DESIGN OF MACHINERY

AN INTRODUCTION TO THE SYNTHESIS AND ANALYSIS OF MECHANISMS AND MACHINES

Second Edition
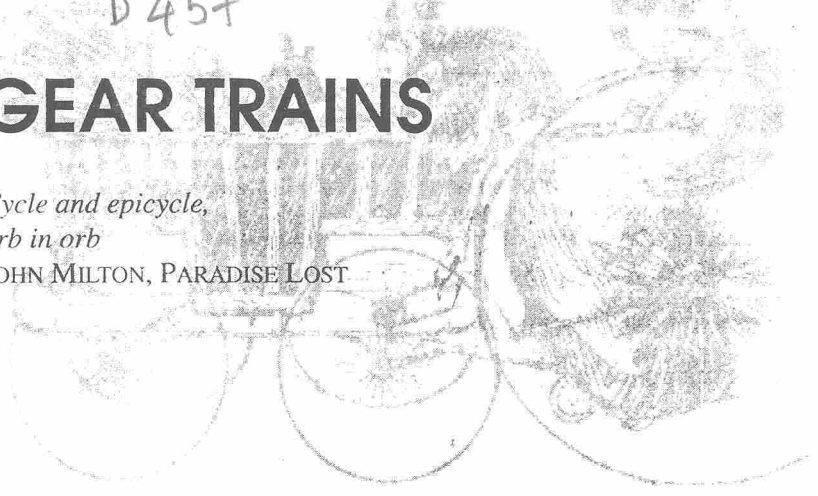
Chapter 9

GEAR TRAINS

Cycle and epicycle,
orb in orb
JOHN MILTON, PARADISE LOST

9.0 INTRODUCTION

The earliest known reference to gear trains is in a treatise by Hero of Alexandria (c. 100 B.C.). Gear trains are widely used in all kinds of mechanisms and machines, from can openers to aircraft carriers. Whenever a change in the speed or torque of a rotating device is needed, a gear train or one of its cousins, the belt or chain drive mechanism, will usually be used. This chapter will explore the theory of gear tooth action and the design of these ubiquitous devices for motion control. The calculations involved are trivial compared to those for cams or linkages. The shape of gear teeth has become quite standardized for good kinematic reasons which we will explore.

Gears of various sizes and styles are readily available from many manufacturers. Assembled gearboxes for particular ratios are also stock items. The kinematic design of gear trains is principally involved with the selection of appropriate ratios and gear diameters. A complete gear train design will necessarily involve considerations of strength of materials and the complicated stress states to which gear teeth are subjected. This text will not deal with the stress analysis aspects of gear design. There are many texts which do. Some are listed in the bibliography at the end of this chapter. This chapter will discuss the kinematics of gear tooth theory, gear types, and the kinematic design of gearsets and gear trains of simple, compound, reverted, and epicyclic types. Chain and belt drives will also be discussed. Examples of the use of these devices will be presented as well.

432

9.1   ROLLING CYLINDERS

The simplest means of transferring rotary motion from one shaft to another is a pair of rolling cylinders. They may be an external set of rolling cylinders as shown in Figure 9-1a or an internal set as in Figure 9-1b. Provided that sufficient friction is available at the rolling interface, this mechanism will work quite well. There will be no slip between the cylinders until the maximum available frictional force at the joint is exceeded by the demands of torque transfer.

A variation on this mechanism is what causes your car or bicycle to move along the road. Your tire is one rolling cylinder and the road the other (very large radius) one. Friction is all that prevents slip between the two, and it works well unless the friction coefficient is reduced by the presence of ice or other slippery substances. In fact, some early automobiles had rolling cylinder drives inside the transmission, as do some present-day snowblowers and garden tractors which use a rubber-coated wheel rolling against a steel disk to transmit power from the engine to the wheels.

A variant on the rolling cylinder drive is the flat or vee belt as shown in Figure 9-2. This mechanism also transfers power through friction and is capable of quite large power levels, provided enough belt cross section is provided. Friction belts are used in a wide variety of applications from small sewing machines to the alternator drive on your car, to multihorsepower generators and pumps. Whenever absolute phasing is not required and power levels are moderate, a friction belt drive may be the best choice. They are relatively quiet running, require no lubrication, and are inexpensive compared to gears and chain drives.

Both rolling cylinders and belt (or chain) drives have effective linkage equivalents as shown in Figure 9-3. These effective linkages are valid only for one instantaneous position but nevertheless show that these devices are just another variation of the four-bar linkage in disguise.
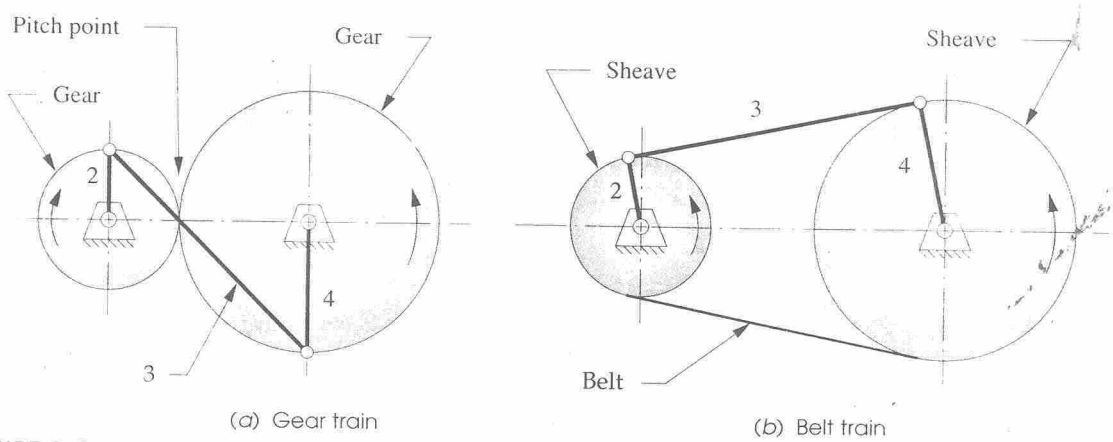

(a) External set

(b) Internal set
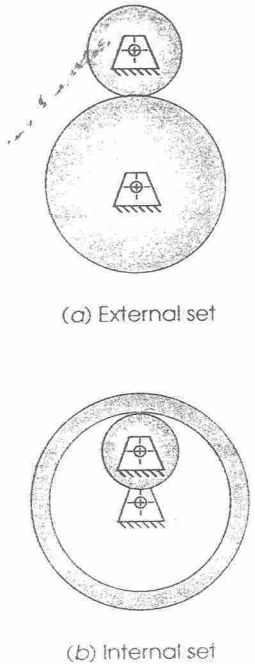
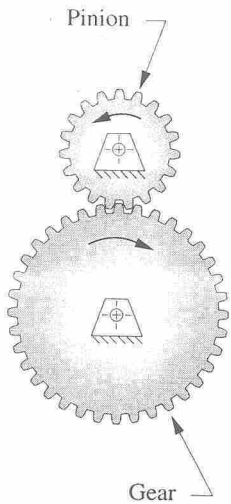**FIGURE 9-1**

Rolling cylinders
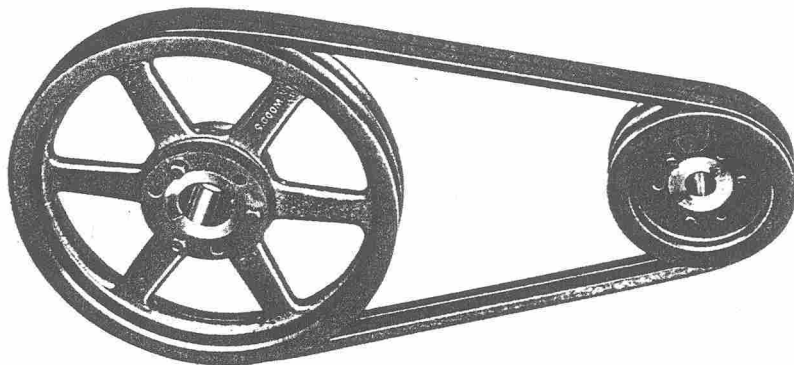
FIGURE 9-2

A two-groove vee belt drive *Courtesy of I. B. Woods Sons Co., Chambersburg, PA*

FIGURE 9-3

Gear and belt trains each have an equivalent fourbar linkage for any instantaneous position

The principal drawbacks to the rolling cylinder drive (or smooth belt) mechanism are its relatively low torque capability and the possibility of slip. Some drives require absolute phasing of the input and output shafts for timing purposes. A common example is the valve train drive in an automobile engine. The valve cams must be kept in phase with the piston motion or the engine will not run properly. A smooth belt or rolling cylinder drive from crankshaft to camshaft would not guarantee correct phasing. In this case some means of preventing slip is needed.

This usually means adding some meshing teeth to the rolling cylinders. They then become gears as shown in Figure 9-4 and are together called a *gearset*. When two gears are placed in mesh to form a gearset such as this one, it is conventional to refer to the smaller of the two gears as the *pinion* and to the other as the *gear*.

9.2   THE FUNDAMENTAL LAW OF GEARING

Conceptually, teeth of any shape will prevent gross slip. Old water-powered mills and windmills used wooden gears whose teeth were merely round wooden pegs stuck into the rims of the cylinders. Even ignoring the crudity of construction of these early examples of gearsets, there was no possibility of smooth velocity transmission because the geometry of the tooth "pegs" violated the **fundamental law of gearing** which, if followed, provides that *the angular velocity ratio between the gears of a gearset remains constant throughout the mesh.* A more complete and formal definition of this law is given on p. 436. The angular velocity ratio (m_V) referred to in this law is the same one that we derived for the fourbar linkage in Section 6.4 and equation 6.10 (p. 257). It is equal to the ratio of the radius of the input gear to that of the output gear.

FIGURE 9-4

An external gearset

$$m_V = \frac{\omega_{out}}{\omega_{in}} = \pm \frac{r_{in}}{r_{out}} = \pm \frac{d_{in}}{d_{out}} \tag{9.1a}$$

$$m_T = \frac{\omega_{in}}{\omega_{out}} = \pm \frac{r_{out}}{r_{in}} = \pm \frac{d_{out}}{d_{in}} \tag{9.1b}$$

The **torque ratio** (m_T) was shown in equation 6:12 (p. 259) to be the reciprocal
f the velocity ratio (m_V); thus a gearset is essentially a device to exchange torque for
elocity or vice versa. Since there are no applied forces as in a linkage, but only applied
orques on the gears, the **mechanical advantage** m_A of a gearset is equal to its torque
atio m_T. The most common application is to reduce velocity and increase torque to drive
ieavy loads as in your automobile transmission. Other applications require an increase
n velocity, for which a reduction in torque must be accepted. In either case, it is usually
lesirable to maintain a constant ratio between the gears as they rotate. Any variation in
atio will show up as oscillation in the output velocity and torque even if the input is
:onstant with time.

The radii in equation 9.1 are those of the rolling cylinders to which we are adding the
eeth. The positive or negative sign accounts for internal or external cylinder sets as de-
ined in Figure 9-1 (p. 433). An external set reverses the direction of rotation between
he cylinders and requires the negative sign. An internal gearset or a belt or chain drive
will have the same direction of rotation on input and output shafts and require the posi-
tive sign in equation 9.1. The surfaces of the rolling cylinders will become the **pitch cir-
cles,** and their diameters the **pitch diameters** of the gears. The contact point between the
cylinders lies on the line of centers as shown in Figure 9-3a, and this point is called the
pitch point.

In order for the fundamental law of gearing to be true, the gear tooth contours on
mating teeth must be conjugates of one another. There is an infinite number of possible
conjugate pairs that could be used, but only a few curves have seen practical application
as gear teeth. The **cycloid** still is used as a tooth form in watches and clocks, but most
other gears use the **involute** curve for their shape.

The Involute Tooth Form

The involute is a curve which can be generated by unwrapping a taut string from a cylin-
der (called the evolute) as shown in Figure 9-5. Note the following about this involute
curve:

The string is always tangent to the cylinder.

*The center of curvature of the involute is always at the point of tangency of the string with
the cylinder.*

*A tangent to the involute is then always normal to the string, the length of which is the
instantaneous radius of curvature of the involute curve.*

Figure 9-6 shows two involutes on separate cylinders in contact or "in mesh." These
represent gear teeth. The cylinders from which the strings are unwrapped are called the
base circles of the respective gears. Note that the base circles are necessarily smaller than
the pitch circles, which are at the radii of the original rolling cylinders, r_p and r_g. The
gear tooth must project both below and above the rolling cylinder surface (pitch circle)
and the *involute only exists outside of the base circle*. The amount of tooth that sticks
out above the pitch circle is the **addendum**, shown as a_p and a_g for pinion and gear, re-
spectively. These are equal for standard, full-depth gear teeth.

The geometry at this tooth-tooth interface is similar to that of a cam-follower
joint as was defined in Figure 8-40 (p. 399). There is a **common tangent** to both
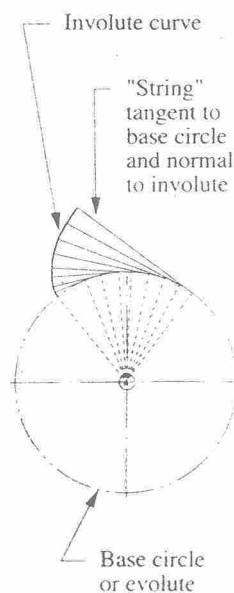
Involute curve

"String"
tangent to
base circle
and normal
to involute

Base circle
or evolute

FIGURE 9-5

Development of the
involute of a circle

Pressure angle rotated in direction of driven gear

Velocity at pitch point

ϕ

Addendum of pinion a_p

Axis of transmission or line of action
(common normal) is tangent to
both base circles

Pitch radius
of pinion r_p

Common tangent

Pitch radius of gear r_g

Pitch point

ω_{gear}

ω_{pinion}

Driven (CCW)

Driving (CW)

Base circle of pinion

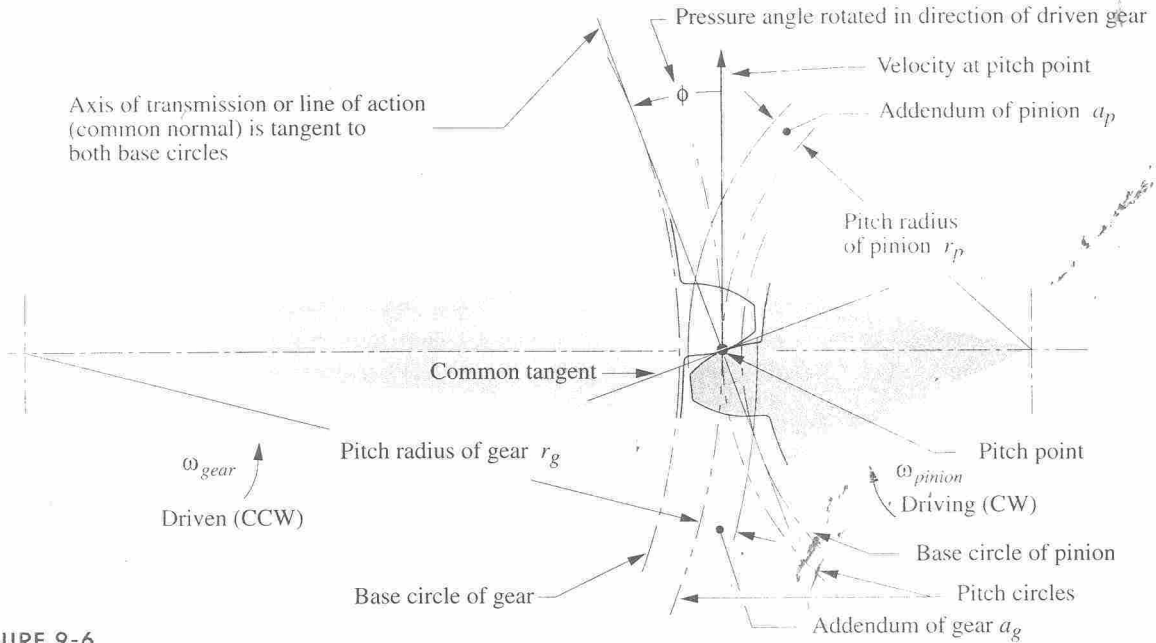Base circle of gear

Pitch circles

Addendum of gear a_g

FIGURE 9-6

Contact geometry and pressure angle of involute gear teeth

curves at the contact point, and a **common normal**, perpendicular to the common tangent. Note that the common normal is, in fact, the "strings" of both involutes, which are colinear. Thus the common normal, which is also the **axis of transmission**, always passes through the pitch point regardless of where in the mesh the two teeth are contacting.

Figure 9-7 shows a pair of involute tooth forms in two positions, just beginning contact and about to leave contact. The common normals of both these contact points still pass through the same pitch point. It is this property of the involute that causes it to obey the fundamental law of gearing. The ratio of the driving gear radius to the driven gear radius remains constant as the teeth move into and out of mesh.

From this observation of the behavior of the involute we can restate the **fundamental law of gearing** in a more kinematically formal way as: *the common normal of the tooth profiles, at all contact points within the mesh, must always pass through a fixed point on the line of centers, called the pitch point.* The gearset's velocity ratio will then be a constant defined by the ratio of the respective radii of the gears to the pitch point.

The points of beginning and leaving contact define the **mesh** of the pinion and gear. The distance along the line of action between these points within the mesh is called the **length of action**, Z, defined by the intersections of the respective addendum circles with the line of action, as shown in Figure 9-7.
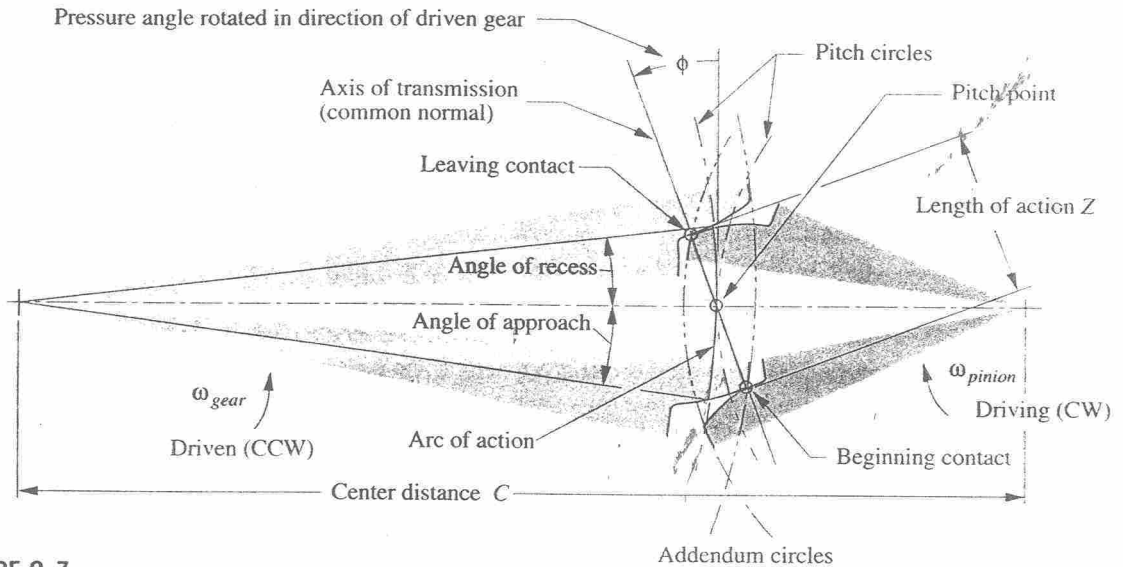
FIGURE 9-7

Pitch point, pitch circles, pressure angle, length of action, arc of action, and angles of approach and recess during the meshing of a gear and pinion

$$Z = \sqrt{(r_p + a_p)^2 - (r_p \cos\phi)^2} + \sqrt{(r_g + a_g)^2 - (r_g \cos\phi)^2} - C\sin\phi \qquad (9.2)$$

The distance along the pitch circle within the mesh is the **arc of action**, and the angles subtended by these points and the line of centers are the **angle of approach** and **angle of recess**. These are shown only on the gear in Figure 9-7 for clarity, but similar angles exist for the pinion. The arc of action on both pinion and gear pitch circles must be the same length for zero slip between the theoretical rolling cylinders.

Pressure Angle

The **pressure angle** in a gearset is similar to that of the cam and follower and is defined as the angle between the axis of transmission or line of action (common normal) and the direction of velocity at the pitch point as shown in Figures 9-6 and 9-7. Pressure angles of gearsets are standardized at a few values by the gear manufacturers. These are defined at the nominal center distance for the gearset as cut. The standard values are 14.5°, 20°, and 25° with 20° being the most commonly used and 14.5° now being considered obsolete. Any custom pressure angle can be made, but its expense over the available stock gears with standard pressure angles would be hard to justify. Special cutters would have to be made. Gears to be run together must be cut to the same nominal pressure angle.

Changing Center Distance

When involute teeth (or any teeth) have been cut into a cylinder, with respect to a particular base circle, to create a single gear, we do not yet have a pitch circle. The pitch circle only comes into being when we mate this gear with another to create a pair of gears, or gearset. There will be some range of center-to-center distances over which we can achieve a mesh between the gears. There will also be an ideal center distance (CD) that will give us the nominal pitch diameters for which the gears were designed. However, limitations of manufacturing processes give a low probability that we will be able to exactly achieve this ideal center distance in every case. More likely, there will be some error in the center distance, even if small.

What will happen to the adherence to the fundamental law of gearing if there is error in the location of the gear centers? If the gear tooth form is **not** an involute, then an error in center distance will violate the fundamental law, and there will be variation, or "ripple," in the output velocity. The output angular velocity will not be constant for a constant input velocity. However, **with an involute tooth form**, *center distance errors do not affect the velocity ratio.* This is the principal advantage of the involute over all other possible tooth forms and the reason why it is nearly universally used for gear teeth. Figure 9-8 shows what happens when the center distance is varied on an involute gearset. Note that the common normal still goes through a pitch point, common to all contact points within the mesh. But the pressure angle is affected by the change in center distance.

Figure 9-8 also shows the pressure angles for two different center distances. As the center distance increases, so will the pressure angle and vice versa. This is one result of a change, or error, in center distance when using involute teeth. Note that the fundamental law of gearing still holds in the modified center distance case. The common normal is still tangent to the two base circles and still goes through the pitch point. The pitch point has moved, but in proportion to the move of the center distance and the gear radii. The velocity ratio is unchanged despite the shift in center distance. In fact, the velocity ratio of involute gears is fixed by the ratio of the base circle diameters, which are unchanging once the gear is cut.

Backlash

Another factor affected by changing center distance is backlash. Increasing the CD will increase the backlash and vice versa. **Backlash** is defined as *the clearance between mating teeth measured along the circumference of the pitch circle.* Manufacturing tolerances preclude a zero clearance, as all teeth cannot be exactly the same dimensions, and all must mesh. So, there must be some small difference between the tooth thickness and the space width (see Figure 9-9). As long as the gearset is run with a nonreversing torque, backlash should not be a problem. But, whenever torque changes sign, the teeth will move from contact on one side to the other. The backlash gap will be traversed and the teeth will impact with noticeable noise. This is the same phenomenon as crossover shock in the form-closed cam. As well as increasing stresses and wear, backlash can cause undesirable positional error in some applications. If the center distance is set exactly to match the theoretical value for the gearset, the tooth-to-tooth composite backlash tolerance is in the range of 0.0001

to 0.0007 in for precision gears. The increase in angular backlash as a function of error in center distance is approximately

$$\theta_B = 43\,200(\Delta C)\frac{\tan\phi}{\pi d} \quad \text{minutes of arc} \qquad (9.3)$$

where ϕ = pressure angle, ΔC = error in center distance, and d = pitch diameter of the gear on the shaft where the backlash is measured.

In servomechanisms, where motors are driving, for example, the control surfaces on an aircraft, backlash can cause potentially destructive "hunting" in which the control system tries in vain to correct positional errors due to backlash "slop" in the mechanical drive system. Such applications need **antibacklash gears** which are really two gears back-to-back on the same shaft which can be rotated slightly at assembly with respect to one another, and then fixed so as to take up the backlash. In less critical applications, such as the propeller drive on a boat, backlash on reversal will not even be noticed.
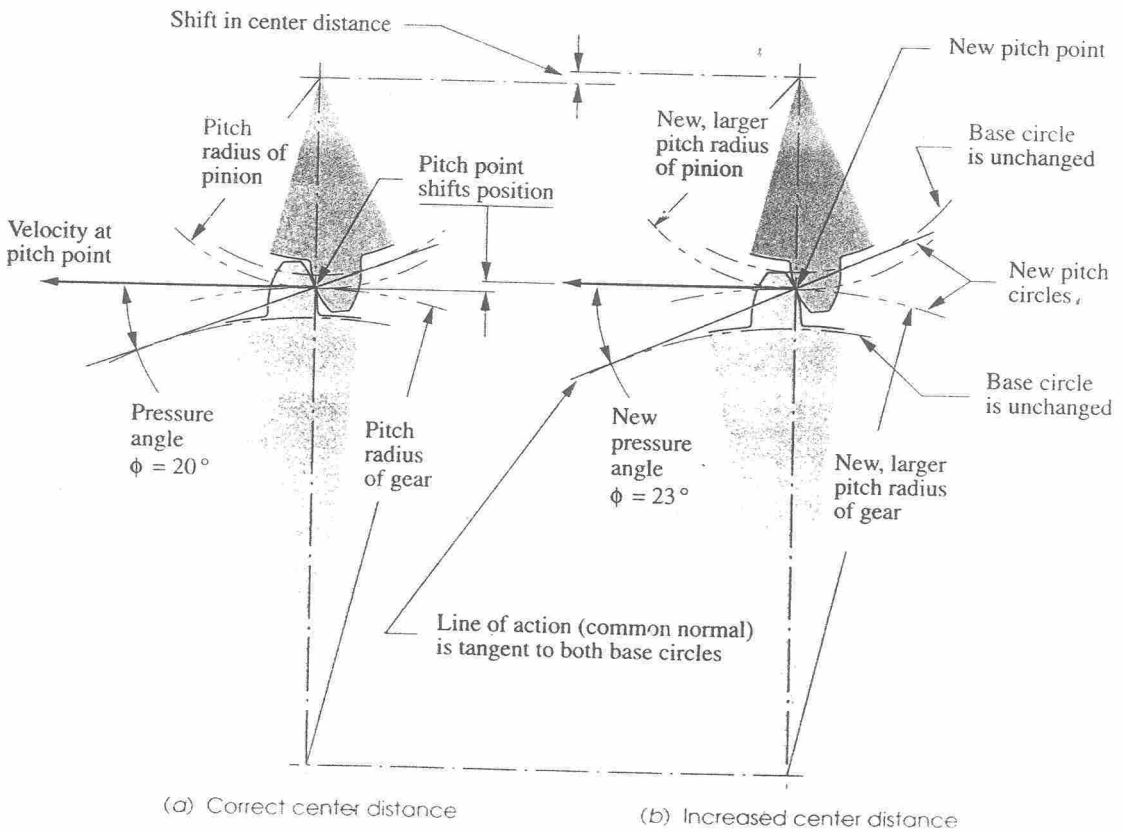

FIGURE 9-8

Changing center distance of involute gears changes the pressure angle and pitch diameters

Bạn đang có trong tay thông tin cơ bản của một trong số những tài liệu có hàm lượng chất xám cao, với những thông tin vô cùng hữu ích cho quá trình học tập và nghiên cứu của bản thân bạn !

Bạn đã và đang trải qua những ưu tư, trăn trở của bản thân mình trước biển kiến thức mênh mông vô hạn và đang tìm hướng đi cho riêng mình bằng việc biến kiến thức thành tài sản tri thức của riêng bạn !

Hãy để Thư viện trường Đại học Sư phạm Kỹ thuật Tp. HCM chia sẻ những khó khăn và trăn trở đó cùng bạn!

Hãy đến với Thư viện trường Đại học Sư phạm Kỹ thuật Tp. HCM để cùng nhau, chúng ta xây dựng vương quốc khoa học và trí tuệ của chính mình !

THƯ VIỆN

Cùng với bạn, Thư viện ĐH Sư phạm Kỹ thuật Tp. HCM mong ước góp phần duy trì và phát triển văn hóa đọc !
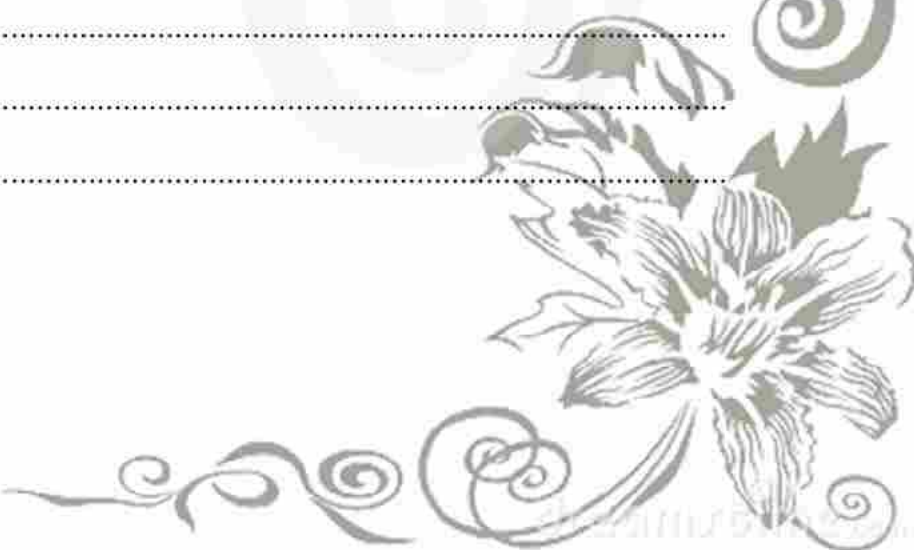
THƯ VIỆN

Hãy đến với chúng tôi - Thư viện trường Đại học Sư phạm Kỹ thuật Tp. HCM để cảm nhận, trải nghiệm và biến ước mơ khoa học của bạn thành hiện thực !

Hân hạnh được đón tiếp và phục vụ bạn tại
Số 1 – 3, Võ Văn Ngân, Phường Linh Chiểu, Quận Thủ Đức, Tp. HCM
ĐT: (08) 3896 9920 – Email: thuvienspkt@hcmute.edu.vn
http://www.thuvienspkt.edu.vn – http://thuvien.hcmute.edu.vn

Thông tin tài trợ!

Bạn đang có trong tay thông tin cơ bản của một trong số những tài liệu có hàm lượng chất xám cao, với những thông tin vô cùng hữu ích cho quá trình học tập và nghiên cứu của bản thân bạn !

Bạn đã và đang trải qua những ưu tư, trăn trở của bản thân mình trước biển kiến thức mênh mông vô hạn và đang tìm hướng đi cho riêng mình bằng việc biến kiến thức thành tài sản tri thức của riêng bạn !

Hãy để Thư viện trường Đại học Sư phạm Kỹ thuật Tp. HCM chia sẻ những khó khăn và trăn trở đó cùng bạn!

Hãy đến với Thư viện trường Đại học Sư phạm Kỹ thuật Tp. HCM để cùng nhau, chúng ta xây dựng vương quốc khoa học và trí tuệ của chính mình !

Cùng với bạn, Thư viện ĐH Sư phạm Kỹ thuật Tp. HCM mong ước góp phần duy trì và phát triển văn hóa đọc !

Hãy đến với chúng tôi - Thư viện trường Đại học Sư phạm Kỹ thuật Tp. HCM để cảm nhận, trải nghiệm và biến ước mơ khoa học của bạn thành hiện thực !

Hân hạnh được đón tiếp và phục vụ bạn tại
Số 1 – 3, Võ Văn Ngân, Phường Linh Chiểu, Quận Thủ Đức, Tp. HCM
ĐT: (08) 3896 9920 – Email: thuvienspkt@hcmute.edu.vn
http://www.thuvienspkt.edu.vn – http://thuvien.hcmute.edu.vn

Thông tin tài trợ!