LECTURE SERIES
on
STRUCTURAL OPTIMIZATION

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LECTURE 2

S.O. PROBLEM FORMULATION

CONTENTS

INTRODUCTION

Structural Design Philosophy

Determisnistic or probabilistic-based?

Kind of failure modes?

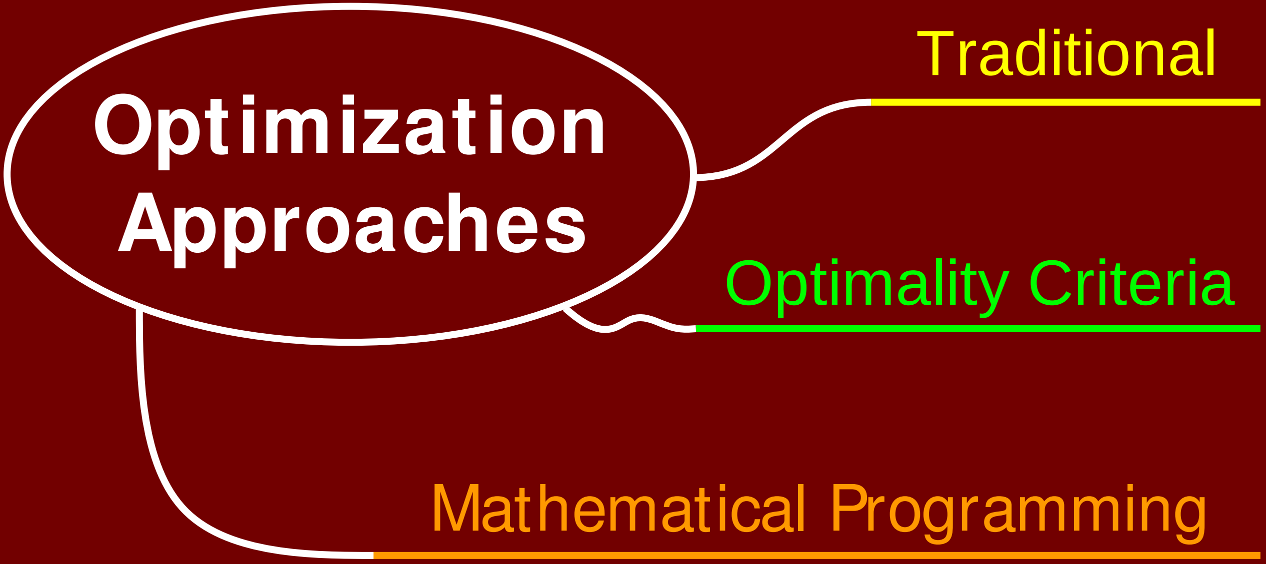
Service load and overload conditions?

Ex.

Design against Initial Yielding under Service Load

Design against Collapse under Overload

Design to Avoid Damage under Service Load **and** Collapse under Overload


Optimization Approaches
Traditional
Optimality Criteria
Mathematical Programming

Traditional Approach

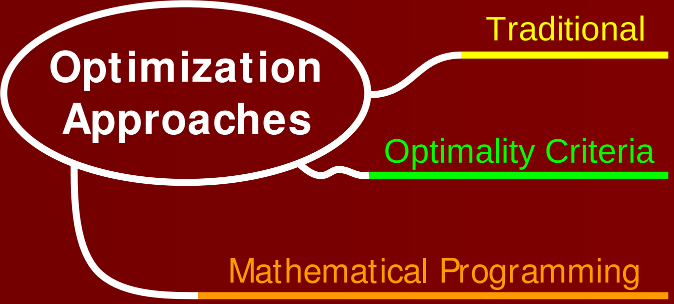
Preselect the set of constraints active at the optimum

Good when?

[critical constraints are easily identified AND
their number equals to the number of variables]

Wrong solution awareness!

[if wrong choice of critical constraints OR
their number is less than that of variables]



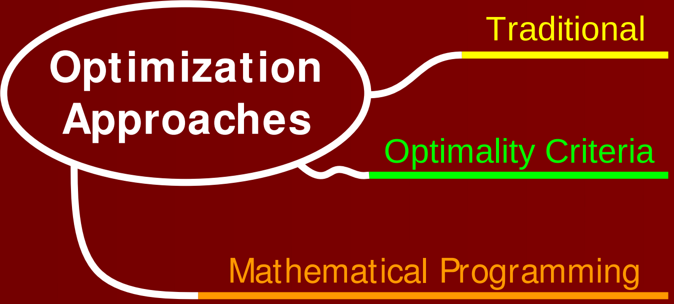
Optimality Criteria Approach

Assume a criterion related to structural behavior is satisfied at the optimum

Ex.
Fully stressed design; displacement constraints; stability …

Iterative solution process, usually!

Characteristics
[simple; may lead to “suboptimal” sol.; not general as of math. programming]



Math. Programming Approach

Oops, which of the constraints will be critical??

[no prediction on the set of active constraints ➔ essentially use inequalities constraints for a proper formulation]

Characteristics

[no single method for all problem; no accurate method for large structures ➔ often based on approximation concepts]

Relative optima may occur

[➔ repeat computations from different starting points]

Structural Analysis

Core of any S.O. formulation and solution

Acceptable analytical model

[must describe physical behavior adequately]

[yet be simple to analyze]

Most common

[elastic analysis of framed structures]

[elastic analysis of continuum structures]

[plastic analysis of framed structures]

ELASTIC ANALYSIS OF FRAMED STRUCTURES

Frame structures

Length >> sizes of cross-section

Typically: beams, grids, trusses, frames

Assumptions for linear elastic analysis

Displacements vary linearly w/ applied forces

All deformations are small

Methods

Force method

Displacement method

Force method

Compatibility equation

$$\mathbf{FX} \quad \Delta_P \quad \Delta_0 \qquad (1)$$

Final displacement **U** and forces **S**

$$\mathbf{U} \quad \mathbf{U}_P \quad \mathbf{F}_U\mathbf{X} \qquad (2)$$

$$\mathbf{S} \quad \mathbf{S}_P \quad \mathbf{F}_S\mathbf{X} \qquad (3)$$

If several loading conditions → all vectors are transformed into matrices

Force method

Brief procedure

(1) Determine degree of statical indeterminacy; Redundant forces and primary structure are chosen.

(2) Compute the entries of **F**, $_{P}$, $\mathbf{F}_{U}$, $\mathbf{U}_{P}$, $\mathbf{F}_{S}$, $\mathbf{U}_{S}$ in the primary structure

(3) Compute unknown redundant forces **X**

(4) Compute desired displacements **U** and forces **S** in the original indeterminate structure

Ex.

Displacement method

Equilibrium equation

$$\mathbf{Kq} \quad \mathbf{R}_{L} \quad \mathbf{R}_{0} \qquad (4)$$

Final displacement **U** (other than those included in **q**) and forces **S**

$$\mathbf{U} \quad \mathbf{U}_{L} \quad \mathbf{K}_{U}\mathbf{q} \qquad (5)$$

$$\mathbf{S} \quad \mathbf{S}_{L} \quad \mathbf{K}_{S}\mathbf{q} \qquad (6)$$

If several loading conditions → all vectors are transformed into matrices

Force method

Brief procedure

(1) Determine the joint displacement degree of freedom; Add restraints (= Establish restrained structure. It is unique.)

(2) Compute the entries of $\mathbf{K}$, $\mathbf{R}_L$, $\mathbf{K}_U$, $\mathbf{U}_L$, $\mathbf{K}_S$, $\mathbf{S}_L$ in the restrained structure

(3) Compute unknown displacements $\mathbf{q}$

(4) Compute desired displacements $\mathbf{U}$ and forces $\mathbf{S}$ in the original structure

Ex.

ELASTIC ANALYSIS OF CONTINUUM STRUCTURES

Continuum structures

Often: plates, shells, membranes, solid bodies

PDE

Analytical solution or Numerical solution?

For small problems

For large problems

Methods

FDM

FEM

and many more …

Finite element method

See the structure as one compased of discrete elements connected together at a number of nodes

ODE
PDE?

Types of unknowns: Nodal displacements / Nodal stresses

If “material nonlinearity” or “geometric nonlinearity” ➔ using FEM with an incremental approach

Finite element method

Brief steps

(1) The structure is divided into finite elements
(2) Compute stiffness matrix and nodal force vector for each element
(3) Transform to the global coordinates
(4) Assemble element stiffness matrices and nodal force vectors to have system stiffness and system force vector
(5) Apply boundary conditions
(6) Solve for unlnown nodal displacements
(7) Compute back internal stresses (or other desired quantities)

Finite element method

Equations at element level

$$\mathbf{K}_e\mathbf{q}_e \quad \mathbf{R}_e \tag{7}$$

$$\mathbf{K}_e \quad {}_V\,\mathbf{B}^T\mathbf{D}\mathbf{B}\,dV \tag{8}$$

$$\mathbf{R}_e \quad \mathbf{Q}_e \quad {}_A\,\mathbf{N}^T\mathbf{p}\,dA \tag{9}$$

$\varepsilon \quad \mathbf{B}\mathbf{q}_e$; appropriate derivatives of **N** (10)

$$\sigma \quad \mathbf{D}\varepsilon \tag{11}$$

$$\mathbf{q} \quad \mathbf{N}\mathbf{q}_e \tag{12}$$

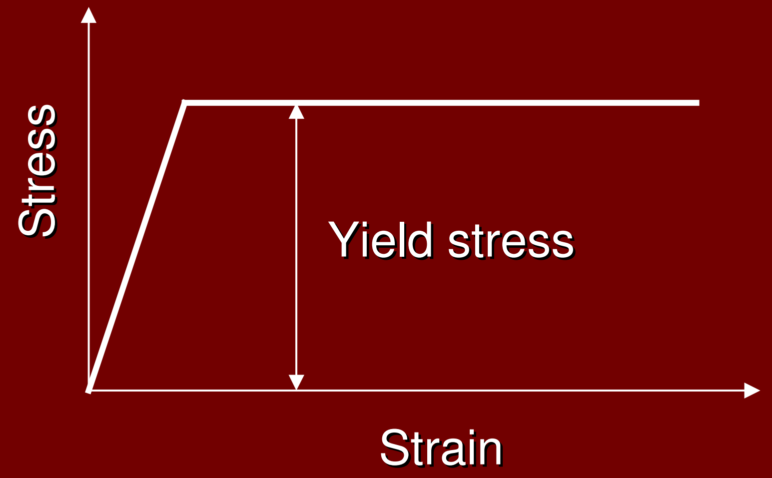
PLASTIC ANALYSIS OF FRAMED STRUCTURES

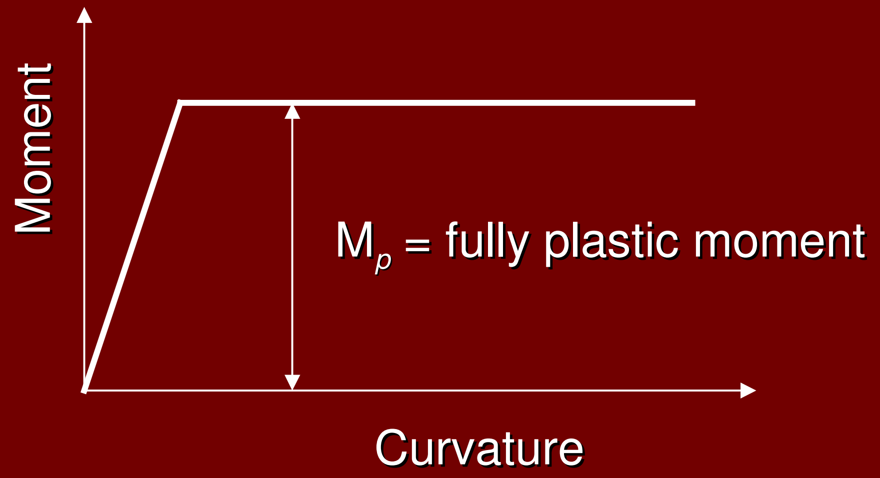
What for?

Supplement elastic analysis by giving useful information about collapse load and the mode of collapse

Limit design (in various codes)

Plastic model





Kinematic Approach

Equilibrium of possible mechanism *i*

$$U_i \qquad {}_i E_i \qquad (13)$$

where

E_i: external work of applied service loads

${}_i$: kinematic multiplier

U_i: total energy dissipated by plastic hinges

Kinematic Approach

$$U_i \qquad {}_{j\ \ 1}^{\ \ J} \quad M_{pj} \quad {}_{ij} \qquad (14)$$

where

$_{ij}$: hinge rotations

M_{pj}: plastic moments

J: number of potential plastic hinges

Kinematic Approach

After the kinematic theorem of plastic analysis

$$\min_{i} \quad {}_{i} \qquad \min_{i} \frac{U_i}{E_i} \qquad \min_{i} \sum_{j\;\;1}^{J} M_{pj} \frac{{}^{ij}}{E_i} \;; \quad i \quad 1..p \tag{15}$$

where

: load factor

p: total number of possible mechanisms

Kinematic Approach

Independent mechanisms

$$m \quad J \quad NR \tag{15}$$

where

NR: degree of statical indeterminacy

All possible collapse mechanisms can be generated by linear combinations of *m* independent mechanisms!

QUESTION?

Thank you for your coming and listening!