LECTURE SERIES
on
STRUCTURAL OPTIMIZATION

Thanh X. Nguyen
Structural Mechanics Division
National University of Civil Engineering
thanhnx@nuce.edu.vn

OUTLINES

ABOUT THE COURSE

OVERVIEW

STRUCTURAL OPTIMIZATION PROBLEM FORMULATION

CLASSICAL PROGRAMMING TECHNIQUES

INTRODUCTION TO SOME MODERN METHODS

INTRODUCTION TO PBO PROBLEMS

ABOUT THE COURSE

OBJECTIVE

To provide students an insight into the **concept** of structural optimization

To equip students with classical and state-of-art **methods** of optimization

To present to students some optimum structural design **applications**

CORE CONTENTS

W1. Overview (concepts, terminologies, classifications, examples)

W2. Structural Optimization Problem Formulation (following force/displacement methods, …)

W3. Classical Programming Techniques (for linear, nonlinear problems with or without constraints, sensitivity analysis)

W4. Introduction to Modern Methods of Optimization (Genetic Algorithms, Simulated Annealing, Ant Colony, Optimization of Fuzzy Systems, Neural-Network-Based Optimization)

W5 and W6. Introduction to Performance-Based Optimization (various topics given later)

GRADING

Homework: 20%; Report: 30%; Exam: 50%

(the right to change the weights is reserved)

TEXTBOOK

Christensen, P. W., Klarbring, A., (2009). An Introduction to Structural Optimization. Springer.

REFERENCE BOOKS

Liang, Q. Q., (2005). *Performance-based Optimization of Structures (Theory and applications)*. Spon.

Spillers, W. R., MacBain, K. M., (2009). *Structural Optimization*. Springer.

Rao, S. S., (2009). *Engineering Optimization (Theory and Practice), 4th Edition*. John Wiley & Sons, Inc.

COURSE REQUIREMENTS

- Regular attendance
- Weekly Homework Submission
- Presentation and Report Assignement
- Final Exam

PREREQUISITES

- Structural Mechanics
- Finite Element Method
- Advanced Mathematics

LECTURE 1
OVERVIEW

CONTENTS

The Basic Idea

The Design Process

General Mathematical Form

Three Types of Structural Optimization Problems

Some Examples

THE BASIC IDEA

Structure

"any assemblage of materials which is intended to sustain loads."

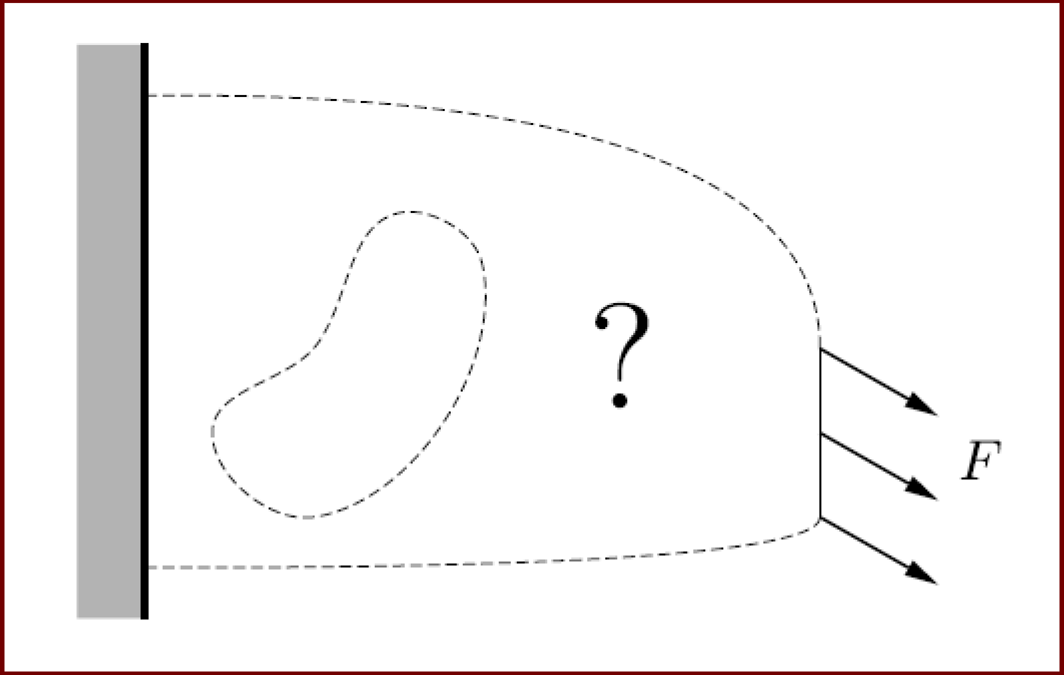
Optimization

making things the best

Structural optimization

the subject of making an assemblage of materials sustain loads in the best way.

But “best” in what sense?



THE DESIGN PROCESS

Function

What is the use of the product?

Conceptual design

What type of construction concept should we use?

Optimization

Within the chosen concept, and within the constraints on function, make the product as good as possible

Details

Usually controlled by market, social or esthetic factors

Think of the design of a bridge!

Ways of realizing step 3

Iterative-intuitive (traditional, still dominant)
Mathematical design optimization

Iterative-intuitive way

(a) A specific design is suggested
(b) Requirements based on the function are investigated (may use FEM)
(c) If not satisfied, a new design must be suggested, and even if such requirements are satisfied the design may not be optimal so we suggest a new design
(d) The suggested new design is brought back to step (b).

In this way an **iterative** process is formed where, on mainly **intuitive** grounds, a series of designs are created which hopefully converges to an acceptable final design.

Mathematical Design Optimization Way

A mathematical optimization problem is formulated

Requirements act as constraints.

Step 3 following mathematical design optimization is much more automatic

Not all factors can be usefully treated. Factors need to be measurable in mathematical form.

GENERAL MATH. FORM OF A STRUCTURAL OPTIMIZATION PROBLEM

Objective function (f)

A function used to **classify** designs: measures weight, displacement in a given direction, effective stress or even cost of production

Design variable (x)

A function or vector that **describes** the design, and which can be changed during optimization.

It may represent geometry or choice of material. When it describes geometry, it may relate to a sophisticated interpolation of shape or it may simply be the area of a bar, or the thickness of a sheet.

State variable (y)

For a given structure, *y* is a function or vector that represents the **response** of the structure (displacement, stress, strain or force…)

Single-Objective Structural Optimization

$$
(\mathbb{SO}) \quad \begin{cases} \text{minimize } f(x, y) \text{ with respect to } x \text{ and } y \\ \text{subject to} \quad \begin{cases} \text{behavioral constraints on } y \\ \text{design constraints on } x \\ \text{equilibrium constraint.} \end{cases} \end{cases}
$$

Multi-Objective Structural Optimization

$$
(\mathbb{SO}) \quad \begin{cases} \text{minimize } (f_1(x,y), f_2(x,y), \ldots, f_l(x,y)), \\ \text{subject to} \quad \begin{cases} \text{behavioral constraints on } y \\ \text{design constraints on } x \\ \text{equilibrium constraint.} \end{cases} \end{cases}
$$

Pareto optimality: a design is Pareto optimal if there does not exist any other design that satisfies all of the objectives better.

Common way to solve → $\sum_{i=1}^{l} w_i f_i(x,y),$

Three types of constraints

(1) *Behavioral constraints:* constraints on the **state variable** *y*. Usually they are written $g(y) \leq 0$, where *g* is a function which represents, e.g., a displacement in a certain direction.

(2) *Design constraints:* constraints involving the **design variable** *x*. Obviously, these two types of constraints can be combined.

(3) *Equilibrium constraint:*

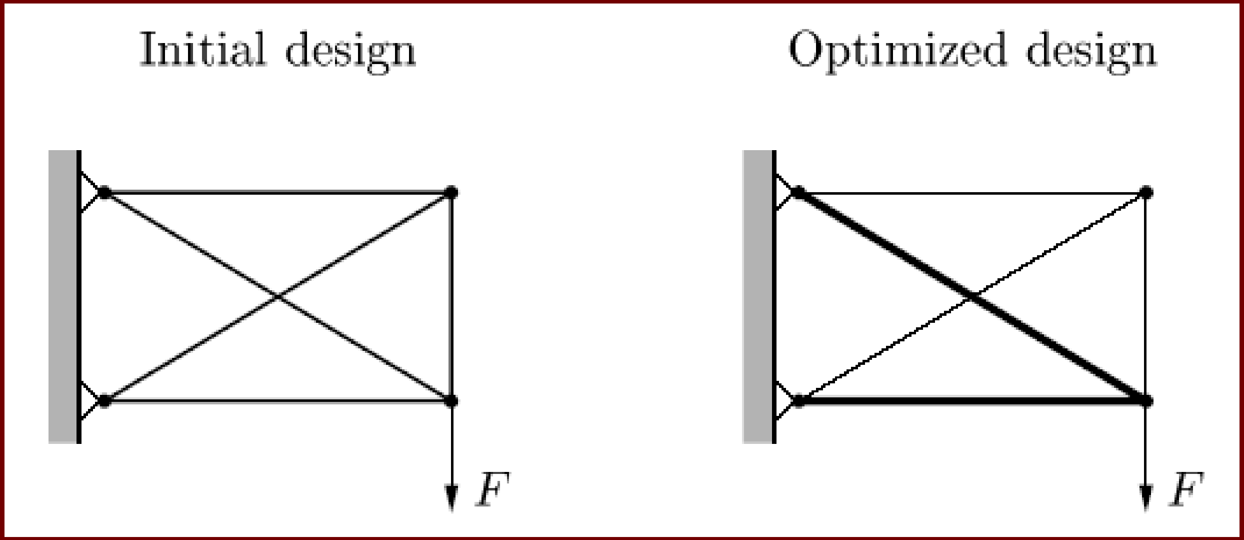
$$K(x)u = F(x),$$

Nested formulation

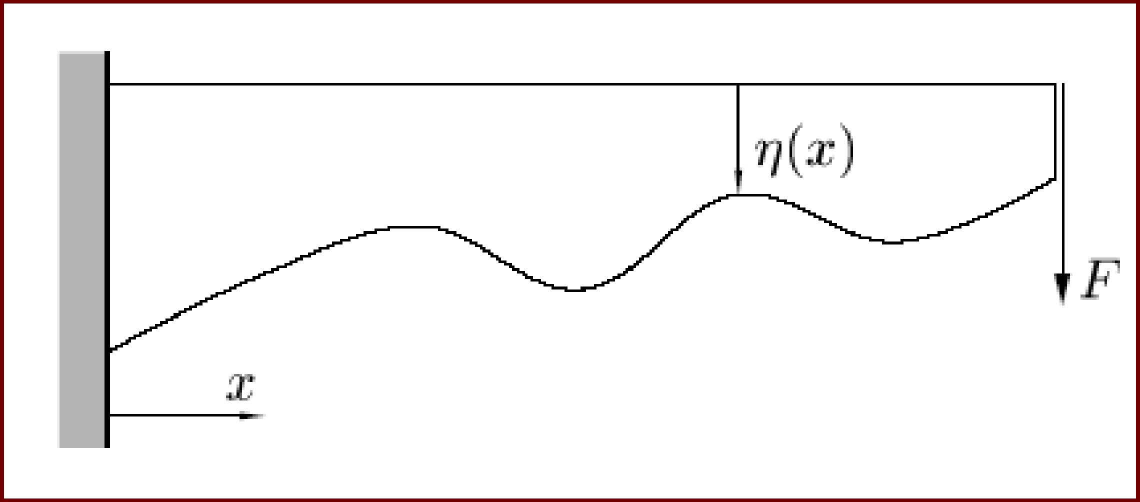
$$(\mathbb{SO})_{\mathrm{nf}} \quad \begin{cases} \min_{x} & f(x, u(x)) \\ \text{s.t.} & g(x, u(x)) \leq 0, \end{cases}$$

THREE TYPES OF STRUCTURAL OPTIMIZATION PROBLEMS

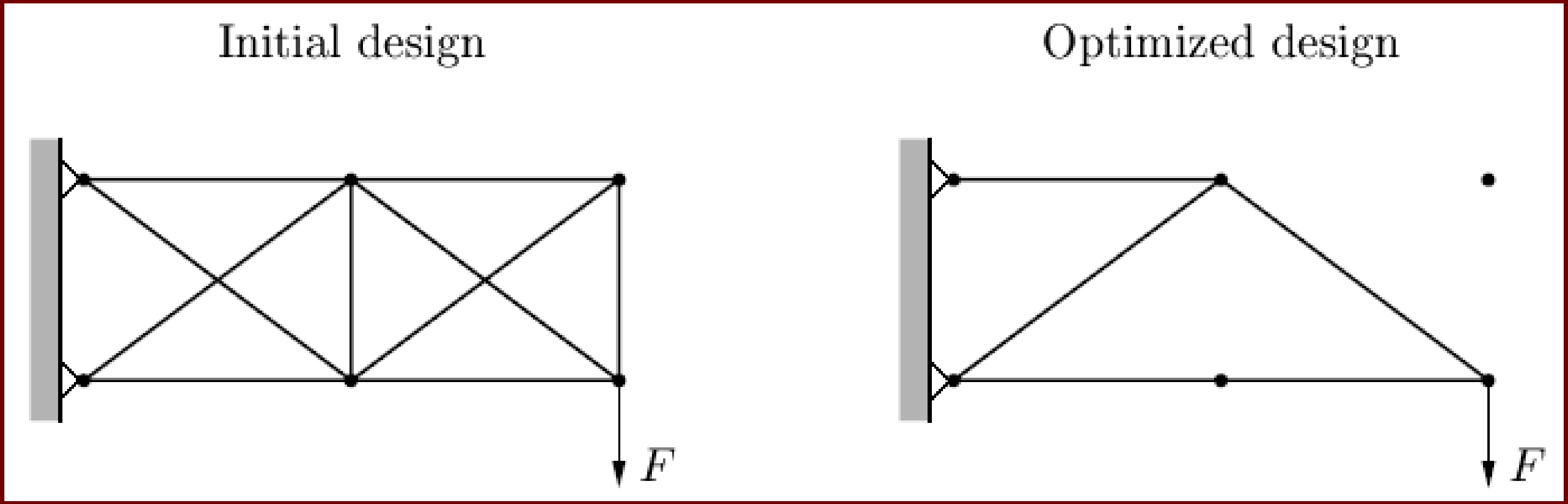
(1) **Sizing optimization:** This is when *x* is some type of structural thickness, i.e., cross- sectional areas of truss members, or the thickness distribution of a sheet.



(2) **Shape optimization:** *x* represents the form or contour of some part of the boundary of the structural domain. The state y is described by a set of partial differential equations. Connectivity of the structure is not changed.



(3) **Topology optimization:** In this way the connectivity of nodes is variable. Design variables x (often present geometric feature) can take the value zero, i.e., bars are removed from the truss.



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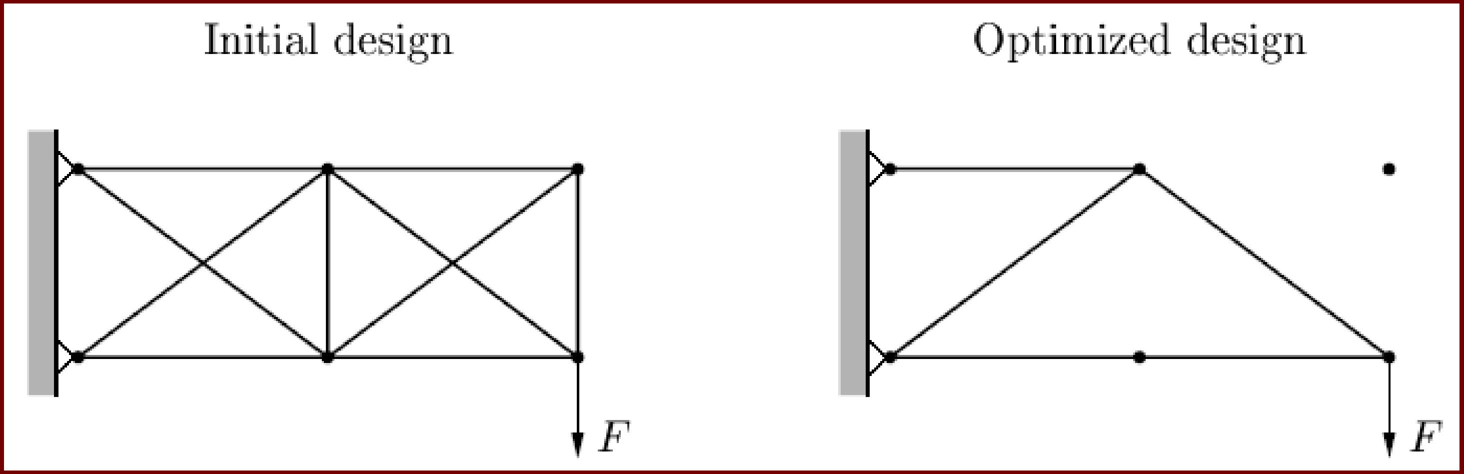
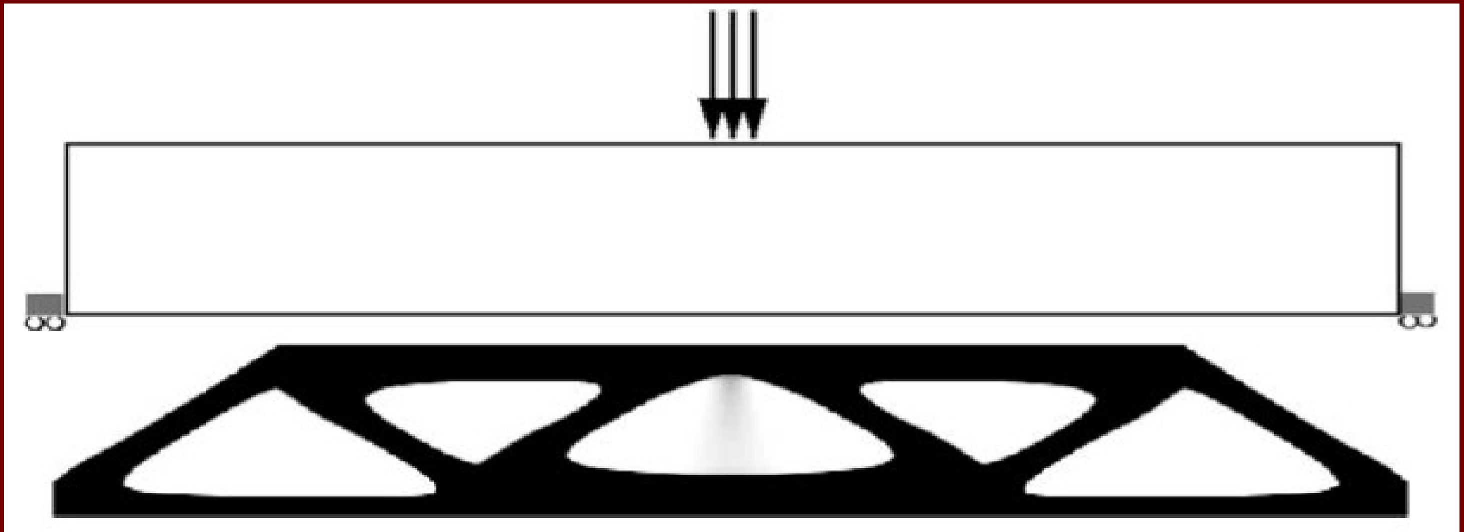
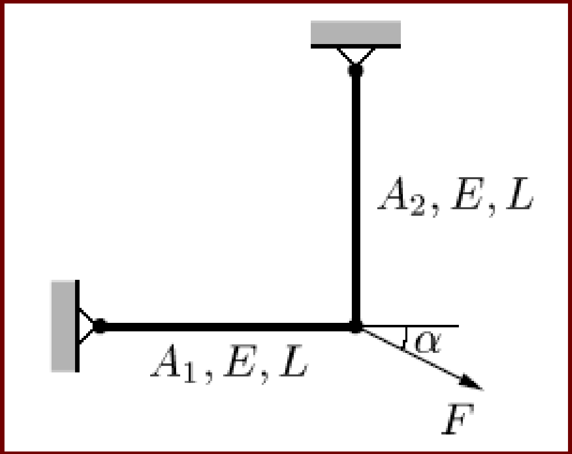


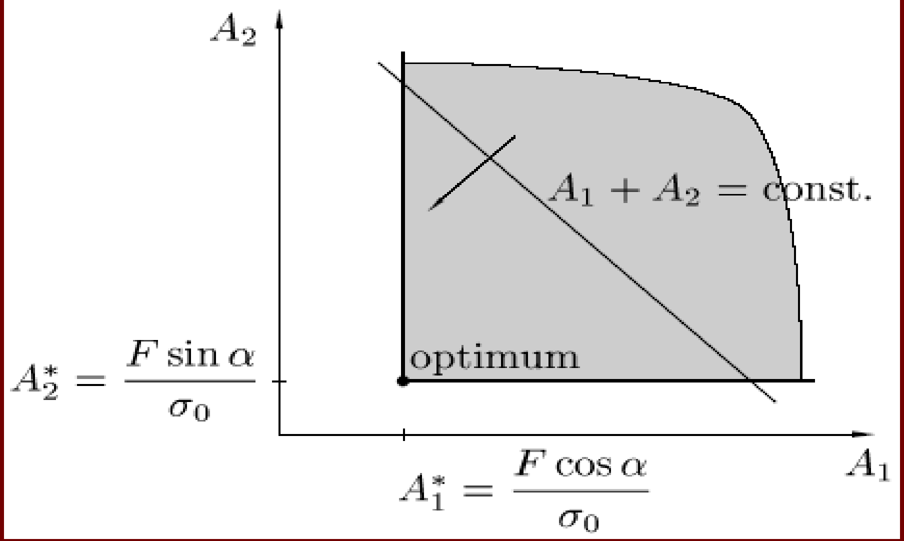


SOME EXAMPLES

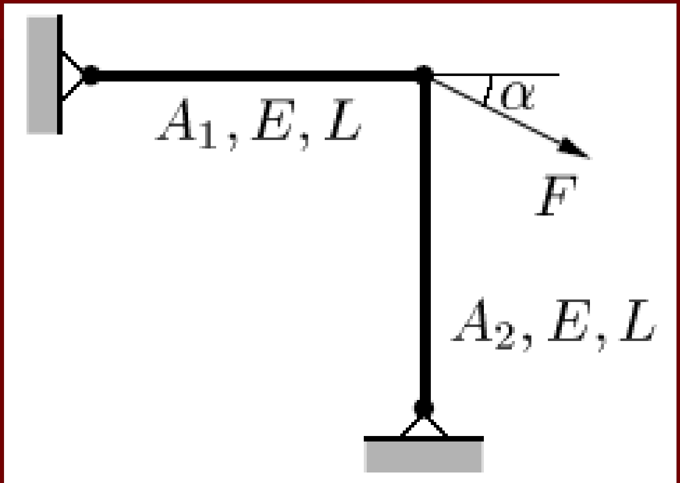
(1) Weight Minimization of a Two-Bar Truss Subject to Stress Constraints



$$(\mathbb{SO})^{1}_{\mathrm{nf}} \quad \begin{cases} \min\limits_{A_1, A_2} \quad A_1 + A_2 \\ \text{s.t.} \quad \begin{cases} A_1 \geq \dfrac{F\cos\alpha}{\sigma_0} \\ A_2 \geq \dfrac{F\sin\alpha}{\sigma_0}, \end{cases} \end{cases}$$



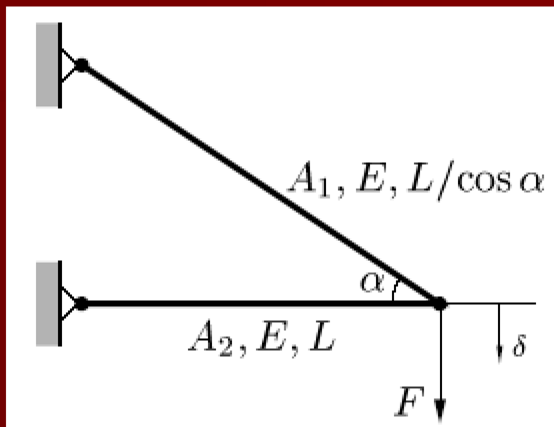
(2) Weight Minimization of a Two-Bar Truss Subject to Stress and Instability Constraints



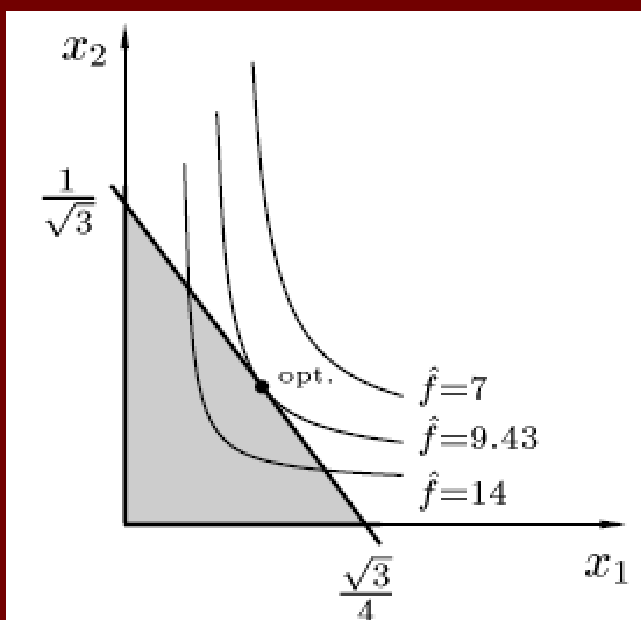
$$
(\mathbb{SO})^2_{\text{nf}} \quad \begin{cases} \min\limits_{A_1, A_2} & A_1 + A_2 \\ \text{s.t.} & \begin{cases} A_1 \geq \dfrac{F}{\sqrt{2}\sigma_0} \\ A_2 \geq \dfrac{F}{\sqrt{2}\sigma_0} \\ A_2^2 \geq \dfrac{16FL^2}{\sqrt{2}\pi E}. \end{cases} \end{cases}
$$

$$
A_1^* = \frac{L^2}{16\sqrt{2}} \approx 0.044L^2, \qquad A_2^* = \frac{L^2}{10\sqrt{\sqrt{2}\pi}} \approx 0.047L^2.
$$

(3) Weight Minimization of a Two-Bar Truss Subject to Stress and Displacement Constraints

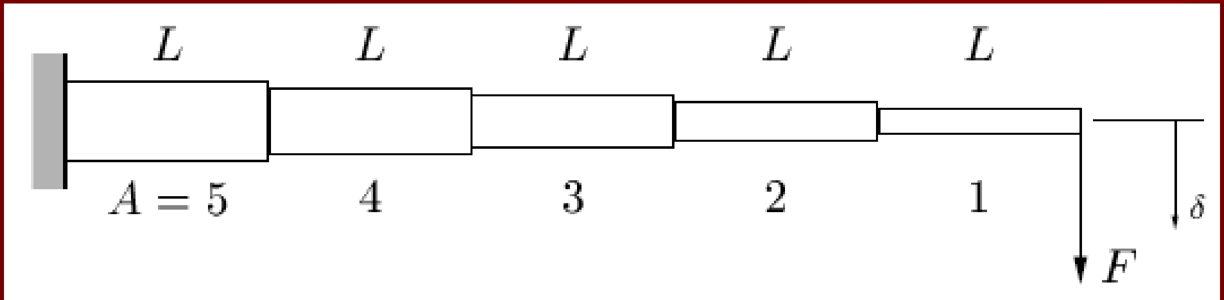


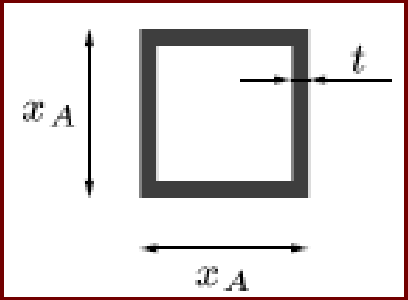
$$(\mathbb{SO})^3_{\mathrm{nf}} \quad \begin{cases} \min\limits_{x_1, x_2} & \hat{f}(x_1, x_2) = \dfrac{4}{3x_1} + \dfrac{1}{x_2} \\ \text{s.t.} & \begin{cases} \dfrac{4}{\sqrt{3}} x_1 + \sqrt{3} x_2 \leq 1 \\ x_1 > 0, \qquad x_2 > 0. \end{cases} \end{cases}$$



$$A_1^* = \frac{14F}{\sqrt{3}\sigma_0} \approx \frac{8.1F}{\sigma_0}, \qquad A_2^* = \frac{7F}{\sigma_0}.$$

(4) Weight Minimization of a Two-Beam Cantilever Subject to a Displacement Constraint





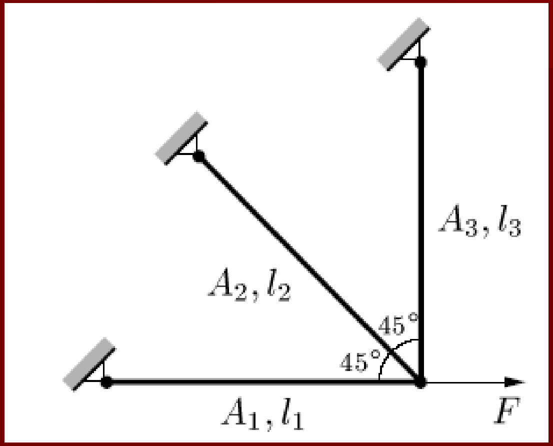
$$f(x_1, \ldots, x_N) = L\rho \sum_{A=1}^{N} \left(x_A^2 - (x_A - 2t)^2 \right) = 4L\rho t \sum_{A=1}^{N} x_A,$$

$$(\mathbb{SO})^4_{\mathrm{nf}} \quad \begin{cases} \min_{x_1, x_2} & f(x_1, x_2) = C_1 (x_1 + x_2) \\ \text{s.t.} & \begin{cases} \dfrac{1}{x_1^3} + \dfrac{7}{x_2^3} \le C_2 \\ x_1 > 0, \qquad x_2 > 0, \end{cases} \end{cases}$$

$$C_1 = 4\rho L t, \qquad C_2 = \frac{2\delta_0 E t}{F L^3}.$$

$$x_1^* = \left(\frac{1 + 7^{1/4}}{C_2} \right)^{1/3}, \qquad x_2^* = 7^{1/4} \left(\frac{1 + 7^{1/4}}{C_2} \right)^{1/3}.$$

(5) Weight Minimization of a Three-Bar Truss Subject to Stress Constraints



$$f(A_1, A_2) = \rho_1 L A_1 + \rho_2 L A_2 + \rho_3 \frac{L}{\beta} A_3 = L\left(\rho_1 + \frac{\rho_3}{\beta}\right) A_1 + L\rho_2 A_2,$$

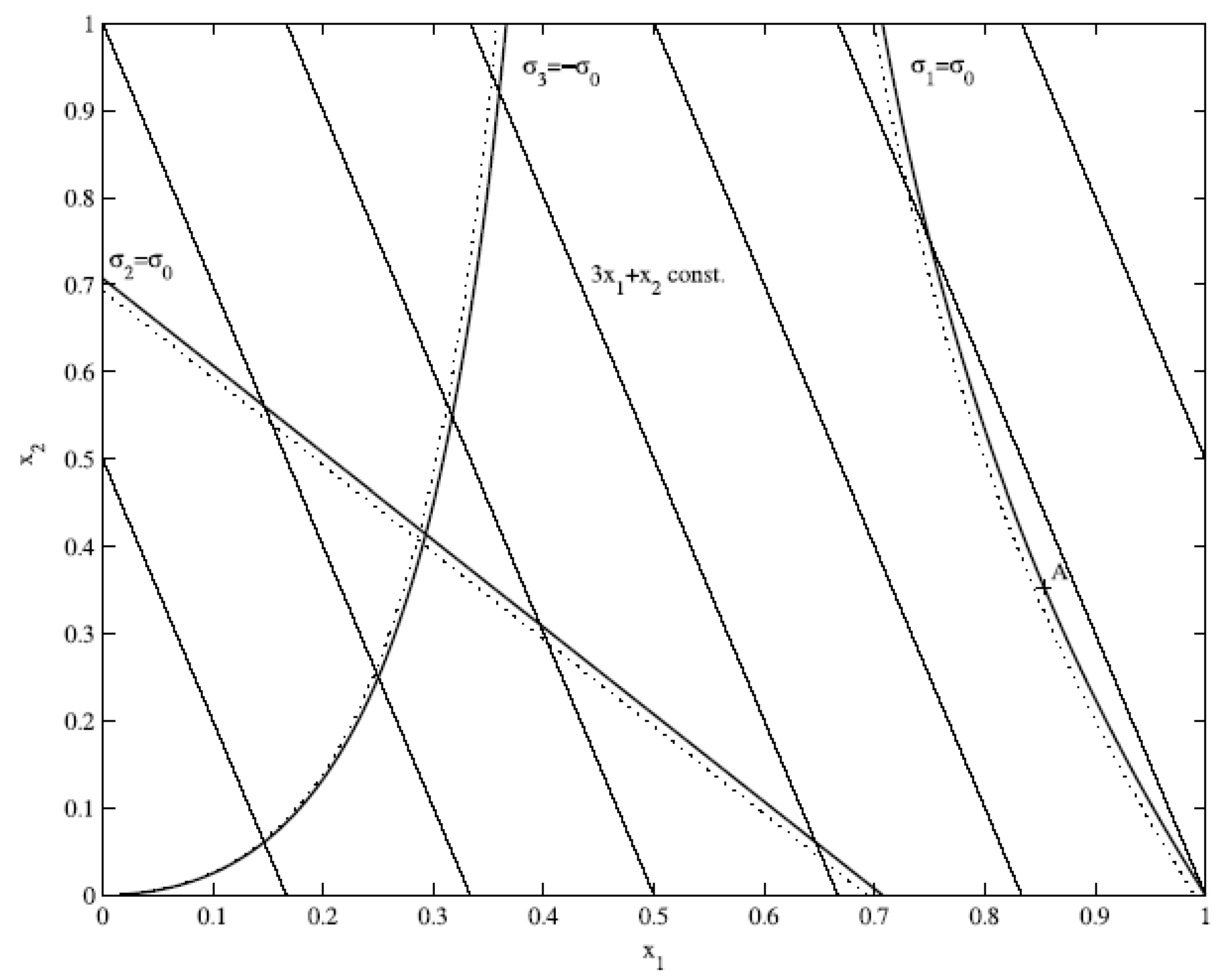
$$(\mathbb{SO})^5_{\mathrm{nf}} \quad \begin{cases} \min\limits_{A_1, A_2} \quad (\rho_1 A_1 + \rho_2 A_2 + \rho_3 A_1)\, L \\ \text{s.t.} \quad \begin{cases} \dfrac{F(2A_1 + A_2)}{2A_1(A_1 + A_2)} - \sigma_1^{\max} \leq 0 \\ \dfrac{F}{\sqrt{2}(A_1 + A_2)} - \sigma_2^{\max} \leq 0 \quad \text{if } A_2 > 0 \\ \dfrac{F A_2}{2A_1(A_1 + A_2)} - \sigma_3^{\max} \leq 0 \\ A_1 > 0, \qquad A_2 \geq 0. \end{cases} \end{cases}$$

(5a)

$$\rho_1 = 2\rho_0,\ \ \rho_2 = \rho_3 = \rho_0,\ \ \sigma_1^{\max} = \sigma_2^{\max} = \sigma_3^{\max} = \sigma_0.$$

$$(\mathbb{SO})^{5a}_{\mathrm{nf}} \quad \left\{ \begin{array}{ll} \displaystyle\min_{x_1,x_2} & 3x_1 + x_2 \\ \text{s.t.} & \left\{ \begin{array}{lll} \dfrac{2x_1 + x_2}{2x_1(x_1+x_2)} - 1 \le 0 & & (\sigma_1) \\ \dfrac{1}{\sqrt{2}(x_1+x_2)} - 1 \le 0 & \text{if } x_2 > 0 & (\sigma_2) \\ \dfrac{x_2}{2x_1(x_1+x_2)} - 1 \le 0 & & (\sigma_3) \\ x_1 > 0, \qquad x_2 \ge 0, & & \end{array} \right. \end{array} \right.$$

(5a)



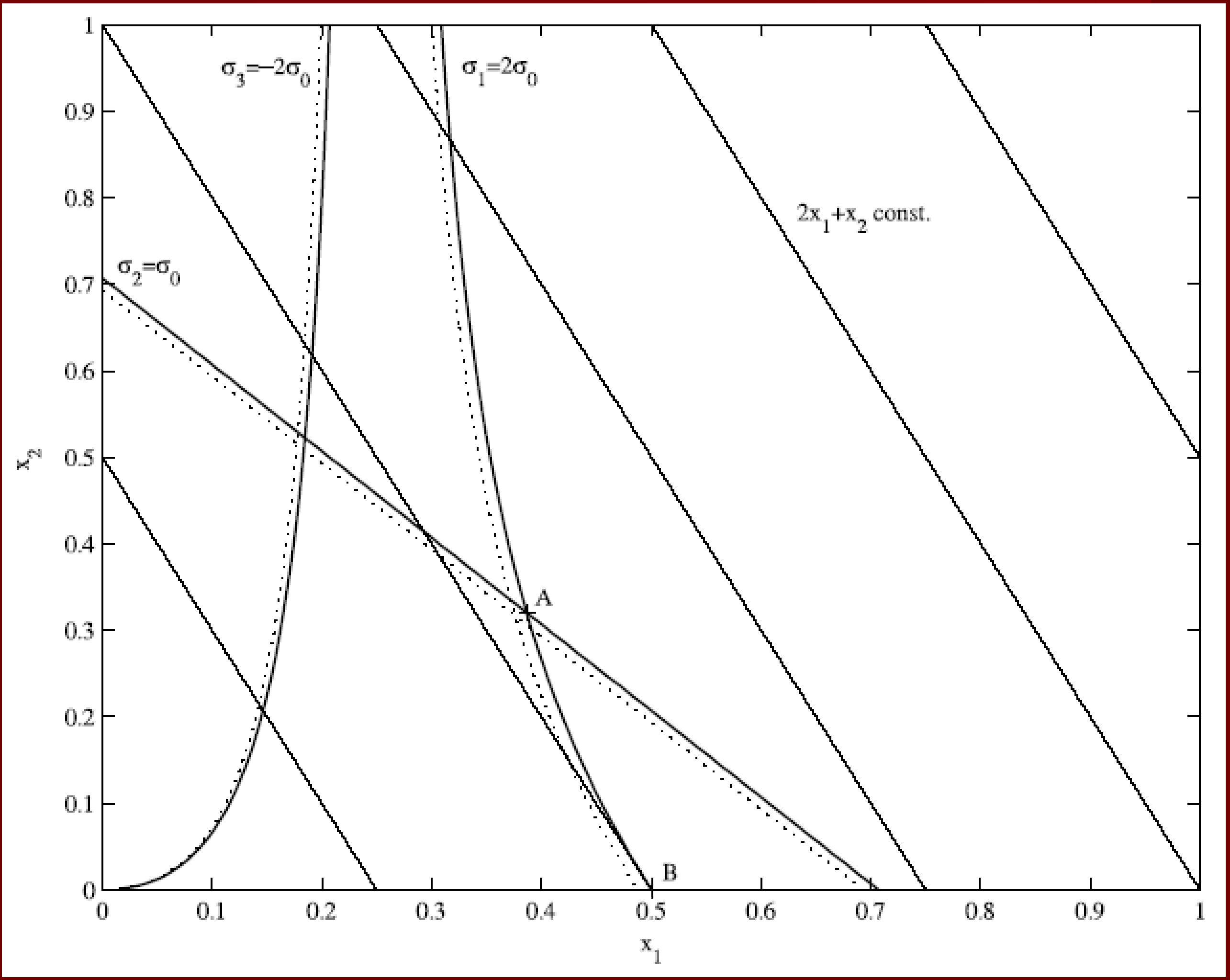
$$A_1^* = \frac{F}{2\sigma_0}\left(1 + \frac{1}{\sqrt{2}}\right), \qquad A_2^* = \frac{F}{2\sqrt{2}\sigma_0},$$

(5b)

$$\rho_1 = \rho_2 = \rho_3 = \rho_0, \ \sigma_1^{\max} = \sigma_3^{\max} = 2\sigma_0, \ \sigma_2^{\max} = \sigma_0.$$

$$(\mathbb{SO})^{5b}_{\mathrm{nf}} \quad \begin{cases} \min\limits_{x_1,x_2} \ 2x_1 + x_2 \\ \text{s.t.} \quad \begin{cases} \dfrac{2x_1 + x_2}{4x_1(x_1+x_2)} - 1 \le 0 & & (\sigma_1) \\ \dfrac{1}{\sqrt{2}(x_1+x_2)} - 1 \le 0 & \text{if } x_2 > 0 & (\sigma_2) \\ \dfrac{x_2}{4x_1(x_1+x_2)} - 1 \le 0 & & (\sigma_3) \\ x_1 > 0, \qquad x_2 \ge 0, \end{cases} \end{cases}$$

(5b)

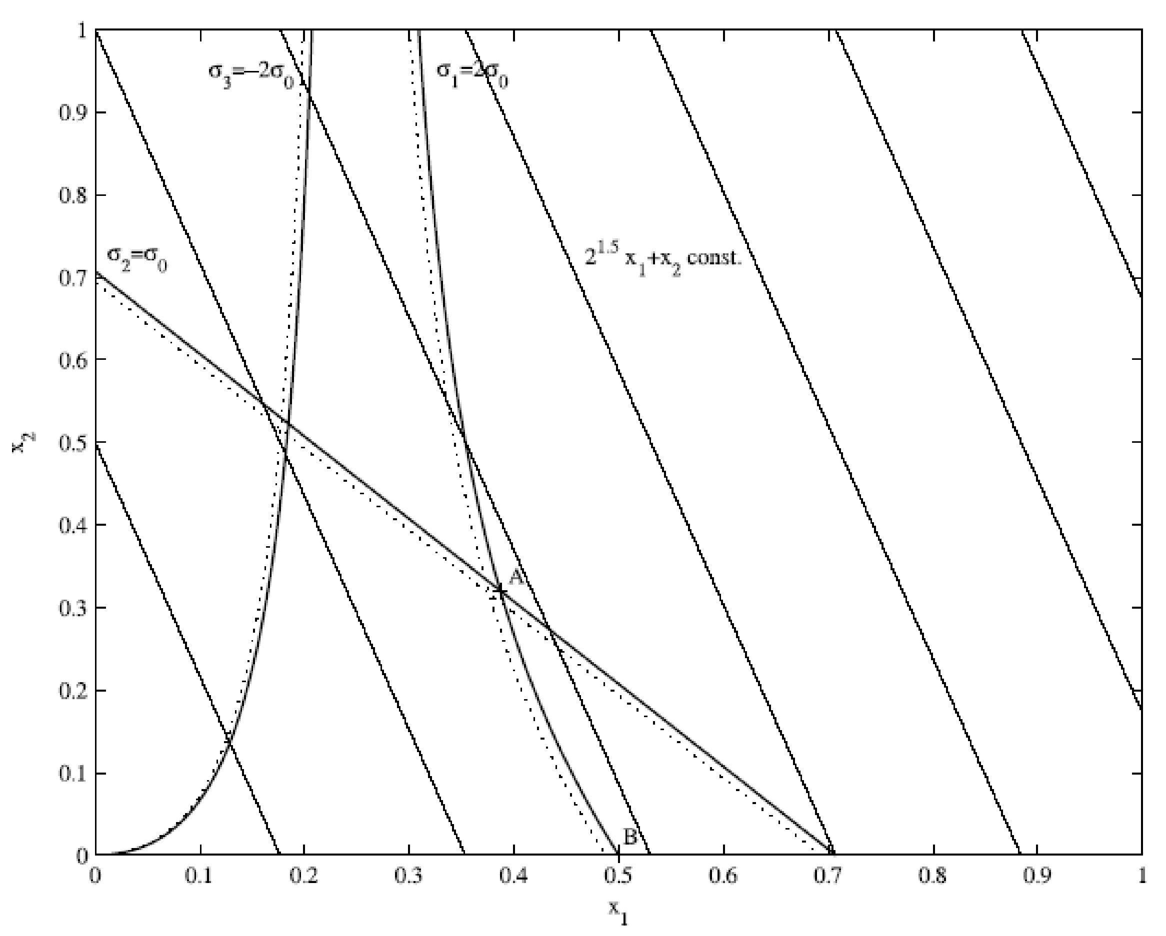


$$A_1^* = \frac{F}{2\sigma_0}, \qquad A_2^* = 0,$$

(5c)

$$\rho_1 = (2\sqrt{2}-1)\rho_0,\ \rho_2 = \rho_3 = \rho_0,\ \sigma_1^{\max} = \sigma_3^{\max} = 2\sigma_0,\ \sigma_2^{\max} = \sigma_0.$$

$$(\mathbb{SO})_{\text{nf}}^{5c} \quad \begin{cases} \min\limits_{x_1,x_2} \ 2\sqrt{2}x_1 + x_2 \\ \text{s.t. the constraints in } (\mathbb{SO})_{\text{nf}}^{5b}, \end{cases}$$



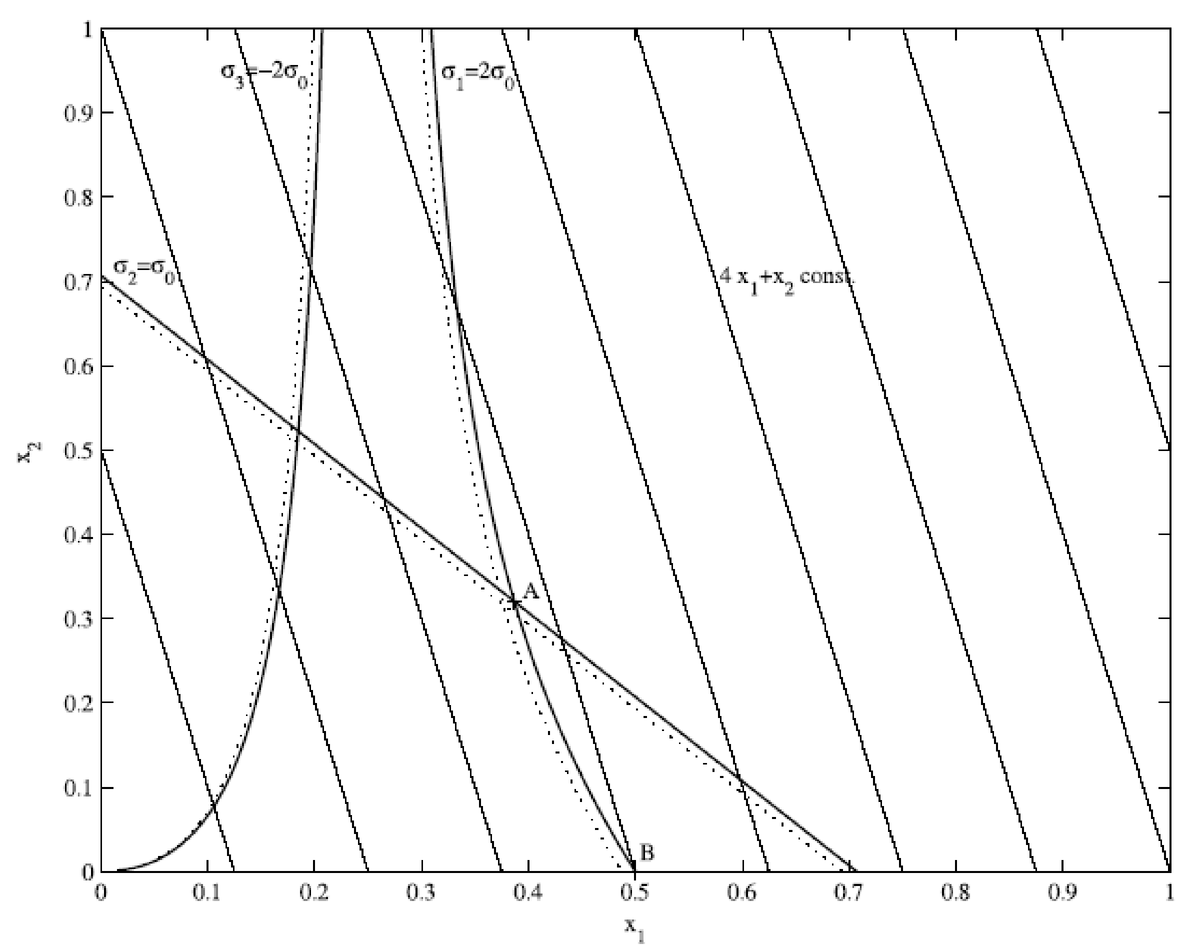
$$A_1^* = \frac{F}{\sigma_0}\left(\frac{4+\sqrt{2}}{14}\right),$$

$$A_2^* = \frac{F}{\sigma_0}\left(\frac{6\sqrt{2}-4}{14}\right),$$

(5d)

$$\rho_1 = 3\rho_0,\ \rho_2 = \rho_3 = \rho_0,\ \sigma_1^{\max} = \sigma_3^{\max} = 2\sigma_0,\ \sigma_2^{\max} = \sigma_0.$$

$$(\mathbb{SO})_{\text{nf}}^{5d} \quad \begin{cases} \min\limits_{x_1, x_2} \ 4x_1 + x_2 \\ \text{s.t. the constraints in } (\mathbb{SO})_{\text{nf}}^{5b}. \end{cases}$$



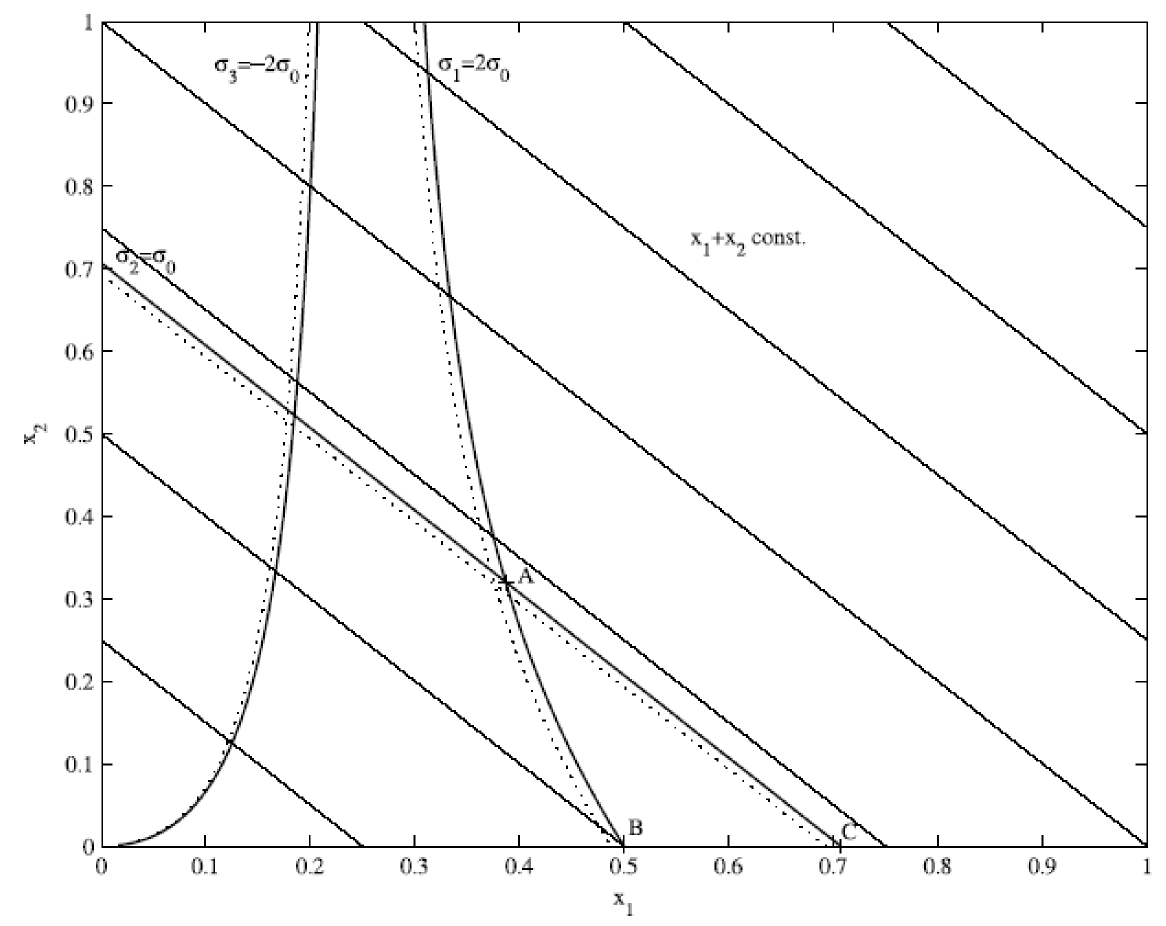
$$A_1^* = \frac{F}{\sigma_0}\left(\frac{4+\sqrt{2}}{14}\right)$$

$$A_2^* = \frac{F}{\sigma_0}\left(\frac{6\sqrt{2}-4}{14}\right),$$

(5e)

$$\rho_1 = \rho_3 = \rho_0,\ \rho_2 = 2\rho_0,\ \sigma_1^{\max} = \sigma_3^{\max} = 2\sigma_0,\ \sigma_2^{\max} = \sigma_0.$$

$$(\mathbb{SO})_{\rm nf}^{5e} \quad \begin{cases} \min\limits_{x_1,x_2} \ x_1 + x_2 \\ \text{s.t. the constraints in } (\mathbb{SO})_{\rm nf}^{5b}, \end{cases}$$



$$A_1^* = \frac{F}{2\sigma_0},$$

$$A_2^* = 0,$$

QUESTION?

Thank you for

your coming and listening!