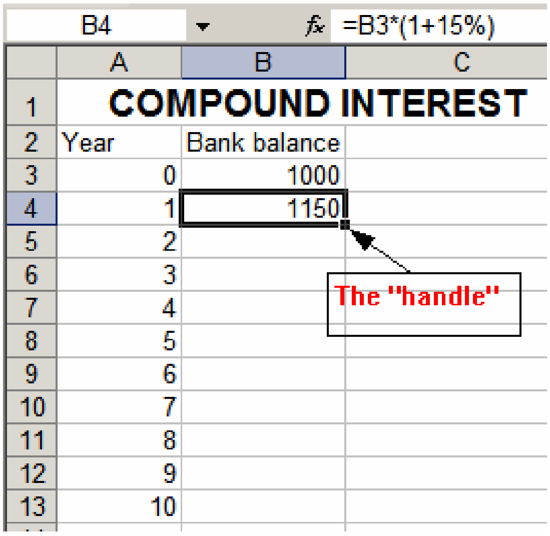

B4 f_x =B3*(1+15%)

	A	B	C
1	COMPOUND INTEREST		
2	Year	Bank balance	
3	0	1000	
4	1	1150	
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		

Copying the formula

So, if you deposit $1,000 in the bank today and the bank gives you 15% interest, you'll have $1,150 at the end of year 1. If you've read Chapter 1 of this book, you know that at the end of year 2 you'll have $1{,}150*(1+15\%)$ in the bank. Instead of typing in this formula, we'll use Excel's *copy* ability to put it in cell B5:

- The lower right-hand corner of the frame around cell B4 has a little black square; we call this the "handle" of the cell.



- Put the cursor on the handle of cell B4. Press on the left mouse button, and drag down until you get to cell B13. At this point your spreadsheet will look like this:

B4 =B3*(1+15%)

	A	B	C
1	COMPOUND INTEREST		
2	Year	Bank balance	
3	0	1000	
4	1	1150	
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		
14			
15			

Release the left mouse button and:

	A	B	C
1	**COMPOUND INTEREST**		
2	Year	Bank balance	
3	0	1000	
4	1	1150	<-- =B3*(1+15%)
5	2	1322.5	<-- =B4*(1+15%)
6	3	1520.875	<-- =B5*(1+15%)
7	4	1749.00625	<-- =B6*(1+15%)
8	5	2011.357188	<-- =B7*(1+15%)
9	6	2313.060766	<-- =B8*(1+15%)
10	7	2660.01988	<-- =B9*(1+15%)
11	8	3059.022863	<-- =B10*(1+15%)
12	9	3517.876292	<-- =B11*(1+15%)
13	10	4045.557736	<-- =B12*(1+15%)

Notice how Excel copied the cell formulas:

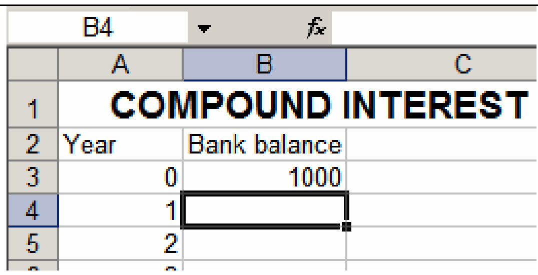
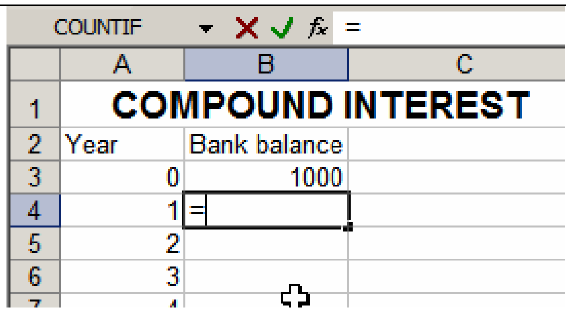
- The formula in cell B4 says: "Take the contents of the cell above and multiply by (1+15%).
- When we *drag down* the cell formula in B5 says: "Take the contents of the cell above and multiply by (1+15%).

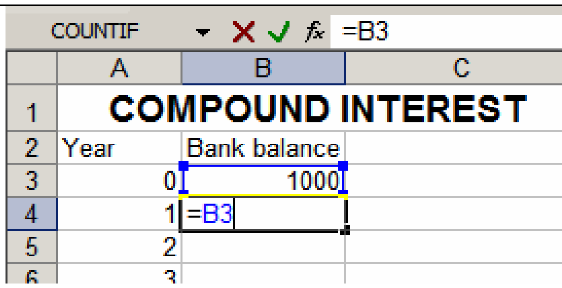
This kind of copying is called *relative copying* in Excel: The cell formulas change in the direction of the copy (that is, in the direction which you dragged the cell handle). There's also *absolute copying*, which we'll explain in Section 27.4 below.

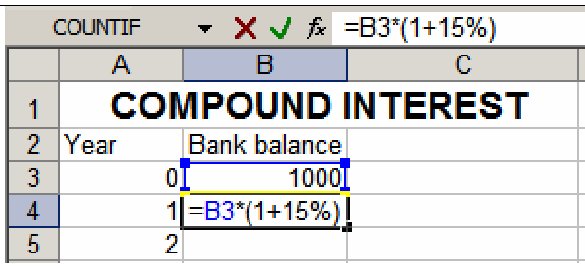
Excel hint
Instead of dragging cell B4, there's an even simpler way to copy. If you put your cursor on the handle and double-click with the left mouse button, the formula in cell B4 will be copied from cell B5 through B13.

Entering formulas by pointing (a better way)

So far we've written the formula in cell B4. But it's usually a better idea to use the mouse and *point* at the relevant cells. Pointing and clicking formulas avoids a lot of mistakes. In the previous example:

Put the cursor on cell B4	
Type in "="	

<table>
<tr>
<td>With the mouse, point at cell B3</td>
<td>
</td>
</tr>
<tr>
<td>Write in the rest of the formula—*(1+15%).

Click the left mouse button or hit [Enter].</td>
<td>
</td>
</tr>
</table>

27.2. Formatting the numbers

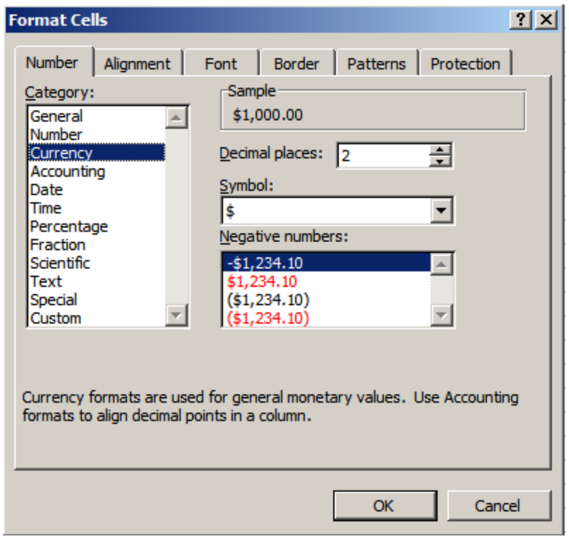
The spreadsheet we've constructed so far is cute but ugly. Why do we need so many decimal places? Why aren't there commas in the numbers? How about indicating that these are *dollar* amounts?

We can make all these changes by using Excel's extensive *formatting* facilities.

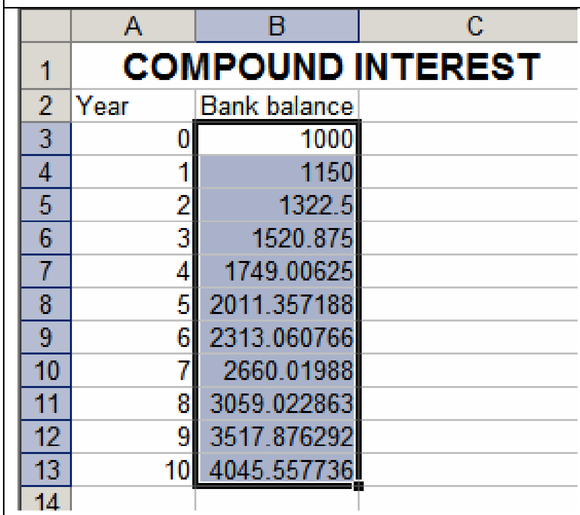
FORMATTING NUMBERS IN EXCEL

Before: Mark the numbers to be formatted. Go to **Format|Cells|Number** on the menu bar and choose something appropriate.

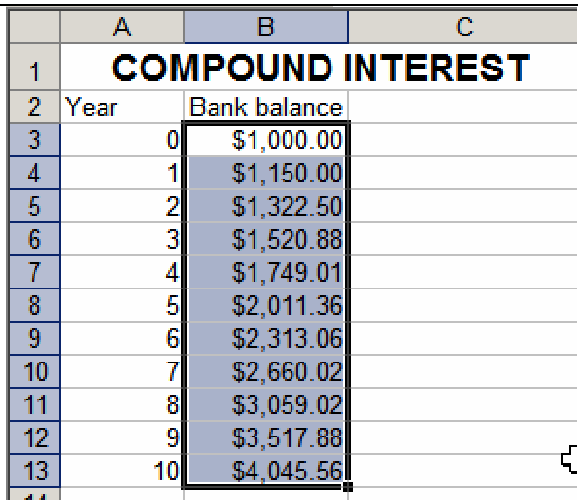
After: Here's what we chose:



Here's how the spreadsheet looks now:



	A	B	C
1	COMPOUND INTEREST		
2	Year	Bank balance	
3	0	1000	
4	1	1150	
5	2	1322.5	
6	3	1520.875	
7	4	1749.00625	
8	5	2011.357188	
9	6	2313.060766	
10	7	2660.01988	
11	8	3059.022863	
12	9	3517.876292	
13	10	4045.557736	
14			



	A	B	C
1	COMPOUND INTEREST		
2	Year	Bank balance	
3	0	$1,000.00	
4	1	$1,150.00	
5	2	$1,322.50	
6	3	$1,520.88	
7	4	$1,749.01	
8	5	$2,011.36	
9	6	$2,313.06	
10	7	$2,660.02	
11	8	$3,059.02	
12	9	$3,517.88	
13	10	$4,045.56	

In other chapters we'll use the **Format|Cells** command to change the way dates and text and fonts appear in Excel. The important thing to note about this command is that *it changes the way cell contents appear, but not the actual cell contents.* For example, suppose your cell contents read 3287.65898992; now suppose that you made them look like dollars with a comma and two decimal places, so that the cell reads $3,287.66." The actual contents of the cell haven't changed—there are still eight decimal places, but it only shows two of them.

27.3. Absolute copying—building a more sophisticated model

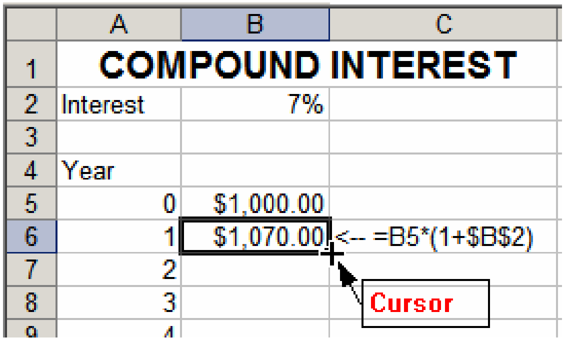
The spreadsheet of the previous section is cute, but it doesn't allow us to change the interest rate at which the money accumulates. We fix this by writing the following spreadsheet; in this spreadsheet we've got a separate cell (B2) to indicate the interest rate. By changing this cell we'll change all the accumulations.

	A	B	C
1	COMPOUND INTEREST		
2	Interest	7%	
3			
4	Year		
5	0	$1,000.00	
6	1		
7	2		
8	3		
9	4		
10	5		
11	6		
12	7		
13	8		
14	9		
15	10		

Go to cell B6 . Type the formula " =B5*(1+$B$2) " in this cell. The dollar signs on $B$2 indicate that when we copy this formula, this particular cell reference will not change. In the jargon of Excel: $B$2 is an *absolute reference*, whereas B5 is a *relative reference*—it will change to B6, B7, … as we go down the column.

	A	B	C
1	COMPOUND INTEREST		
2	Interest	7%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,070.00	<-- =B5*(1+$B$2)
7	2		
8	3		
9	4		
10	5		
11	6		
12	7		
13	8		
14	9		
15	10		

Copying as we did in the previous section (click on B6, put the cursor on the B6 handle and drag):



The result is a table much like that of the previous section:

	A	B	C
1	COMPOUND INTEREST		
2	Interest	7%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,070.00	<-- =B5*(1+$B$2)
7	2	$1,144.90	<-- =B6*(1+$B$2)
8	3	$1,225.04	<-- =B7*(1+$B$2)
9	4	$1,310.80	<-- =B8*(1+$B$2)
10	5	$1,402.55	<-- =B9*(1+$B$2)
11	6	$1,500.73	<-- =B10*(1+$B$2)
12	7	$1,605.78	<-- =B11*(1+$B$2)
13	8	$1,718.19	<-- =B12*(1+$B$2)
14	9	$1,838.46	<-- =B13*(1+$B$2)
15	10	$1,967.15	<-- =B14*(1+$B$2)

(We've formatted the numbers as currency.)

The difference between this spreadsheet and the previous one is that we can change the interest rate simply by changing the contents of cell B2. In this example the interest rate is 10%:

	A	B	C
1	**COMPOUND INTEREST**		
2	Interest	10%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,100.00	<-- =B5*(1+$B$2)
7	2	$1,210.00	<-- =B6*(1+$B$2)
8	3	$1,331.00	<-- =B7*(1+$B$2)
9	4	$1,464.10	<-- =B8*(1+$B$2)
10	5	$1,610.51	<-- =B9*(1+$B$2)
11	6	$1,771.56	<-- =B10*(1+$B$2)
12	7	$1,948.72	<-- =B11*(1+$B$2)
13	8	$2,143.59	<-- =B12*(1+$B$2)
14	9	$2,357.95	<-- =B13*(1+$B$2)
15	10	$2,593.74	<-- =B14*(1+$B$2)

Excel hint

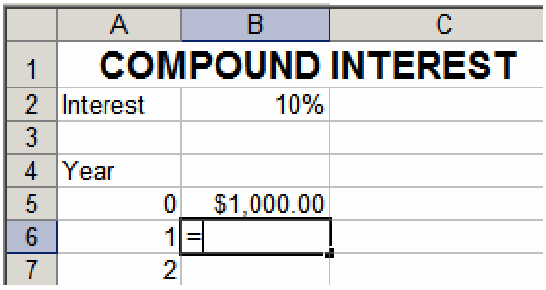
Never use a number if you can use a cell reference! Compare the previous example with this one: If, as in the previous section, you "hard-wire" the 15% interest rate in cells B6:B15, you have to change each of these cells in order to change the interest rate assumption. On the other hand, if you put the interest rate in a cell (as in this section's example), you need only change the contents of that cell in order to recalculate the whole spreadsheet.

In Excel, numbers are always inferior to formulas!

Pointing and using the F4 key

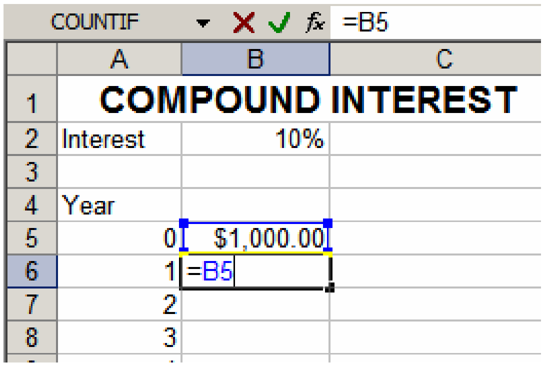
Let's go back to the stage in this example where we were putting the formula "=B5*(1+$B$2)" into cell B5. We've already suggested that it's better to enter formulas by pointing and clicking than by typing. Now we'll teach you another little trick, the use of the F4 key to “dollarize” cell references—that is, to make them absolute references instead of relative references. Here’s what you do:

- Put the cursor in cell B6. Type “=”.



	A	B	C
1	COMPOUND INTEREST		
2	Interest	10%	
3			
4	Year		
5	0	$1,000.00	
6	1	=	
7	2		

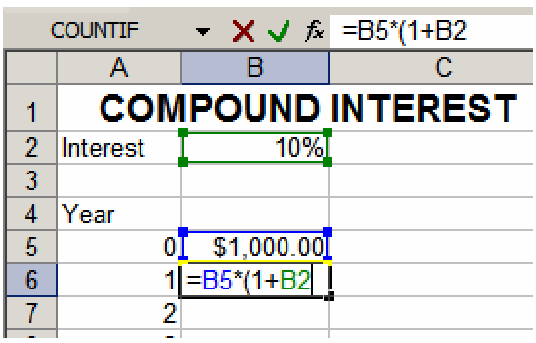
Now *point* at cell B5, the one that contains $1,000.00. You can point with either the mouse (clicking when you’re on B5), or you can point with the arrow keys.



COUNTIF =B5

	A	B	C
1	COMPOUND INTEREST		
2	Interest	10%	
3			
4	Year		
5	0	$1,000.00	
6	1	=B5	
7	2		
8	3		

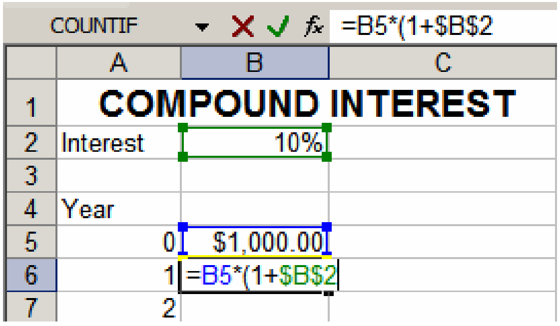
- Now type a star, the opening of a parenthesis, and a 1, and a + : *(1+ . Then point at cell B2 containing the interest rate:



COUNTIF =B5*(1+B2

	A	B	C
1	COMPOUND INTEREST		
2	Interest	10%	
3			
4	Year		
5	0	$1,000.00	
6	1	=B5*(1+B2	
7	2		

- Next hit function key **F4**. This puts the dollar signs into the cell reference B2 in cell B6.

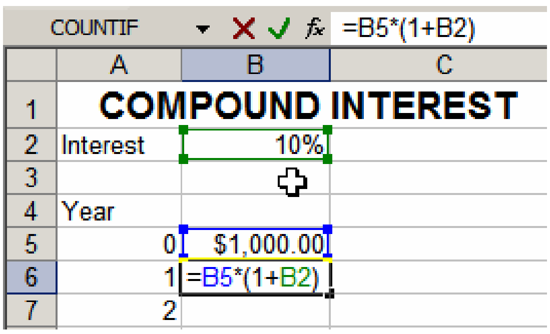


- Finally, close the parentheses by typing a ")". Hit [Enter].
- Copy cell B6 as before

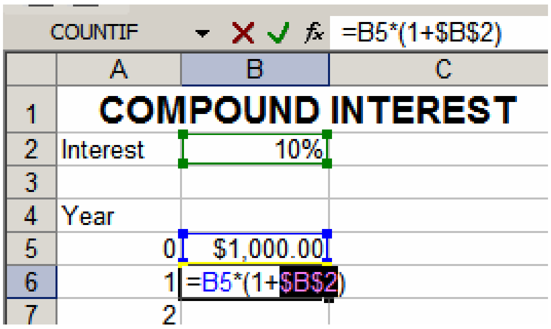
Correcting errors—editing the cell

Suppose you made a mistake and forgot to "dollarize" the B2 cell reference, so that the contents of cell B6 are "=B5*(1+B2)." This isn't good—the cell contents should read "=B5*(1+$B$2)." To make the appropriate change, we edit the formula in cell B6 and we use the F4 key:

- Put the cursor on B6 and click the left mouse key twice. This opens the formula for editing.



- Move the cursor until it's somewhere on the B2 in the formula (it doesn't matter where). Hit the F4 key and your cell reference will be "dollarized."

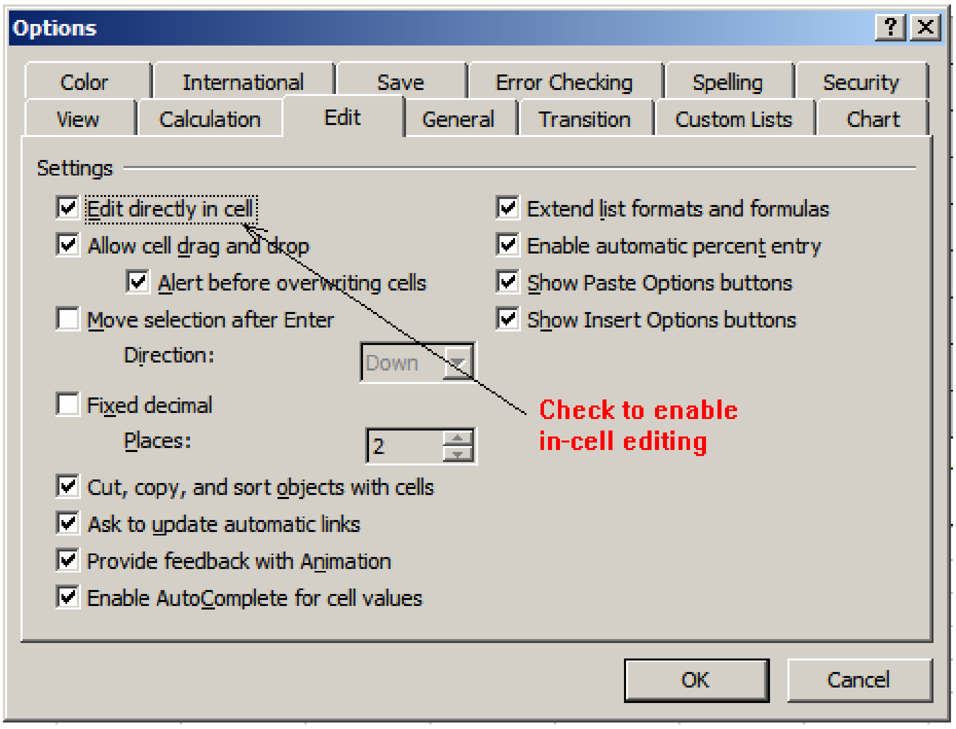


- Now hit [Enter] and copy as before.

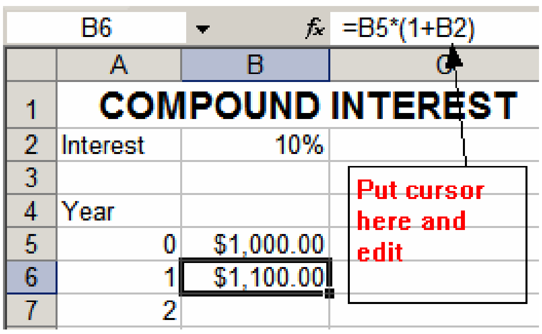
Three Excel hints about editing

1. You can also edit the cell contents by putting the cursor on the cell and hitting the **F2** function button.

2. If you can't edit the formula in the cell, someone may have changed the default settings on your Excel spreadsheet. Go to **Tools|Options**, click the **Edit** tab and check the "Edit directly in cell" box:



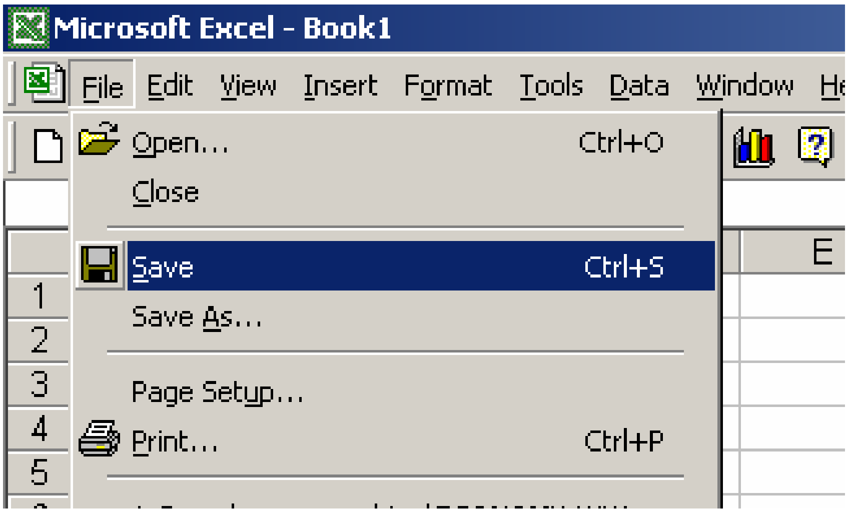
3. You can always edit a cell formula in the formula bar:



27.4. Saving the spreadsheet

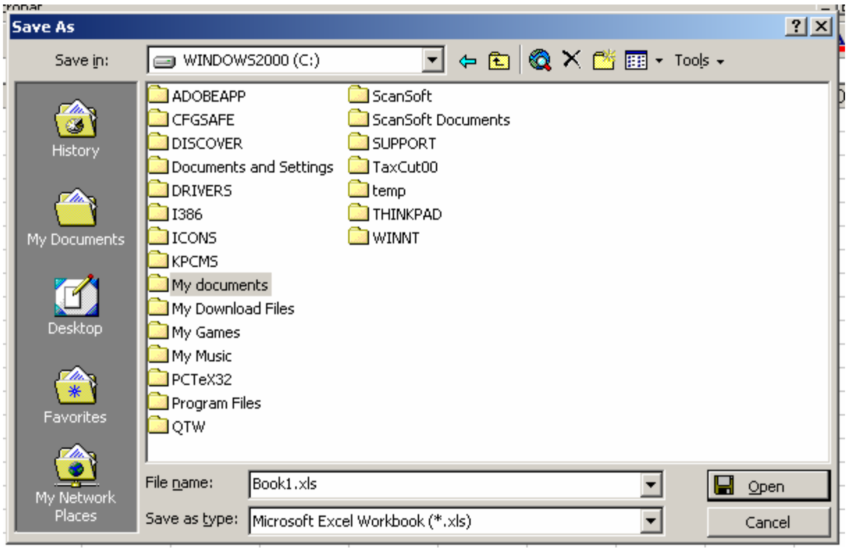
What's the next step? We suggest that you *save* the spreadsheet.[1] An appropriate place to save it is in that **Junk** directory that you're going to create right now.

- Go to **File|Save**

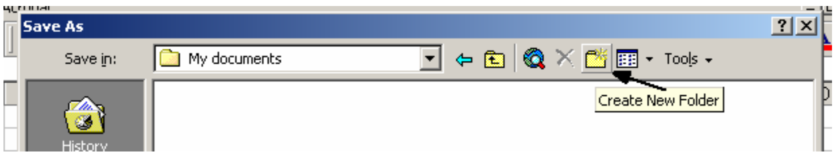


- Excel will probably suggest a directory called **My Documents**:

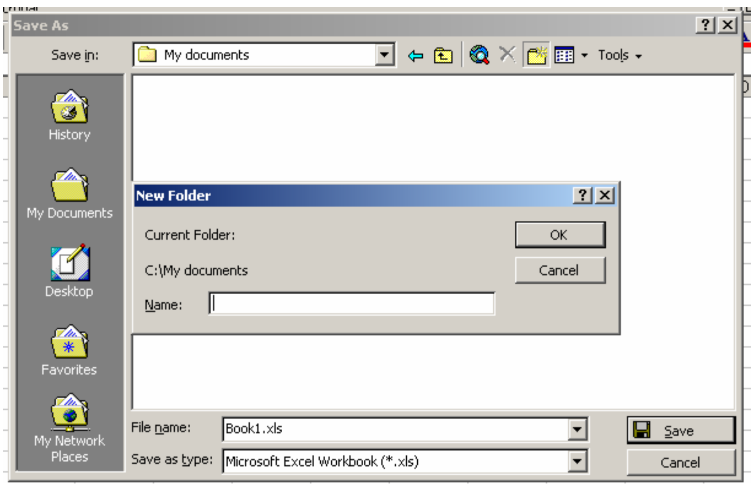
[1] As a rule of thumb, we suggest that you save *all the time*. Someday, your computer is going to crash *right after* you've spent a long time working and *before* you've saved your work.



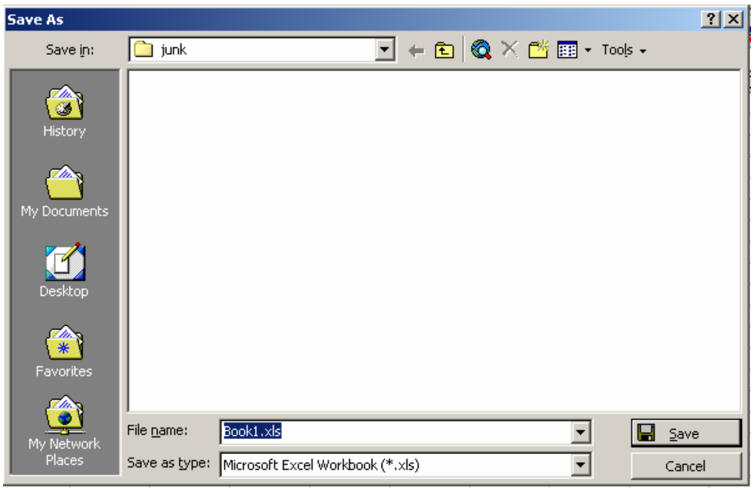
- Click on **My documents**, and then click on the "Create New Folder" icon which looks like this:



- When you click on the "Create New Folder" icon, you'll get a dialogue box:

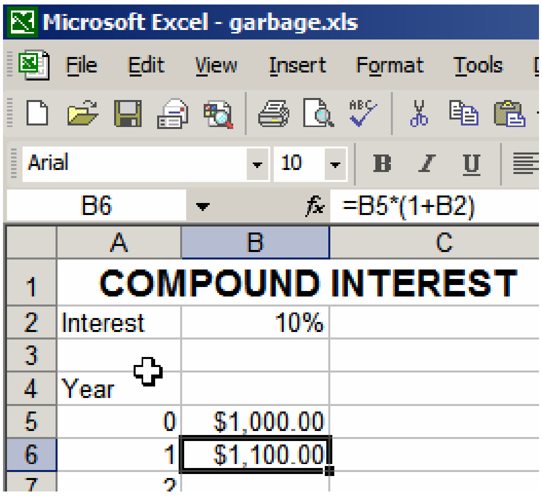


In the Name box, type "junk." The author's computer always has a directory called "junk"—it's the directory containing all the files which you can get rid of without thinking twice (a file called "junk" in the "junk" directory is a double whammy—absolutely worthless!). Now you'll find yourself in the Junk subdirectory:



Type something clever in the box called **File Name**. We'll call our spreadsheet "garbage."

Now you'll see the name of the spreadsheet in the upper left-hand corner of the sheet:

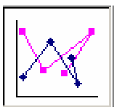


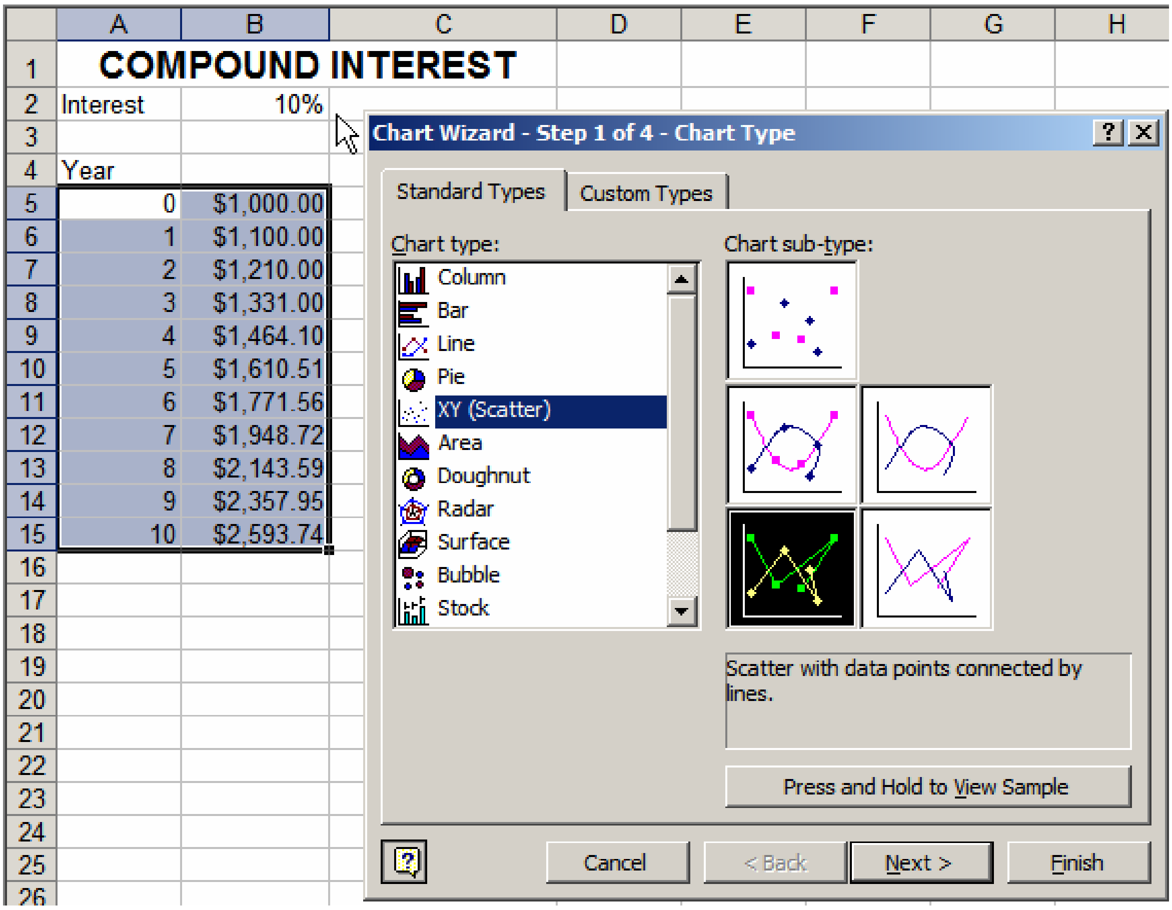
Every time you subsequently save the workbook (either by **File|Save** or by pressing [Ctrl]+S or by clicking on the save icon in the form of the little disk ), the workbook with all its changes will be saved under the same name in the same place.

27.5. Your first Excel graph

You're going to want to graph the compound interest example. Take your mouse, put it in cell A5; push the left button and move down until you get to cell B15:

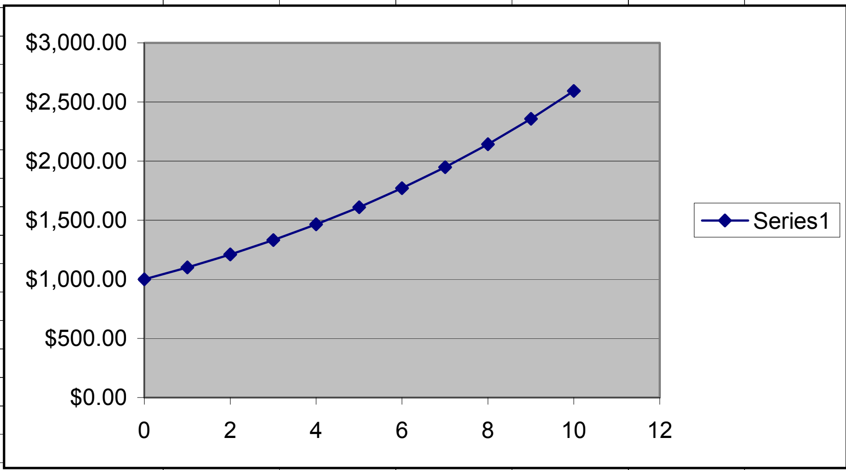
	A	B	C
1	**COMPOUND INTEREST**		
2	Interest	10%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,100.00	
7	2	$1,210.00	
8	3	$1,331.00	
9	4	$1,464.10	
10	5	$1,610.51	
11	6	$1,771.56	
12	7	$1,948.72	
13	8	$2,143.59	
14	9	$2,357.95	
15	10	$2,593.74	
16			

Now go to the chart icon on the toolbar (). Click on this icon and choose a chart type. Our favorite chart type (the one used most in this book) is **XY (Scatter)**. We also like the *connected* XY chart, so we press on the **Chart sub-type**  :



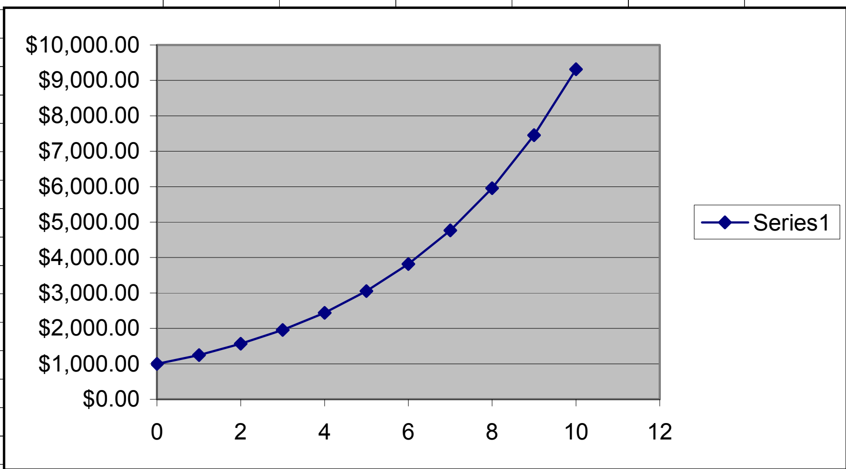
At this point there's lots we can do in terms of formatting the chart, but we'll explain that to you later in Chapter 28. Just press the **Finish** button at the bottom of the Chart Wizard, and you'll get a reasonable graph:

	A	B
1	**COMPOUND INTEREST**	
2	Interest	10%
3		
4	Year	
5	0	$1,000.00
6	1	$1,100.00
7	2	$1,210.00
8	3	$1,331.00
9	4	$1,464.10
10	5	$1,610.51
11	6	$1,771.56
12	7	$1,948.72
13	8	$2,143.59
14	9	$2,357.95
15	10	$2,593.74



This graph has lots of features we don't like, but they can all be fixed (Chapter 28 again). Instead of fixing things, *play* with the spreadsheet—change the interest rate and see what happens:

	A	B
1	**COMPOUND INTEREST**	
2	Interest	25%
3		
4	Year	
5	0	$1,000.00
6	1	$1,250.00
7	2	$1,562.50
8	3	$1,953.13
9	4	$2,441.41
10	5	$3,051.76
11	6	$3,814.70
12	7	$4,768.37
13	8	$5,960.46
14	9	$7,450.58
15	10	$9,313.23

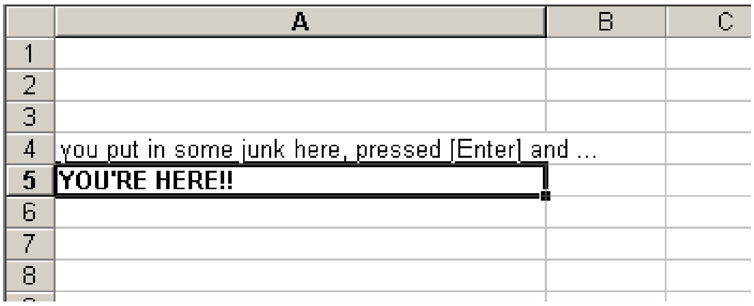


27.6. Initial settings

Before you make intensive use of Excel, it's worthwhile to change a few of the initial settings to suit your needs and preferences. In this section we'll show you our suggestions (they're all reversible).

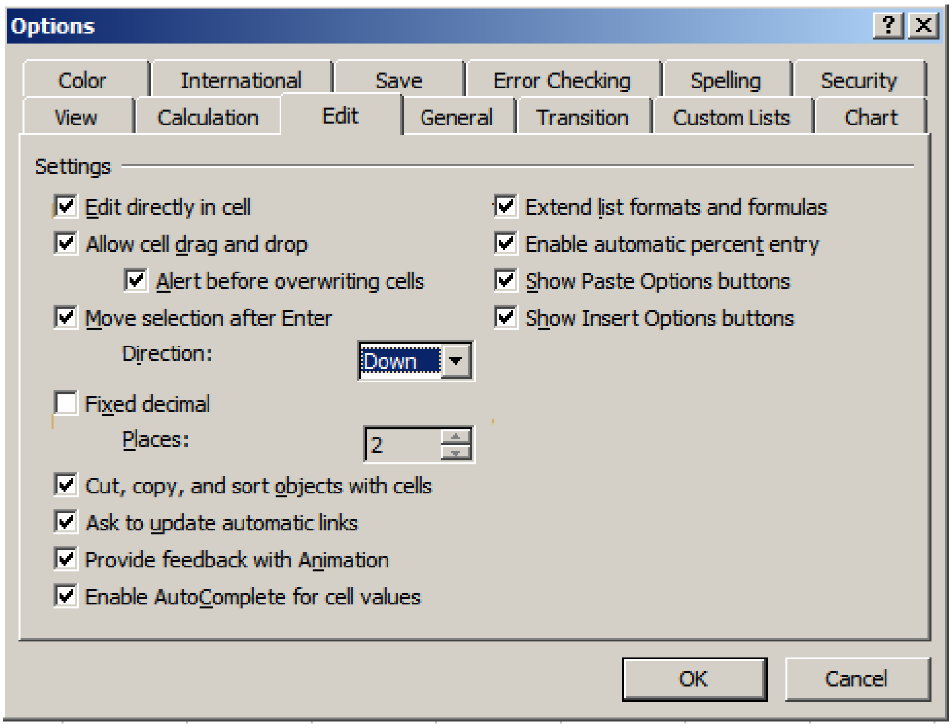
Make Excel less jumpy

The default installation of Excel has the cursor go down one cell each time you press [Enter].



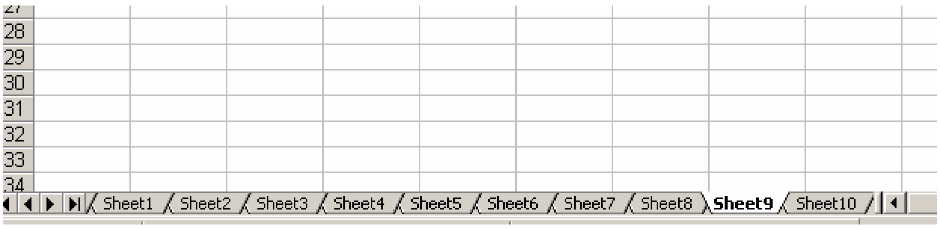
This is great for accountants, who have to enter lots of data. But we're finance people, and we make lots of mistakes! We want to stay on the cell we just entered, so we can correct it, and so we want to turn this feature off.

How? Press **Tools|Options** on the menu bar. Then go to the **Edit** tab and *unclick* the **Move selection after Enter** box. In the picture below, this box is still clicked (this is the default):



The number of sheets in a workbook

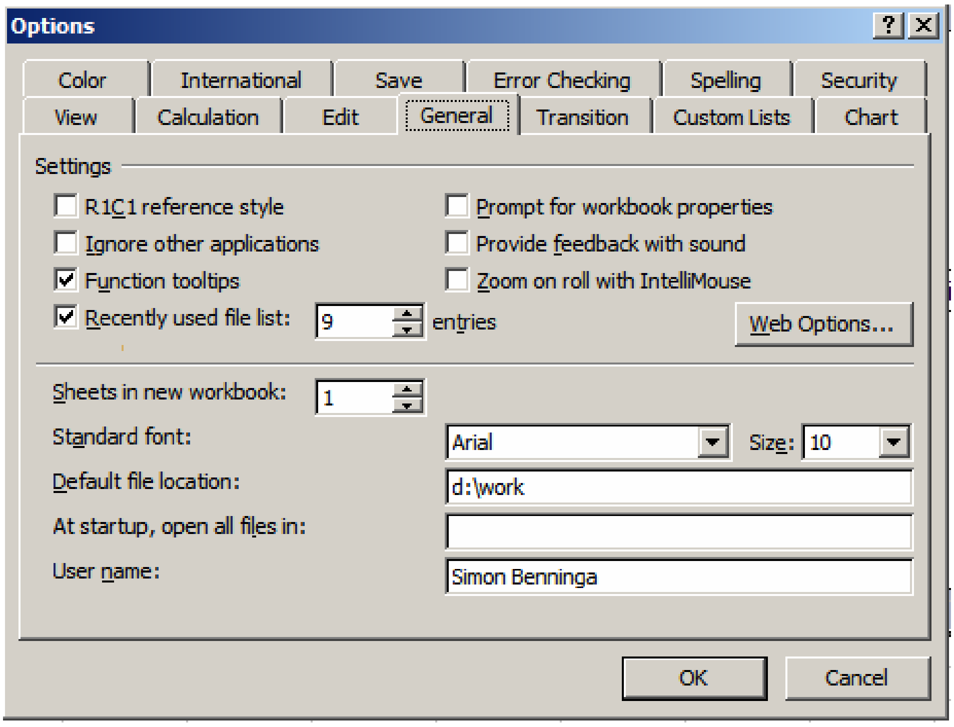
The default installation for Excel starts each new workbook with three spreadsheets.[2] This means that the bottom of your screen looks like:



Each of these sheets can be separately programmed and also separately named (see below). But the fact remains that most users use only one sheet per workbook. We suggest that you change

[2] Nomenclature: Microsoft calls an Excel file (the thing you saved as “Garbage.xls”) a *workbook*. The individual sheets of the workbook are called *spreadsheets* or *worksheets*. Like many Excel users, we often mix up this nomenclature.

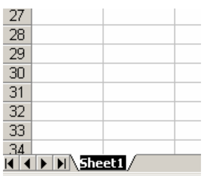
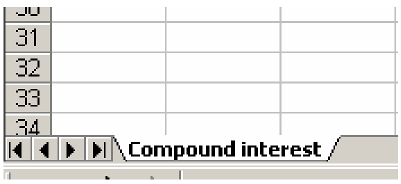
the defaults so that Excel starts a new workbook with only one spreadsheet (you can always add more). To do this, go to **Tools|Options** and click on the **General** tab:



In the picture above we've changed the **Sheets in new workbook** to "1."

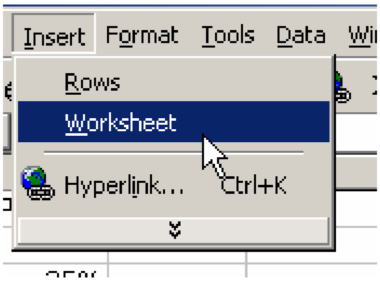
Naming a sheet

To name a sheet, double click on the sheet tab. You can now type in the name you want for the sheet:

Before	After
	

Adding more sheets

To add more sheets, go to **Insert|Worksheet**



You can also delete a sheet (**Edit|Delete sheet**). This is an *irreversible* action, so we suggest you save the workbook before you doing this.

27.7. Using a function

Excel contains many functions. In this section we illustrate a few of these.[3] We'll go back to the spreadsheet in Section 27.3. In cell B17 we'll calculate the average value of the cells B5:B15 (this has very little economic meaning ...). The final product will look like this:

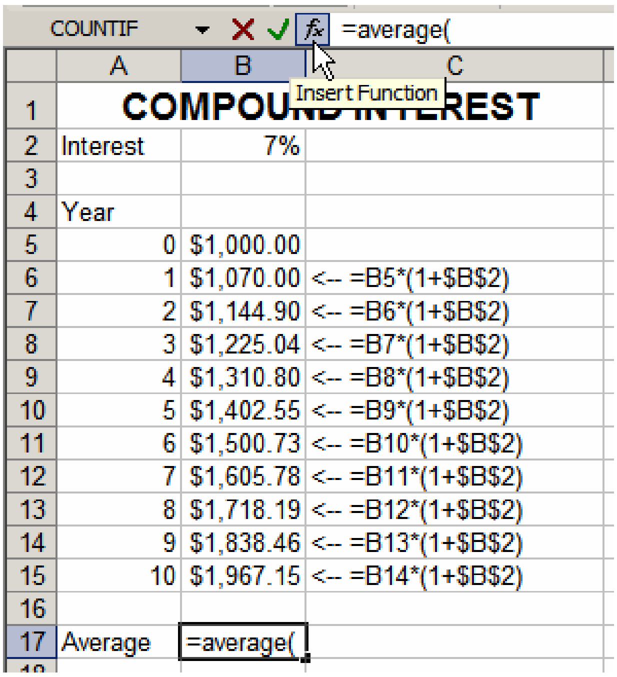
[3] The discussion in this section is really preliminary and intended to give you a taste of how Excel functions work. In this book we use many Excel functions. Chapter 29 discusses most of the functions used in the book and Chapter 31 discusses Excel's date functions.

	A	B	C
1	COMPOUND INTEREST		
2	Interest	7%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,070.00	<-- =B5*(1+$B$2)
7	2	$1,144.90	<-- =B6*(1+$B$2)
8	3	$1,225.04	<-- =B7*(1+$B$2)
9	4	$1,310.80	<-- =B8*(1+$B$2)
10	5	$1,402.55	<-- =B9*(1+$B$2)
11	6	$1,500.73	<-- =B10*(1+$B$2)
12	7	$1,605.78	<-- =B11*(1+$B$2)
13	8	$1,718.19	<-- =B12*(1+$B$2)
14	9	$1,838.46	<-- =B13*(1+$B$2)
15	10	$1,967.15	<-- =B14*(1+$B$2)
16			
17	Average	$1,434.87	<-- =AVERAGE(B5:B15)

To do this:

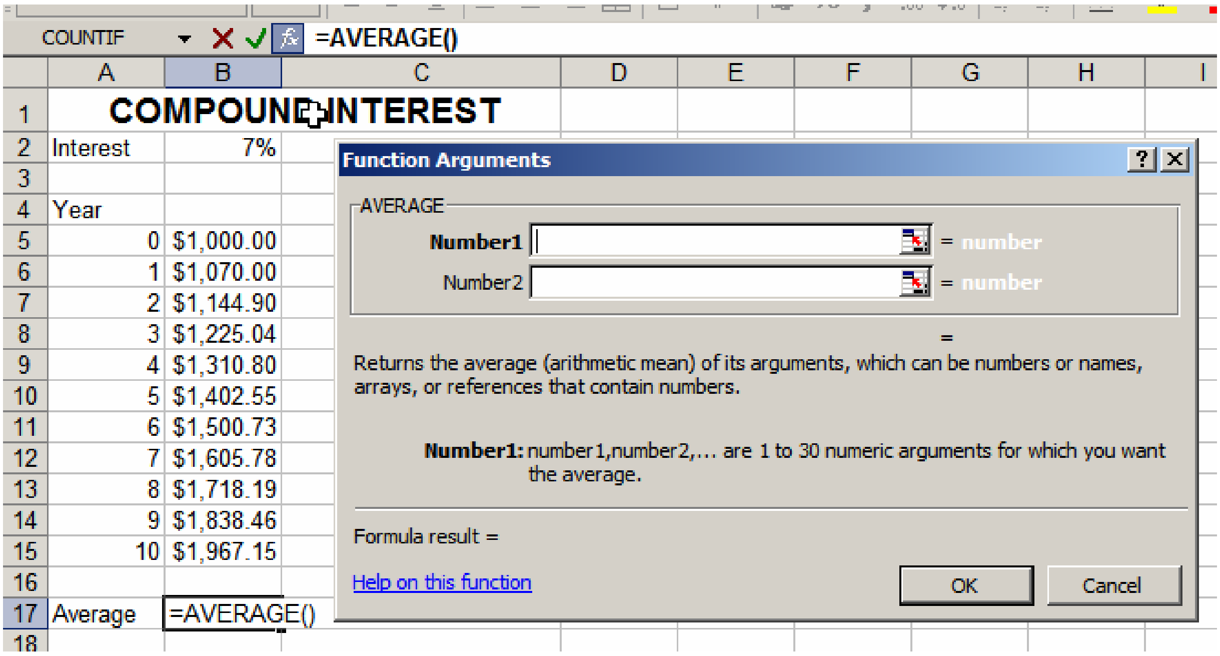
- In cell A17 we type "Average." This is known as "annotating the spreadsheet." In simple English—tell yourself what you're doing, because otherwise you'll forget.

  In cell B17, we type "**=Average(**", and then hit the f_x sign on the toolbar:

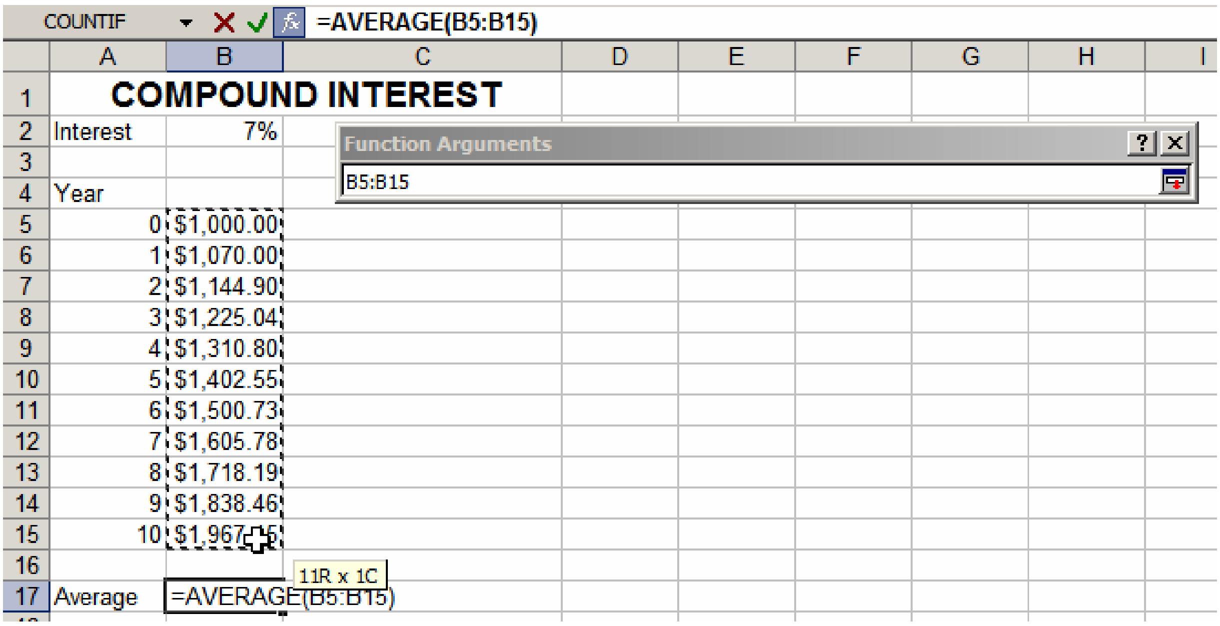


	A	B	C
1	COMPOUND INTEREST		
2	Interest	7%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,070.00	<-- =B5*(1+$B$2)
7	2	$1,144.90	<-- =B6*(1+$B$2)
8	3	$1,225.04	<-- =B7*(1+$B$2)
9	4	$1,310.80	<-- =B8*(1+$B$2)
10	5	$1,402.55	<-- =B9*(1+$B$2)
11	6	$1,500.73	<-- =B10*(1+$B$2)
12	7	$1,605.78	<-- =B11*(1+$B$2)
13	8	$1,718.19	<-- =B12*(1+$B$2)
14	9	$1,838.46	<-- =B13*(1+$B$2)
15	10	$1,967.15	<-- =B14*(1+$B$2)
16			
17	Average	=average(	

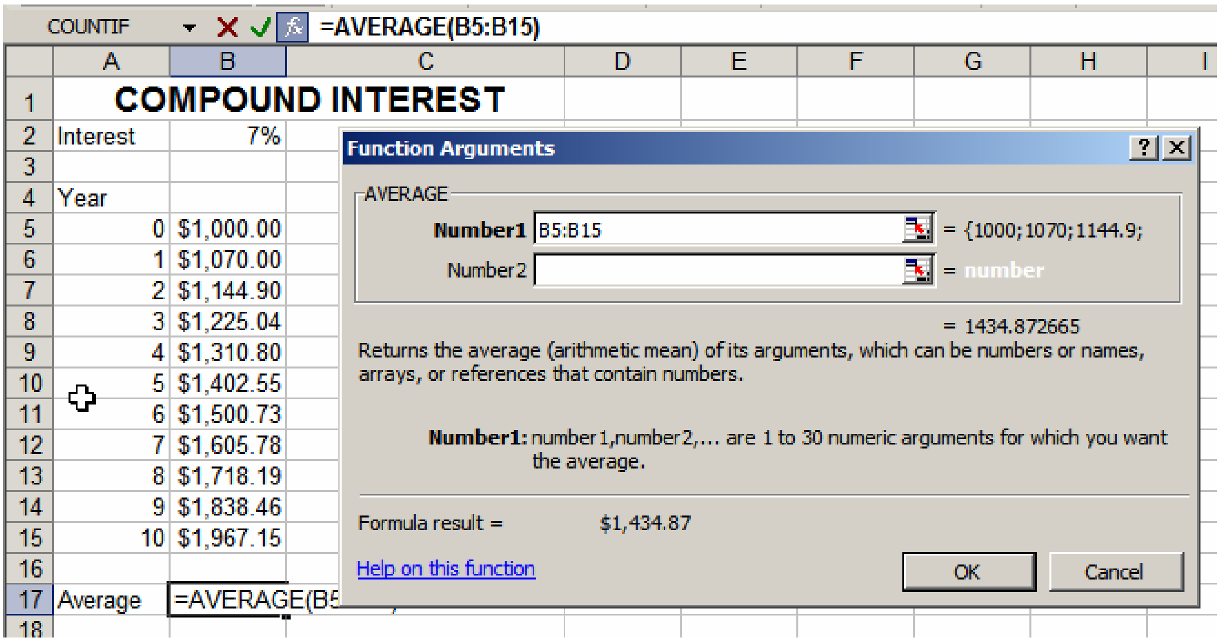
You'll see a *function dialogue box:*



You cursor is already in a box labeled **Number1**. Put the mouse on cell B5, push down the left mouse button, and drag the cursor to B15. Here's what you'll see:



Now let the cursor go:



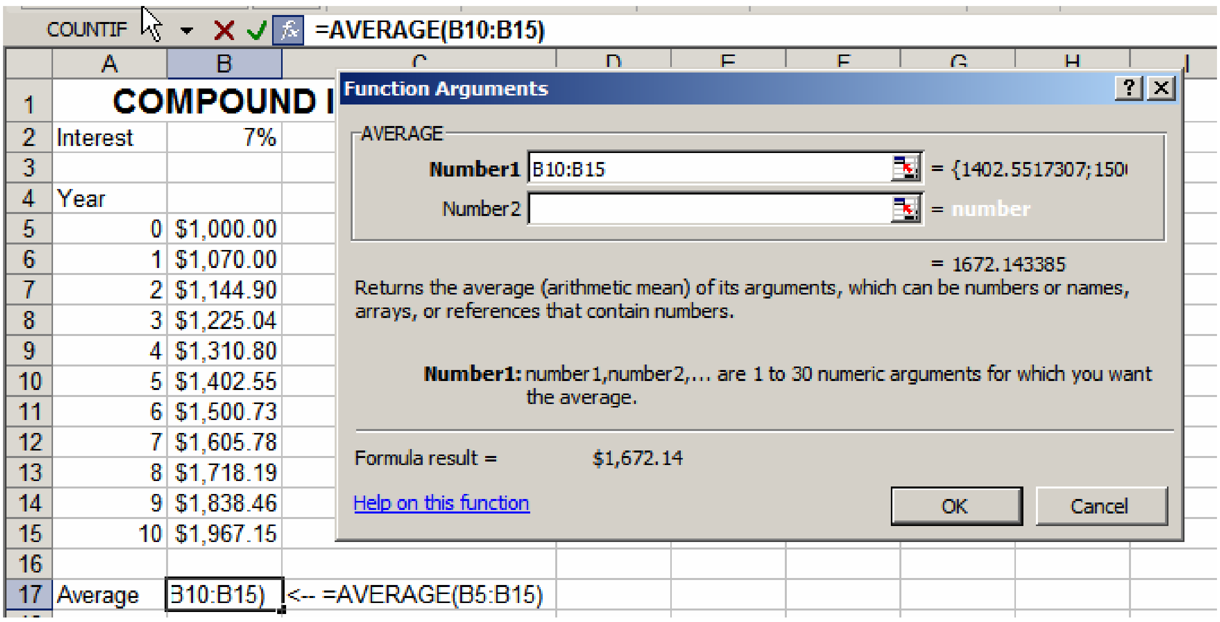
Now press **OK** in the dialog box. Here's the result:

	A	B	C
1	COMPOUND INTEREST		
2	Interest	7%	
3			
4	Year		
5	0	$1,000.00	
6	1	$1,070.00	
7	2	$1,144.90	
8	3	$1,225.04	
9	4	$1,310.80	
10	5	$1,402.55	
11	6	$1,500.73	
12	7	$1,605.78	
13	8	$1,718.19	
14	9	$1,838.46	
15	10	$1,967.15	
16			
17	Average	$1,434.87	<-- =AVERAGE(B5:B15)

Suppose you didn't want to average all the numbers, but only those from years 5-10. There are two ways to do this:

- You can double click on cell B17, and change the range in the formula to **=Average(B10:B15)**.

- You can double-click B17, and re-click the f_x sign on the toolbar. This reopens the dialog box. Now click the next to the range currently being averaged:



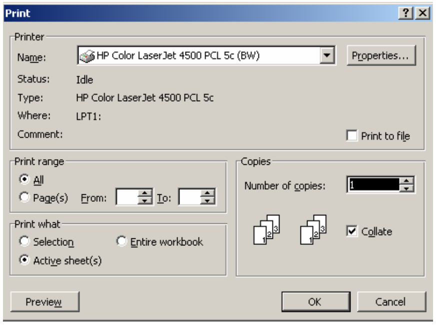
You can now indicate the range (B10:B15) you want to average. A couple of [Enters] should give you the result.

Practice makes perfect

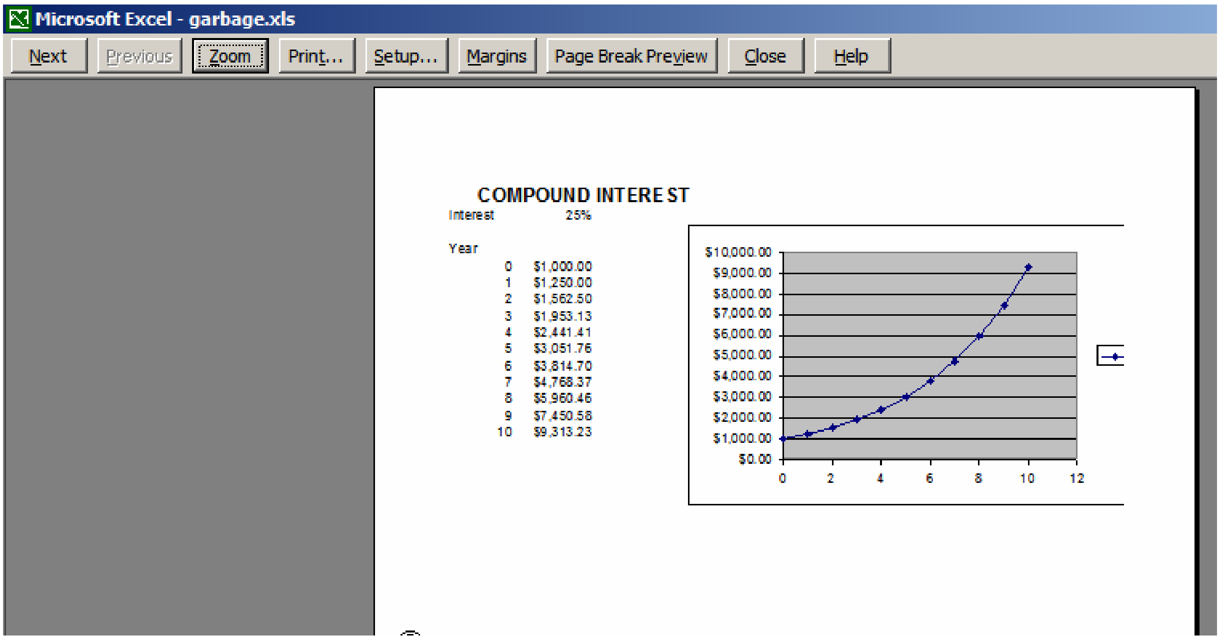
The exercises to this chapter let you practice with a few functions that work like **Average**.

27.8. Printing

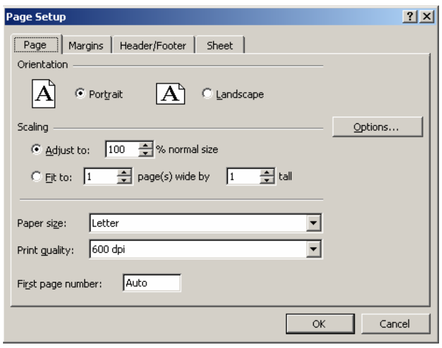
You've just completed your beautiful first spreadsheet and you want to print it. Press **File|Print**. This brings up the following screen:

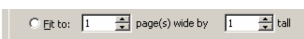


Before printing, press the **Preview** box:

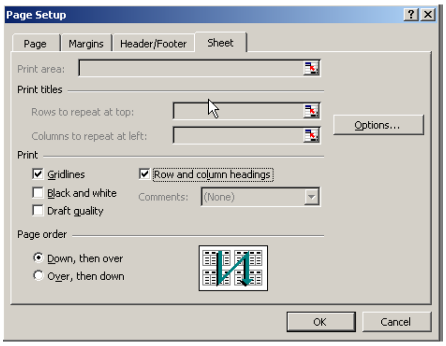


Notice that the graph is a bit cut off at the right edge. Press **Setup** and explore the various tabs:

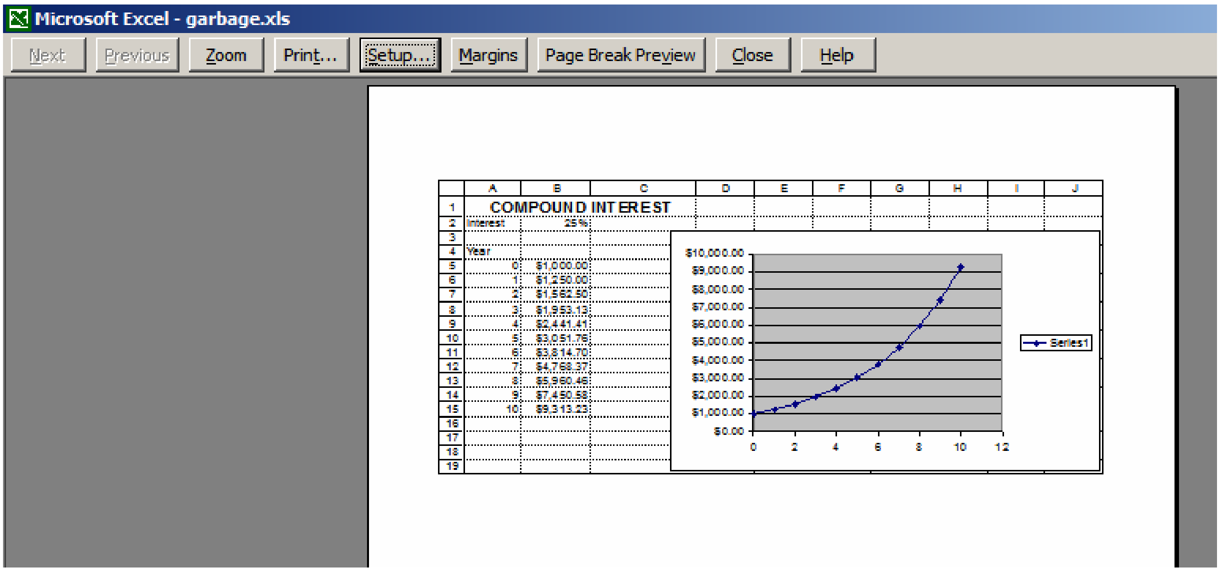


In the **Page** tab, choose  to fit everything onto one sheet. (You could also put it on **Landscape** paper, using the  button on the same tab.)

On the **Sheet** tab, you can choose to print the spreadsheet using **Gridlines** and **Row and column headings** (these are the settings we've used for most of the spreadsheets in this book).



Now click **OK** to see what the printing will look like:



If this suits your purpose, press **Print**.

Summary

In this chapter we've explored the preliminaries of Excel—how to set up a spreadsheet, save it, type in a formula, use a function, and print your results. The following chapters explore more advanced Excel techniques.

Exercises

Here are three functions which work just like **Average**:

- **Sum**—this function adds numbers in a range of cells
- **Count**—counts the number of non-blank cells in a range
- **Countblank**—counts the number of blank cells in a range

Play with these functions and see how they work.

CHAPTER 28: GRAPHS AND CHARTS IN EXCEL*

This version: August 25, 2004

Chapter contents

Overview

Excel has extensive facilities to do graphs.[1] If you're like most Excel users and finance majors, you'll be using these facilities a lot.

In this short chapter, we'll discuss the basics of graphing, assuming that—by and large—you already know how to make a chart in Excel. We will also discuss some less well-known techniques that have to do with charts:

- Making a graph with non-contiguous data series
- Changing the axis parameters of a chart
- Making a chart where the title changes when the data changes

28.1. The basics of Excel charts

Every Excel chart has its origins in the data on a spreadsheet:

	A	B	C	D
1	**MERCK & CO. 1991-2000**			
2		**Dividends**	**Purchase of treasury stock**	**Proceeds from exercise of stock options**
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641

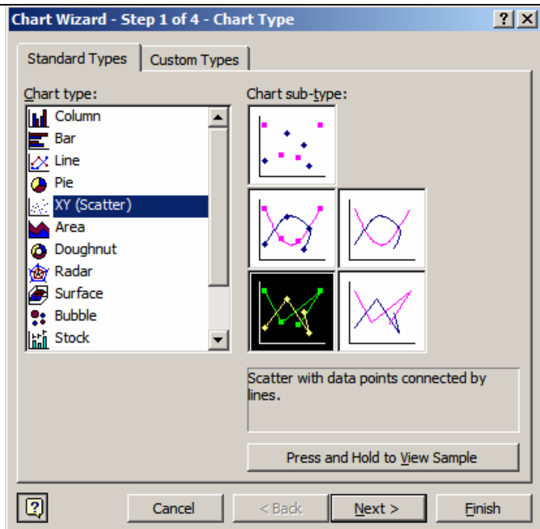
[1] In "Excelese" graphs are called "charts." We will use both words interchangeably.

To create a graph that shows the dividends paid each year, we mark the relevant data:

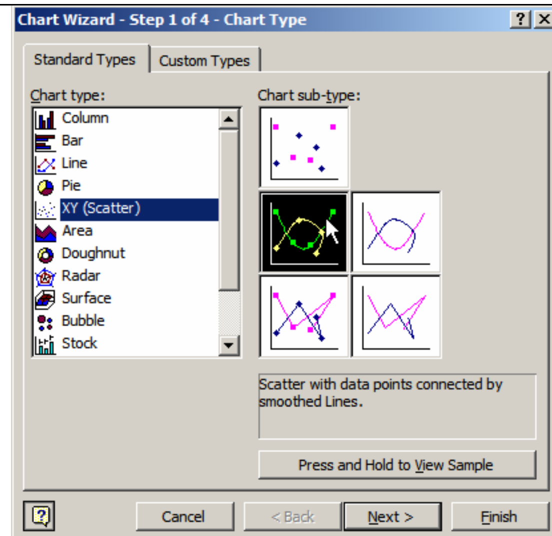
	A	B	C	D
1	MERCK & CO. 1991-2000			
2		Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641

Clicking on the chart icon  on the toolbar brings up the chart menu, which gives a bewildering variety of chart options. Being finance people, we're primarily interested in the XY (Scatter) chart option. We usually want to draw a connected line (shown here as the chosen "Chart sub-type"):

Two options for "connected" Excel XY Charts

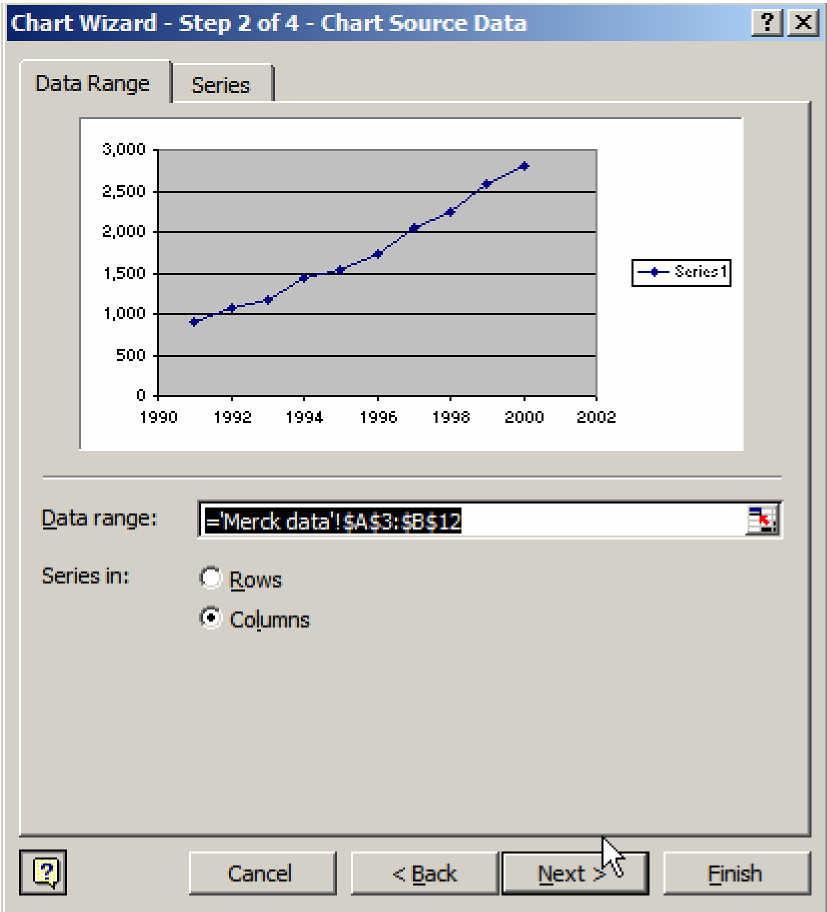


This creates a "jagged" XY chart (the points are connected by line segments. It is the option we generally use in this book

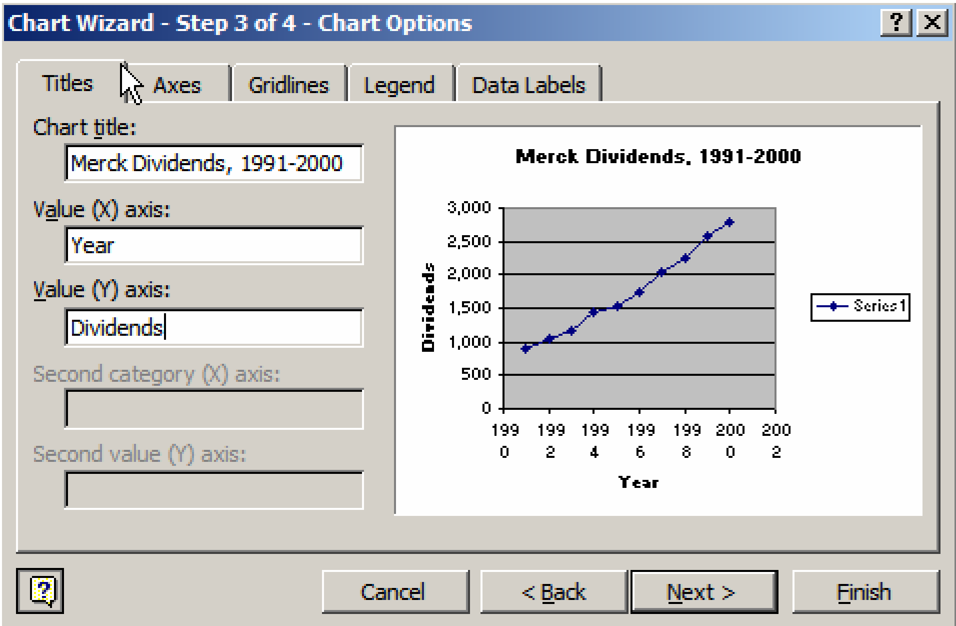


This smooths the lines connecting the points.

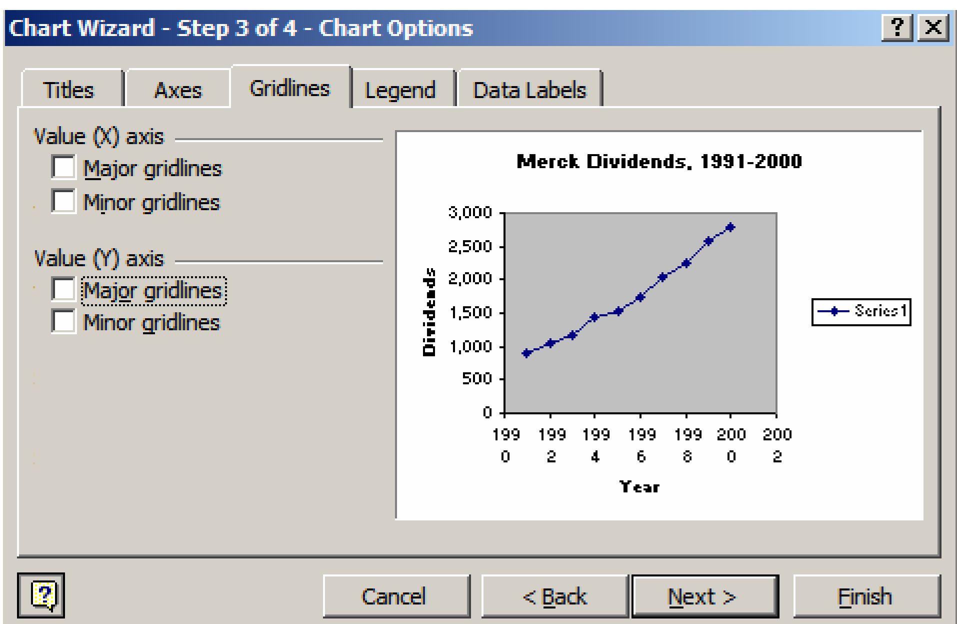
Going to the next step in the chart wizard, you'll see:



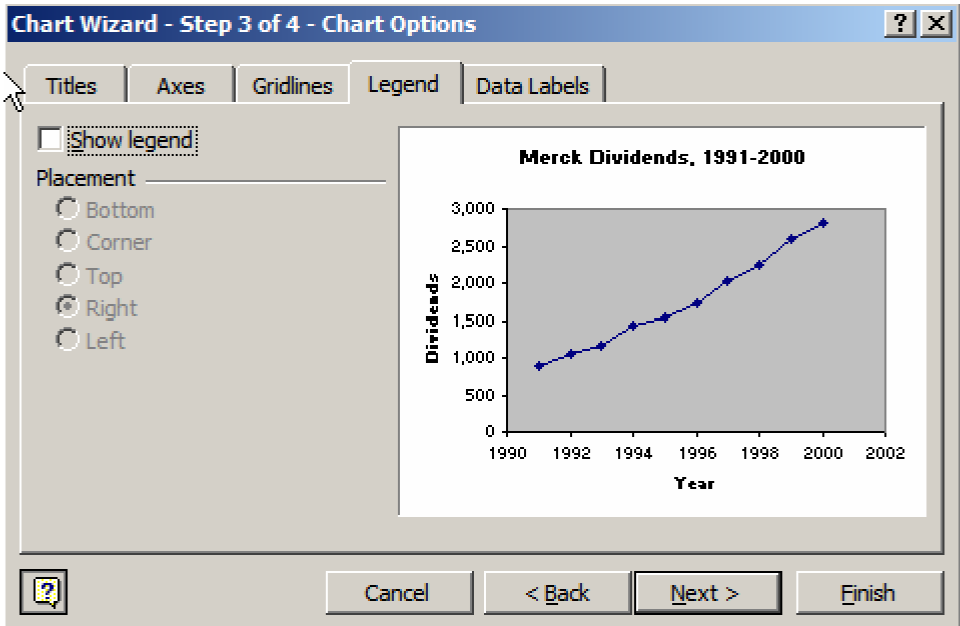
There's nothing much to do here, so press **Next** and go on to the next step, which allows you to annotate the graph with titles:



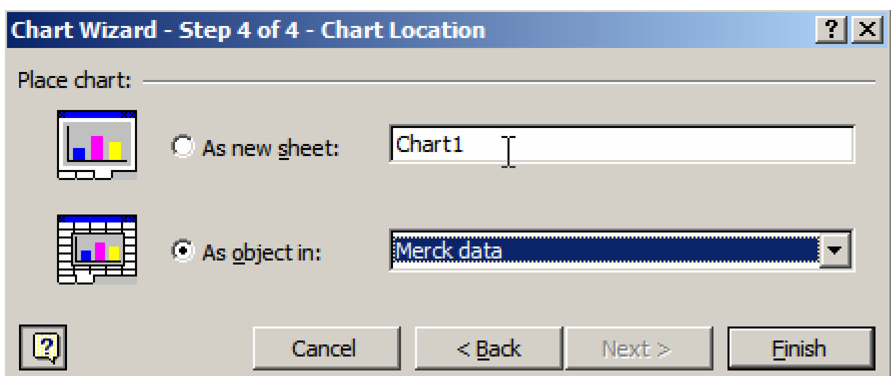
This author doesn't like gridlines of any sort!:



Nor does he like legends very much ... though sometimes there's room for one (see Section ???):

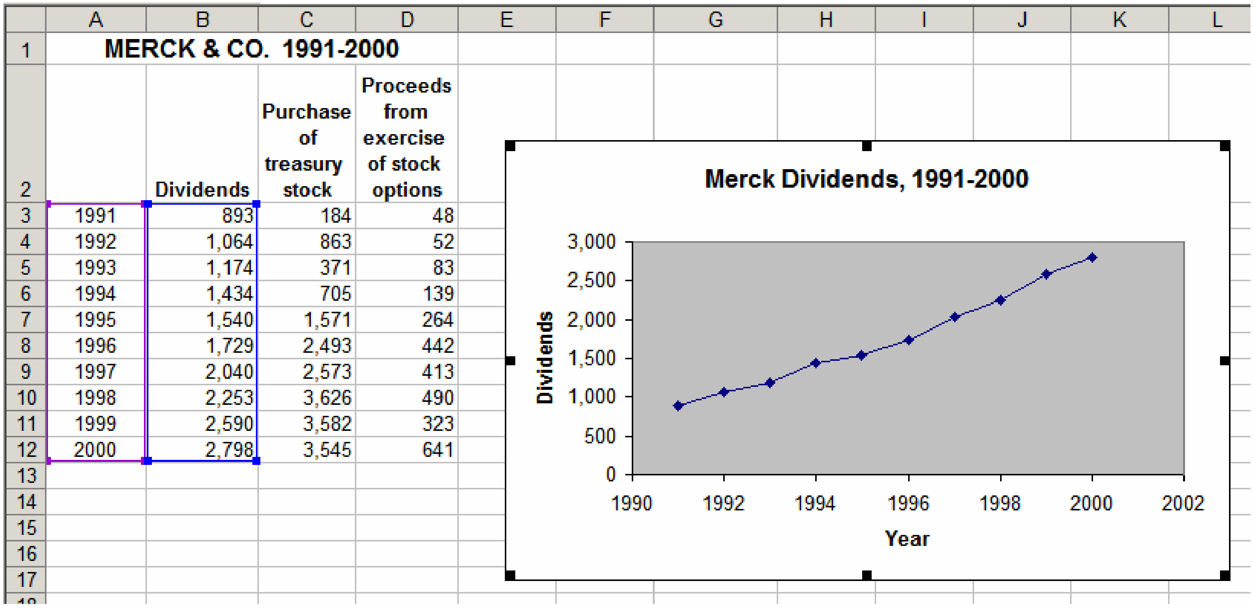


Pressing [Enter] gets you to:



Tell Excel where to put the graph (in this case, on the spreadsheet labeled "Merck data", which is also the spreadsheet where our data is stored.

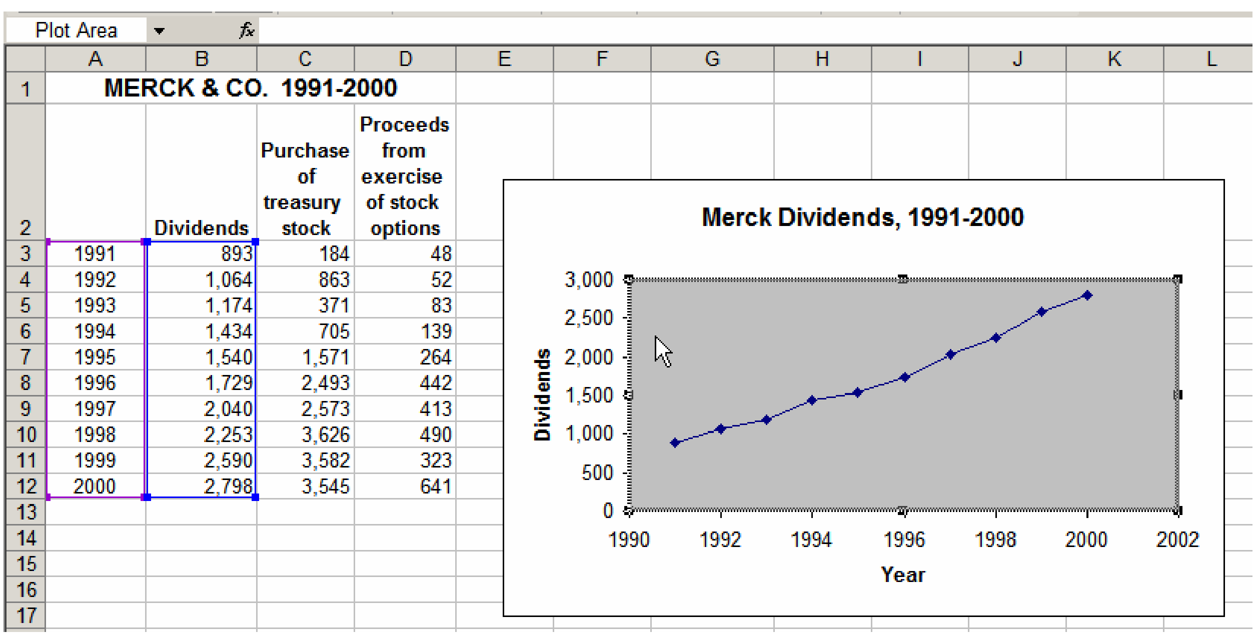
Pushing **Finish** gives the following graph:



MERCK & CO. 1991-2000			
	Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
1991	893	184	48
1992	1,064	863	52
1993	1,174	371	83
1994	1,434	705	139
1995	1,540	1,571	264
1996	1,729	2,493	442
1997	2,040	2,573	413
1998	2,253	3,626	490
1999	2,590	3,582	323
2000	2,798	3,545	641

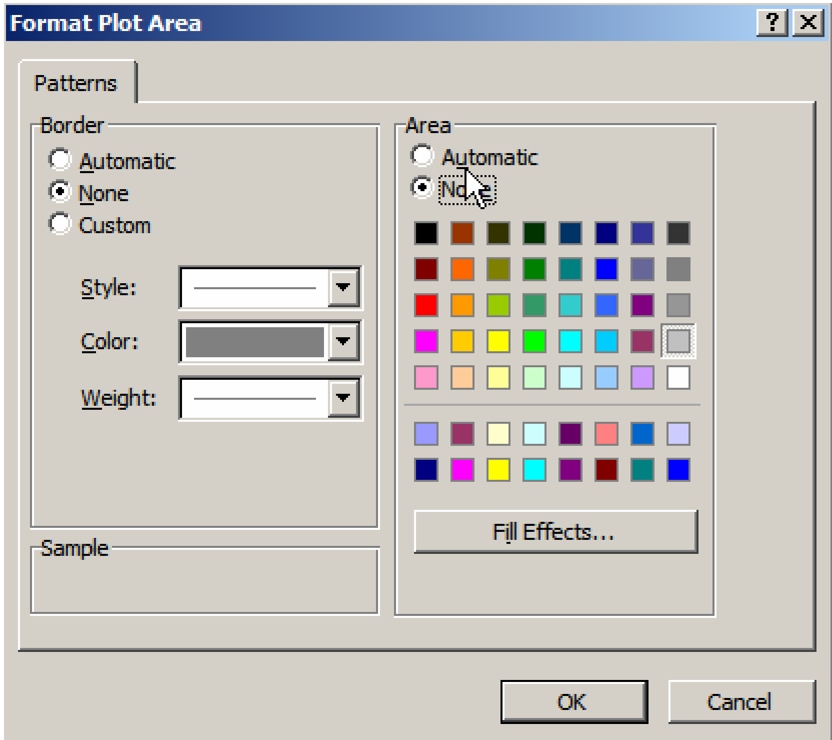
More changes

Change 1: The Excel default graph has a murky gray area where the data is graphed. This looks alright on the screen, but it looks terrible when you print it. All the graphs in this book have this -gray graph area blanked out. To do this, mark the graph area:



MERCK & CO. 1991-2000			
	Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
1991	893	184	48
1992	1,064	863	52
1993	1,174	371	83
1994	1,434	705	139
1995	1,540	1,571	264
1996	1,729	2,493	442
1997	2,040	2,573	413
1998	2,253	3,626	490
1999	2,590	3,582	323
2000	2,798	3,545	641

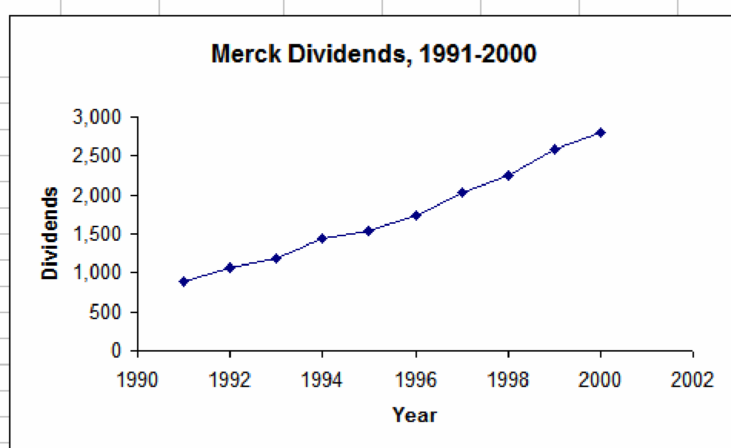
Now double-click on the graph area; this brings up the following box:



In the **Format Plot Area** box above, we always mark **Border—None** and **Area—None.**

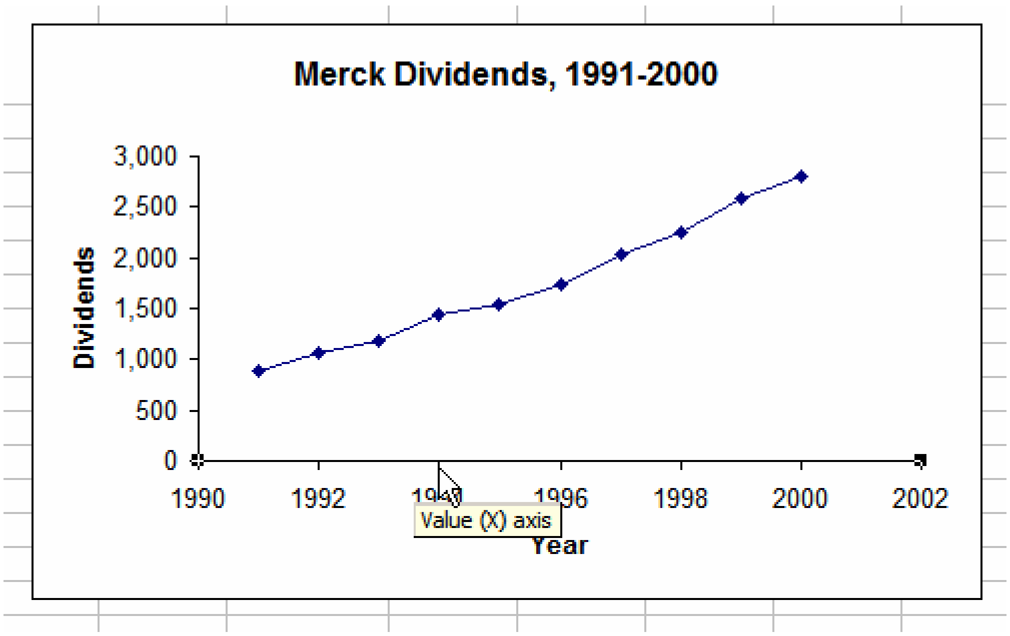
Here's the result:

MERCK & CO. 1991-2000			
	Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
1991	893	184	48
1992	1,064	863	52
1993	1,174	371	83
1994	1,434	705	139
1995	1,540	1,571	264
1996	1,729	2,493	442
1997	2,040	2,573	413
1998	2,253	3,626	490
1999	2,590	3,582	323
2000	2,798	3,545	641

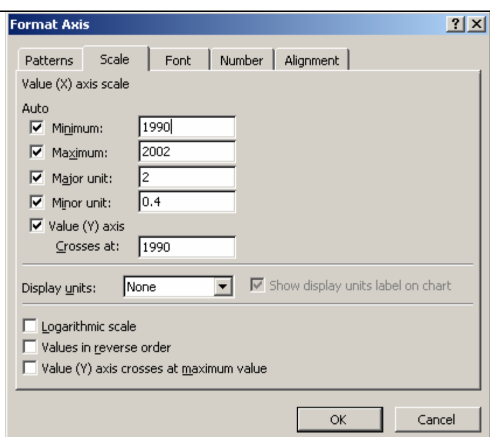


One more change

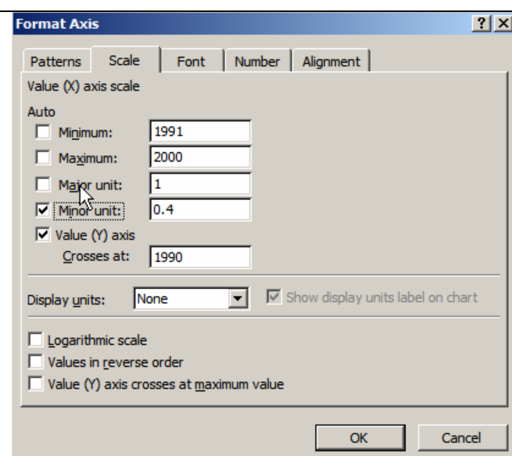
Although our data only goes from 1991 – 2000, the x-axis on our chart goes from 1990 to 2002. To change this, mark the x-axis of the graph with a gentle click on the left mouse button:



(Notice the square marks at either end of the x-axis.) Now right-click with the mouse and

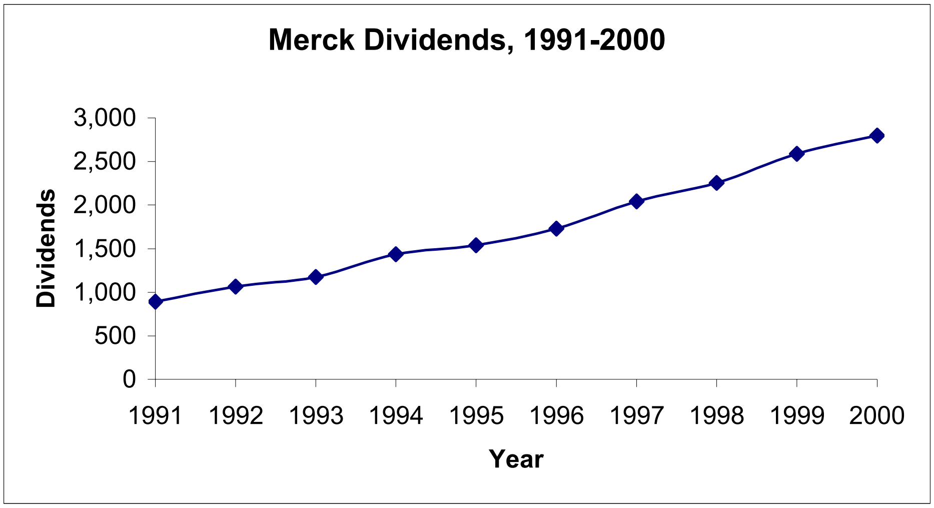


Before: A checked box indicates the Excel defaults. At this point the chart is set to show every other year on the x-axis (**Major unit** = 2). **Minor unit** indicates the number of ticks between the major units (not relevant here).



After. Note that we've changed both the **Minimum** and the **Maximum**, as well as the **Major Unit**.

Here's the result:

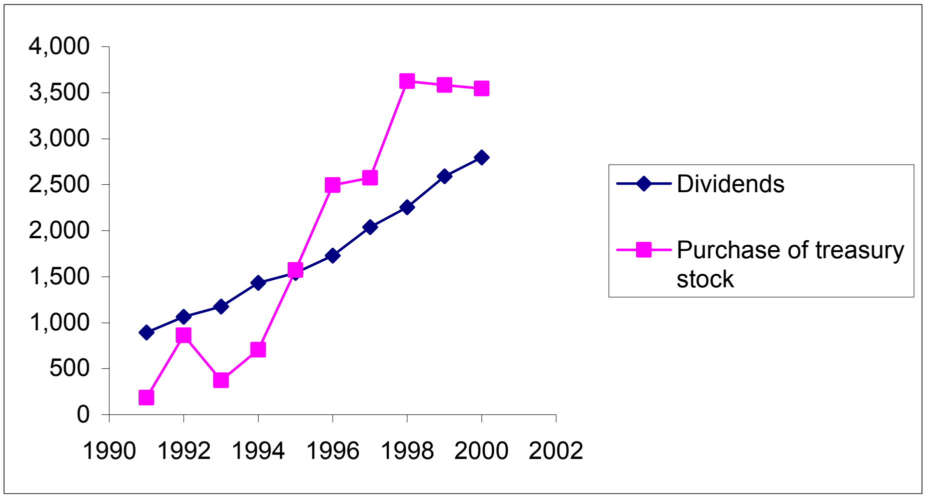


28.2. Creative use of legends

If you build your XY chart with data that includes legends, then Excel will generally transfer them in the proper way to the graph. Here's an example: We've marked the data to include the column headings:

	A	B	C	D
1	MERCK & CO. 1991-2000			
2		Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641
13				
14				

Here's the resulting graph:



28.3. Graphing non-contiguous data

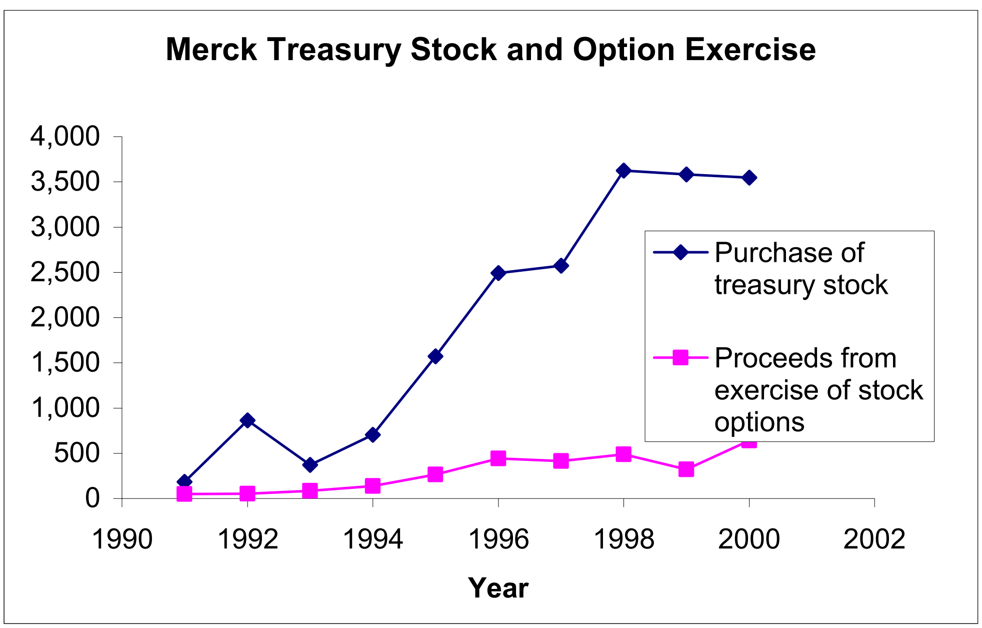
Suppose you want to make a graph of columns A, C and D of the Merck data. To mark these three columns:

- Mark the first column (that is, press the left mouse button and "paint" cells A3:A12)
- Press the [Ctrl] key and mark columns C and D (again, pressing the left mouse button).

At this point your spreadsheet looks like this:

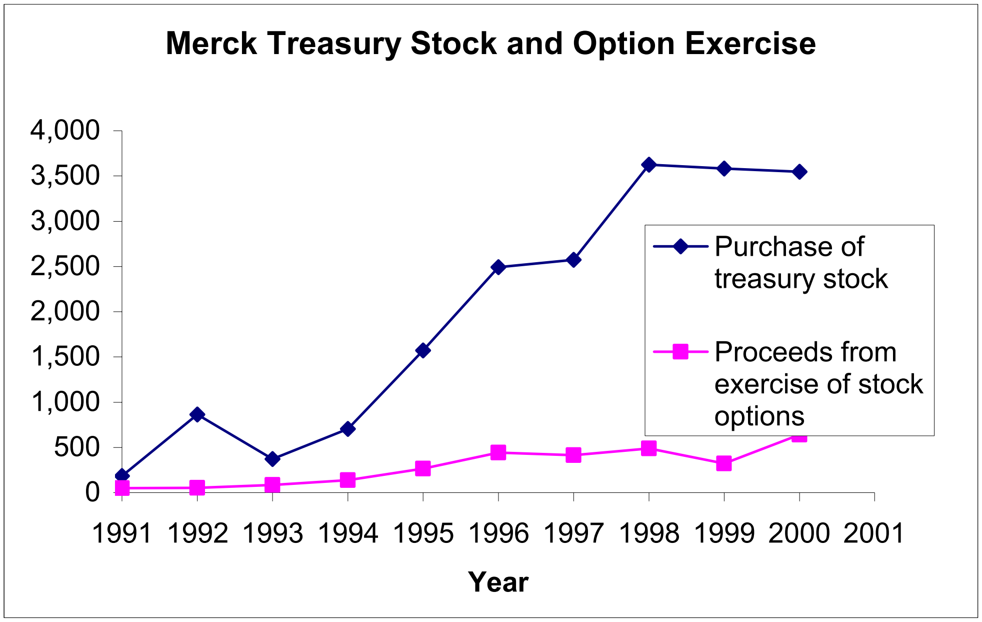
	A	B	C	D
1	**MERCK & CO. 1991-2000**			
2		Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641
13				

You can now follow the regular graphing procedure to create the following chart:

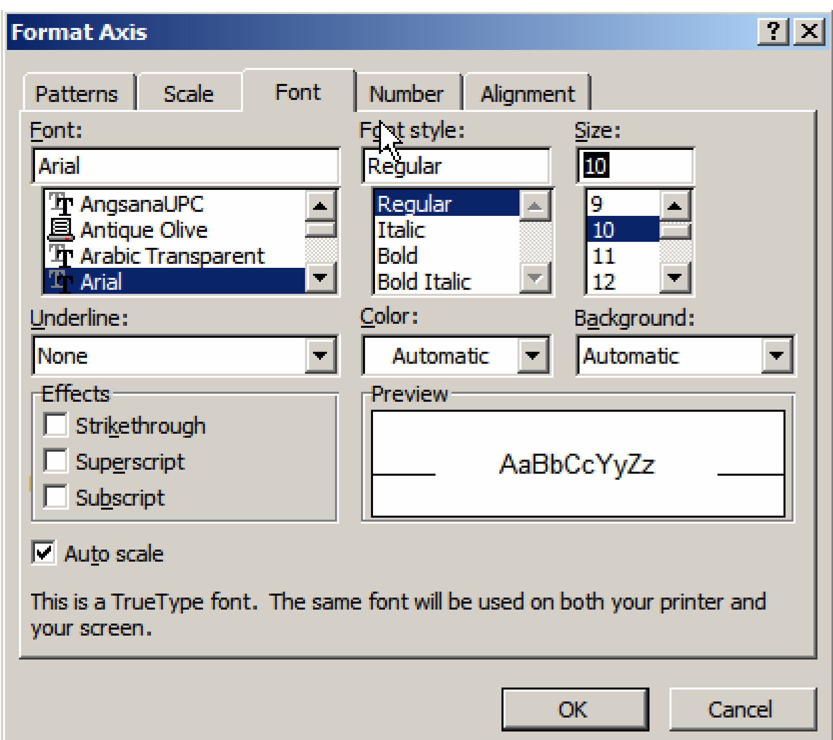


Fine-tuning—changing font size so that the axis labels fit

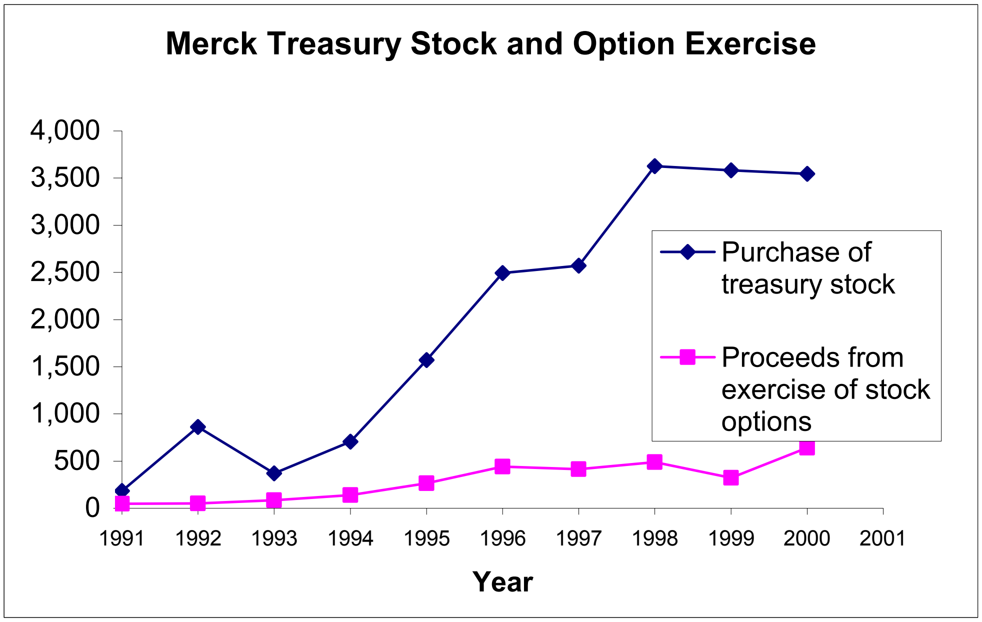
Look at the *x*-axis above: It goes from 1990 to 2002 even though the data only goes from 1991 – 2000. This often happens when Excel creates an *x*-axis for a graph. We've already shown how to use the **Format axis** menu to change the axis. But this time when we do this, the x-axis labels don't fit properly:



Go back into the dialog box and hit the **Font** tab to change the size of the *x*-axis font:



Now the graph looks fine:



(There are other ways to accomplish this trick also—if you make the chart bigger, for example.)

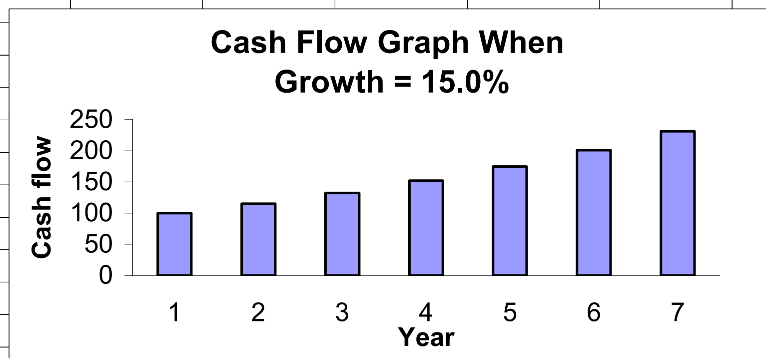
28.4. Graph titles that update[2]

You want to have the graph title change when a parameter on the spreadsheet changes. For example, in the next spreadsheet, you want the graph title to indicate the growth rate.

---

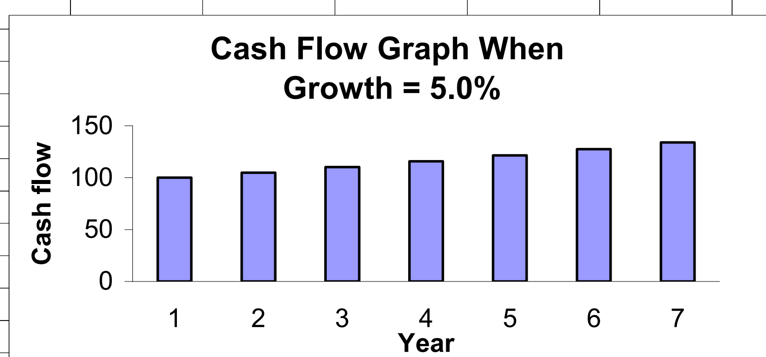
[2] This section makes (largely self-explanatory) use of the **Text** function, which is discussed in Chapter 27.

	A	B	C	D	E	F	G	H	I
1	**GRAPH TITLES THAT UPDATE AUTOMATICALLY**								
2	Growth	15%							
3									
4	Year	Cash flow							
5	1	100.00							
6	2	115.00	<-- B6*(1+$B$3)						
7	3	132.25							
8	4	152.09							
9	5	174.90							
10	6	201.14							
11	7	231.31							
12									
13									
14									
15									
16									
17									
18									



Once we have completed the necessary steps explained below, a change in the growth rate will change both the graph *and* its title:

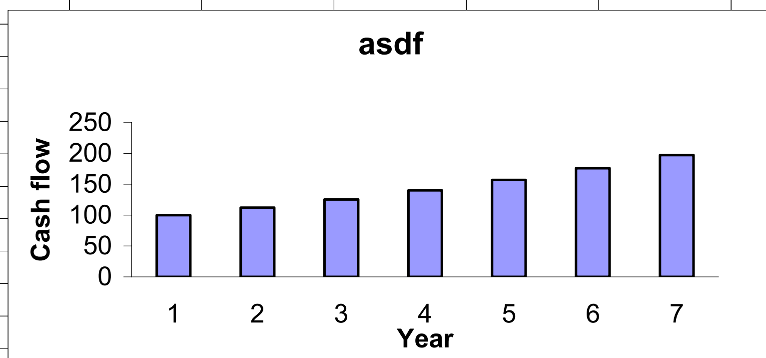
	A	B	C	D	E	F	G	H	I
1	**GRAPH TITLES THAT UPDATE AUTOMATICALLY**								
2	Growth	5%							
3									
4	Year	Cash flow							
5	1	100.00							
6	2	105.00	<-- B6*(1+$B$3)						
7	3	110.25							
8	4	115.76							
9	5	121.55							
10	6	127.63							
11	7	134.01							
12									
13									
14									
15									
16									
17									
18									



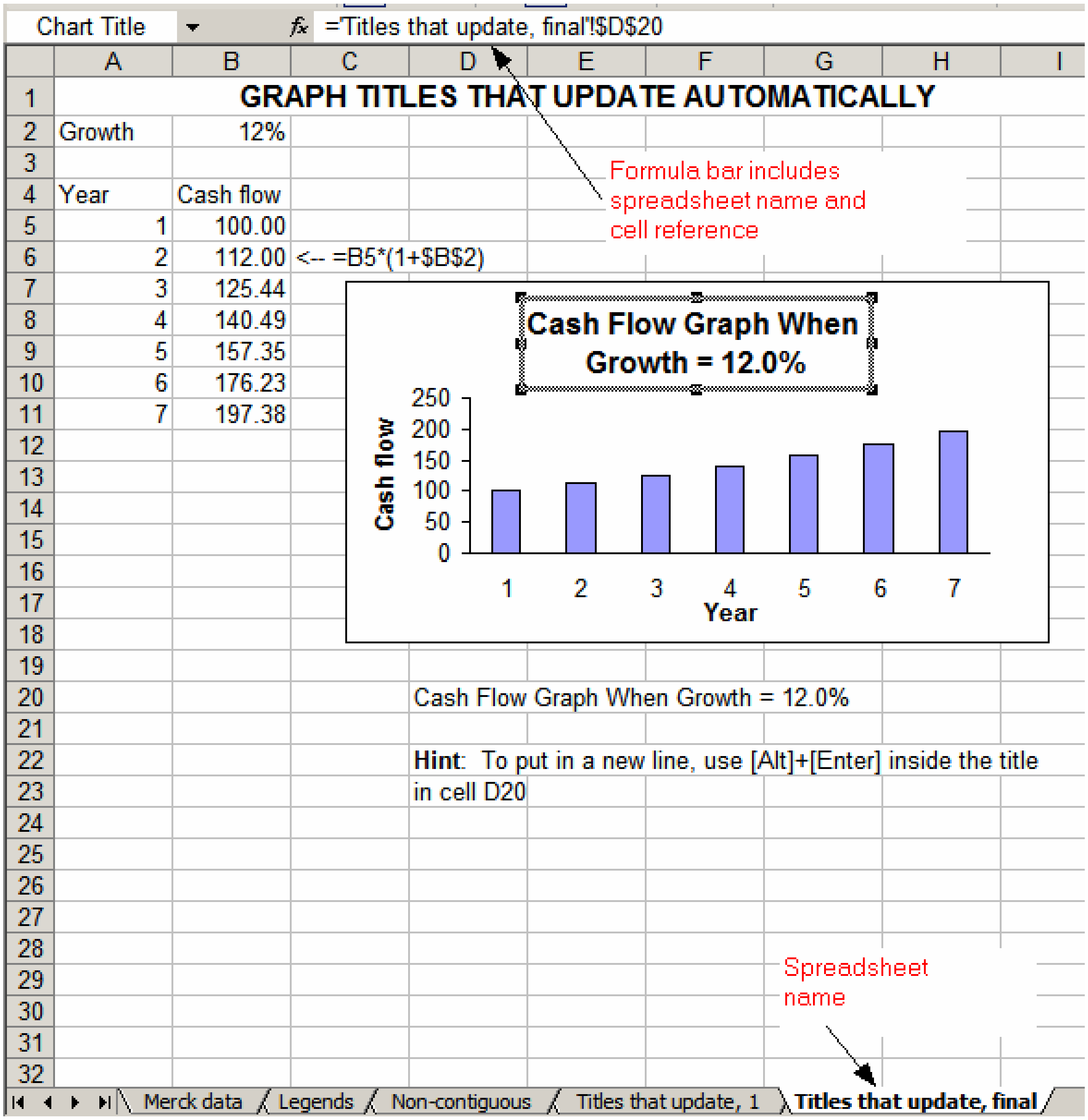
To make graph titles update automatically, carry out the following steps:

- Create the graph you want in the format you want it. Give the graph a "proxy title." (It makes no difference what, you're going to eliminate it soon.) At this stage your graph might look like:

	A	B	C	D	E	F	G	H	I
1	**GRAPH TITLES THAT UPDATE AUTOMATICALLY**								
2	Growth	12%							
3									
4	Year	Cash flow							
5	1	100.00							
6	2	112.00	<-- =B5*(1+$B$2)						
7	3	125.44							
8	4	140.49							
9	5	157.35							
10	6	176.23							
11	7	197.38							
12									
13									
14									
15									
16									
17									
18									
19									
20				Cash Flow Graph When Growth = 12.0%					



- Create the title you want in a cell. In the example above, cell D20 contains the formula: ="Cash Flow Graph when Growth = "&TEXT(B2,"0.0%").
- Click on the graph title to mark it, and then go to the formula bar and insert an equal sign to indicate a formula. Then **point** at cell D20 with the formula and click [Enter]. In the picture below, you see the chart title highlighted and in the formula bar "=Titles that update!$D$20" indicating the title of the graph. Note that "Titles that update" is the name of the spreadsheet.



Summary

There's lots more you can do with Excel charts, but we've covered the essentials. The exercises to this chapter will show you some more variations.

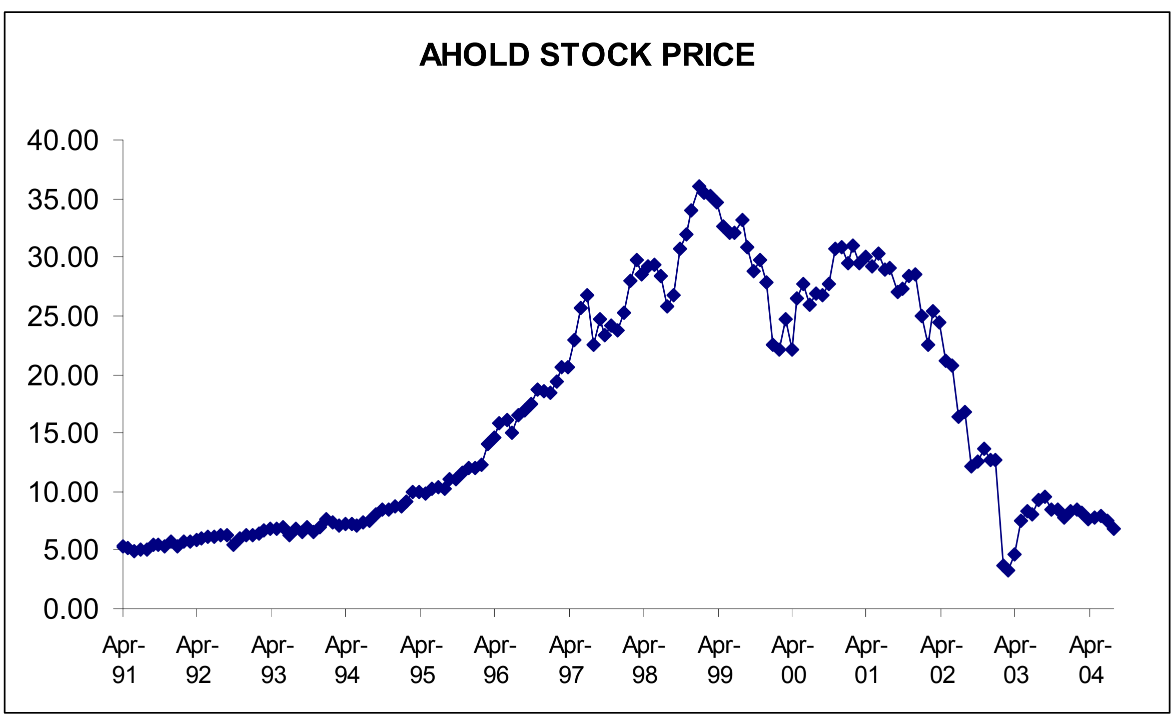
Exercises

Note: All the data for the exercises is on the CD-ROM which accompanies *Principles of Finance with Excel*.

1. The CD gives the monthly prices for the Dutch grocery chain Ahold from April 1991 through August 2004. Graph these prices.

	A	B
1	**PRICE OF AHOLD STOCK April 1991 - August 2004**	
2	**Date**	**Stock price**
3	8-Apr-91	5.32
4	1-May-91	5.23
5	3-Jun-91	4.94
6	1-Jul-91	5.03
7	1-Aug-91	5.09
8	3-Sep-91	5.43
9	1-Oct-91	5.40
10	1-Nov-91	5.37
11	2-Dec-91	5.68
12	2-Jan-92	5.39

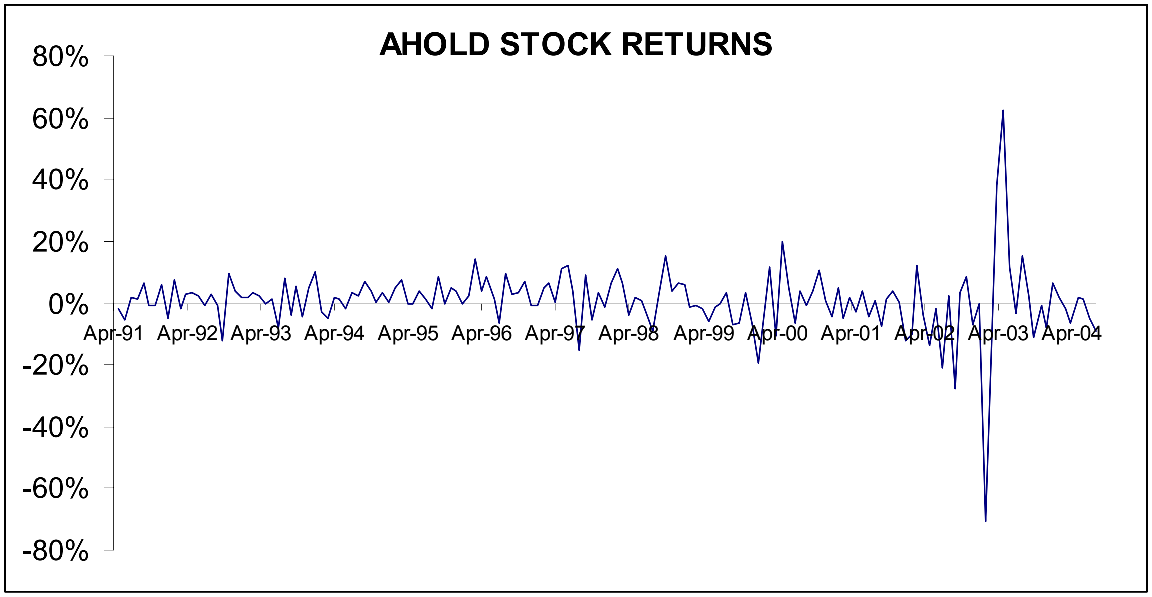
Your graph should look like this:



2. Using the data for Ahold from the previous exercise, determine the monthly stock returns and graph them. The monthly return for a stock which has price P_t in month t and price P_{t-1} in month t-1 is $\frac{P_t}{P_{t-1}}-1$. (When you compute the returns, you'll have "non-contiguous data," so that you'll have to use the technique described in Section 28.3.)

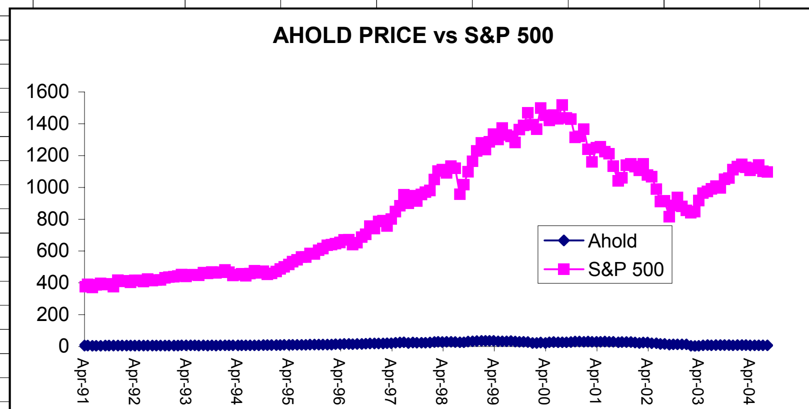
	A	B	C	D
1	**RETURNS ON AHOLD STOCK April 1991 - August 2004**			
2	**Date**	**Stock price**	**Monthly return**	
3	8-Apr-91	5.32		
4	1-May-91	5.23	-1.69%	<-- =B4/B3-1
5	3-Jun-91	4.94	-5.54%	<-- =B5/B4-1
6	1-Jul-91	5.03	1.82%	<-- =B6/B5-1
7	1-Aug-91	5.09	1.19%	
8	3-Sep-91	5.43	6.68%	
9	1-Oct-91	5.40	-0.55%	

Your graph should look like this:



3. The CD with the book gives the prices for Ahold and for the S&P 500. Use this data to produce the following graph (see note following the graph):

	A	B	C
1	**AHOLD'S STOCK PRICE VERSUS THE S&P 500**		
2			
3	**Date**	**Ahold**	**S&P 500**
4	8-Apr-91	5.32	375.34
5	1-May-91	5.23	389.83
6	3-Jun-91	4.94	371.16
7	1-Jul-91	5.03	387.81
8	1-Aug-91	5.09	395.43
9	3-Sep-91	5.43	387.86
10	1-Oct-91	5.40	392.45
11	1-Nov-91	5.37	375.22
12	2-Dec-91	5.68	417.09
13	2-Jan-92	5.39	408.78
14	3-Feb-92	5.80	412.70
15	2-Mar-92	5.69	403.69
16	1-Apr-92	5.86	414.95
17	1-May-92	6.06	415.35
18	1-Jun-92	6.19	408.14
19	1-Jul-92	6.14	424.21
20	3-Aug-92	6.32	414.03
21	1-Sep-92	6.26	417.80

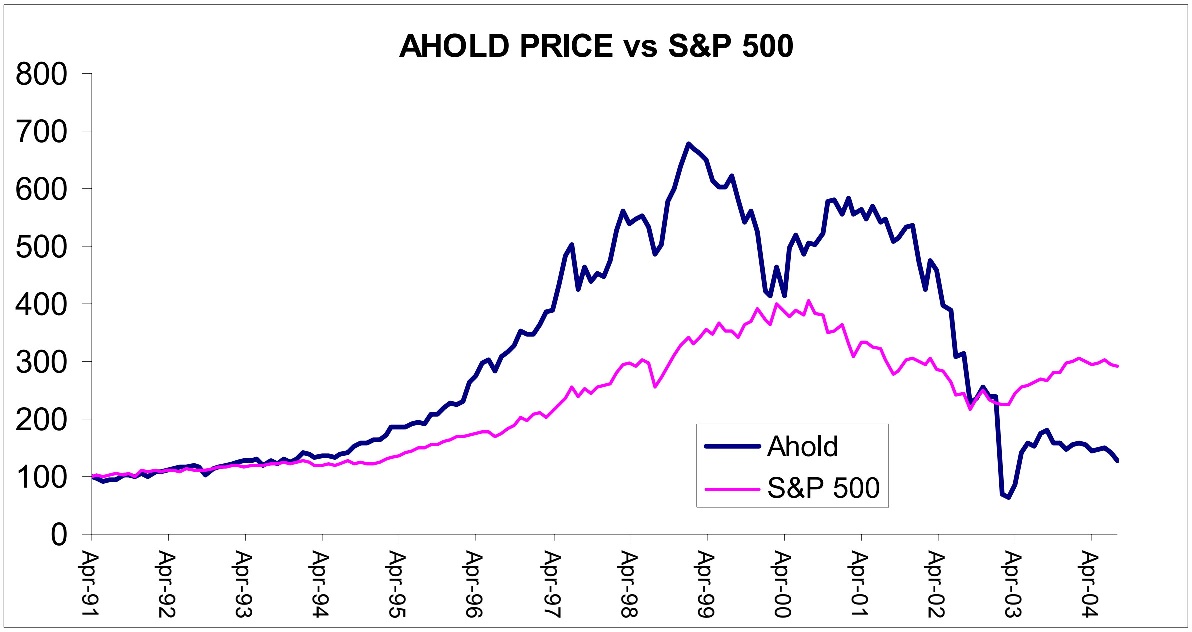


Note: This graph is obviously unsatisfactory—Ahold's price is so much less than the S&P's that the Ahold price series appears to be zero. See the next exercise for one solution to this problem.

4. Transform the S&P and Ahold price data so that the beginning price of each is 100 and graph these series:

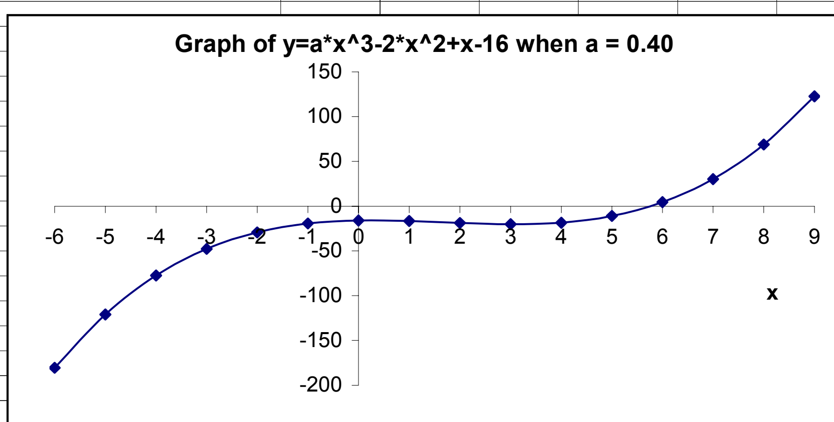
	A	B	C	D	E	F	G
1	**AHOLD'S STOCK PRICE VERSUS THE S&P 500**						
2							
3	**Date**	**Ahold**	**S&P 500**		**Ahold adjusted**	**S&P adjusted**	
4	8-Apr-91	5.32	375.34		100.00	100.00	
5	1-May-91	5.23	389.83		98.31	103.86	<-- =F4*C5/C4
6	3-Jun-91	4.94	371.16		92.86	98.89	<-- =F5*C6/C5
7	1-Jul-91	5.03	387.81		94.55	103.32	<-- =F6*C7/C6
8	1-Aug-91	5.09	395.43		95.68	105.35	

The final result should look like this:



5. You have been asked to graph the function $y = ax^3 - 2x^2 + x - 16$. The variable a can take on a variety of values (in the example below, $a = 0.4$). Make a graph of this function with a title that indicates the value of a, as illustrated below. (You may want to refer to Section 28.4.)

	A	B	C	D	E	F	G	H	I
1	a	0.4							
2									
3	x	y=a*x^3-2*x^2+x-16							
4	-6	-180.4	<-- =$B$1*A4^3-2*A4^2+A4-16						
5	-5	-121.0							
6	-4	-77.6							
7	-3	-47.8							
8	-2	-29.2							
9	-1	-19.4							
10	0	-16.0							
11	1	-16.6							
12	2	-18.8							
13	3	-20.2							
14	4	-18.4							
15	5	-11.0							
16	6	4.4							
17	7	30.2							
18	8	68.8							
19	9	122.6							
20									
21									
22									



CHAPTER 28: GRAPHS AND CHARTS IN EXCEL*

This version: February 1, 2003

Chapter contents

Introduction

Excel has extensive facilities to do graphs.[1] If you're like most Excel users and finance majors, you'll be using these facilities a lot.

In this short chapter, we'll discuss the basics of graphing, assuming that—by and large—you already know how to make a chart in Excel. We will also discuss some less well-known techniques that have to do with charts:

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[1] In "Excelese" graphs are called "charts." We will use both words interchangeably.

- Making a graph with non-contiguous data series
- Changing the axis parameters of a chart
- Making a chart where the title changes when the data changes

28.1. The basics of Excel charts

Every Excel chart has its origins in the data on a spreadsheet:

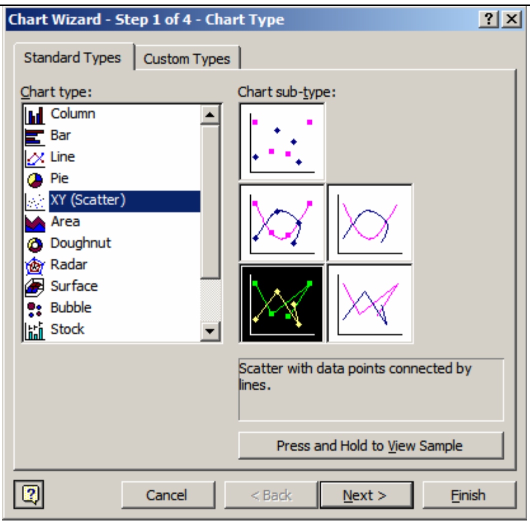
	A	B	C	D
1	**MERCK & CO. 1991-2000**			
2		**Dividends**	**Purchase of treasury stock**	**Proceeds from exercise of stock options**
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641

To create a graph that shows the dividends paid each year, we mark the relevant data:

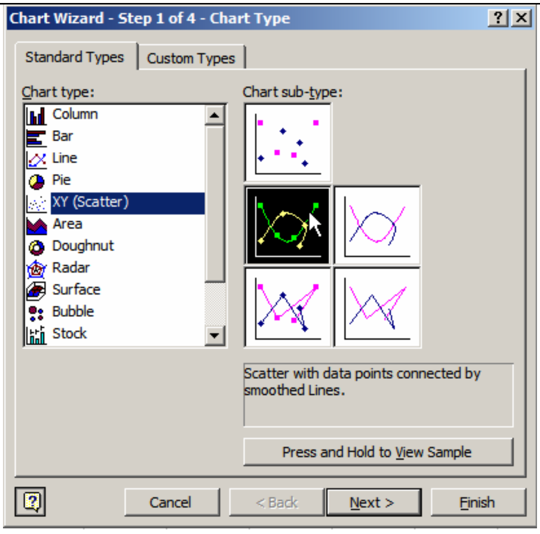
	A	B	C	D
1	**MERCK & CO. 1991-2000**			
2		**Dividends**	**Purchase of treasury stock**	**Proceeds from exercise of stock options**
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641

Clicking on the chart icon  on the toolbar brings up the chart menu, which gives a bewildering variety of chart options. Being finance people, we're primarily interested in the XY (Scatter) chart option. We usually want to draw a connected line (shown here as the chosen "Chart sub-type"):

Two options for “connected” Excel XY Charts

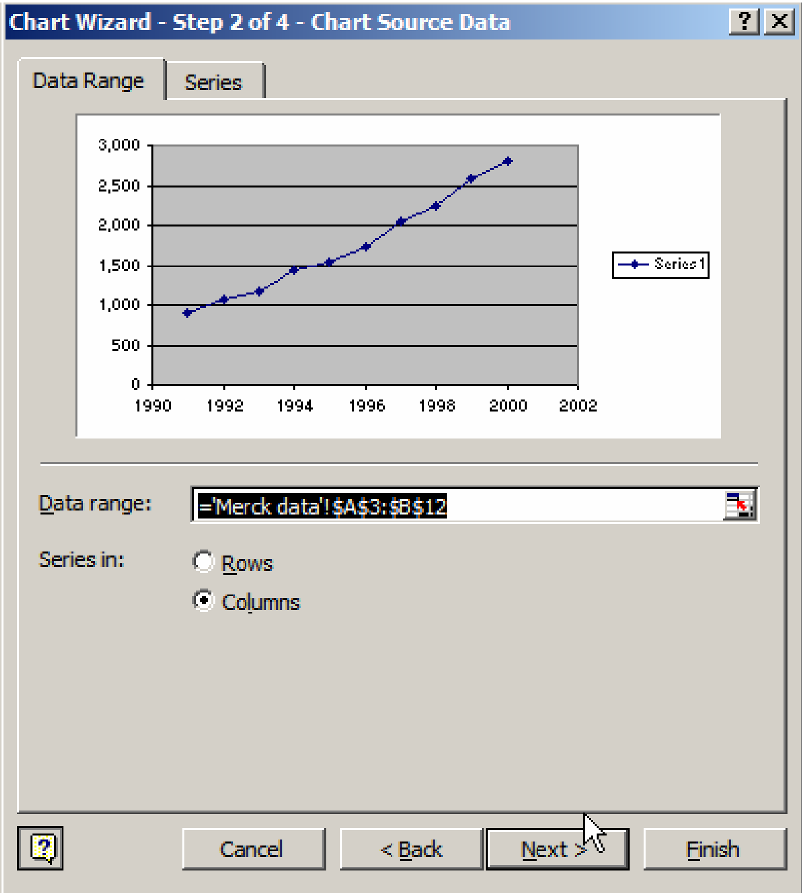


This creates a “jagged” XY chart (the points are connected by line segments. It is the option we generally use in this book

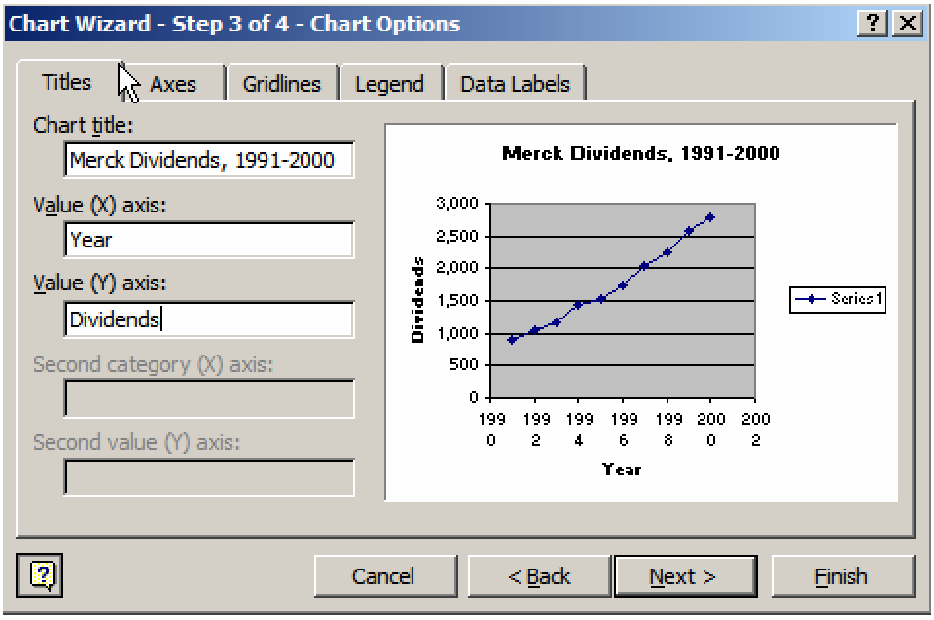


This smooths the lines connecting the points.

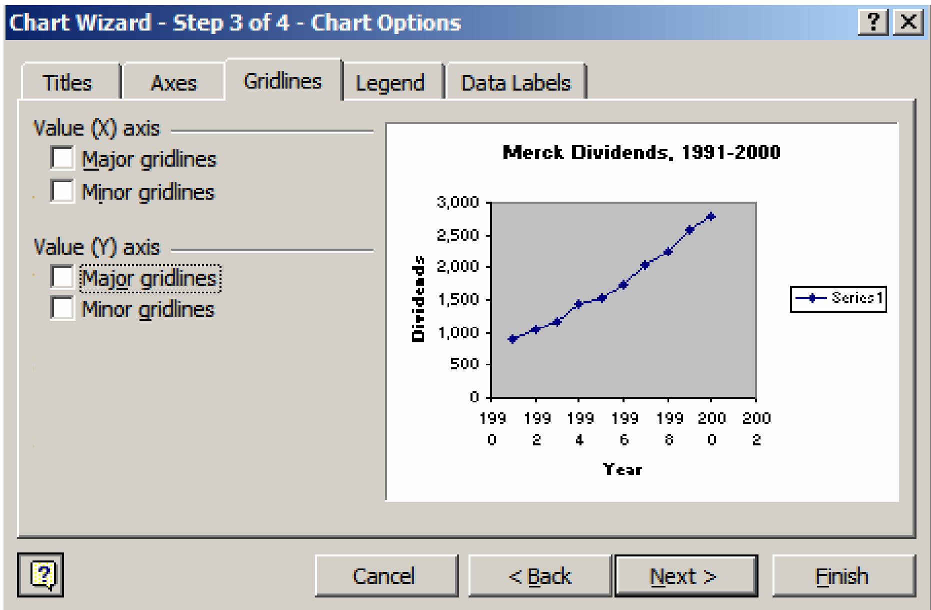
Going to the next step in the chart wizard, you’ll see:



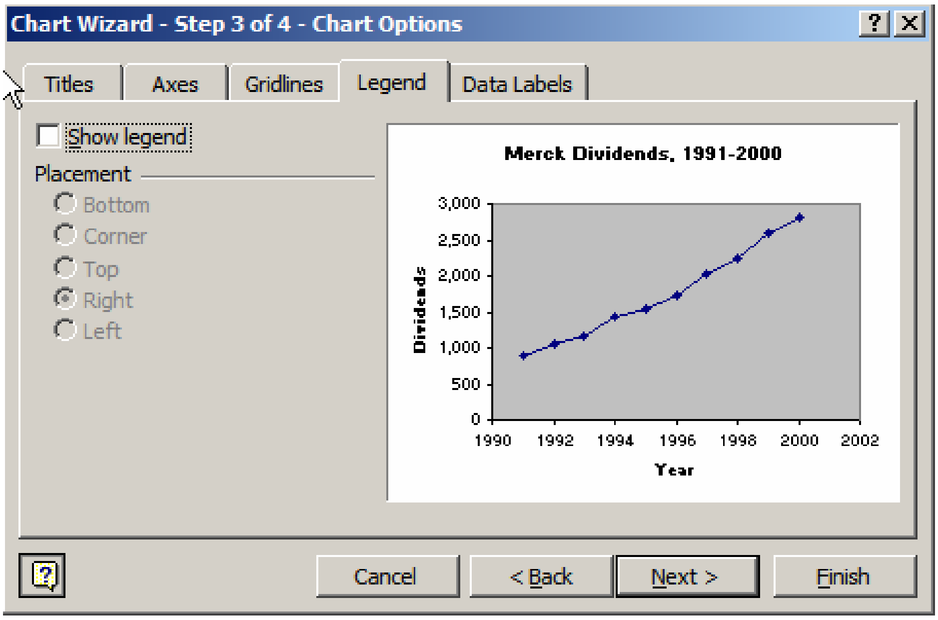
There's nothing much to do here, so press **Next** and go on to the next step, which allows you to annotate the graph with titles:



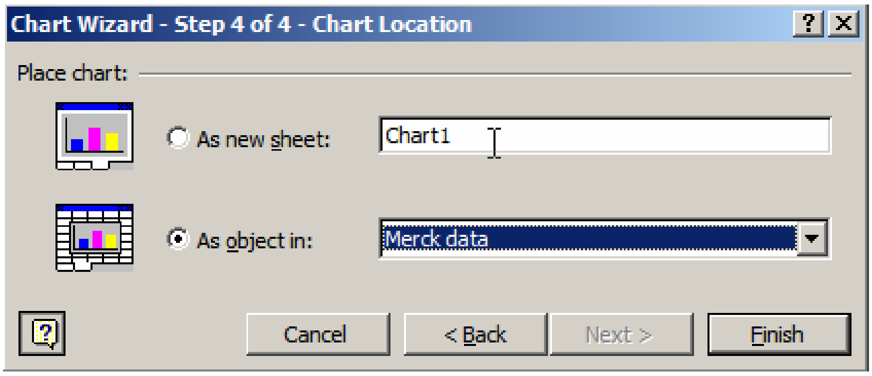
This author doesn’t like gridlines of any sort!:



Nor does he like legends very much ... though sometimes there’s room for one (see Section ???):

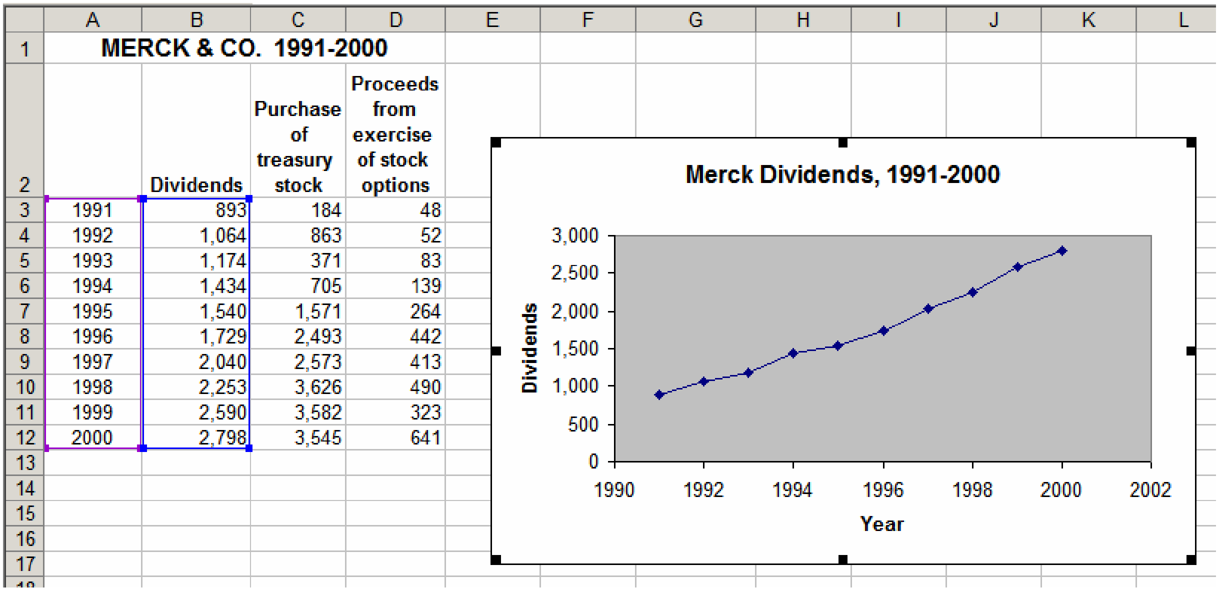


Pressing [Enter] gets you to:



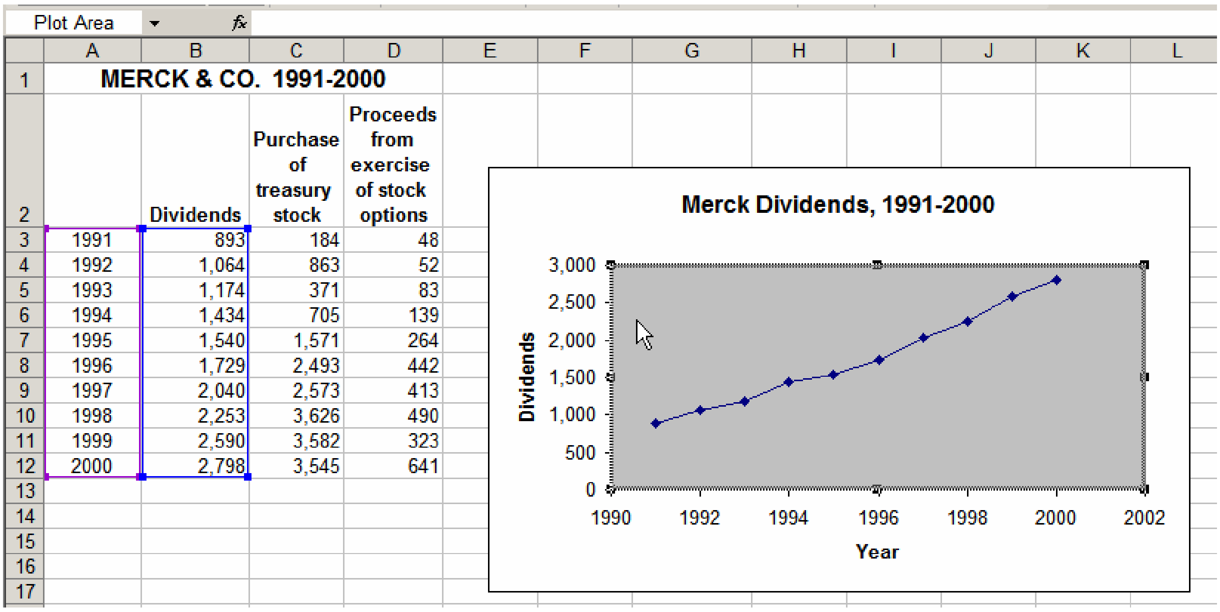
Tell Excel where to put the graph (in this case, on the spreadsheet labeled "Merck data", which is also the spreadsheet where our data is stored.

Pushing **Finish** gives the following graph:

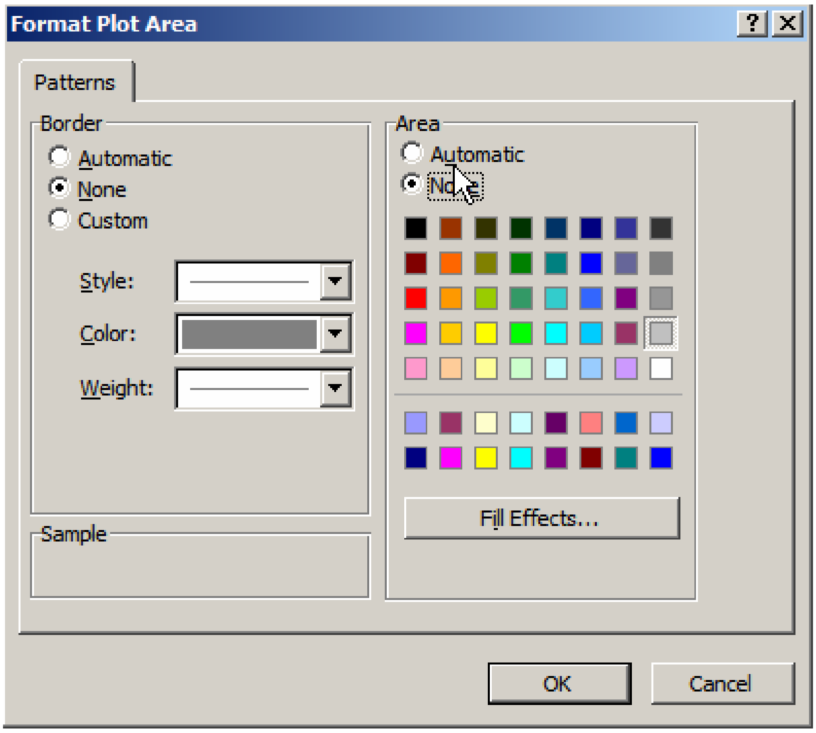


More changes

Change 1: The Excel default graph has a murky gray area where the data is graphed. This looks alright on the screen, but it looks terrible when you print it. All the graphs in this book have this -gray graph area blanked out. To do this, mark the graph area:



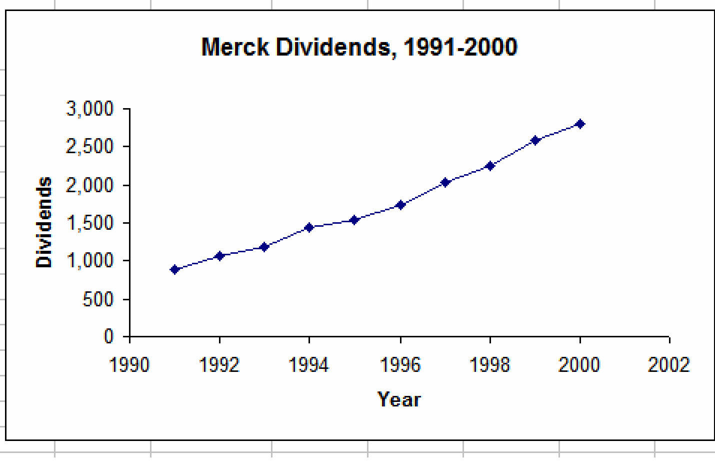
Now double-click on the graph area; this brings up the following box:



In the **Format Plot Area** box above, we always mark **Border—None** and **Area—None.**

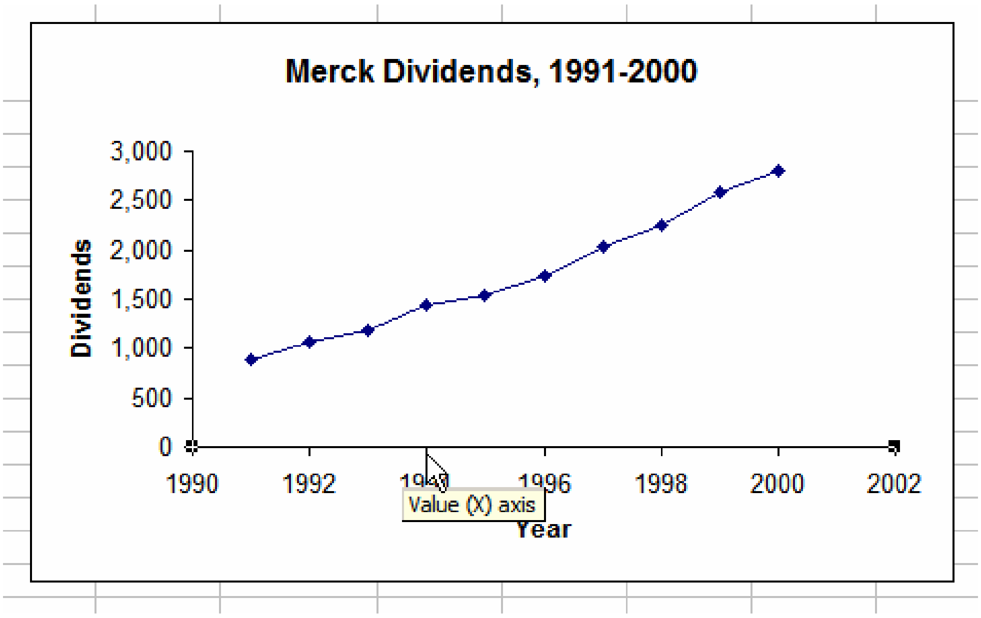
Here's the result:

	A	B	C	D
1	MERCK & CO. 1991-2000			
2		Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641

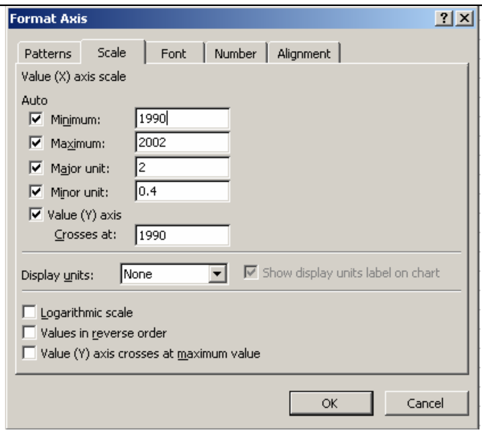


One more change

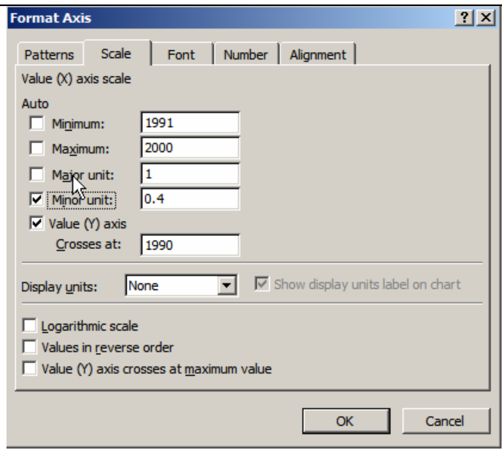
Although our data only goes from 1991 – 2000, the x-axis on our chart goes from 1990 to 2002. To change this, mark the x-axis of the graph with a gentle click on the left mouse button:



(Notice the square marks at either end of the x-axis.) Now right-click with the mouse and

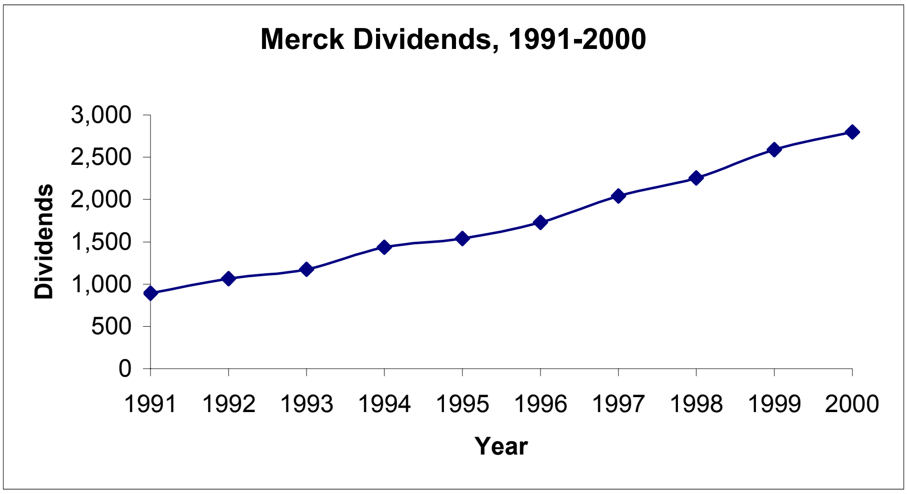


Before: A checked box indicates the Excel defaults. At this point the chart is set to show every other year on the x-axis (**Major unit** = 2). **Minor unit** indicates the number of ticks between the major units (not relevant here).



After. Note that we've changed both the **Minimum** and the **Maximum**, as well as the **Major Unit**.

Here's the result:

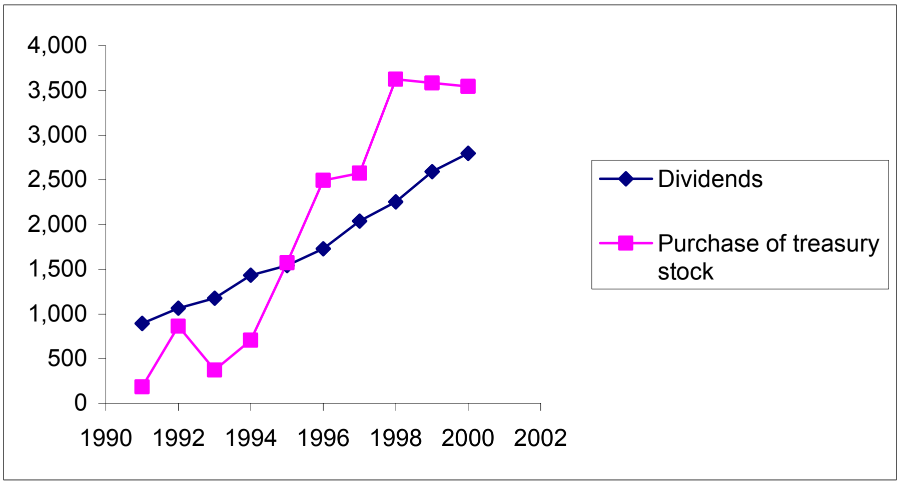


28.2. Creative use of legends

If you build your XY chart with data that includes legends, then Excel will generally transfer them in the proper way to the graph. Here's an example: We've marked the data to include the column headings:

	A	B	C	D
1	MERCK & CO. 1991-2000			
2		Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641
13				
14				

Here's the resulting graph:



28.3. Graphing non-contiguous data

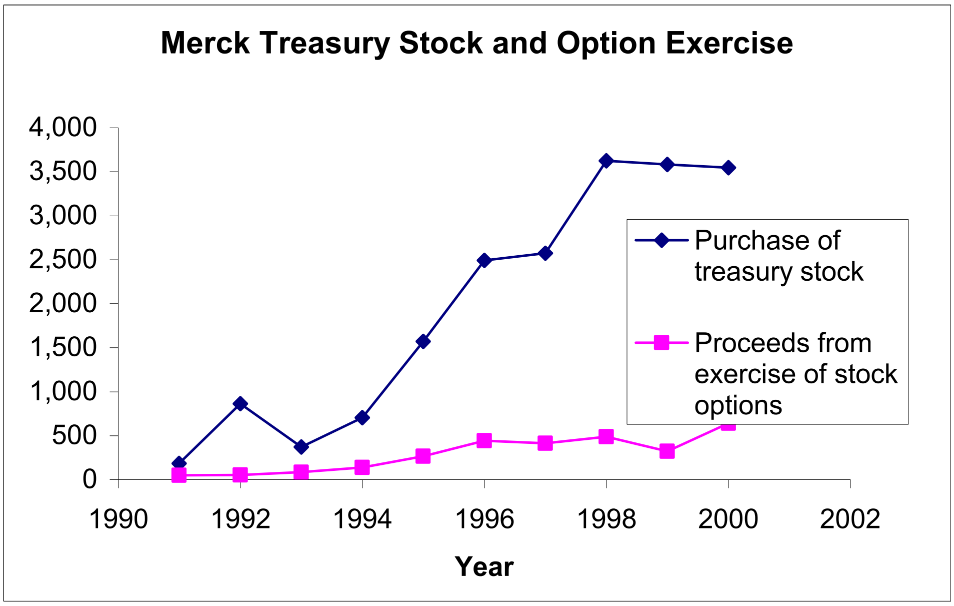
Suppose you want to make a graph of columns A, C and D of the Merck data. To mark these three columns:

- Mark the first column (that is, press the left mouse button and "paint" cells A3:A12)
- Press the [Ctrl] key and mark columns C and D (again, pressing the left mouse button).

At this point your spreadsheet looks like this:

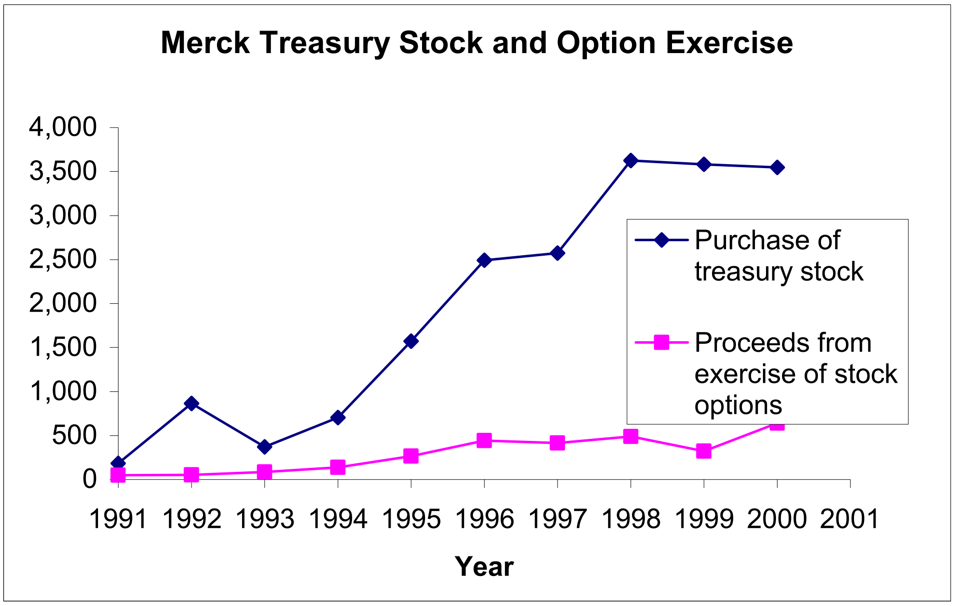
	A	B	C	D
1	MERCK & CO. 1991-2000			
2		Dividends	Purchase of treasury stock	Proceeds from exercise of stock options
3	1991	893	184	48
4	1992	1,064	863	52
5	1993	1,174	371	83
6	1994	1,434	705	139
7	1995	1,540	1,571	264
8	1996	1,729	2,493	442
9	1997	2,040	2,573	413
10	1998	2,253	3,626	490
11	1999	2,590	3,582	323
12	2000	2,798	3,545	641
13				

You can now follow the regular graphing procedure to create the following chart:

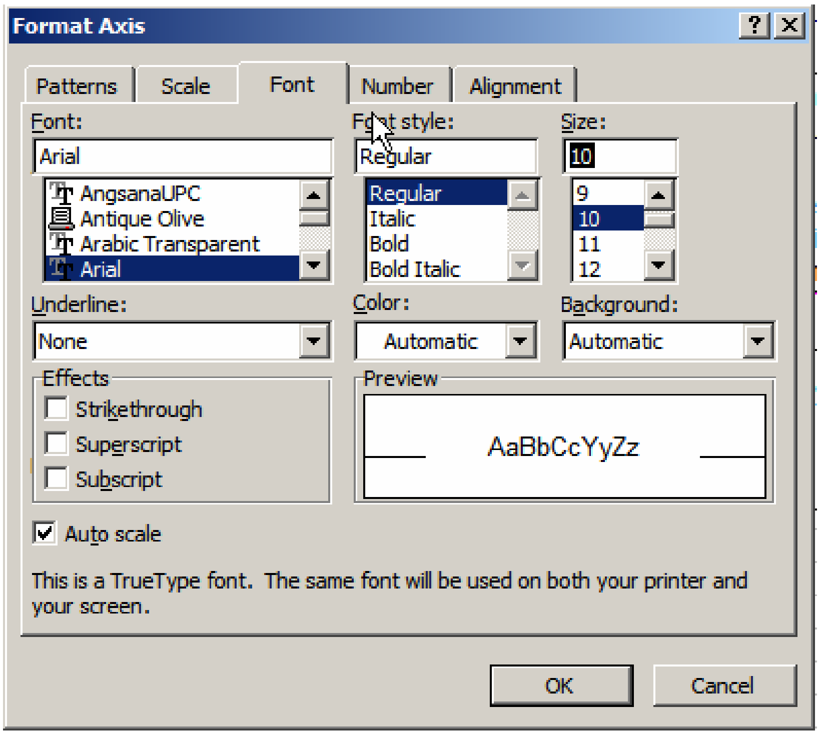


Fine-tuning—changing font size so that the axis labels fit

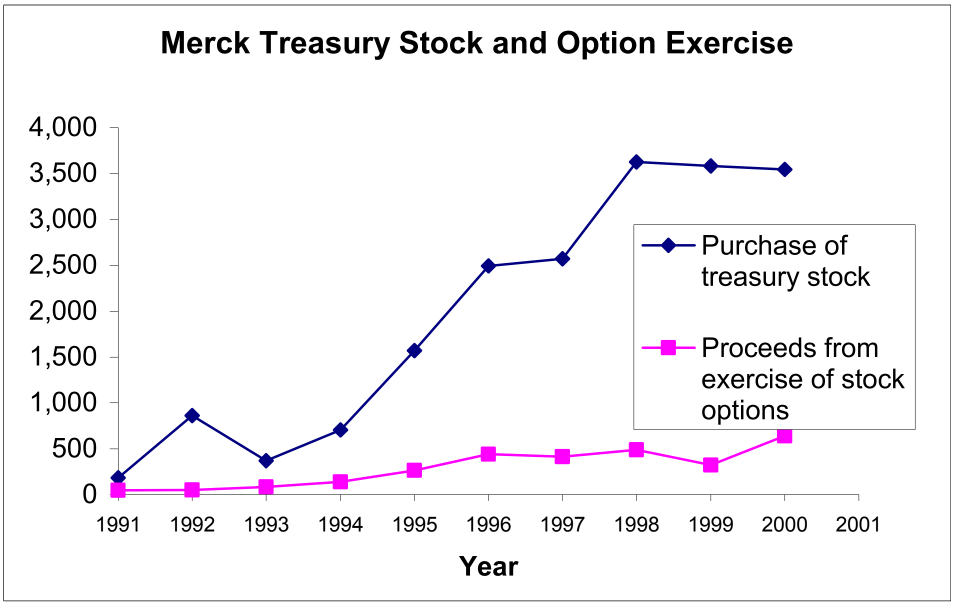
Look at the *x*-axis above: It goes from 1990 to 2002 even though the data only goes from 1991 – 2000. This often happens when Excel creates an *x*-axis for a graph. We've already shown how to use the **Format axis** menu to change the axis. But this time when we do this, the x-axis labels don't fit properly:



Go back into the dialog box and hit the **Font** tab to change the size of the *x*-axis font:



Now the graph looks fine:



(There are other ways to accomplish this trick also—if you make the chart bigger, for example.)

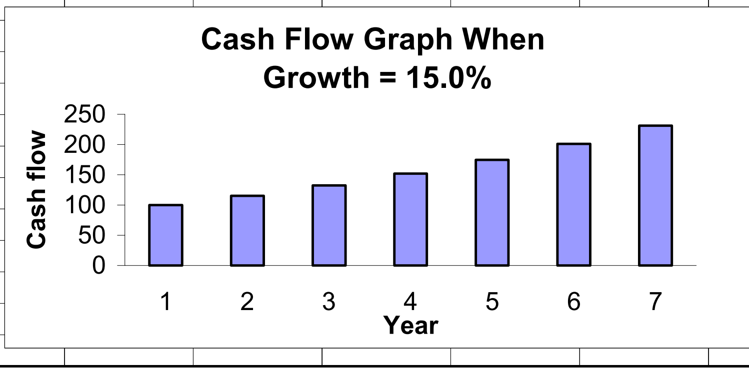
28.4. Graph titles that update[2]

You want to have the graph title change when a parameter on the spreadsheet changes. For example, in the next spreadsheet, you want the graph title to indicate the growth rate.

---

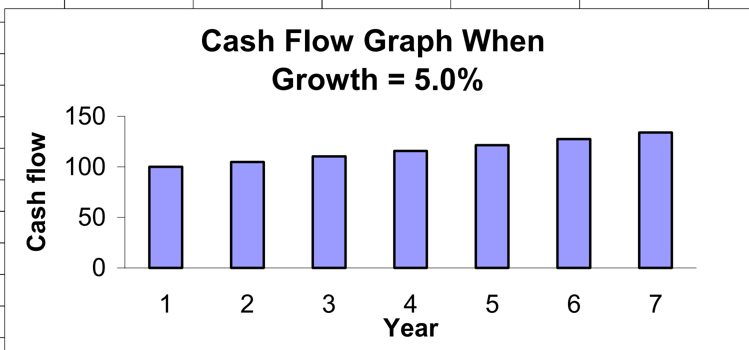
[2] This section makes (largely self-explanatory) use of the **Text** function, which is discussed in Chapter 29.

	A	B	C	D	E	F	G	H	I
1	GRAPH TITLES THAT UPDATE AUTOMATICALLY								
2	Growth	15%							
3									
4	Year	Cash flow							
5	1	100.00							
6	2	115.00	<-- B6*(1+$B$3)						
7	3	132.25							
8	4	152.09							
9	5	174.90							
10	6	201.14							
11	7	231.31							
12									
13									
14									
15									
16									
17									
18									



Once we have completed the necessary steps explained below, a change in the growth rate will change both the graph *and* its title:

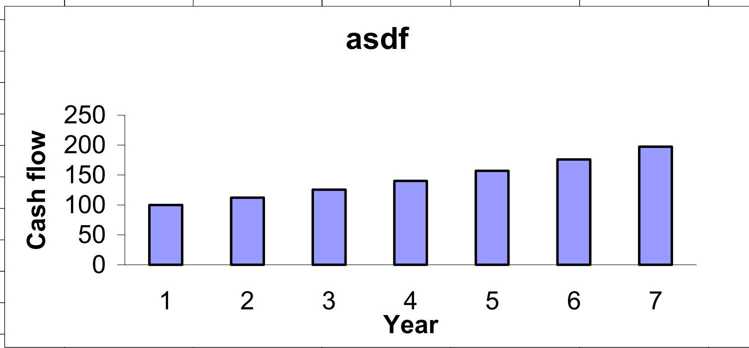
	A	B	C	D	E	F	G	H	I
1	GRAPH TITLES THAT UPDATE AUTOMATICALLY								
2	Growth	5%							
3									
4	Year	Cash flow							
5	1	100.00							
6	2	105.00	<-- B6*(1+$B$3)						
7	3	110.25							
8	4	115.76							
9	5	121.55							
10	6	127.63							
11	7	134.01							
12									
13									
14									
15									
16									
17									
18									



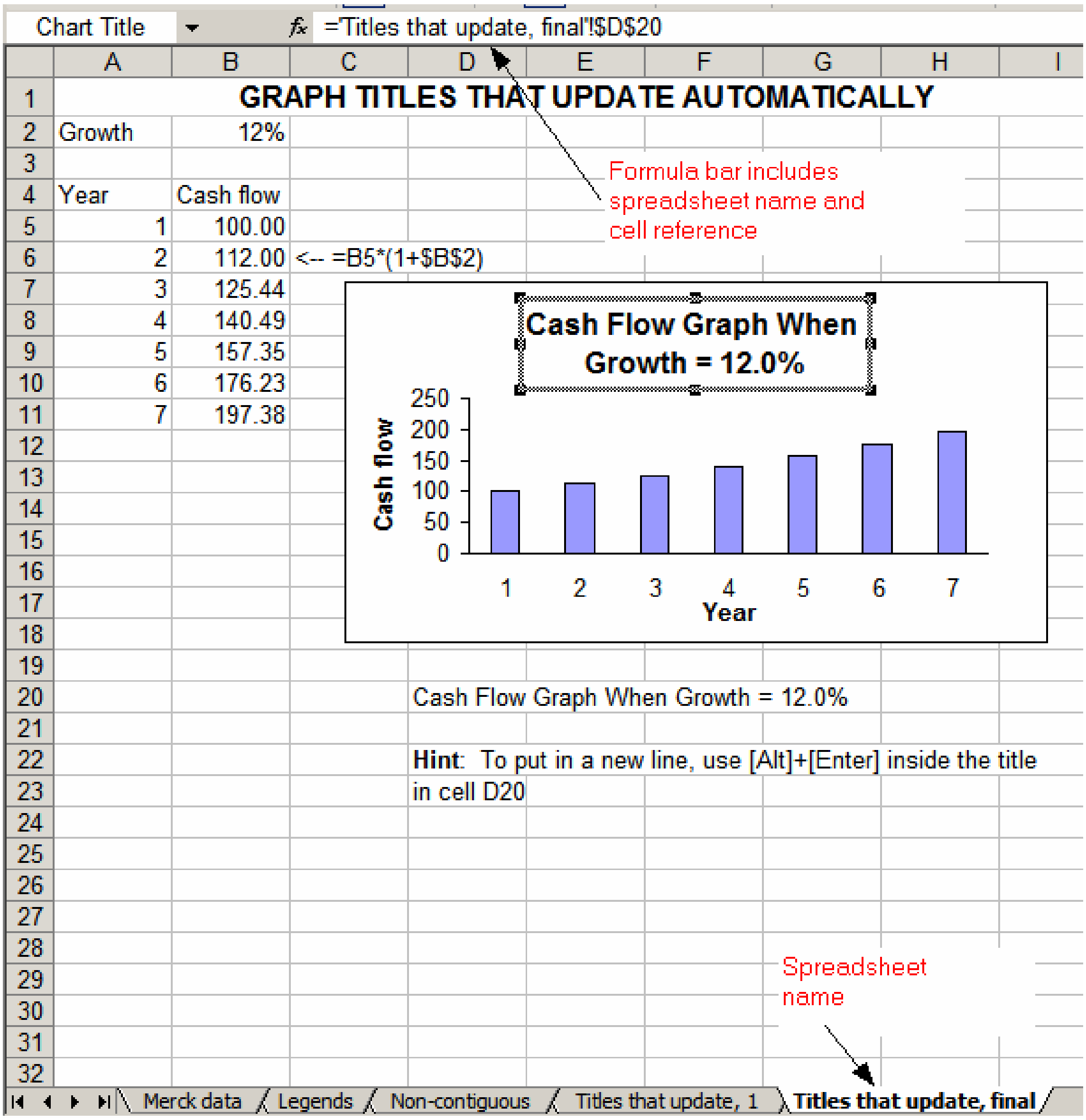
To make graph titles update automatically, carry out the following steps:

- Create the graph you want in the format you want it. Give the graph a "proxy title." (It makes no difference what, you're going to eliminate it soon.) At this stage your graph might look like:

	A	B	C	D	E	F	G	H	I
1	GRAPH TITLES THAT UPDATE AUTOMATICALLY								
2	Growth	12%							
3									
4	Year	Cash flow							
5	1	100.00							
6	2	112.00	<-- =B5*(1+$B$2)						
7	3	125.44							
8	4	140.49							
9	5	157.35							
10	6	176.23							
11	7	197.38							
12									
13									
14									
15									
16									
17									
18									
19									
20				Cash Flow Graph When Growth = 12.0%					



- Create the title you want in a cell. In the example above, cell D20 contains the formula: ="Cash Flow Graph when Growth = "&TEXT(B2,"0.0%").
- Click on the graph title to mark it, and then go to the formula bar and insert an equal sign to indicate a formula. Then **point** at cell D20 with the formula and click [Enter]. In the picture below, you see the chart title highlighted and in the formula bar "=Titles that update!$D$20" indicating the title of the graph. Note that "Titles that update" is the name of the spreadsheet.



Summary

There's lots more you can do with Excel charts, but we've covered the essentials. The exercises to this chapter will show you some more variations.

CHAPTER 29: EXCEL FUNCTIONS*

This version: March 27, 2003

Chapter contents

Introduction

In this chapter we discuss the principal Excel functions a financial analyst needs to know. There is some overlap between the discussion here and in other chapters (for example, the **NPV** function is discussed in Chapter 1). We also discuss some functions that are not used in this book, but that are so handy that we include them for reference.

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Here are the functions discussed in this book. Not all are in this chapter; the table below indicates the functions and where they are discussed (if not in this chapter).

Financial functions	**Date and time functions**	**Math functions**
FV IRR NPV PMT PV RATE NPER (XIRR and XNPV are discussed in Chapter 31 on Date and Time functions)	Discussed in Chapter 31	LN Exp Round RoundDown RoundUp Truncate Sqrt Sum SumIf SumProduct
Statistical functions (all these functions are discussed in Chapter ?? on doing statistics in Excel) Average Correl Count, CountA, CountIf Covar Frequency Intercept, Slope, Rsq Max, Min Median Stdev, StdevP Var, VarP Large() and Rank()	**Lookup functions** HLookup VLookup	**Database functions** DAverage DSum DCount DStdev DStdevp DVar DVarp DProduct (these functions are discussed in Chapter 33 on data manipulation in Excel)
Text functions Text Left, Right, Mid Combining text in cells	**Logical functions** If	

A word about nomenclature: In order to differentiate an Excel function from the surrounding text, we usually (though not in the table above!) denote it with boldface. Most Excel functions depend on some variable, but we do not always indicate these variables. For example the variables for the **NPV** function are the interest rate and the range to be discounted; when we want to make this explicit, we write **NPV(interest,range)** .

One more note: The functions in each class are not always discussed alphabetically. Where there's a logical order, we use this (for example: We discuss **NPV** before **IRR** .).

29.1. Financial functions

NPV( )

This function is extensively discussed in Chapter 1. The Excel definition of **NPV( )** differs somewhat from the standard finance definition. In the finance literature, the net present value of a sequence of cash flows $C_0, C_1, C_2, \ldots, C_n$ at a discount rate r refers to the expression:

$$\sum_{t=0}^{n} \frac{C_t}{(1+r)^t} \quad or \quad C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} \quad .$$

In many cases C_0 represents the cost of the asset purchased and is therefore negative.

The Excel definition of **NPV( )** always assumes that the first cash flow occurs after one period. The user who wants the standard finance expression must therefore calculate **NPV(r,{C$_1$, ... , C$_n$}) + C$_0$**. Here is an example:

	A	B	C	D	E	F	G
1	**EXCEL'S NPV FUNCTION**						
2	Discount rate	10%					
3	Year	0	1	2	3	4	5
4	Cash flow	-100	35	33	34	25	16
5							
6	NPV	$11.65	<-- =NPV(B2,C4:G4)+B4				

IRR( )

The internal rate of return (IRR) of a sequence of cash flows C_0, C_1, C_2, ..., C_n is an interest rate r such that the net present value of the cash flows is zero:

$$\sum_{t=0}^{n} \frac{C_t}{(1+r)^t} = 0 \ .$$

The Excel syntax for the **IRR( )** function is **IRR(cash flows**, **guess**). Here **cash flows** represents the whole sequence of cash flows, including the first cash flow C_0, and **guess** is a starting point for the algorithm that calculates the IRR.

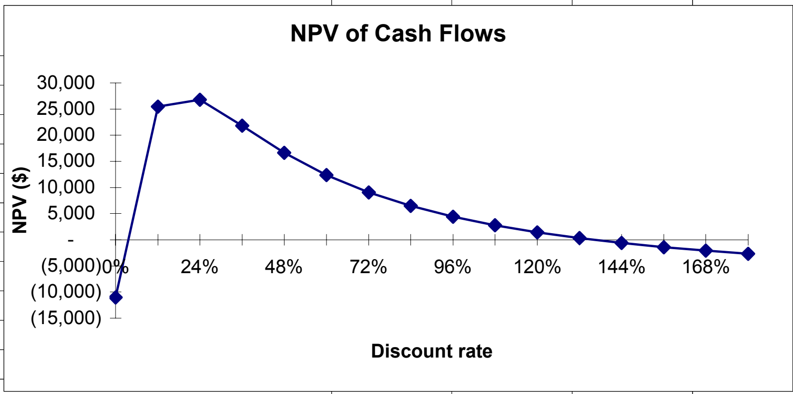
First a simple example—consider the cash flows given above:

	A	B	C	D	E	F	G
8	**EXCEL'S IRR FUNCTION**						
9	Year	0	1	2	3	4	5
10	Cash flow	-100	35	33	34	25	16
11							
12	IRR	15.00%	<-- =IRR(B10:G10,0)				
13		15.00%	<-- =IRR(B10:G10)				

Note that **guess** is not necessary when there is only one IRR. Thus in cell B13 (where we haven't indicated a **guess**) we get the same answer as in cell B12 (**guess** = 0).

The choice of **guess** can, however, make a difference when there is more than one IRR. Consider, for example, the following cash flows:

	A	B	C	D	E	F	G
1	**MULTIPLE IRRs**						
2	**Year**	**Cash flow**					
3	0	-11,000					
4	1	15,000					
5	2	15,000					
6	3	15,000					
7	4	15,000					
8	5	15,000					
9	6	15,000					
10	7	15,000					
11	8	15,000					
12	9	15,000					
13	10	-135,000					
14							
15	IRR	1.86%	<-- =IRR(B3:B13,0), guess = 0				
16	IRR	135.99%	<-- =IRR(B3:B13,2), guess = 2				



The graph (created from table that is not shown) shows that there are two IRRs, since the NPV curve crosses the *x*-axis twice. To find both these IRRs, we have to change the **guess** (though the precise value of guess is still not critical). In the example below we have changed both guesses, but still get the same answer:

	A	B	C
15	IRR	1.86%	<-- =IRR(B3:B13,0.1)
16	IRR	135.99%	<-- =IRR(B3:B13,0.8)

Note: A given set of cash flows typically has more than one IRR if there is more than one change of sign in the cash flows—in the above example, the initial cash flow is negative, and $CF_1 - CF_9$ are positive (this accounts for one change of sign); but then CF_{10} is negative—making a second change of sign. If you suspect that a set of cash flows has more than one IRR, the first thing to do is to use Excel to make a graph of the NPVs, as we did above. The number of times that the NPV graph crosses the *x*-axis identifies the number of IRRs (and also their approximate values).[1]

[1] For more examples of multiple IRRs, see Chapter 5.

PV( )

This function calculates the present value of an annuity (a series of fixed periodic payments). For example:

	A	B	C
1	**THE PV FUNCTION**		
2	**Payments made at the end of the period**		
3	Rate	10%	
4	Number of periods	10	
5	Payment	100	
6	Present value	(614.46)	<-- =PV(B3,B4,B5)

Thus $\$614.46 = \sum_{t=1}^{10} \frac{100}{(1.10)^t}$. Here are two things to note about the **PV( )** function:

- Writing **PV(B3,B4,B5)** assumes that payments are made at dates 1, 2, ..., 10. If the payments are made at dates 0, 1, 2, ..., 9, you should write:

	A	B	C
8	**Payments made at the beginning of the period**		
9	Rate	10%	
10	Number of periods	10	
11	Payment	100	
12	Present value	(675.90)	<-- =PV(B9,B10,B11,,1)

- Irritatingly, when the payments are positive as in the above example, the **PV( )** function (and the **PMT( )** function—see below) gives the present value as a negative number (there is a logic here, but it's not worth explaining). To get a positive present value in cell B12, we would either write **-PV(B3,B4,B5)** or let the payment be negative by writing **PV(B3,B4,-B5)**.

PMT( )

This function calculates the payment necessary to pay off a loan with equal payments over a fixed number of periods. For example, the first calculation below shows that a loan of $1000, to be paid off over 10 years at an interest rate of 8% will require equal annual payments of interest and principal of $149.03. The calculation performed is the solution of the following equation:

$$\sum_{t=1}^{n} \frac{X}{(1+r)^t} = initial\ loan\ principal,$$

Where X is the payment

	A	B	C
1	**THE PMT FUNCTION**		
2	**Payments made at the end of the period**		
3	Rate	8%	
4	Number of periods	10	
5	Principal	1000	
6	Payment	($149.03)	<-- =PMT(B3,B4,B5)
7			
8	**Payments made at the beginning of the period**		
9	Rate	8%	
10	Number of periods	10	
11	Principal	1000	
12	Payment	($137.99)	<-- =PMT(B9,B10,B11,,1)

Loan tables can be calculated using the **PMT( )** function. These tables—explained in detail in Chapter 1—show what part of each payment is interest and what part is repayment of the loan principal. In each period, the payment on the loan (calculated with **PMT( )**) is split:

- We first calculate the interest owing for that period on the principal outstanding at the beginning of the period. In the table below, at the end of year 1, we owe $80 (= 8% * $1000) of interest on the loan principal outstanding at the beginning of the year.
- The remainder of the payment (for year 1: $69.03) goes to reduce the principal outstanding.

	A	B	C	D	E
1	**Loan Table**				
2	Interest	8%			
3	Number of periods	10			
4	Principal	1,000.00			
5	Annual payment	149.03	<-- =-PMT(B2,B3,B4)		
6					
7				**Split of payment into**	
8	**Year**	**Principal at beginning of year**	**Payment at end of year**	**Interest**	**Repayment of principal**
9	1	1,000.00	149.03	80.00	69.03
10	2	930.97	149.03	74.48	74.55
11	3	856.42	149.03	68.51	80.52
12	4	775.90	149.03	62.07	86.96
13	5	688.95	149.03	55.12	93.91
14	6	595.03	149.03	47.60	101.43
15	7	493.60	149.03	39.49	109.54
16	8	384.06	149.03	30.73	118.30
17	9	265.76	149.03	21.26	127.77
18	10	137.99	149.03	11.04	137.99

Note that the repayment of principal at the end of year 10 is exactly equal to the principal outstanding at the beginning of the year (i.e., the loan has been paid off).

RATE( )

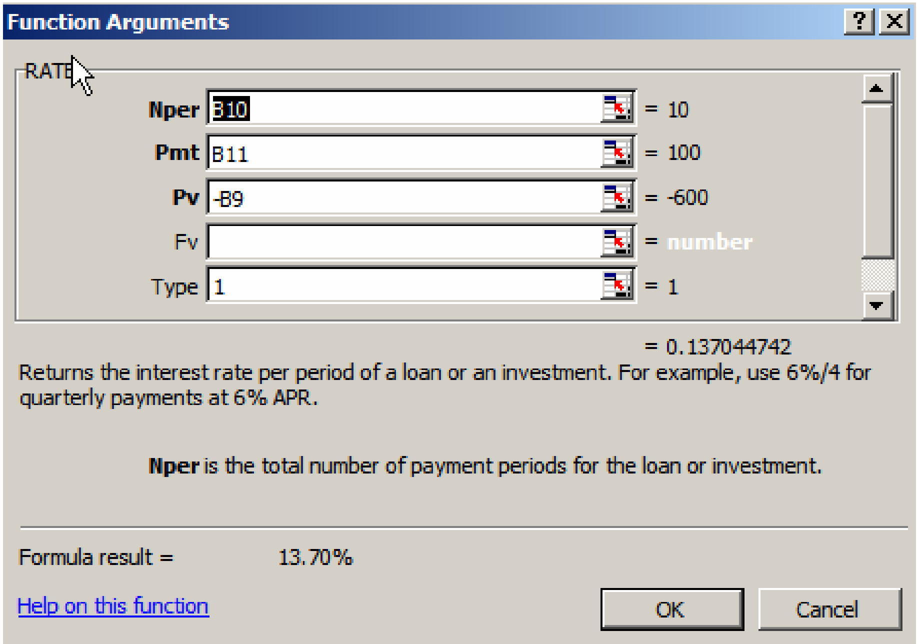
RATE calculates the internal rate of return of a series of constant payments. In the example below **RATE(B4,B5,-B3)** in cell B6 computes 10.56%, which is the internal rate of return:

$$-600+\frac{100}{(1.1056)}+\frac{100}{(1.1056)^2}+\ldots+\frac{100}{(1.1056)^{10}}=0$$

	A	B	C	D
1	**THE RATE FUNCTION** should be compared to IRR			
2	**RATE used for payments made at the end of the period**			
3	Initial payment	600		
4	Number of periods	10		
5	Annual payment	100		
6	Rate of return	10.56%	<-- =RATE(B4,B5,-B3)	
7				
8	**RATE used for payments made at the beginning of the period**			
9	Initial payment	600		
10	Number of periods	10		
11	Annual payment	100		
12	Rate of return	13.70%	<-- =RATE(B10,B11,-B9,,1,20%)	
13				
14	**What does RATE do? Computing the IRR**			
15	**Year**	**Payment at end of period**	**Payment at beginning of period**	
16	0	-600	-500	
17	1	100	100	
18	2	100	100	
19	3	100	100	
20	4	100	100	
21	5	100	100	
22	6	100	100	
23	7	100	100	
24	8	100	100	
25	9	100	100	
26	10	100		
27				
28	**IRR**	10.56%	13.70%	<-- =IRR(C16:C26)

Like **PV** and **PMT**, **RATE** gives the possibility of specifying whether the cash flows occur at the end of the period (the default) or its beginning. If you look in cell B12, **RATE(B10,B11,-B9,,1,20%)** computes 13.70%; this is the internal rate of return of an initial payment of $600 and 10 payments of $100 *made at the beginning of the period* (the beginning of the period is indicated by the "1" at the end of the formula. The **20%** in the function is a **Guess** like that which is also allowed in the IRR function.

Here's the dialog box which created this result:



Think for a second what this means for an internal rate of return:

$$-600+\underbrace{100}_{\substack{\uparrow \\ \text{First payment} \\ \text{made at "beginning"} \\ \text{of period--meaning,} \\ \text{made at time 0}}}+\frac{100}{(1.1370)}+\frac{100}{(1.1370)^2}+\frac{100}{(1.1370)^3}+\ldots+\frac{100}{(1.1370)^9}=0$$

Effectively, then **RATE(B10,B11,-B9,,1,20%)** refers to an initial payment of $500 and 9 subsequent payments of 100.

RATE versus **IRR**

If you look at the above example, you will see (rows 16-28) that **IRR** and **RATE** give the same values. There are, of course, tradeoffs:

- **RATE** is shorter; **IRR** requires you to specify all the cash flows.
- On the other hand, **IRR** can handle cash flows which vary over time.

NPER( )

This function calculates the number of periods to repay a loan given a fixed amount. Example: You borrow $1,000 from the bank, which charges you a 10% annual interest rate. You intend to repay the loan with $250 per year. How long will it take you to repay the loan?

	A	B	C	D	E
1	**HOW LONG TO PAY OFF THIS LOAN?**				
2	Loan amount	1,000.00			
3	Interest rate	10%			
4	Annual payment	250			
5	How long to pay off the lo	5.3596	<-- =NPER(B3,B4,-B2)		
6					
7	**Year**	**Principal at beginning of year**	**Payment at end of year**	**Interest**	**Repayment of principal**
8	1	1,000.00	250.00	100.00	150.00
9	2	850.00	250.00	85.00	165.00
10	3	685.00	250.00	68.50	181.50
11	4	503.50	250.00	50.35	199.65
12	5	303.85	250.00	30.39	219.62
13	6	84.24	250.00	8.42	241.58

As you can see from the loan table, it takes somewhere between 5 and 6 years to repay the loan.[2] **NPER(B3,B4,-B2)** gives the exact number of periods as 5.3596.

29.2. Math functions

Using Exp to calculate future values

Suppose you invest $100 at 10% for 3 years. As explained in Chapter 2, if interest is compounded annually, the future value after 3 years will be

[2] Why? At the end of year 5 (which is also the beginning of year 6), there's still $84.24 of principal outstanding. But if you pay back $250 at the end of year 6, then you've paid back too much.

	A	B	C
1	**ANNUAL COMPOUNDING**		
2	Initial investment	100	
3	Years invested, t	3	
4	Interest rate, r	10%	
5	Future value, FV	133.1	<-- =B2*(1+B4)^B3

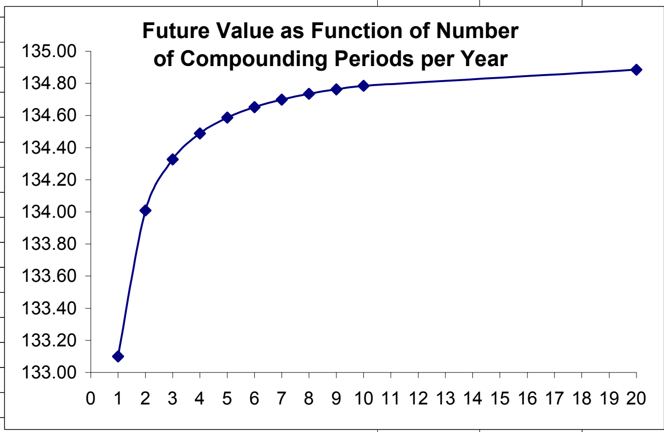
Suppose the 10% is compounded semi-annually (meaning: you get 5% each half year). Then there will be 6 compounding periods—3 years * 2 periods/year. Your future value will be $InitialInvestment*(1+5\%)^{6}=134.0096$:

	A	B	C
7	Initial investment	100	
8	Years invested, t	3	
9	Compounding periods per year, n	2	
10	Interest rate, r	10%	
11	Future value, FV	134.0096	<-- =B7*(1+B10/B9)^(B8*B9)

Denote the number of years by *t*, the interest rate by *r*, and the number of compounding periods per year by *n*. As the number of compounding periods increases, the future value tends towards 100*e^{r*t}, where *e* is the number 2.71828.[3] In Excel this is written as 100***Exp(r*t)** . This is illustrated in the table and graph below:

[3] In mathematical notation: $\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{nt}=e^{rt}$.

	A	B	C	D	E	F
15	Years invested, t	3				
16	Interest rate, r	10%				
17						
18	**Number of compounding periods per year, n**	**Future value**				
19	1	133.100	<-- =$B$14*(1+$B$16/A19)^($B$15*A19)			
20	2	134.010	<-- =$B$14*(1+$B$16/A20)^($B$15*A20)			
21	3	134.327	<-- =$B$14*(1+$B$16/A21)^($B$15*A21)			
22	4	134.489				
23	5	134.587				
24	6	134.653				
25	7	134.700				
26	8	134.735				
27	9	134.763				
28	10	134.785				
29	20	134.885				
30						
31						
32						
33						
34						
35						
36						
37						
38						
39						
40	**As n gets large, this converges to**	134.9859	<-- =B14*EXP(B16*B15)			



Nomenclature: When the number of compounding periods becomes infinite, the investment is said to be *continuously compounded*. Otherwise (that is, when there are a finite number of compounding periods per year), the investment is said to be *discretely compounded*.

Using Exp to calculate present values

Above we illustrated how $100 grows to 100***Exp(r*t)** when it is compounded continuously for t years at interest rate r. Suppose you're going to get $100 in 3 years. What is the present value of this $100 if the relevant interest rate is r? The answer depends on the number of compounding periods:

- If the investment is discretely compounded n times per year, then its present value is

$$\frac{100}{\left(1+\frac{r}{n}\right)^{n*t}} = 100*\left(1+\frac{r}{n}\right)^{-n*t}$$

- If the investment will be continuously compounded, then its present value is

$$\frac{100}{\exp(r*t)} = 100*\exp(-r*t)$$

In Excel:

	A	B	C
43	**Discounting--discrete versus continuous**		
44	Future value	100	
45	What year received, t	3	
46	Compounding periods per year, n	2	
47	Interest rate, r	10%	
48			
49	Present value, discrete discounting	74.62154	<-- =B44/(1+B47/B46)^(B46*B45)
50			
51	Present value, continuous discounting	74.08182	<-- =B44*EXP(-B47*B45)

You can use the above spreadsheet to show that as n gets very large, the two values in B56 and B58 converge. For example, when $n = 100$:

	A	B	C
43	**Discounting--discrete versus continuous**		
44	Future value	100	
45	What year received, t	3	
46	Compounding periods per year, n	100	
47	Interest rate, r	10%	
48			
49	Present value, discrete discounting	74.09293	<-- =B44/(1+B47/B46)^(B46*B45)
50			
51	Present value, continuous discounting	74.08182	<-- =B44*EXP(-B47*B45)

LN

This function (the "natural logarithm" to differentiate it from the "logarithm base 10" that you learned in high school) is often used to calculate continuously compounded rates of return.[4] Suppose you invest in a stock that is worth $25 and suppose that one year later the stock is worth

[4] In this book we've used it extensively in the option chapters, ?????.

\$40. What rate of return r have you earned? If you use *discrete compounding*, the rate of return is $r = \frac{P_1}{P_0} - 1 = \frac{40}{25} - 1 = 60\%$.

Now suppose that your alternative is to earn *continuously compounded interest r*. Then the rate of return has to solve the equation

$$P_0 \exp(r) = P_1 \Rightarrow \exp(r) = \frac{P_1}{P_0}.$$

The function which solves this equation is the natural logarithm ln:

$$r = \ln\left(\frac{P_1}{P_0}\right).$$

In Excel:

	A	B	C
1	**USING LN TO COMPUTE CONTINUOUSLY COMPOUNDED RATES OF RETURN**		
2	Price of stock, t=0	25	
3	Price of stock, t=1	40	
4	Discretely compounded rate of return, r	60.00%	<-- =B3/B2-1
5	Continously compounded rate of return, r	47.00%	<-- =LN(B3/B2)

When $t \neq 1$, the problem looks like this:

$$P_0 \exp(r * t) = P_t \Rightarrow \exp(r * t) = \frac{P_t}{P_0}$$

has solution:

$$r = \frac{1}{t}\ln\left(\frac{P_t}{P_0}\right)$$

For example: Suppose you invested in Intel stock on 25 October 1999, buying the stock for its closing price of \$38.6079, and suppose you sold it at the end of the day, 24 July 2000, for \$64.4379. As the calculation below shows, you would have earned a continuously compounded return of 68.49% on your stock.

	A	B	C	D
7	**Intel stock**			
8	Purchase date and price	25-Oct-99	38.6079	
9	Sale date and price	24-Jul-00	64.4379	
10				
11	Elapsed time, t	0.7479	<-- =(B9-B8)/365	
12	Continuously compounded rate of return, r	68.49%	<-- =1/B11*LN(C9/C8)	

Note that this calculation is easier than the calculation of the *annualized daily return*—it has one fewer step:

	A	B	C	D
14	**Daily return, annualized**			
15	Purchase date and price	25-Oct-99	38.6079	
16	Sale date and price	24-Jul-00	64.4379	
17				
18	Elapsed days	273	<-- =(B16-B15)	
19	Daily return	0.1878%	<-- =(C16/C15)^(1/B18)-1	
20	Annualized	98.35%	<-- =(1+B19)^365-1	

A short finance note

We can't resist a short finance note on the difference between the continuously compounded annual return of 68.49% and the discretely-compounded annual return of 98.35%.

- Both of these returns cause \$38.6079 to grow over a period of 273 days to \$64.4379. So they're both—in an economic sense—the same number.
- The *daily* returns are very close: The continuously compounded daily return is calculated by $\frac{annual\ continuously\text{-}compounded\ return}{365}$ and the discretely-compounded daily return is calculated by $\left(\frac{Stock\ \ price,\ day\ 273}{Stock\ \ price,\ day\ 0}\right)^{1/273}-1$. These numbers are very close:

	A	B	C
22	**Note**		
23	Daily, continuously-compounded return	0.1876%	<-- =B12/365
24	Daily, discretely-compounded return	0.1878%	<-- =B19

However, when you compound them for 365 days, the differences are very large.

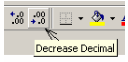
Round, RoundDown, RoundUp, Trunc

The Excel functions **Round**, **RoundDown**, **RoundUp** do exactly what they say. All 3 of these functions require you to specify the number of decimal places to which you want to round off the number. The function **Trunc** cuts off a number after a specified number of places (if you do not specify, **Trunc** gives you the integer part of a number). Here are examples using the Excel function **Pi** as a basis:

	A	B	C
1	**ROUNDING NUMBERS IN EXCEL**		
2	Number	3.1415926535898	<-- =PI()
3			
4	Round, no decimal places	3.00000000	<-- =ROUND(B2,0)
5	Round, 3 decimal places	3.14200000	<-- =ROUND(B2,3)
6			
7	RoundDown, no decimal places	3.00000000	<-- =ROUNDDOWN(B2,0)
8	RoundDown, 3 decimal places	3.14100000	<-- =ROUNDDOWN(B2,3)
9			
10	RoundUp, no decimal places	4.00000000	<-- =ROUNDUP(B2,0)
11	RoundUp, 4 decimal places	3.14160000	<-- =ROUNDUP(B2,4)
12			
13	Truncate, no decimal places	3.00000000	<-- =TRUNC(B2)
14	Truncate, 5 decimal places	3.14159000	<-- =TRUNC(B2,5)

There's a difference between using these functions and merely formatting a number so that it looks rounded or truncated. Here's an example:

	A	B	C
16	Number	4.5632	
17	Rounded to 2 decimals	4.56	<-- =ROUND(B16,2)
18	Formatted to 2 decimals	4.56	<-- =B16
19			
20	10 times cell B20	45.6	<-- =10*B17
21	10 times cell B21	45.632	<-- =10*B18

In cell B21 we used the "decrease decimal" button 

to change the representation of the number. However, as you can see in cell B24, this button does not change the number, whereas **Round** actually changes the number.

Sqrt

This function calculates the square root of a number. In this book, we've used square roots to calculate the standard deviation (see Chapter ???) of returns.

	A	B	C
1	**SQRT**		
2	Number	3	
3	Square root	1.732051	<-- =SQRT(B2)
4	Equivalent way	1.732051	<-- =B2^(1/2)

Note that you can use the carat (^) as an alternative way of calculating the square root. In Excel's notation, *a*^*b* raises *a* to the power *b* (meaning $a \wedge b = a^b$). Since a square root is equivalent to the power ½, you can also use this notation (see cell B4 above).

Sum

The Excel function **Sum** adds numbers in a range of cells:

	A	B
1	**SUM**	
2	1	
3	2	
4	3	
5	4	
6	5	
7	15	<-- =SUM(A2:A6)

SumIf

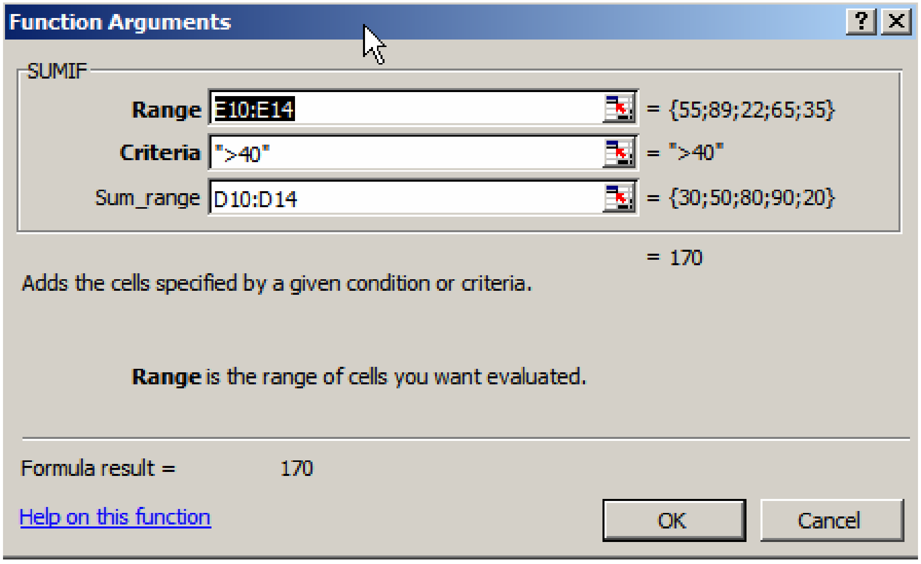
SumIf allows you to add only numbers that fulfill some specific condition. Here's an example in which we add only those scores that are greater than 30:

	A	B
9	**Score**	
10	30	
11	50	
12	80	
13	90	
14	20	
15	220	<-- =SUMIF(A10:A14,">30")

The function **SumIf** also allows you to have the conditional column some other place. In the following example, we add the numbers in D10:D14 for which the corresponding number in E10:E14 is greater than 40 (highlighted here):

	D	E	F
9	**Score 1**	**Score 2**	
10	30	55	
11	50	89	
12	80	22	
13	90	65	
14	20	35	
15	170	<-- =SUMIF(E10:E14,">40",D10:D14)	

The function wizard really helps when you use this function. Here it is for the above example. You'll notice that **Range** is the column of criteria ("Score 2") and **Sum_range** is the column to be added. If you don't specify **Sum_range**, Excel assumes that it's the same as **Range**:



SumProduct

This function pairwise multiplies the entries in two columns and adds the results. It's sometimes useful in statistics (do we have an example?). Here's a simple example that calculates the expected return of a portfolio. There are four assets, each with a different expected return. To calculate the expected portfolio return, we have to multiply the expected return in column B by the portfolio proportion of each asset (column C). **SumProduct** does this nicely:

	A	B	C	D	E	F
18	**Asset**	**Expected return**	**Portfolio proportion**			
19	1	20%	15%			
20	2	8%	22%			
21	3	15%	38%			
22	4	12%	25%			
23						
24		Expected portfolio return	13.46%	<-- =SUMPRODUCT(B19:B22,C19:C22)		

29.3. Conditional functions

If( ), **VLookup( )**, and **HLookup( )** are three functions that allow you to put in conditional statements.

The syntax of Excel's **If** statement is: **If(condition,output if condition is true, output if condition is false)**. In the example below, if the initial number in B3 $\leq$ 3, then the desired output is 15. If B3 > 3, then the output is 0:

	A	B	C
1	**THE IF FUNCTION**		
2	Initial number	2	
3	If statement	15	<-- =IF(B2<=3,15,0)

You can make **If** print text also, by enclosing the desired text in double quotes:

	A	B	C
5	Initial number	2	
6	If statement	Less than or equal to 3	<-- =IF(B5<=3,"Less than or equal to 3","More than 3")

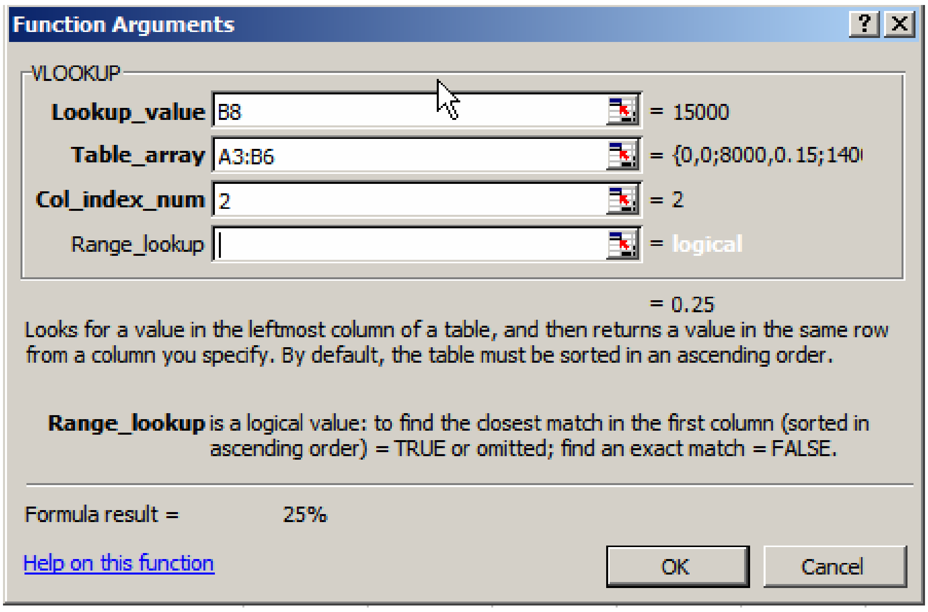
VLookup and HLookup

Since **VLookup( )**, and **HLookup( )** both have the same structure, we will concentrate on **VLookup( )** and leave you to figure out **HLookup( )** for yourself. **VLookup( )** is a way to introduce a table search in your spreadsheet. Here is an example: Suppose the marginal tax rates on income are given by the table below—for income less than $8,000, the marginal tax rate is 0%; for income above $8,000, the marginal tax rate is 15%, etc. Cell B9 illustrates how the function **VLookup** is used to lookup the marginal tax rate.

	A	B	C
1	**VLOOKUP FUNCTION**		
2	**Income**	**Tax rate**	
3	0	0%	
4	8,000	15%	
5	14,000	25%	
6	25,000	38%	
7			
8	Income	15,000	
9	Tax rate	25%	<-- =VLOOKUP(B8,A3:B6,2)

The syntax of this function is **VLookup(lookup_value,table,column)**. The first column of the lookup table, A3:A6, must be arranged in ascending (increasing) order. The **lookup_value** (in this case the income of 15,000) is used to determine the applicable row of the **table**. The row is the first row whose value is ≤ the **lookup_value**; in this case, this is the row that starts with 14,000. The **column** entry determines from which column of the applicable row the answer is taken; in this case the marginal tax rates are in column 2.

Here's the Excel function wizard for this table:



The first column of VLookup must be sorted

The first column of the **VLookup** table must be *sorted*, meaning it must be in increasing order (either numerical or alphabetical). To see what this means, we have a slightly complicated example: The data in columns A and B below were imported from a data base; column A gives the date and column B gives an interest rate on a particular date.

	A	B	C	D	E	F	G
1	FIRST COLUMN OF VLOOKUP MUST BE SORTED						
2	**Date**	**Interest rate**		**Month**	**Day**	**Year**	
3	JAN. 07,1991	6.721		JAN	07	1991	
4	FEB. 07,1991	6.145		Feb	07	1991	
5	FEB. 11,1991	6.03		FEB	11	1991	
6	MAR. 04,1991	6.287		MAR	04	1991	
7	APR. 01,1991	5.985		APR	01	1991	
8	JUN. 08,1991	5.777		JUN	08	1991	
9	AUG. 15,1991	5.744		AUG	15	1991	
10	OCT. 22,1991	5.868		OCT	22	1991	
11							
12							
13		=LEFT(A10,3)		=MID(A10,6,2)		=RIGHT(A10,4)	
14							

We would like to give each date a standard Excel value. That is, instead of "Jan. 07, 1991," we'd like to write

	A	B
18	**Standard Excel date format**	**Number equivalent**
19	7-Jan-91	33245

If this is somewhat unclear, refer to Chapter ???.

In order to write the date in the standard Excel format, we use the functions **Left**, **Mid**, and **Right** to *parse* the dates in column A into month, day and year (see next section). We now need to identify each month with its number (i.e., Jan = 1, Feb = 2, etc.). We can use **VLookup** to do this, but only if the VLookup table has its left column in alphabetical order.

	A	B	C	D	E	F	G	H	I	J	K
1	**FIRST COLUMN OF VLOOKUP MUST BE SORTED**										
2	**Date**	**Interest rate**		**Month**	**Day**	**Year**		**Which month?**		**Date value**	
3	JAN. 07,1991	6.721		JAN	07	1991		1	<-- =VLOOKUP(D3,$J$6:$K$17,2)	1/7/1991	<-- =DATE(F3,H3,E3)
4	FEB. 07,1991	6.145		Feb	07	1991		2			
5	FEB. 11,1991	6.03		FEB	11	1991		2		**VLookup table**	
6	MAR. 04,1991	6.287		MAR	04	1991		3		Apr	4
7	APR. 01,1991	5.985		APR	01	1991		4		Aug	8
8	JUN. 08,1991	5.777		JUN	08	1991		6		Dec	12
9	AUG. 15,1991	5.744		AUG	15	1991		8		Feb	2
10	OCT. 22,1991	5.868		OCT	22	1991		10		Jan	1
11										Jul	7
12										Jun	6
13		=LEFT(A10,3)		=MID(A10,6,2)		=RIGHT(A10,4)				Mar	3
14										May	5
15										Nov	11
16										Oct	10
17										Sep	9

This gives a rather strange-looking table (cells J6:K17), but you can convince yourself that this works.

29.4. Text functions

Excel distinguishes between numbers and *text*. To sound stupid: You can add, subtract, etc. numbers, but you can't do this for text. On the other hand, Excel allows you to *concatenate* text (if this sounds mysterious, read on).

Concatenation: combining text from several cells

Here's an example. In the example below, we've written "twelve" in cell A2 and "cows" in cell B2. In cell A4, we tried to write **=A3+B3**; we intended this to come out "Twelvecows", but Excel won't accept this, because neither the contents of A2 ("Twelve") nor those of B2 ("cows") is a number. We can combine the text as in cell A5, by writing **=A3&B3**.

	A	B
2	Twelve	cows
3		
4	#VALUE!	<-- =A2+B2
5	Twelvecows	<-- =A2&B2
6		
7	Twelve blue cows	<-- =A2&" blue "&B2

In cell A7, we've added the word "blue" plus some spaces, putting the additional text/spaces inside quotation marks.

TEXT

Now look at the example below:

	A	B	C
10	Number of cows	1200	
11			
12	Text	1200 cows	<-- =TEXT(B10,"0")&" cows"
13			
14		1200.00 cows	<-- =TEXT(B10,"0.00")&" cows"
15		1,200.0 cows	<-- =TEXT(B10,"0,000.0")&" cows"
16		120,000.00% cows	<-- =TEXT(B10,"0,000.00%")&" cows"

In cell B12 we want to create a text that contains the number of cows (cell B10) and the word " cows". The Excel function **Text(B10,"0")** turns the number 1200 into a text form which can then be used in the formula in cell B12. The second part of the **Text** function—where we've currently written "0"—is used to indicate the appearance of the text. Cells B14:B16 give some other examples.

LEFT, RIGHT, MID, LEN

The first three functions allow you to pick out parts of texts. In the example below, we've used these functions to pick out parts of the text in cell A18:

	A	B
18	15 pink flamingos went to the zoo	
19		
20	15	<-- =LEFT(A18,2)
21	pink flamingos	<-- =MID(A18,4,14)
22	zoo	<-- =RIGHT(A18,3)
23	33	<-- =LEN(A18)

The function **=Left(A19,2)** picks out the 2 left-most characters of cell A19. The function **=Mid(A19,4,14)** picks out the 14 characters of cell A19, starting with the 4th character. And the function **=Right(A19,3)**, well ... you'll figure that one out yourself.

As illustrated in cell A23, the function **Len** tells you the number of characters in the text.

You might ask why a finance book needs to consider these functions. Here's an example that arose in the writing of this book: In Chapter ??? we discuss the prices of options on General Motors stock. This data was originally downloaded from the website of the Chicago Board of Options Exchange (CBOE). When we downloaded the data, here's what it looked like:

	A	B	C	D
1	**GENERAL MOTORS OPTION DATA** **Downloaded from Chicago Board of Options Exchange Web Site**			
2	Calls	Last Sale	Puts	Last Sale
3	01 Aug 60.00 (GM HL-E)	3.5	01 Aug 60.00 (GM TL-E)	0.5
4	01 Aug 60.00 (GM HL-A)	3.4	01 Aug 60.00 (GM TL-A)	0.4
5	01 Aug 60.00 (GM HL-P)	3	01 Aug 60.00 (GM TL-P)	0.4
6	01 Aug 60.00 (GM HL-X)	2.9	01 Aug 60.00 (GM TL-X)	0.6
7	01 Aug 60.00 (GM HL-8)	3.4	01 Aug 60.00 (GM TL-8)	0.5
8	01 Aug 65.00 (GM HM-E)	0.45	01 Aug 65.00 (GM TM-E)	2.85
9	01 Aug 65.00 (GM HM-A)	0.45	01 Aug 65.00 (GM TM-A)	1.8
10	01 Aug 65.00 (GM HM-P)	0.45	01 Aug 65.00 (GM TM-P)	2.4
11	01 Aug 65.00 (GM HM-X)	1.15	01 Aug 65.00 (GM TM-X)	2.25
12	01 Aug 65.00 (GM HM-8)	0.4	01 Aug 65.00 (GM TM-8)	2.7
13	01 Aug 70.00 (GM HN-E)	0.05	01 Aug 70.00 (GM TN-E)	7.9
14	01 Aug 70.00 (GM HN-A)	0.05	01 Aug 70.00 (GM TN-A)	6.3
15	01 Aug 70.00 (GM HN-P)	0.05	01 Aug 70.00 (GM TN-P)	0
16	01 Aug 70.00 (GM HN-X)	0.2	01 Aug 70.00 (GM TN-X)	7.5
17	01 Aug 70.00 (GM HN-8)	0.05	01 Aug 70.00 (GM TN-8)	6.8
18				
19				
20			Other information	
21				
22	Option expiration year and month			
23			Option exercise price	

The information in columns A and C gives information about the option, including the expiration year and month, the exercise price, and a parenthetical item that shows you the stock on which the option is written, the option symbol, and the exchange on which the option traded. For example:

GM HN-E a General Motors call option with exercise price 70 expiring in August 2001 and trading on the Chicago Board of Options Exchange

GM TL-A is the stock symbol for a General Motors put option with exercise price 60, expiring in August 2001 and trading on the American Stock Exchange

Now suppose we want to separate the dates, the option's symbol, and the exchange on which the option traded:

	C	D	E	F	G	H	I	J	K
2	Puts	Last Sale		Date	Symbol	Exchange			
3	01 Aug 60.00 (GM TL-E)	0.5		01Aug	TL	E			
4	01 Aug 60.00 (GM TL-A)	0.4							
5	01 Aug 60.00 (GM TL-P)	0.4							
6	01 Aug 60.00 (GM TL-X)	0.6							
7	01 Aug 60.00 (GM TL-8)	0.5							
8	01 Aug 65.00 (GM TM-E)	2.85							
9	01 Aug 65.00 (GM TM-A)	1.8							
10	01 Aug 65.00 (GM TM-P)	2.4							
11	01 Aug 65.00 (GM TM-X)	2.25							
12	01 Aug 65.00 (GM TM-8)	2.7							
13	01 Aug 70.00 (GM TN-E)	7.9							

=LEFT(C3,2)&MID(C3,4,3)

=MID(C3,LEN(C3)-4,2)

=MID(C3,LEN(C3)-1,1)

In Chapter ??? (which explains how to use times and dates in Excel), we use this information to design a function that gives us the option's expiration date.

29.5. Statistical functions

Many of Excel's statistical functions have already been discussed in previous chapters:

Average	Finds the average of a range of cells	Chapter 11
Covar	The covariance of two sets of data	Chapter 11
Correl	The correlation coefficient of two sets of data	Chapter 11
Frequency	An array function that computes the frequency distribution.	Chapter 10
Intercept, Slope, Rsq	Compute the intercept, slope and R^2 of a regression	Chapters 11 and 13
Max, Min	The maximum and minimum of a set of numbers	Chapter 10, option chapters
Stdev, StdevP	The standard deviation	Chapters 10 and 11
Var, VarP	The variance	Chapters 10 and 11

Median, Large, and Rank

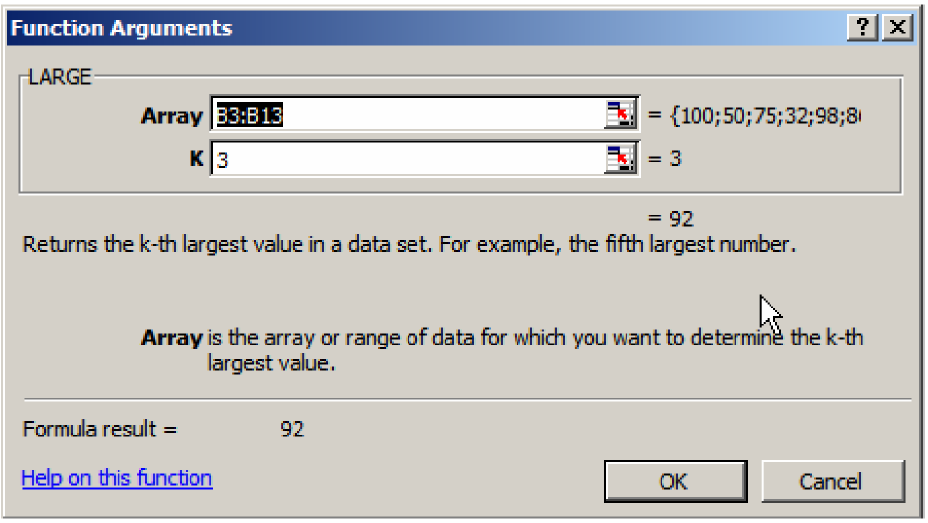
In this section we discuss a three more statistical functions: **Median**, **Large**, and **Rank**.

We illustrate the following example, which gives the grades for 11 students:

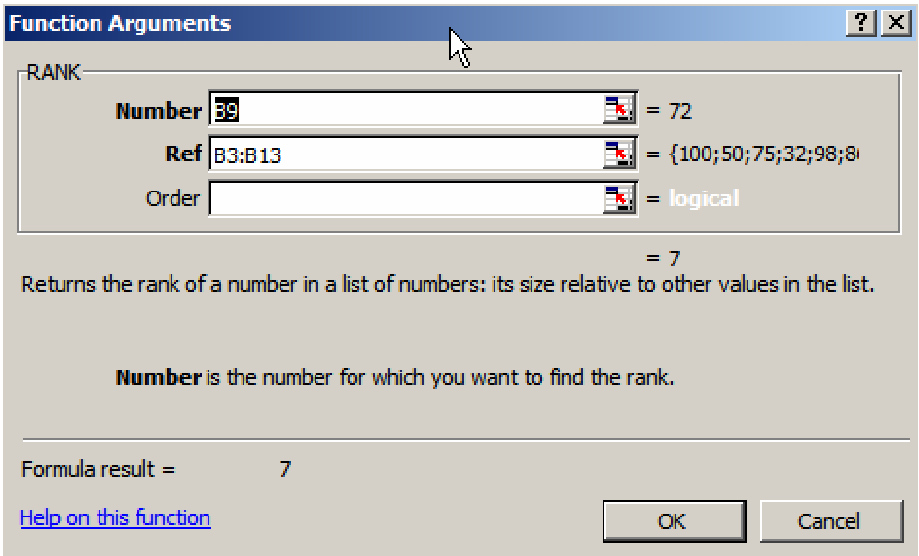
	A	B	C
1	**Median, Large, Rank**		
2	**Student**	**Grade**	
3	1	100	
4	2	50	
5	3	75	
6	4	32	
7	5	98	
8	6	86	
9	7	72	
10	8	63	
11	9	41	
12	10	88	
13	11	92	
14			
15	Average	72.45	<-- =AVERAGE(B3:B13)
16	Median	75	<-- =MEDIAN(B3:B13)
17	Large	92	<-- =LARGE(B3:B13,3)
18	Rank	7	<-- =RANK(B9,B3:B13)

The median is the grade which splits the list in 2: There are 5 grades higher than 75 and 5 lower. The median is different from the average, as you can see.

The Excel function **Large** tells you the k[th] largest number in the set of grades:



The Excel function **Rank** tells you where a particular number places in the range of grades. In the example given the grade 72 is the 7[th] among the set of grades in B3:B13:



Count, CountIf, CountA

All three of these functions *count* cells. The difference (we'll illustrate) is that:

- **Count** counts the number of cells which contain values and ignores the cells that contain text.
- **CountA** counts all non-blank cells in a range, whether they contain values or text.
- **CountIf** counts cells which fulfill a particular condition.

Now we'll illustrate:

	A	B	C
1	**COUNT, COUNTIF, COUNTA**		
2	**List**		
3	1		
4	2		
5	3		
6	4		
7	Terry		
8	Oliver		
9	Noah		
10	Sara		
11	Zvi		
12			
13	Count	4	<-- =COUNT(A3:A11)
14	CountA	9	<-- =COUNTA(A3:A11)
15	CountIf	2	<-- =COUNTIF(A3:A11,">2")

Summary

Excel has hundreds of functions. This chapter has illustrated the major functions used in this book (and then some). We rely on you, as an educated reader, to figure the rest out for yourself.

Exercises

IRR: Use the IRR function to compute the internal rate of return of the project whose cash flows are given below:

	A	B
3	**Year**	**Cash flow**
4	0	-500
5	1	100
6	2	150
7	3	200
8	4	150
9	5	100

IRR: Use **Goal Seek** or **Solver** (see Chapter 32) to determine the IRR of the project whose cash flows are given below. (Recall that IRR is the discount rate for which the NPV equals zero.)

	A	B	C
1	Discount rate	15%	
2			
3	**Year**	**Cash flow**	
4	0	-1,000	
5	1	200	
6	2	400	
7	3	500	
8	4	400	
9	5	200	
10			
11	NPV	133.27	<-- =B4+NPV(B1,B5:B9)

Datevalue as in text.

In the exercise with the dates and the rates and VLookup, some of the dates were not 4 spaces (like the date in A11). Ask them to use **Left** and **Right** as below to fix the Day:

	A	B	C	D	E	F	G
3	**Date**	**Interest rate**		**Month**	**Day**	**Year**	
4	JAN. 07,1991	6.721		JAN	07	1991	
5	FEB. 07,1991	6.145		Feb	07	1991	
6	FEB. 11,1991	6.03		FEB	11	1991	
7	MAR. 04,1991	6.287		MAR	04	1991	
8	APR. 01,1991	5.985		APR	01	1991	
9	JUN. 08,1991	5.777		JUN	08	1991	
10	AUG. 15,1991	5.744		AUG	15	1991	
11	SEPT. 27,1991	5.868		SEP	27	1991	
12							
13							
14		=LEFT(A11,3)				=RIGHT(A11,4)	
15				=LEFT(RIGHT(A11,7),2)			
16							

CHAPTER 30, DATA TABLES*

This version: February 8, 2004

Chapter contents

* This is a preliminary draft of a chapter of *Principles of Finance with Excel*. (benninga@wharton.upenn.edu).

Overview

Data tables are Excel's most sophisticated way of doing sensitivity analysis. They are a bit tricky to implement, but the effort of learning them is well worth it!

30.1. A simple example

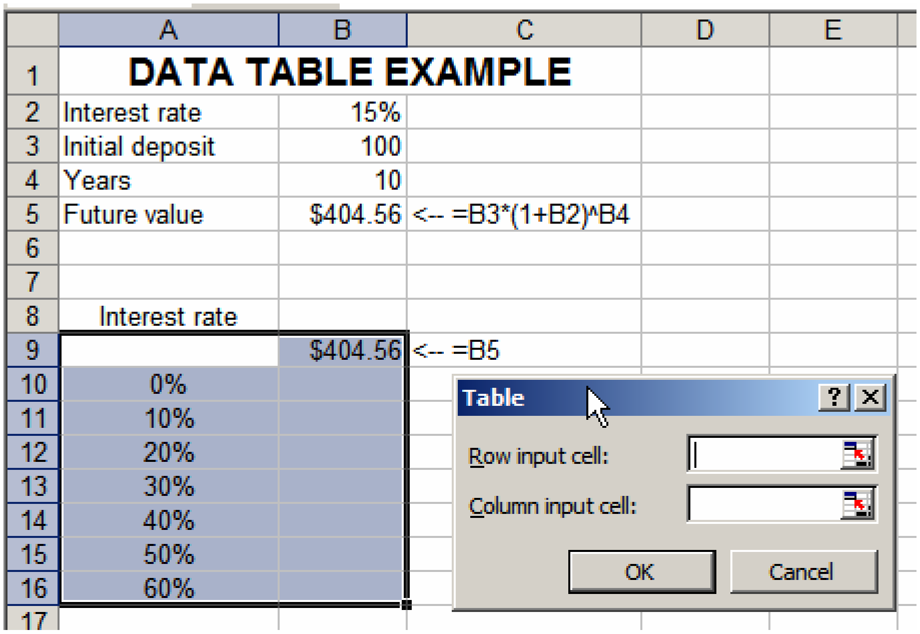
If we deposit $100 today and leave it in a bank drawing 15% interest for 10 years, what will be its future value? As the example below shows, the answer is $404.56:

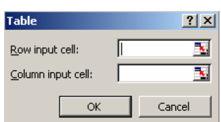
	A	B	C
1	DATA TABLE EXAMPLE		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4

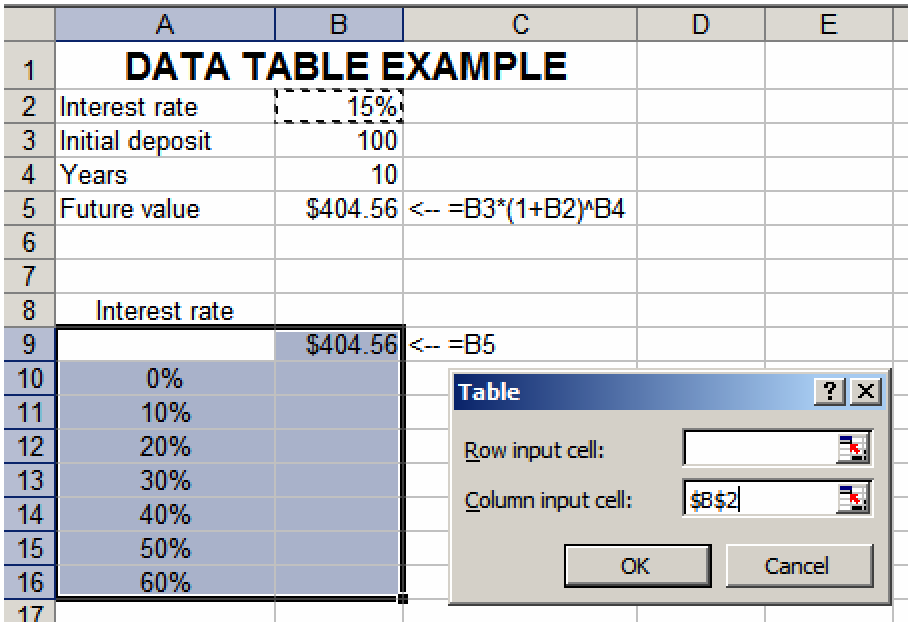
Now suppose we want show the sensitivity of the future value to the interest rate. In cells A10:A16 we have put interest rates varying from 0% to 60%, and in cell B9 we have put **=B5**, which refers to the initial calculation of the future value.

	A	B	C
1	DATA TABLE EXAMPLE		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4
6			
7			
8	Interest rate		
9		$404.56	<-- =B5
10	0%		
11	10%		
12	20%		
13	30%		
14	40%		
15	50%		
16	60%		

To use the data table technique we mark the range A9:B16 and then use the command **Data|Table**. Here's the way the screen looks at this point:



The dialog box  asks whether the parameter to be varied is in a *row* or a *column* of the marked table. In our case, the interest rate to be varied is in column A of the table, so we move the cursor from **Row input cell** to **Column input cell** and indicate *where in the original example the interest rate occurs*:



	A	B	C	D	E
1	DATA TABLE EXAMPLE				
2	Interest rate	15%			
3	Initial deposit	100			
4	Years	10			
5	Future value	$404.56	<-- =B3*(1+B2)^B4		
6					
7					
8	Interest rate				
9		$404.56	<-- =B5		
10	0%				
11	10%				
12	20%				
13	30%				
14	40%				
15	50%				
16	60%				
17					

When you press **OK** you get the result:

	A	B	C
1	DATA TABLE EXAMPLE		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4
6			
7			
8	Interest rate		
9		$404.56	<-- =B5
10	0%	100	
11	10%	259.3742	
12	20%	619.1736	
13	30%	1378.585	
14	40%	2892.547	
15	50%	5766.504	
16	60%	10995.12	

30.2. Summary: How to do a one-dimensional data table

- Create an initial example
- Set up a range with:

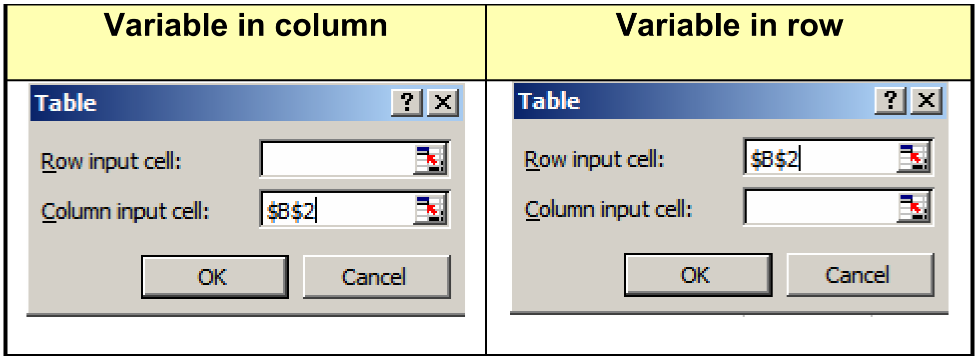
- Some variable in the initial example that will be changed (like the interest rate in the above example)
- A reference to the initial example (like the **=B5** in the above). Note that you will always have a *blank cell* next to this reference. Note the blank cells when the variable is in a column:

	A	B	C	D
1	**DATA TABLE EXAMPLE**			
2	Interest rate	15%		
3	Initial deposit	100		
4	Years	10		
5	Future value	$404.56	<-- =B3*(1+B2)^B4	
6				
7		Blank cell when variable is in column		
8	Interest rate			
9		$404.56	<-- =B5	
10	0%			
11	5%			
12	10%			
13	15%			
14	20%			
15	25%			
16	30%			

Here's the blank cell when the variable is in a row:

	E	F	G	H	I	J	K	L
6								
7		Blank cell when variable is in row						
8								
9		0%	5%	10%	15%	20%	25%	30%
10	$404.56							
11								
12	=B5							
13								

- Bring up the **Data|Table** command and indicate in the dialog box:
  - Whether the variable is in a column or a row
  - Where in the initial example the variable occurs:



Either way the result will be a sensitivity table:

	A	B	C	D	E	F	G	H	I	J	K	L
1	DATA TABLE EXAMPLE											
2	Interest rate	15%										
3	Initial deposit	100										
4	Years	10										
5	Future value	$404.56	<-- =B3*(1+B2)^B4									
6												
7			Blank cell when variable is in column			Blank cell when variable is in row						
8	Interest rate											
9		$404.56	<-- =B5			0%	5%	10%	15%	20%	25%	30%
10	0%	100			$404.56	100	162.8895	259.3742	404.5558	619.1736	931.3226	1378.585
11	5%	162.8895										
12	10%	259.3742			=B5							
13	15%	404.5558										
14	20%	619.1736										
15	25%	931.3226										
16	30%	1378.585										

30.3. Some notes on data tables

Data tables are dynamic

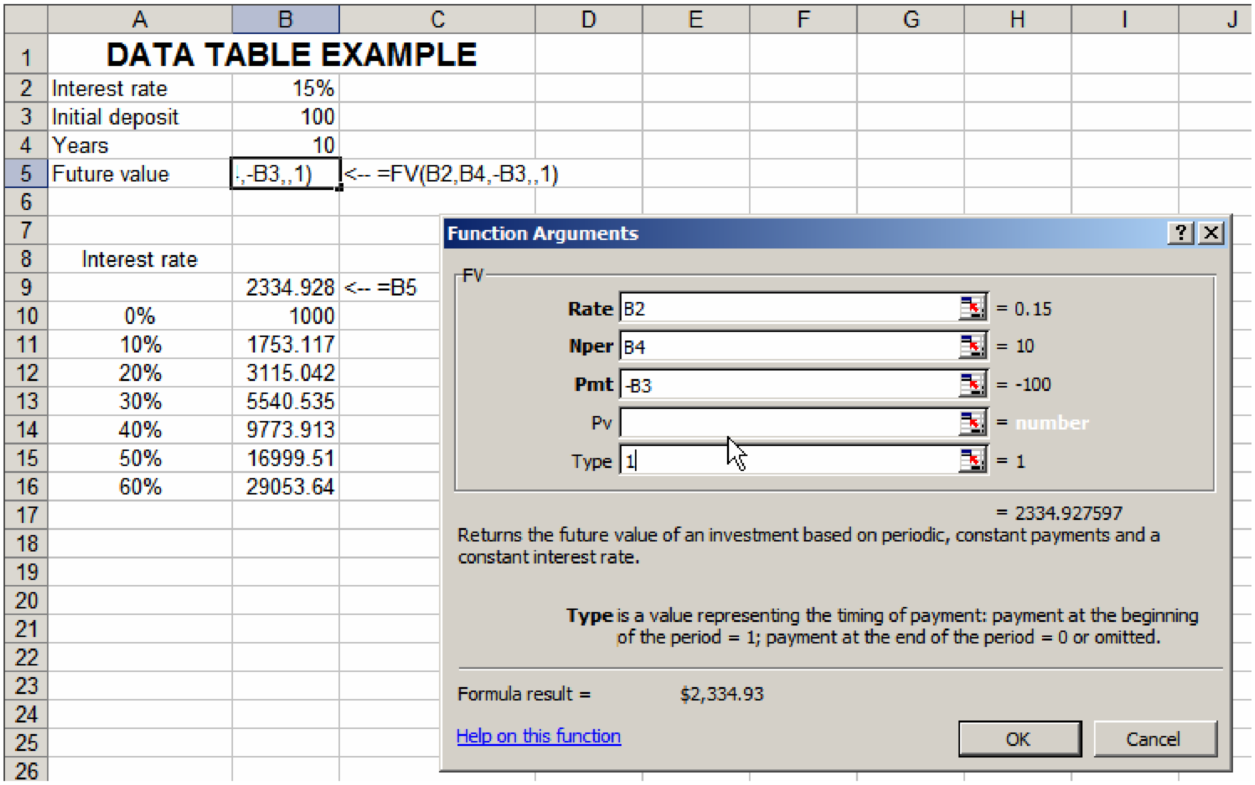
You can change either your initial example or the variables and the table will adjust. Here's an example where we've changed the interest rates we want to vary (compare to the previous example):

	A	B	C
1	DATA TABLE EXAMPLE		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4
6			
7			
8	Interest rate		
9		$404.56	<-- =B5
10	0%	100	
11	10%	259.3742	
12	20%	619.1736	
13	30%	1378.585	
14	40%	2892.547	
15	50%	5766.504	
16	60%	10995.12	

Here's another example: We change the function we're calculating, putting **=FV(B2,B4,-B3,,1)** in cell B5, as explained in Chapter 1, this function calculates the future value of 10 annual $100 deposits starting today and accumulating interest at 15% for 10 years.[1] Note that we've also changed the text in cell A5 from "initial deposit" to "annual deposit" to reflect what's now happening.

---

[1] As we also explained in Chapters 1 and 29, we put the minus sign before **B3** because otherwise—for reasons beyond logic—Excel produces a negative future value. Note that if we had typed **FV(B2,B4,-B3)** the assumption is that there are 10 deposits starting one year from now.

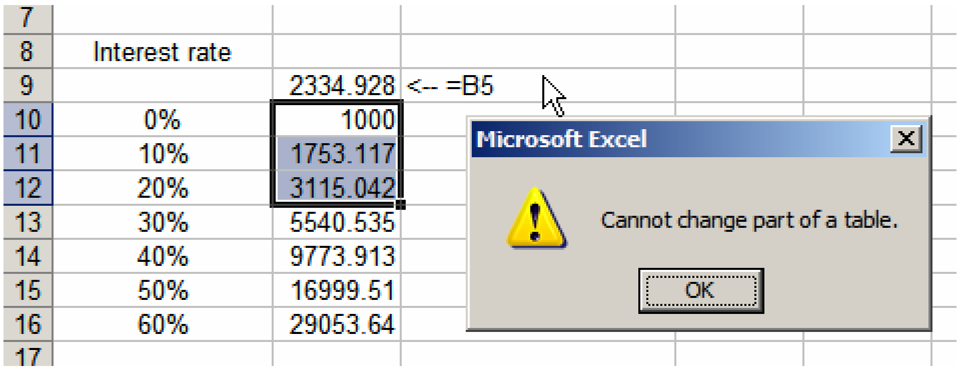


When we press **OK**, both the example and the data table update:

	A	B	C
1	**DATA TABLE EXAMPLE**		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$2,334.93	<-- =FV(B2,B4,-B3,,1)
6			
7			
8	Interest rate		
9		2334.928	<-- =B5
10	0%	1000	
11	10%	1753.117	
12	20%	3115.042	
13	30%	5540.535	
14	40%	9773.913	
15	50%	16999.51	
16	60%	29053.64	

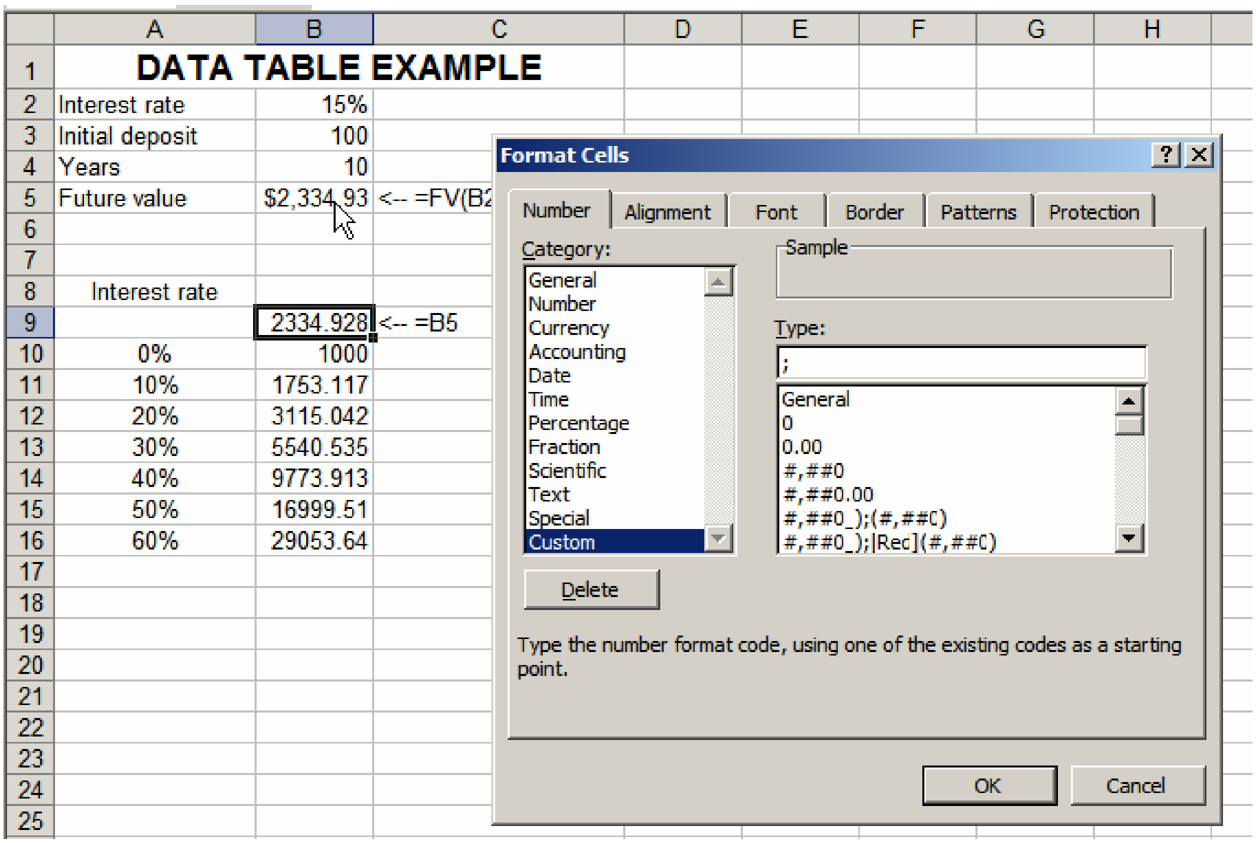
You can only erase the whole table but you cannot erase part of a table

If you try to erase part of a data table, you'll get an error message:



You can hide the cell header but not erase it

The formula at the top of the table's second column (cell B9 in our case, containing the reference to cell B5) is called the "column header." This formula controls what the data table calculates. If you want to print a table, you often want to hide the column header. In the example below, we've put the cursor on cell B9. We then use the command **Format|Cells** and go to **Number|Custom**. Typing a semicolon in the **Type** box hides the cell:

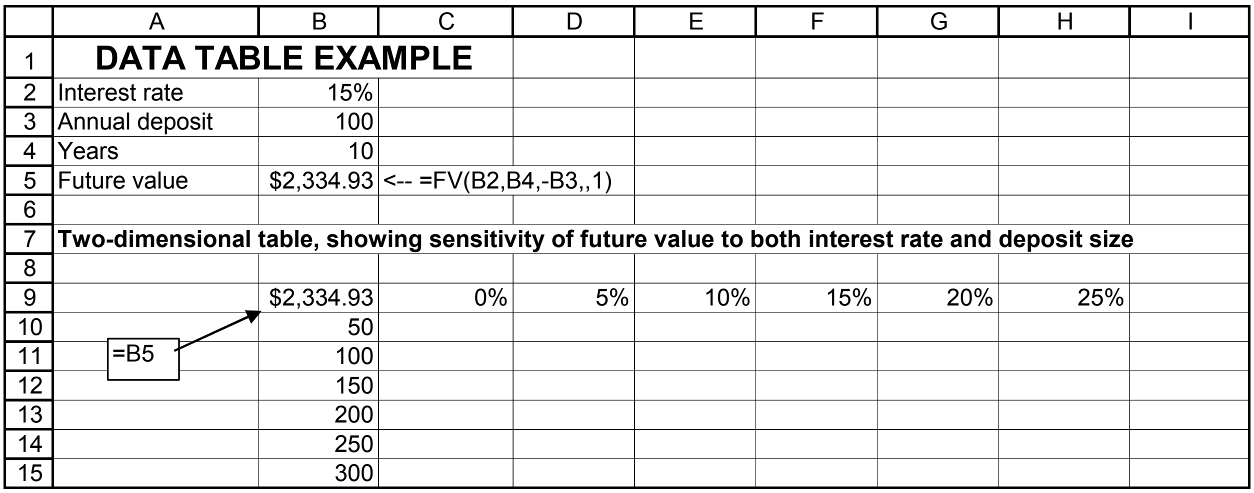


Here's the result:

	A	B	C
8	Interest rate		
9			<-- =B5
10	0%	1000	
11	10%	1753.117	
12	20%	3115.042	
13	30%	5540.535	
14	40%	9773.913	
15	50%	16999.51	
16	60%	29053.64	

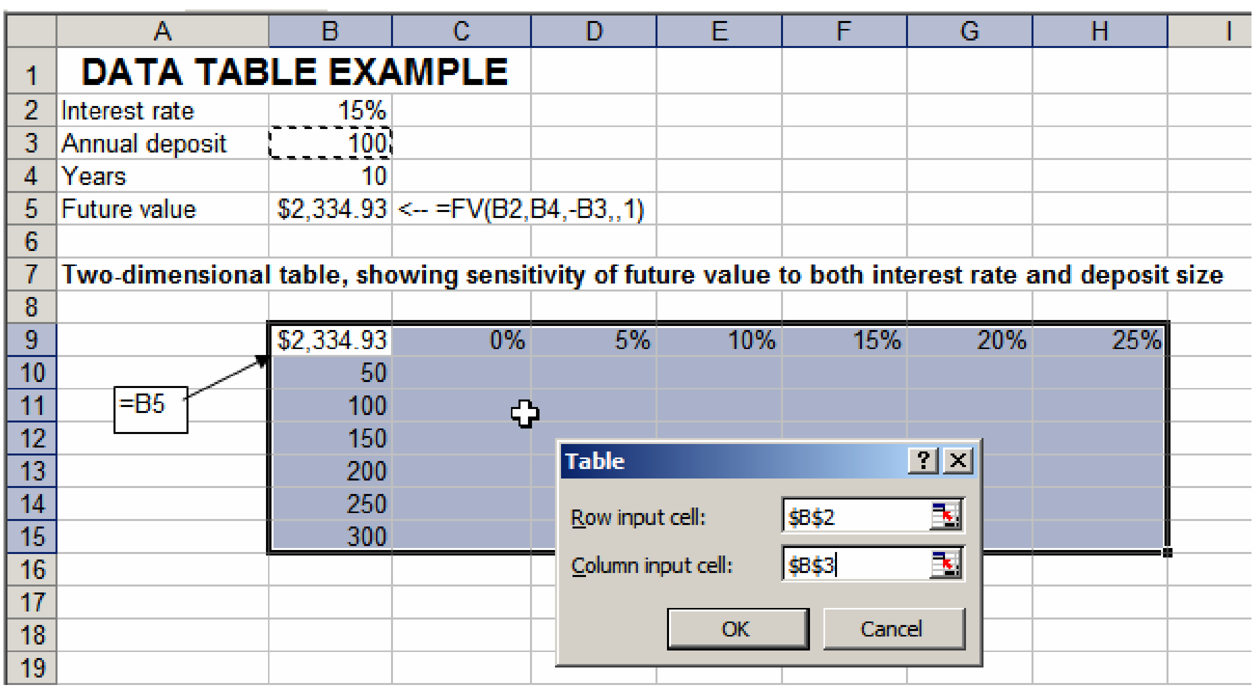
30.4. Two dimensional data tables

In the example below we return to the **FV** example discussed above. We want to vary our initial example with respect to both the interest rate and the initial deposit. The data table is set up in cells B9:H15:



	A	B	C	D	E	F	G	H	I
1	DATA TABLE EXAMPLE								
2	Interest rate	15%							
3	Annual deposit	100							
4	Years	10							
5	Future value	$2,334.93	<-- =FV(B2,B4,-B3,,1)						
6									
7	Two-dimensional table, showing sensitivity of future value to both interest rate and deposit size								
8									
9		$2,334.93	0%	5%	10%	15%	20%	25%	
10		50							
11	=B5	100							
12		150							
13		200							
14		250							
15		300							

This time we indicate in the **Data|Table** command that there are two variables:



	A	B	C	D	E	F	G	H	I
1	DATA TABLE EXAMPLE								
2	Interest rate	15%							
3	Annual deposit	100							
4	Years	10							
5	Future value	$2,334.93	<-- =FV(B2,B4,-B3,,1)						
6									
7	Two-dimensional table, showing sensitivity of future value to both interest rate and deposit size								
8									
9		$2,334.93	0%	5%	10%	15%	20%	25%	
10		50							
11	=B5	100							
12		150							
13		200							
14		250							
15		300							
16									
17									
18									
19									

This creates a two-dimensional table:

	B	C	D	E	F	G	H
9	$2,334.93	0%	5%	10%	15%	20%	25%
10	50	500.00	660.34	876.56	1,167.46	1,557.52	2,078.31
11	100	1,000.00	1,320.68	1,753.12	2,334.93	3,115.04	4,156.61
12	150	1,500.00	1,981.02	2,629.68	3,502.39	4,672.56	6,234.92
13	200	2,000.00	2,641.36	3,506.23	4,669.86	6,230.08	8,313.23
14	250	2,500.00	3,301.70	4,382.79	5,837.32	7,787.60	10,391.53
15	300	3,000.00	3,962.04	5,259.35	7,004.78	9,345.13	12,469.84

Exercises

1. The spreadsheet below shows the value of the function $f(x) = x^2 + 3x - 16$ for *x=3*. Create the indicated data table and use it to graph the function in the range *(-10,14)*.

	A	B	C	D
3	x	3		
4	f(x)	2	<-- =B3^2+3*B3-16	
5				
6				
7	**Data table**			
8		2	<-- =B4	
9	-10			
10	-8			
11	-6			
12	-4			
13	-2			
14	0			
15	2			
16	4			
17	6			
18	8			
19	10			
20	12			
21	14			

2. The example below calculates the NPV and IRR for an investment.

    2.a. Create a one-dimensional data table showing the sensitivity of the NPV and IRR to the year-1 cash flow (currently $10,000). Use a range of $9,000 - $12,000 in increments of $500.

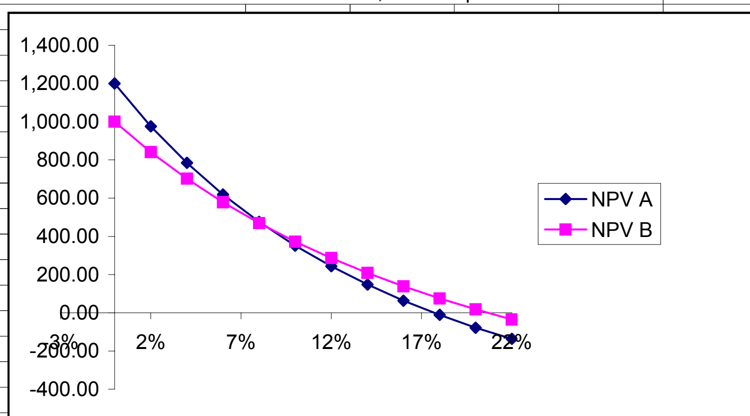
    2.b. Create a two-dimension data table showing the sensitivity of NPV to the year-1 cash flow and to the discount rate. Use the same range for the cash flow as above and use discount rates from 8% to 20%, with increments of 2%.

	A	B	C	D	E
3	Discount rate	15%			
4	Cost	50,000			
5	Cash flow growth	6%			
6					
7	**Year**	**Cash flow**			
8	0	(50,000.00)	<-- =-B4		
9	1	10,000.00			
10	2	10,600.00	<-- =B9*(1+$B$5)		
11	3	11,236.00	<-- =B10*(1+$B$5)		
12	4	11,910.16			
13	5	12,624.77			
14	6	13,382.26			
15	7	14,185.19			
16	8	15,036.30			
17	9	15,938.48			
18	10	16,894.79			
19					
20	NPV	11,925.54	<-- =NPV(B3,B9:B18)+B8		
21	IRR	20.41%	<-- =IRR(B8:B18)		

3. Project A and Project B cash flows are given in the spreadsheet below. Recreate the **Data Table** in cells A21:C37 and create the graph. Notice that the Data Table headers in cells B21:C21 have been hidden (see Section 30.3 for details on how to do this).

What is the crossover point of the two lines? (You can use the data table to do this, but you can also refer to Chapter 3 for a better solution.)

	A	B	C	D
1	**TWO INVESTMENTS AND THEIR NPVs**			
2	Discount rate	15%		
3				
4	**Year**	**Project A cash flow**	**Project B cash flow**	
5	0	-1,000	-1,000	
6	1	220	300	
7	2	220	300	
8	3	220	300	
9	4	220	300	
10	5	220	300	
11	6	220	100	
12	7	220	100	
13	8	220	100	
14	9	220	100	
15	10	220	100	
16				
17	NPV	104.13	172.31	<-- =NPV($B$2,C6:C15)+C5
18	IRR	17.68%	20.64%	<-- =IRR(C5:C15)
19				
20		**NPV A**	**NPV B**	
21				<-- The data table headers have been hidden; see Chapter 30 for details
22	0%	1,200.00	1,000.00	
23	2%	976.17	840.95	
24	4%	784.40	701.45	
25	6%	619.22	578.48	
26	8%	476.22	469.55	
27	10%	351.80	372.61	
28	12%	243.05	285.98	
29	14%	147.55	208.23	
30	16%	63.31	138.18	
31	18%	-11.30	74.84	
32	20%	-77.66	17.37	
33	22%	-136.90	-34.95	
34	24%	-189.99	-82.74	
35	26%	-237.74	-126.51	
36	28%	-280.84	-166.71	
37	30%	-319.86	-203.73	
38				



4. Finance texts always have tables which give the present value factor for an annuity:

$$PV \;\; factor \;\; for \;\; annuity \;\; of \;\; \$1 \;\; for \;\; N \;\; years = \sum_{t=1}^{N} \frac{1}{(1+r)^t}.$$

As illustrated below in Excel these present value factors are created with the **PV function**:

	A	B	C	D	E	F	G	H	I	J	K
1					ANNUITY TABLE						
2											
3	r	9%									
4	T	5									
5	PV factor	3.8897	<-- =PV(B3,B4,-1)								
6											
7											
8	**Number of periods**		**PRESENT VALUE OF AN ANNUITY OF $1 FOR N PERIODS**								
9		**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
10	**1**										
11	**2**										
12	**3**										
13	**4**										
14	**5**										
15	**6**										
16	**7**										
17	**8**										
18	**9**										
19	**10**										

Use **Data Table** to create the table in the template above.

5. (Do this example only if you've studied Chapter 22 on option pricing.) The Black-Scholes option pricing model, defined in Chapter 24, prices call and put options based on 5 parameters:

- *S*, the stock price today
- *X*, the option's exercise price (also called the option's *strike price*)
- *T*, the option's expiration date
- *r*, the interest rate
- σ ("Sigma"), the riskiness of the stock

These inputs and the resulting call and put prices are highlighted below.

Your assignment: Use **Data Table** to create tables showing the sensitivity of the call and put prices to the various inputs. Here are some suggestions:

5.a. Using the parameters shown below, what are the call and put prices given σ = 10%, 15%, 20%, … , 80%?

5.b. Using the parameters shown below, what are the call and put prices when $T = 0.1$, 0.2, 0.3, … , 1?

	A	B	C
1	**The Black-Scholes Option-Pricing Formula**		
2	S	100	Current stock price
3	X	90	Exercise price
4	T	0.50000	Time to maturity of option (in years)
5	r	4.00%	Risk-free rate of interest
6	Sigma	35%	Stock volatility
7			
8	d_1	0.6303	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))
9	d_2	0.3828	<-- d_1-sigma*SQRT(T)
10			
11	N(d_1)	0.7357	<-- Uses formula NormSDist(d_1)
12	N(d_2)	0.6491	<-- Uses formula NormSDist(d_2)
13			
14	Call price	16.32	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)
15	Put price	4.53	<-- call price - S + X*Exp(-r*T): by Put-Call parity

CHAPTER 30, DATA TABLES*

slight bug fix: July 12, 2003

Chapter contents

Overview

Data tables are Excel's most sophisticated way of doing sensitivity analysis. They are a bit tricky to implement, but the effort of learning them is well worth it!

---

* **Notice**: This is a preliminary draft of a chapter of *Principles of Finance* by Simon Benninga (benninga@wharton.upenn.edu). Check with the author before distributing this draft (though you will probably get permission). Make sure the material is updated before distributing it. All the material is copyright and the rights belong to the author.

30.1. A simple example

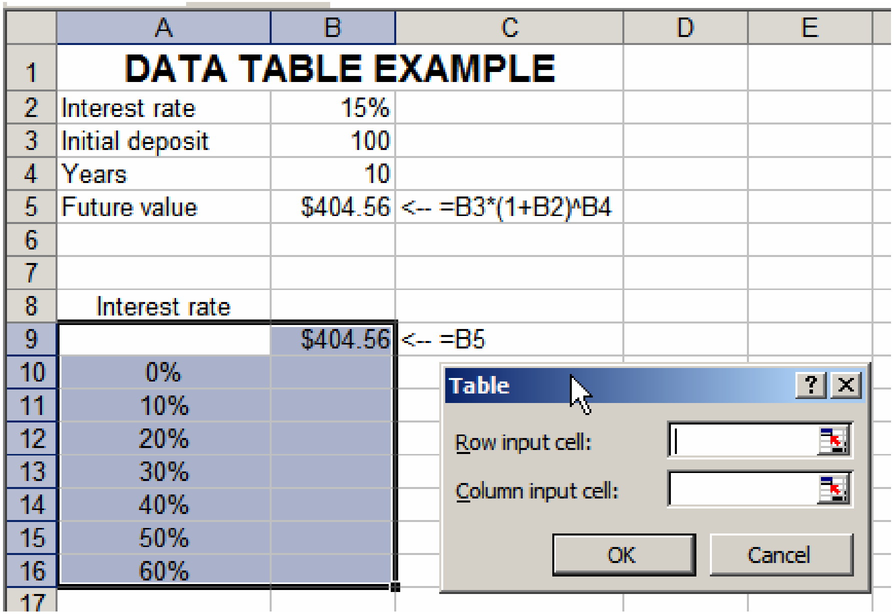
If we deposit $100 today and leave it in a bank drawing 15% interest for 10 years, what will be its future value? As the example below shows, the answer is $404.56:

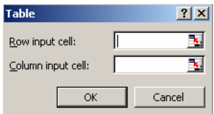
	A	B	C
1	**DATA TABLE EXAMPLE**		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4

Now suppose we want show the sensitivity of the future value to the interest rate. In cells A10:A16 we have put interest rates varying from 0% to 60%, and in cell B9 we have put **=B5**, which refers to the initial calculation of the future value.

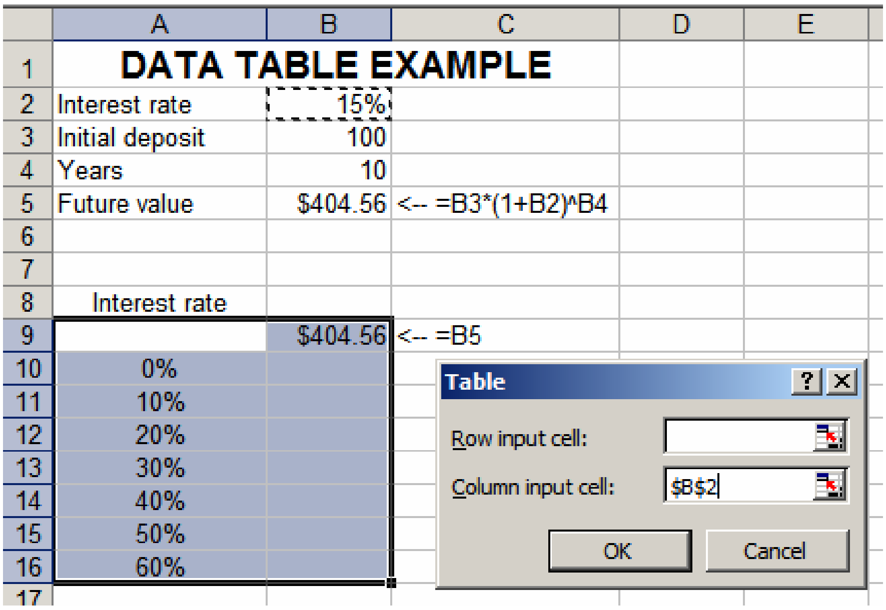
	A	B	C
1	**DATA TABLE EXAMPLE**		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4
6			
7			
8	Interest rate		
9		$404.56	<-- =B5
10	0%		
11	10%		
12	20%		
13	30%		
14	40%		
15	50%		
16	60%		

To use the data table technique we mark the range A9:B16 and then use the command **Data|Table**. Here's the way the screen looks at this point:



The dialog box 

asks whether the parameter to be varied is in a *row* or a *column* of the marked table. In our case, the interest rate to be varied is in column A of the table, so we move the cursor from **Row input cell** to **Column input cell** and indicate *where in the original example the interest rate occurs*:



When you press **OK** you get the result:

	A	B	C
1	DATA TABLE EXAMPLE		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4
6			
7			
8	Interest rate		
9		$404.56	<-- =B5
10	0%	100	
11	10%	259.3742	
12	20%	619.1736	
13	30%	1378.585	
14	40%	2892.547	
15	50%	5766.504	
16	60%	10995.12	

30.2. Summary: How to do a one-dimensional data table

- Create an initial example
- Set up a range with:

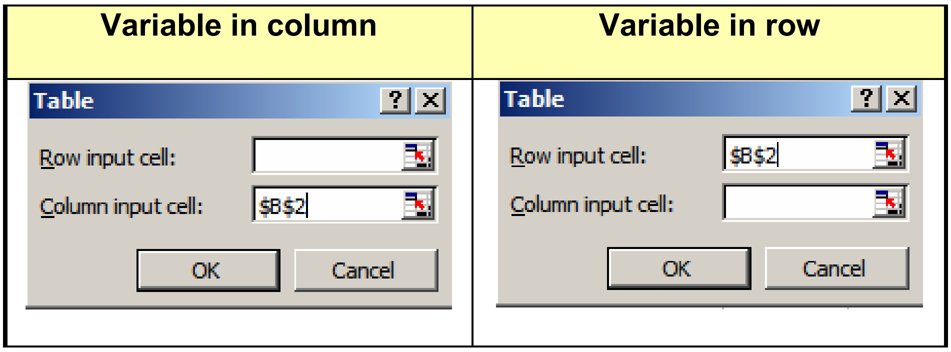
- Some variable in the initial example that will be changed (like the interest rate in the above example)
- A reference to the initial example (like the **=B5** in the above). Note that you will always have a *blank cell* next to this reference. Note the blank cells when the variable is in a column:

	A	B	C	D
1	**DATA TABLE EXAMPLE**			
2	Interest rate	15%		
3	Initial deposit	100		
4	Years	10		
5	Future value	$404.56	<-- =B3*(1+B2)^B4	
6				
7		Blank cell when variable is in column		
8	Interest rate			
9		$404.56	<-- =B5	
10	0%			
11	5%			
12	10%			
13	15%			
14	20%			
15	25%			
16	30%			

Here's the blank cell when the variable is in a row:

	E	F	G	H	I	J	K	L
6								
7		Blank cell when variable is in row						
8								
9		0%	5%	10%	15%	20%	25%	30%
10	$404.56							
11								
12	=B5							
13								

- Bring up the **Data|Table** command and indicate in the dialog box:
  - Whether the variable is in a column or a row
  - Where in the initial example the variable occurs:



Either way the result will be a sensitivity table:

	A	B	C	D	E	F	G	H	I	J	K	L
1	**DATA TABLE EXAMPLE**											
2	Interest rate	15%										
3	Initial deposit	100										
4	Years	10										
5	Future value	$404.56	<-- =B3*(1+B2)^B4									
6												
7		Blank cell when variable is in column				Blank cell when variable is in row						
8	Interest rate											
9		$404.56	<-- =B5			0%	5%	10%	15%	20%	25%	30%
10	0%	100			$404.56	100	162.8895	259.3742	404.5558	619.1736	931.3226	1378.585
11	5%	162.8895										
12	10%	259.3742			=B5							
13	15%	404.5558										
14	20%	619.1736										
15	25%	931.3226										
16	30%	1378.585										

30.3. Some notes on data tables

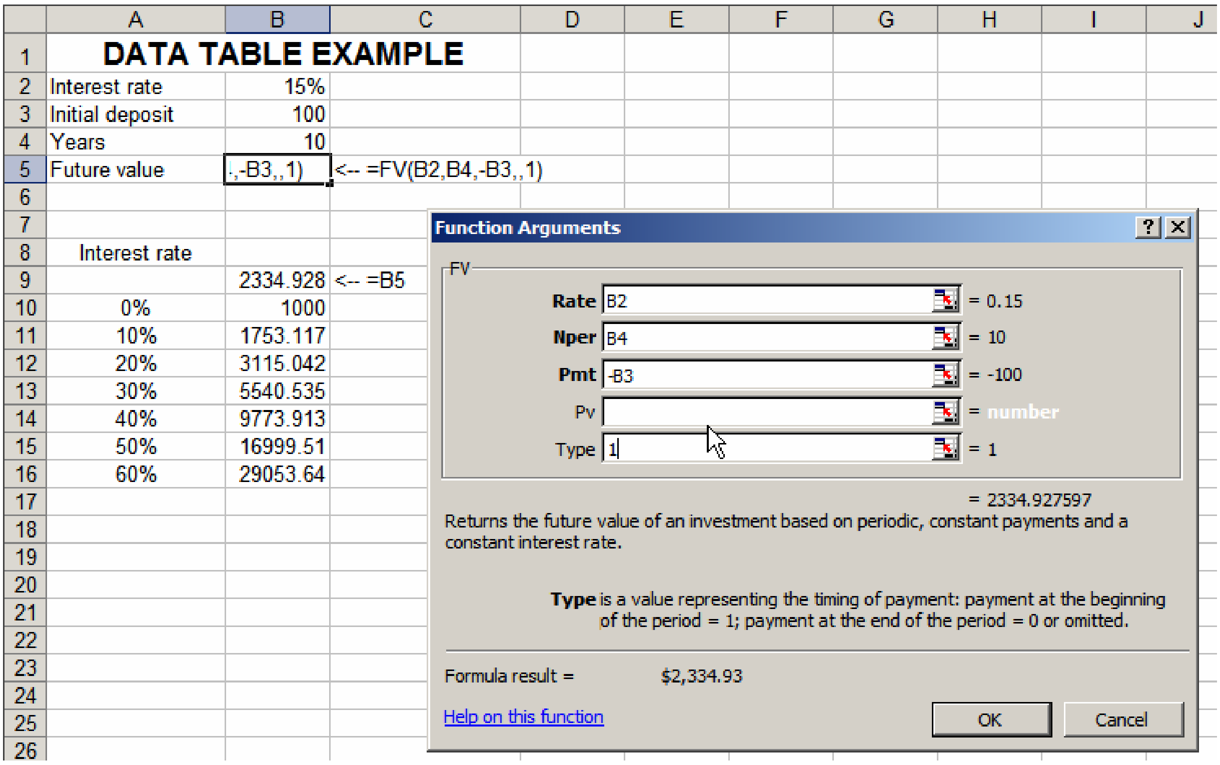
Data tables are dynamic

You can change either your initial example or the variables and the table will adjust. Here's an example where we've changed the interest rates we want to vary (compare to the previous example):

	A	B	C
1	**DATA TABLE EXAMPLE**		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$404.56	<-- =B3*(1+B2)^B4
6			
7			
8	Interest rate		
9		$404.56	<-- =B5
10	0%	100	
11	10%	259.3742	
12	20%	619.1736	
13	30%	1378.585	
14	40%	2892.547	
15	50%	5766.504	
16	60%	10995.12	

Here's another example: We change the function we're calculating, putting **=FV(B2,B4,-B3,,1)** in cell B5, as explained in Chapter 1, this function calculates the future value of 10 annual $100 deposits starting today and accumulating interest at 15% for 10 years.[1] Note that we've also changed the text in cell A5 from "initial deposit" to "annual deposit" to reflect what's now happening.

[1] As we also explained in Chapters 1 and 29, we put the minus sign before **B3** because otherwise—for reasons beyond logic—Excel produces a negative future value. Note that if we had typed **FV(B2,B4,-B3)** the assumption is that there are 10 deposits starting one year from now.

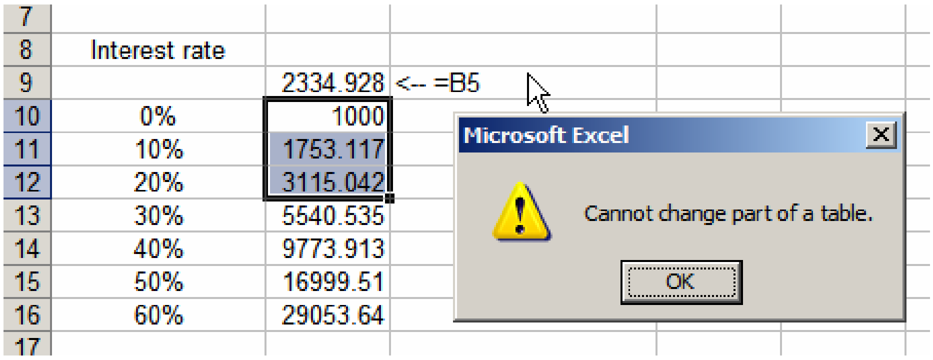


When we press **OK**, both the example and the data table update:

	A	B	C
1	DATA TABLE EXAMPLE		
2	Interest rate	15%	
3	Initial deposit	100	
4	Years	10	
5	Future value	$2,334.93	<-- =FV(B2,B4,-B3,,1)
6			
7			
8	Interest rate		
9		2334.928	<-- =B5
10	0%	1000	
11	10%	1753.117	
12	20%	3115.042	
13	30%	5540.535	
14	40%	9773.913	
15	50%	16999.51	
16	60%	29053.64	

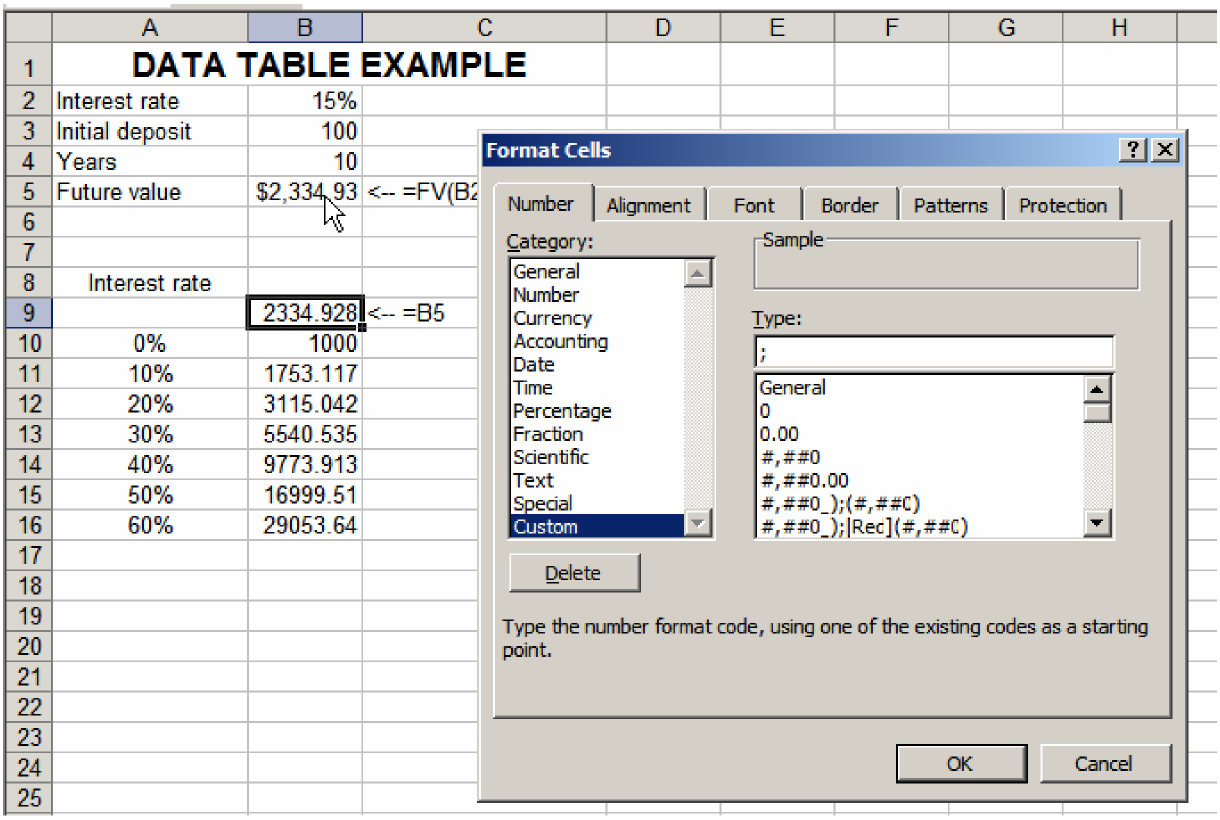
You can only erase the whole table but you cannot erase part of a table

If you try to erase part of a data table, you'll get an error message:



You can hide the cell header but not erase it

The formula at the top of the table's second column (cell B9 in our case, containing the reference to cell B5) is called the "column header." This formula controls what the data table calculates. If you want to print a table, you often want to hide the column header. In the example below, we've put the cursor on cell B9. We then use the command **Format|Cells** and go to **Number|Custom**. Typing a semicolon in the **Type** box hides the cell:



Here's the result:

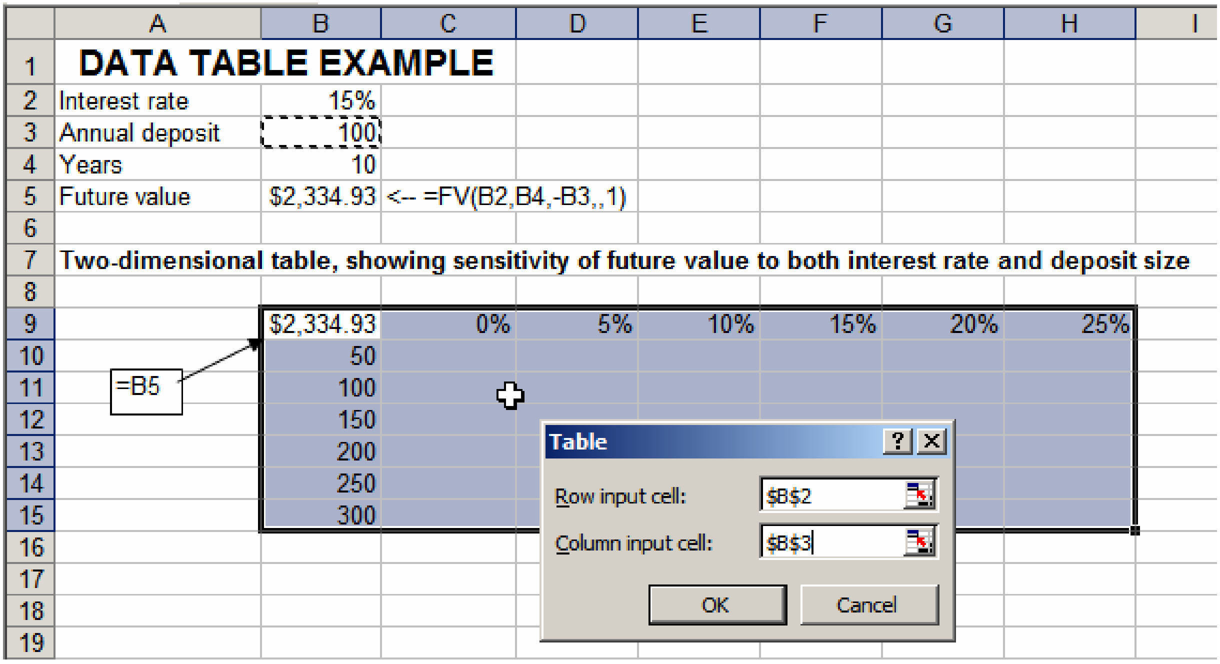
	A	B	C
8	Interest rate		
9			<-- =B5
10	0%	1000	
11	10%	1753.117	
12	20%	3115.042	
13	30%	5540.535	
14	40%	9773.913	
15	50%	16999.51	
16	60%	29053.64	

30.4. Two dimensional data tables

In the example below we return to the **FV** example discussed above. We want to vary our initial example with respect to both the interest rate and the initial deposit. The data table is set up in cells B9:H15:

	A	B	C	D	E	F	G	H	I
1	**DATA TABLE EXAMPLE**								
2	Interest rate	15%							
3	Annual deposit	100							
4	Years	10							
5	Future value	$2,334.93	<-- =FV(B2,B4,-B3,,1)						
6									
7	**Two-dimensional table, showing sensitivity of future value to both interest rate and deposit size**								
8									
9		$2,334.93	0%	5%	10%	15%	20%	25%	
10		50							
11	=B5	100							
12		150							
13		200							
14		250							
15		300							

This time we indicate in the **Data|Table** command that there are two variables:



This creates a two-dimensional table:

	B	C	D	E	F	G	H
9	$2,334.93	0%	5%	10%	15%	20%	25%
10	50	500.00	660.34	876.56	1,167.46	1,557.52	2,078.31
11	100	1,000.00	1,320.68	1,753.12	2,334.93	3,115.04	4,156.61
12	150	1,500.00	1,981.02	2,629.68	3,502.39	4,672.56	6,234.92
13	200	2,000.00	2,641.36	3,506.23	4,669.86	6,230.08	8,313.23
14	250	2,500.00	3,301.70	4,382.79	5,837.32	7,787.60	10,391.53
15	300	3,000.00	3,962.04	5,259.35	7,004.78	9,345.13	12,469.84

EXERCISES

1. The spreadsheet below shows the value of the function $f(x) = x^2 + 3x - 16$ for $x=3$. Create the indicated data table and use it to graph the function in the range *(-10,14)*.

	A	B	C	D
3	x	3		
4	f(x)	2	<-- =B3^2+3*B3-16	
5				
6				
7	**Data table**			
8		2	<-- =B4	
9	-10			
10	-8			
11	-6			
12	-4			
13	-2			
14	0			
15	2			
16	4			
17	6			
18	8			
19	10			
20	12			
21	14			

2. The example below calculates the NPV and IRR for an investment.

a. Create a one-dimensional data table showing the sensitivity of the NPV and IRR to the year-1 cash flow (currently $10,000). Use a range of $9,000 - $12,000 in increments of $500.

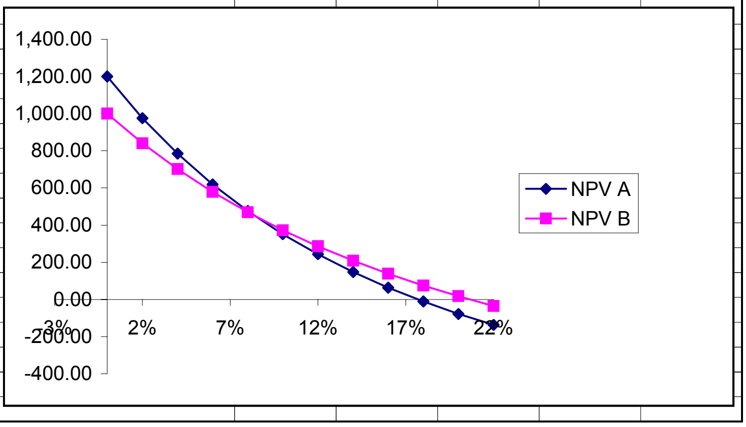
b. Create a two-dimension data table showing the sensitivity of NPV to the year-1 cash flow and to the discount rate. Use the same range for the cash flow as above and use discount rates from 8% to 20%, with increments of 2%.

	A	B	C	D	E
3	Discount rate	15%			
4	Cost	50,000			
5	Cash flow growth	6%			
6					
7	**Year**	**Cash flow**			
8	0	(50,000.00)	<-- =-B4		
9	1	10,000.00			
10	2	10,600.00	<-- =B9*(1+$B$5)		
11	3	11,236.00	<-- =B10*(1+$B$5)		
12	4	11,910.16			
13	5	12,624.77			
14	6	13,382.26			
15	7	14,185.19			
16	8	15,036.30			
17	9	15,938.48			
18	10	16,894.79			
19					
20	NPV	11,925.54	<-- =NPV(B3,B9:B18)+B8		
21	IRR	20.41%	<-- =IRR(B8:B18)		

3. Project A and Project B cash flows are given in the spreadsheet below. Recreate the **Data Table** in cells A21:C37 and create the graph. Notice that the Data Table headers in cells B21:C21 have been hidden (see Section 30.3 for details on how to do this).

What is the crossover point of the two lines? (You can use the data table to do this, but you can also refer to Chapter 3 for a better solution.)

	A	B	C	D	E	F	G	H	I
1	**TWO INVESTMENTS AND THEIR NPVs**								
2	Discount rate	15%							
3									
4	**Year**	**Project A cash flow**	**Project B cash flow**						
5	0	-1,000	-1,000						
6	1	220	300						
7	2	220	300						
8	3	220	300						
9	4	220	300						
10	5	220	300						
11	6	220	100						
12	7	220	100						
13	8	220	100						
14	9	220	100						
15	10	220	100						
16									
17	NPV	104.13	172.31	<-- =NPV($B$2,C6:C15)+C5					
18	IRR	17.68%	20.64%	<-- =IRR(C5:C15)					
19									
20		**NPV A**	**NPV B**						
21				<-- The data table headers have been hidden; see Chapter 30 for details					
22	0%	1,200.00	1,000.00						
23	2%	976.17	840.95						
24	4%	784.40	701.45						
25	6%	619.22	578.48						
26	8%	476.22	469.55						
27	10%	351.80	372.61						
28	12%	243.05	285.98						
29	14%	147.55	208.23						
30	16%	63.31	138.18						
31	18%	-11.30	74.84						
32	20%	-77.66	17.37						
33	22%	-136.90	-34.95						
34	24%	-189.99	-82.74						
35	26%	-237.74	-126.51						
36	28%	-280.84	-166.71						
37	30%	-319.86	-203.73						
38									



4. Finance texts always have tables which give the present value factor for an annuity:

$$PV\ \ factor\ \ for\ \ annuity\ \ of\ \ \$1\ \ for\ \ N\ \ years = \sum_{t=1}^{N} \frac{1}{(1+r)^t}.$$

As illustrated below in Excel these present value factors are created with the **PV function**:

	A	B	C	D	E	F	G	H	I	J	K
1	ANNUITY TABLE										
2											
3	r	9%									
4	T	5									
5	PV factor	3.8897	<-- =PV(B3,B4,-1)								
6											
7											
8	**Number of periods**		**PRESENT VALUE OF AN ANNUITY OF $1 FOR N PERIODS**								
9		**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
10	**1**										
11	**2**										
12	**3**										
13	**4**										
14	**5**										
15	**6**										
16	**7**										
17	**8**										
18	**9**										
19	**10**										

Use **Data Table** to create the table in the template above.

5. (Do this example only if you've studied Chapter 24 on option pricing.) The Black-Scholes option pricing model, defined in Chapter 24, prices call and put options based on 5 parameters:

- *S*, the stock price today
- *X*, the option's exercise price (also called the option's *strike price*)
- *T*, the option's expiration date
- *r*, the interest rate
- σ ("Sigma"), the riskiness of the stock

These inputs and the resulting call and put prices are highlighted below.

Your assignment: Use **Data Table** to create tables showing the sensitivity of the call and put prices to the various inputs. Here are some suggestions:

a. Using the parameters shown below, what are the call and put prices given σ = 10%, 15%, 20%, … , 80%?

b. Using the parameters shown below, what are the call and put prices when $T = 0.1, 0.2, 0.3, \ldots, 1$?

	A	B	C
1	**The Black-Scholes Option-Pricing Formula**		
2	S	100	Current stock price
3	X	90	Exercise price
4	T	0.50000	Time to maturity of option (in years)
5	r	4.00%	Risk-free rate of interest
6	Sigma	35%	Stock volatility
7			
8	d_1	0.6303	<-- (LN(S/X)+(r+0.5*sigma^2)*T)/(sigma*SQRT(T))
9	d_2	0.3828	<-- d_1-sigma*SQRT(T)
10			
11	$N(d_1)$	0.7357	<-- Uses formula NormSDist(d_1)
12	$N(d_2)$	0.6491	<-- Uses formula NormSDist(d_2)
13			
14	Call price	16.32	<-- S*N(d_1)-X*exp(-r*T)*N(d_2)
15	Put price	4.53	<-- call price - S + X*Exp(-r*T): by Put-Call parity