Exercise 1 A repair man fixes broken televisions. The repair time is exponentially distributed with a mean of 30 minutes. Broken televisions arrive at his repair shop according to a Poisson stream, on average 10 broken televisions per day (day=24 hours). The repairman does not need to sleep, so he works 24hours/day.

a) Suggest a mathematical (queueing) model for the above problem?

b) What is the percentage of time that the repair man has no work to do?

c) How many televisions are, on average, at his repair shop?

d) What is the mean sojourn time time (waiting time plus repair time) of a television?

Exercise 2 N trucks drive back and forth between a loading platform and an unloading platform of a transshipment terminal. Trucks queue up at the loading platform to be loaded by a (single) crane, with exponentially distributed loading times (per truck) with parameter λ, all loading times being independent of each other. Similarly, items are removed from the trucks by a single crane at the unloading platform, with (independent) exponentially distributed unloading times with parameter μ. It is assumed that $\lambda \neq \mu$. Truck driving times between the two platforms may be neglected (with respect to loading and unloading times) and it is further assumed that there is an abundance of items at the loading platform and that the system is in equilibrium.

a) The dynamics of this model is identical to that of a well known queueing model. Describe that queueing model.

b) What is the probability that k of the N lorries are at the loading platform?

Help. Define state X as the number of trucks in the unloading platform and draw the transition diagram for X.

Exercise 3 A repair facility for automatic copiers has three repair men. Repair requests occur according to a Poisson process with rate $\lambda = 5$ daily requests. Repair times are exponentially distributed with mean repair time $1/\mu = 0.5$ day. The system is assumed to be in equilibrium.

a) Determine the transition diagram for the possible states with the corresponding transition rates.

b) What is the corresponding queueing model?

c) What is the mean number of copiers at the repair facility?

d) What is the mean out-of-order time (sum of waiting time and repair time) of a copier at the repair facility?

e) What is the mean number of active repair men?

f) What is the usage degree of a repair man (i.e., the fraction of time that the repair man is busy)?

Exercise 4 Two types of customers arrive at a post office with a single counter. Customers of type 1 are patient and always join the waiting queue (if any) and wait for service. Customers of type 2 are less patient and only queue up if (upon arrival) there are less than K customers at the post office. Type-1 customers arrive according to a Poisson process with rate λ and type-2 customers arrive according to an independent Poisson process with rate γ. Customer service times are type-independent and exponentially distributed with mean $1/\mu$.

a) Determine the set of equilibrium equations and calculate the equilibrium probabilities of all possible states.

b) Determine an expression for the fraction of customers that impatiently leave the post office without having received service.

Exercise 5 Consider the regular $M/G/1$ queue with arrival intensity λ, mean service time $E[B]$ and second moment of the service times $E[B^2]$.

a) Suppose $\lambda = 3/2$, $E[B] = 1/2$ and $E[B^2] = 1/2$. Determine the expected values of the waiting time, the sojourn time, the queue length, the number of customers in system and the busy period.

b) Suppose that from measurements it is observed that the expected waiting time equals 5, that the traffic load equals 2/3 and that the mean number of customers in system is 8. Determine the arrival rate and the first two moments of the service time distribution. What are the expectations of the queue length and the sojourn time?

Exercise 6 At a post office with a single counter customers arrive according to a Poisson process with a rate of 60 customers per hour. Half of the customers have a service time that is the sum of a fixed time of 15 seconds and an exponentially distributed time with a mean of 15 seconds. The other half have an exponentially distributed service time with a mean of 1 minute. Determine the mean waiting time and the mean number of customers waiting in the queue.

Exercise 7 Consider the $M/G/1$ queue with arrival rate $\lambda = 1/3$ and with the following hyper-exponential service time distribution:

$$B(x) = \frac{1+a}{2}\left(1 - e^{-\frac{1+a}{2}x}\right) + \frac{1-a}{2}\left(1 - e^{-\frac{1-a}{2}x}\right), \quad x \geq 0,$$

with $0 \leq a < 1$.

a) What is the mean and the variance of the service times?

b) For which values of a is the expected waiting time of a customer larger that the expected duration of a busy period?

c) Verify that when $a \to 1$ this queue behaves very different from the case $a = 1$.

Exercise 8 On average ten customers per hour arrive to a shoe polishing station. The polishing of shoes takes 6 minutes on average. There are two stools, one for the person being served and the other one for a waiting customer. If both stools are occupied, then the arriving customer leaves.

a) Draw the state diagram of the system and solve the balance equations, when the arrival process is assumed to be Poissonian and the service times are exponentially distributed.

b) How many customers are served in an hour on average?

c) What happens if the shoe polisher has an assistant, i.e. when customers on both stools are served at same time and there are no waiting places. Answer to a) and b) in this case.

Exercise 9 Consider an $M/G/1$ queue. Arrivals are Poisson with rate λ. Service times are generally distributed with mean β. By using Little's theorem, prove that

$$P(N > 0) = \rho,$$

where $\rho = \lambda\beta$.

Exercise 10 Consider a simplified model for TCP link. Assume that TCP packets arrive according to a Poisson process with arrival intensity of $\lambda = 100$ pkt/s to a 2 Mbit/s DSL-modem acting as a router. The packet length distribution and respective service times are the following:

length	proportion	time/ms
40	0.1	0.16
576	0.3	2.3
1500	0.6	5.9

Determine the mean waiting time of a packet in the queue when the service discipline is FCFS.

Exercise 11 Consider the Jackson queueing network depicted in Figure 2 below. Packets from outside arrive to the nodes 1, 2 and 5 as a Poisson stream with rate $\lambda = 2$packets/s. In every node each link has its own buffer. The incoming packet stream to each node is randomly directed with the depicted probabilities. The link from node 4 has capacity of $\mu = 8$ packets/s, while the capacity of the other links are $\mu = 3$ packets/s.

a) What are the mean delays of packets taking the routes $1 - 2 - 3$ and $1 - 5 - 4$?

b) How many packets are there on average in the network?

c) What is the mean sojourn time in the network?

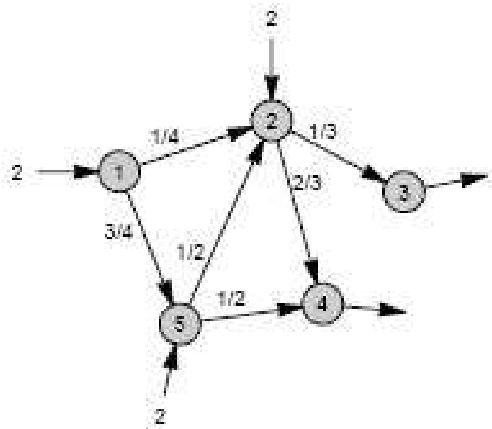

Figure 1: Figure for exercise 11

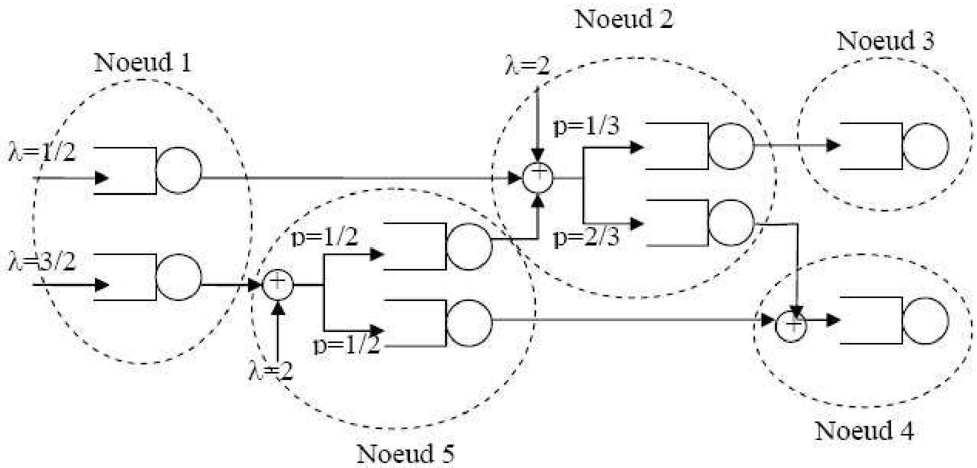

Figure 2: Network for exercise 11