
THEORY OF MACHINES AND MECHANISMS

Second Edition

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**TÀI LIỆU LƯU HÀNH NỘI BỘ
KHÔNG MUA BÁN, SAO CHÉP**



V.2

CHAPTER 11

SYNTHESIS OF LINKAGES¹

In previous chapters we have concentrated primarily on the analysis of mechanisms. By *kinematic synthesis* we mean the design or the creation of a mechanism to yield a desired set of motion characteristics. Because of the very large number of techniques available, some of which may be quite frustrating, we present here only a few of the more useful of the approaches to illustrate the application of the theory.²

¹Extensive bibliographies may be found in K. Hain (transl. by H. Kuenzel, T. P. Goodman, et al.), "Applied Kinematics," 2d ed., pp. 639-727, McGraw-Hill, New York, 1967, and F. Freudenstein and G. N. Sandor, *Kinematics of Mechanism*, in H. A. Rothbart (ed.), "Mechanical Design and Systems Handbook," 2d ed., pp. 4-56 to 4-68, McGraw-Hill, New York, 1985.

²The following are some of the most useful references on kinematic synthesis in the English language: R. Beyer (transl. by H. Kuenzel) *Kinematic Synthesis of Linkages*, McGraw-Hill, New York, 1964; A. G. Erdman and G. N. Sandor, *Mechanism Design: Analysis and Synthesis*, Prentice-Hall, Englewood Cliffs, NJ, 1991; R. E. Gustavson, "Linkages," chap. 41 in J. E. Shigley and C. R. Mischke (eds.), "Standard Handbook of Machine Design," McGraw-Hill, New York, 1986; R. E. Gustavson, "Linkages," chap. 3 in J. E. Shigley and C. R. Mischke (eds.), *Mechanisms—A Mechanical Designer's Workbook*, McGraw-Hill, New York, 1990; Hain, *op. cit.*; A. S. Hall, *Kinematics and Linkage Design*, Prentice-Hall, Englewood Cliffs, NJ, 1961; R. S. Hartenberg and J. Denavit, *Kinematic Synthesis of Linkages*, McGraw-Hill, New York, 1964; J. Hirschhorn, *Kinematics and Dynamics of Plane Mechanisms*, McGraw-Hill, New York, 1962; K. H. Hunt, *Kinematic Geometry of Mechanisms*, Oxford Univ. Press, Oxford, 1978; A. H. Soni, *Mechanism Synthesis and Analysis*, McGraw-Hill, New York, 1974; C. H. Suh and C. W. Radcliffe, *Kinematics and Mechanism Design*, Wiley, New York, 1978; D. C. Tao, *Fundamentals of Applied Kinematics*, Addison-Wesley, Reading, MA, 1967.



11-1 TYPE, NUMBER, AND DIMENSIONAL SYNTHESIS

Type synthesis refers to the kind of mechanism selected; it might be a linkage, a geared system, belts and pulleys, or a cam system. This beginning phase of the total design problem usually involves design factors such as manufacturing processes, materials, safety, reliability, space, and economics. The study of kinematics is usually only slightly involved in type synthesis.

Number synthesis deals with the number of links and the number of joints or pairs that are required to obtain a certain mobility (see Sec. 1-6). Number synthesis is the second step in design following type synthesis.

The third step in design, determining the dimensions of the individual links, is called *dimensional synthesis*. This is the subject of the balance of this chapter.

11-2 FUNCTION GENERATION, PATH GENERATION, AND BODY GUIDANCE

A frequent requirement in design is that of causing an output member to rotate, oscillate, or reciprocate according to a specified function of time or function of the input motion. This is called *function generation*. A simple example is that of synthesizing a four-bar linkage to generate the function $y = f(x)$. In this case x would represent the motion (crank angle) of the input crank and the linkage would be designed so that the motion (angle) of the output rocker would approximate the function y . Other examples of function generation are:

1. In a conveyor line the output member of a mechanism must move at the constant velocity of the conveyor while performing some operation, for example, bottle capping, return, pick up the next cap, and repeat the operation.
2. The output member must pause or stop during its motion cycle to provide time for another event. The second event might be a sealing, stapling, or fastening operation of some kind.
3. The output member must rotate at a specified nonuniform velocity function because it is geared to another mechanism that requires such a rotating motion.

A second type of synthesis problem is that in which a coupler point is to generate a path having a prescribed shape. Common requirements are that a portion of the path be a circle arc, elliptical, or a straight line. Sometimes it is required that the path cross over itself, as in a figure-of-eight.

The third general class of synthesis problems is called body guidance. Here we are interested in moving an object from one position to another. The problem may call for a simple translation or a combination of translation and rotation. In the construction industry, for example, heavy parts such as scoops and bulldozer blades must be moved through a series of prescribed positions.

11-3 TWO-POSITION SYNTHESIS OF SLIDER-CRANK MECHANISMS

The centered slider-crank mechanism of Fig. 11-1a has a stroke B_1B_2 equal to twice the crank radius r_2 . As shown, the extreme positions B_1 and B_2 , also called limited positions of the slider are found by constructing circle arcs through O_2 of length $r_3 - r_2$ and $r_3 + r_2$, respectively.

In general, the centered slider-crank mechanism must have r_3 larger than r_2 . However, the special case of $r_1 = r_2$ results in the *isosceles slider-crank mechanism*, in which the slider reciprocates through O_2 and the stroke is 4 times the crank radius. All points on the coupler of the isosceles slider crank generate elliptic paths. The paths generated by points on the coupler of the slider crank of Fig. 11-1a are not elliptical, but they are always symmetrical about the sliding axis O_2B .

The linkage of Fig. 11-1b is called the *general or offset slider-crank mechanism*. Certain special effects can be obtained by changing the offset distance e . For example, the stroke B_1B_2 is always greater than twice the crank radius. Also, the crank angle required to execute the forward stroke is different from that for the backward stroke. This feature can be used to synthesize quick-return mechanisms where a slower-working stroke is desired. Note from Fig. 11-1b that the limiting positions B_1 and B_2 of the slider are found in the same manner as for the centered slider-crank.

11-4 TWO-POSITION SYNTHESIS OF CRANK-AND-ROCKER MECHANISMS

The limiting positions of the rocker in a crank-and-rocker mechanism are shown as points B_1 and B_2 in Fig. 11-2. Note that these positions are found the same

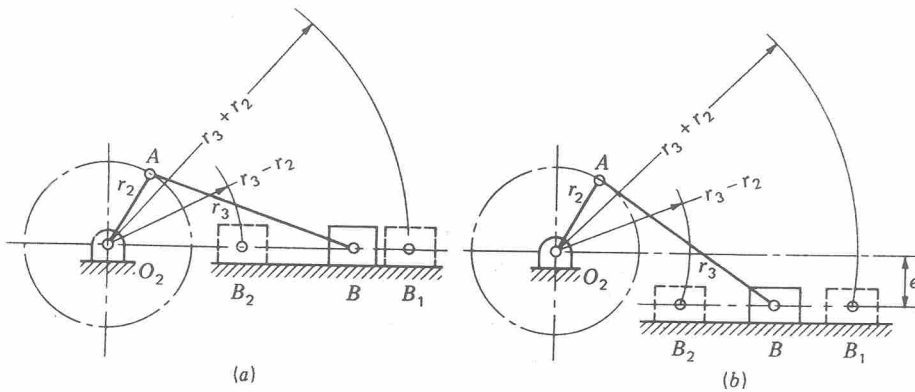


FIGURE 11-1
(a) Centered slider-crank mechanism; (b) general or offset slider-crank mechanism.

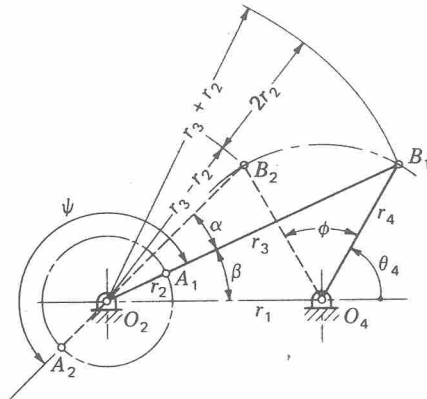


FIGURE 11-2
The extreme positions of the crank-and-rocker mechanism.

as for the slider-crank linkage. Note, too that the crank and coupler form a single straight line at each extreme position.

In this particular case the crank executes the angle ψ while the rocker moves from B_1 to B_2 through the angle ϕ . Note, on the back stroke the rocker swings from B_2 back to B_1 through the same angle ϕ but the crank moves through the angle $360^\circ - \psi$.

There are many cases in which a crank-and-rocker mechanism is superior to a cam-and-follower system. Among the advantages over cam systems are smaller forces involved, the elimination of the retaining spring, and the closer clearances because of the use of revolute pairs.

If $\psi > 180^\circ$ in Fig. 11-2, then $\alpha = \psi - 180^\circ$, where α can be obtained from the equation for the *time ratio* (see Sec. 1-7)

$$Q = \frac{180^\circ + \alpha}{180^\circ - \alpha} \quad (11-1)$$

of the forward and backward motions of the rocker. The first problem that arises in the synthesis of crank-and-rocker linkages is how to obtain the dimensions or geometry that will cause the mechanism to generate a specified output angle ϕ when the time ratio is specified.³

To synthesize a crank-and-rocker mechanism for specified values of ϕ and α , locate point O_4 in Fig. 11-3a and choose any desired rocker length r_4 . Then draw the two positions O_4B_1 and O_4B_2 of link 4 separated by the angle ϕ as given. Through B_1 construct any line X . Then, through B_2 construct the line Y at the given angle α to X . The intersection of these two lines defines the location of the crank pivot O_2 . Since any line X was originally chosen, there are an infinite number of solutions to this problem.

³The method to be described appears in Hall, op. cit., p. 33, and Soni, op. cit., p. 257. Both Tao, op. cit., p. 241, and Hain, op. cit., p. 317, describe another method that gives different results.

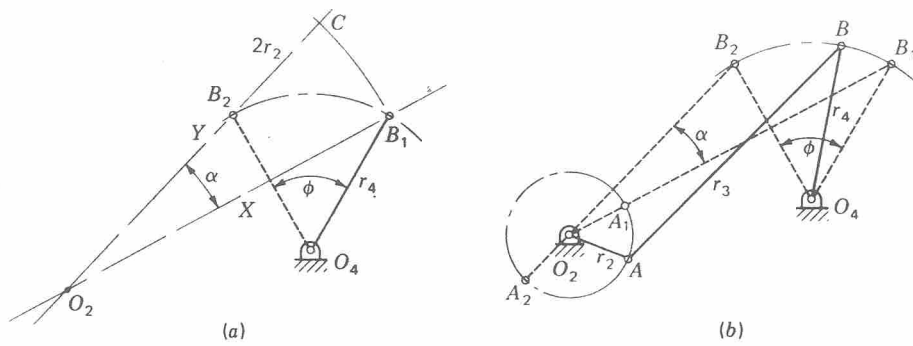


FIGURE 11-3
Synthesis of a four-bar linkage to generate the rocker angle ϕ .

Next, as shown in Figs. 11-2 and 11-3a, the distance B_2C is $2r_2$, twice the crank length. So, bisect this distance to find r_2 . Then the coupler length is $r_3 = O_2B_1 - r_2$. The completed linkage is shown in Fig. 11-3b.

11-5 CRANK-ROCKER MECHANISMS WITH OPTIMUM TRANSMISSION ANGLE

Brodell and Soni⁴ have developed an analytical method of synthesizing the crank-rocker linkage in which the time ratio is $Q = 1$. The design also satisfies the condition

$$\gamma_{\min} = 180^\circ - \gamma_{\max} \quad (a)$$

where γ is the transmission angle (see Sec. 1-10).

To develop the method, use Fig. 11-2 and the law of cosines to write the two equations,

$$\cos(\theta_4 + \phi) = \frac{r_1^2 + r_4^2 - (r_3 - r_2)^2}{2r_1r_4} \quad (b)$$

$$\cos \theta_4 = \frac{r_1^2 + r_4^2 - (r_3 + r_2)^2}{2r_1r_4} \quad (c)$$

⁴R. Joe Brodell and A. H. Soni. "Design of the Crank-Rocker Mechanism with Unit Time Ratio," *J. Mech.*, vol. 5, no. 1, p. 1, 1970.

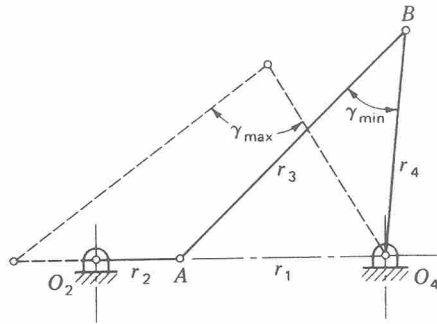


FIGURE 11-4

Then, from Fig. 11-4

$$\cos \gamma_{\min} = \frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4} \quad (d)$$

$$\cos \gamma_{\max} = \frac{r_3^2 + r_4^2 - (r_1 + r_2)^2}{2r_3r_4} \quad (e)$$

Equations (a)–(e) are now solved simultaneously; the results are the link ratios

$$\frac{r_3}{r_1} = \sqrt{\frac{1 - \cos \phi}{2 \cos^2 \gamma_{\min}}} \quad (11-2)$$

$$\frac{r_4}{r_1} = \sqrt{\frac{1 - (r_3/r_1)^2}{1 - (r_3/r_1)^2 \cos^2 \gamma_{\min}}} \quad (11-3)$$

$$\frac{r_2}{r_1} = \sqrt{\left(\frac{r_3}{r_1}\right)^2 + \left(\frac{r_4}{r_1}\right)^2 - 1} \quad (11-4)$$

Brodell and Soni plot these results as a design chart, as shown in Fig. 11-5. They state that the transmission angle should be larger than 30° for a good “quality” motion and even larger if high speeds are involved.

The synthesis of a crank-rocker mechanism for optimum transmission angle when the time ratio is not unity is more difficult. An orderly method of accomplishing this is explained by Hall⁵ and by Soni.⁶ The first step in the approach is illustrated in Fig. 11-6. Here the two points O_2 and O_4 are selected and the points C and C' , symmetrical about O_2O_4 and defined by the angles $(\phi/2) - \alpha$ and $\phi/2$, are found. Then, using C as a center and the distance from C to O_2 as the radius, draw the circle arc which is the locus of B_2 . Next,

⁵Op. cit., pp. 36–42.

⁶Op. cit., p. 258.

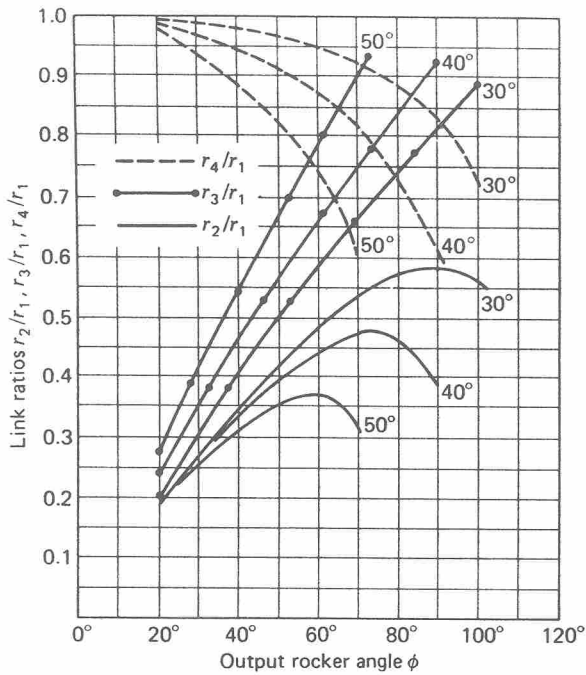


FIGURE 11-5
The Brodell-Soni chart for the design of the crank-rocker linkage with optimum transmission angle and unity time ratio. The angles shown on the graph are γ_{min} .

using C' as the center and the same radius, draw another circle arc which is the locus of B_1 .

One of the many possible crank-rocker linkages has been synthesized in Fig. 11-7. To get the dimensions, choose any point B_1 on the locus of B_1 and swing an arc about O_4 to locate B_2 on the locus of B_2 . With these two points defined, the methods of the preceding section are used to locate points A_1 and A_2 together with the link lengths r_2 and r_3 .

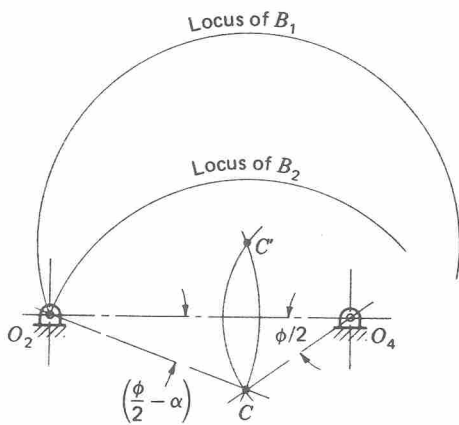


FIGURE 11-6
Layout showing all possible locations of B_1 and B_2 .

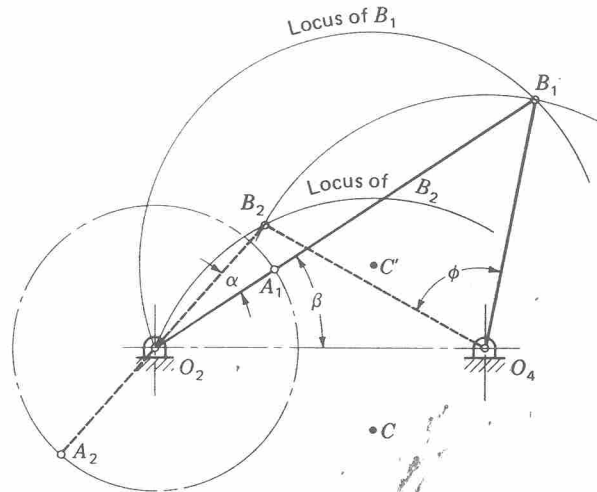


FIGURE 11-7
Determination of the link lengths for one of the possible crank-rocker mechanisms.

The resulting linkages should always be checked to ensure that crank 2 is capable of rotating in a complete circle.

To obtain a linkage with an optimum transmission angle, choose a variety of points B_1 on the locus of B_1 , synthesizing a linkage for each point. Determine the angles $|90^\circ - \gamma_{\min}|$ and $|90^\circ - \gamma_{\max}|$ for each of these linkages. Then plot these data on a chart using the angle β (Fig. 11-7) as the abscissa to obtain two curves. The mechanism having the best transmission angle is then defined by the low point on one of the curves.

11-6 THREE-POSITION SYNTHESIS

In Fig. 11-8a motion of the input rocker O_2A through the angle ψ_{12} causes a motion of the output rocker O_4B through the angle ϕ_{12} . To define inversion as a technique of synthesis, let us hold O_4B stationary and permit the remaining links, including the frame, to occupy the same relative positions as in Fig. 11-8a. The result (Fig. 11-8b) is called *inverting on the output rocker*. Note that A_1B_1 is positioned the same in Fig. 11-8a and 11-8b. Therefore the inversion is made on the O_4B_1 position. Since O_4B_1 is to be fixed, the frame will have to move in order to get the linkage to the A_2B_2 position. In fact, the frame will have to move *backward* through the angle ϕ_{12} . The second position is therefore $O'_2A_2B_2O_4$.

Figure 11-9 illustrates a problem and the synthesized linkage in which it is desired to determine the dimensions of a linkage in which the output lever is to occupy three specified positions corresponding to three given positions of the input lever. In Fig. 11-9 the starting angle of the input lever is θ_2 ; and ψ_{12} , ψ_{23} , and ψ_{13} are the swing angles, respectively, between the design positions 1 and 2, 2 and 3, and 1 and 3. Corresponding angles of swing ϕ_{12} , ϕ_{23} , and ϕ_{13} are

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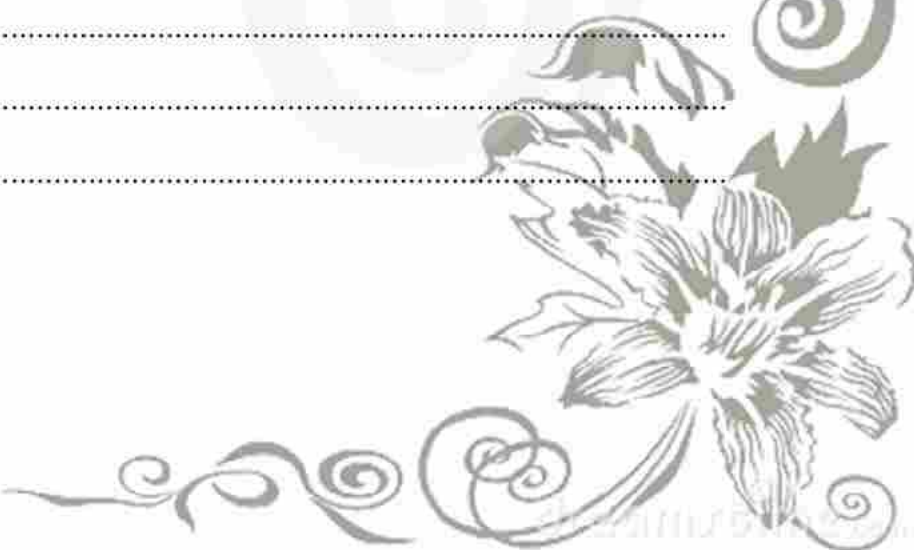
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Thông tin tài trợ!



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