VIBRATION ANALYSIS OF STIFFENED FOLDED COMPOSITE PLATES USING EIGHT NODDED ISOPARAMETRIC QUADRILATERAL ELEMENTS

PHÂN TÍCH DAO ĐỘNG TẤM COMPOSITE LỚP GẤP NẾP CÓ GÂN GIA CƯỜNG BẰNG CÁCH SỬ DỤNG PHẦN TỬ TỨ GIÁC ĐẲNG THAM SỐ TÁM NÚT

Bui Van Binh

Electric Power University

Tóm tắt: Bài báo trình bày một số kết quả tính tần số dao động riêng, phân tích đáp ứng tức thời của chuyển vị, phân tích dạng dao động riêng của tấm composite lớp gấp nếp có và không có gân gia cường bằng phương pháp phần tử hữu hạn. Ảnh hưởng của góc gấp nếp, góc sợi, cách sắp xếp gân, số gân của tấm được làm rõ qua các kết quả số. Chương trình tính bằng Matlab được thiết lập dựa trên lý thuyết tấm bậc nhất có kể đến biến dạng cắt ngang của Mindlin. Các kết quả số thu được có tính tương đồng cao khi so sánh với các kết quả của các tác giả khác đã công bố trên các tạp chí có uy tín.

Từ khóa: Phân tích dao động, đáp ứng động lực học, tấm composite gấp nếp có gân gia cường, phương pháp phần tử hữu hạn.

Abstract: This paper presents several numerical results of natural frequencies, transient displacement responses, and mode shape analysis of unstiffened and stiffened folded laminated composite plates using finite element method. The effects of folding angle, fiber orientations, stiffeners, and position of stiffeners of the plates are illustrated. The program is computed by Matlab using isoparametric rectangular plate elements with five degree of freedom per node based on Mindlin plate theory. The calculated results are correlative in comparison with other authors' outcomes published in prestigious journals.

Keywords: Vibration analysis, dynamic response; stiffeners, stiffened folded laminated composite plates, finite element method.

INTRODUCTION

Folded laminate composite plates have been found almost everywhere in various branches of engineering, such as in roofs, ship hulls, sandwich plate cores and cooling towers, etc. Because of their high strength-to-weight ratio, easy to form, economical, and have much higher load carrying capacities than fat plates, which ensures their popularity and has attracted constant research interest since they were introduced. Because the laminated plates with stiffeners become more and more important in the aerospace industry and other modern engineering fields, wide attention has been paid on the experimental, theoretical and numerical analysis for the static and dynamic problems of such structures in recent years.

The flat plate with stiffeners based on the finite element model and were presented in [1, 2, 3, 5, 6, 7, 8…]. In those studies, the Kirchhoff, Mindlin and higher-order plate theories are used. Those researches used the assumption of eccentricity (or concentricity) between plate and stiffeners: a stiffened plate is divided into plate element and beam element. Behavior of unstiffened isotropic folded plates has been studied previously by a host of investigators using a variety of approaches. Goldberg and Leve [9] developed a method based on elasticity. According to this method, there are four components of displacements at each point along the joints: two components of translation and a rotation, all lying in the plane normal to the joint, and a translation in the direction of the joint. The stiffness matrix is derived from equilibrium equations at the joints, while expanding the displacements and loadings into the Fourier series considering boundary conditions. Bar-Yoseph and Herscovitz [10] formulated an approximate solution for folded plates based on Vlassov's theory of thin-walled beams. According to this work, the structure is divided into longitudinal beams connected to a monolithic structure. Cheung [11] was the first author developed the finite strip method for analyzing isotropic folded plates. Additional works in the finite strip method have been presented. The difficulties encountered with the intermediate supports in the finite strip method [12] were overcome and subsequently Maleki [13] proposed a new method, known as compound strip method. Irie et al. in [14] used Ritz method for the analysis of free vibration of an isotropic cantilever folded plate. Perry et al. in [15] presented a rectangular hybrid stress element for analyzing a isotropic folded plate structures in bending cases. In this, they used a four-node element, which is based on the classical hybrid stress method, is called the hybrid coupling element and is generated by a combination of a hybrid plane stress element and a hybrid plate bending element. Darılmaz et al. in [16] presented an 8-node quadrilateral assumed-stress hybrid shell element. Their formulation is based on Hellinger

- Reissner variational principle for bending and free vibration analyses of structures, which have isotropic material properties. Haldar and Sheikh [17] presented a free vibration analysis of isotropic and composite folded plate by using a sixteen nodes triangular element. Suresh and Malhotra [18] studied the free vibration of damped composite box beams using four node plate elements with five degrees of freedom per node. Niyogi et al. in [19] reported the analysis of unstiffened and stiffened symmetric cross-ply laminate composite folded plates using first-order transverse shear deformation theory and nine nodes elements. In their works, only in axis symmetric cross-ply laminated plates were considered. So that, there is uncoupling between the normal and shear forces, and also between the bending and twisting moments, then besides the above uncoupling, there is no coupling between the forces and moment terms. In [20-23], Bui Van Binh and Tran Ich Thinh presented a finite element method to analyze of bending, free vibration and time displacement response of V-shape; W-shape sections and multi-folding laminate plate. In these studies, the effects of folding angles, fiber orientations, loading conditions, boundary condition have been investigated.

In this paper, the theoretical formulation for calculated natural frequencies and investigating the mode shapes, transient displacement response of the composite plates with and without stiffeners are presented. The eight-noded isoparametric rectangular plate elements were used to analyze the stiffened folded laminate composite plate with in-axis configuration and off-axis configuration. The stiffeners are modeled as laminated plate elements. Thus, this paper did not use any assumption of eccentricity (or concentricity) between plate and stiffeners. The home-made Matlab code based on those formulations has been developed to compute some numerical results for natural frequencies, and dynamic responses of the plates under various fiber orientations, stiffener orientations, and boundary conditions. In transient analysis, the Newmark method is used with parameters that control the accuracy and stability of $\alpha = 0.25$ and $\delta = 0.5$ (see ref. [24, 26]).

2. THEORETICAL FORMULATION

2.1 Displacement and strain field

According to the Reissner-Mindlin plate theory, the displacements (*u, v, w*) are referred to those of the mid-plane (u_0, v_0, w_0) as [25]:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (1)$$

Where: t is time; θ_x and θ_y are the bending slopes in the *xz* - and *yz*-plane, respectively.

The *z*-axis is normal to the *xy*-plane that coincides with the mid-plane of the laminate positive downward and clockwise with *x* and *y*.

The generalized displacement vector at the mid - plane can thus be defined as

$$\{d\} = \{u_0, v_0, w_0, \theta_x, \theta_y\}^T$$

The strain-displacement relations can be taken as:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + z\kappa_x;$$

$$\varepsilon_{yy} = \varepsilon_{yy}^0 + z\kappa_y;$$

$$\varepsilon_{zz} = 0$$

$$\gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy};$$

$$\gamma_{yz} = \gamma_{yz}^0;$$

$$\gamma_{xz} = \gamma_{xz}^0 \qquad (2)$$

Where

$$\{\varepsilon^0\} = \{\varepsilon_{xx}^0, \varepsilon_{yy}^0, \gamma_{xy}^0\}^T = \left\{\frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T$$

$$\{\kappa\} = \{\kappa_x, \kappa_y, \kappa_{xy}\}^T = \left\{\frac{\partial \theta_x}{\partial x}, \frac{\partial \theta_y}{\partial y}, \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right\}^T \qquad (3)$$

$$\{\gamma^0\} = \{\gamma_{yz}^0, \gamma_{xz}^0\}^T = \left\{\frac{\partial w_0}{\partial y} + \theta_y, \frac{\partial w_0}{\partial x} + \theta_x\right\}^T$$

and T represents transpose of an array.

In laminated plate theories, the membrane $\{N\}$, bending moment $\{M\}$ and shear stress $\{Q\}$ resultants can be obtained by integration of stresses over the laminate thickness. The stress resultants-strain relations can be expressed in the form:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [F] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \\ \{\gamma^0\} \end{Bmatrix} \qquad (4)$$

Where

$$\left([A_{ij}], [B_{ij}], [D_{ij}]\right) = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \left([Q'_{ij}]_k\right)(1, z, z^2)\,dz$$

$$i, j = 1, 2, 6 \qquad (5)$$

$$[F] = \sum_{k=1}^{n} f \int_{h_{k-1}}^{h_k} \left([C'_{ij}]_k\right)dz \qquad f = 5/6;$$

$$i, j = 4, 5 \qquad (6)$$

n: number of layers, h_{k-1}, h_k: the position of the top and bottom faces of the k^{th} layer.

$[Q'_{ij}]_k$ and $[C'_{ij}]_k$: reduced stiffness matrices of the k^{th} layer (see [25]).

2.2 Finite element formulations

The governing differential equations of motion can be derived using Hamilton's principle [26]:

$$\int_{t_1}^{t_2} \delta\left(\frac{1}{2}\int_V \rho\{\dot{u}\}^T\{\dot{u}\}dV - \frac{1}{2}\int_V \{\varepsilon\}^T\{\sigma\}dV - \left(\int_V \{u\}^T\{f_b\}dV + \int_S \{u\}^T\{f_s\}dS + \{u\}^T\{f_c\}\right)\right)dt = 0 \qquad (7)$$

In which:

$$T = \frac{1}{2}\int_V \rho\{\dot{u}\}^T\{\dot{u}\}dV;$$

$$U = \frac{1}{2}\int_V \{\varepsilon\}^T\{\sigma\}dV;$$

$$W = \int_V \{u\}^T\{f_b\}dV + \int_S \{u\}^T\{f_s\}dS + \{u\}^T\{f_c\}$$

U, T are the potential energy, kinetic ene1rgy; W is the work done by externally applied forces.

In the present work, eight nodded isoparametric quadrilateral element with five degrees of freedom per nodes is used. The displacement field of any point on the mid-plane given by:

$$u_0 = \sum_{i=1}^{8} N_i(\xi,\eta).u_i;$$

$$v_0 = \sum_{i=1}^{8} N_i(\xi,\eta).v_i;$$

$$w_0 = \sum_{i=1}^{8} N_i(\xi,\eta).w_i;$$

$$\theta_x = \sum_{i=1}^{8} N_i(\xi,\eta).\theta_{xi};$$

$$\theta_y = \sum_{i=1}^{8} N_i(\xi,\eta).\theta_{yi} \qquad (8)$$

Where: $N_i(\xi,\eta)$ are the shape function associated with node i in terms of natural coordinates (ξ,η).

The element stiffness matrix given by:

$$[k]_e = \int_{V_e} \left([B]^T\right)[H][B]dV_e \qquad (9)$$

Where $[H]$ is the material stiffness matrix given by:

$$[H] = \begin{bmatrix} [A] & [B] & 0 \\ [B] & [D] & 0 \\ 0 & 0 & [F] \end{bmatrix}$$

The element mass matrix given by:

$$[m]_e = \int_{A_e} \rho[N_i]^T[N_i]dA_e \qquad (10)$$

With ρ is mass density of material.

Nodal force vector is expressed as:

$$\{f\}_e = \int_{A_e} [N_i]^T q dA_e \qquad (11)$$

Where q is the intensity of the applied load.

For free and forced vibration analysis, the damping effect is neglected, the governing equations are:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$

or $\left\{[M] - \omega^2[K]\right\} = \{0\}$ (12)

And $[M]\{\ddot{u}\} + [K]\{u\} = f(t)$ (13)

In which $\{u\}$, $\{\ddot{u}\}$ are the global vectors of unknown nodal displacement, acceleration, respectively. $[M]$, $[K]$, $f(t)$ are the global mass matrix, stiffness matrix, applied load vectors, respectively.

Where

$$[M] = \sum_1^n [m]_e;\ [K] = \sum_1^n [k]_e;$$

$$\{f(t)\} = \sum_1^n \{f_e(t)\} \qquad (14)$$

With n is the number of element.

When folded plates are considered, the membrane and bending terms are coupled, as can be clearly seen in Fig.1. Even more, since the rotations of the normal appear as unknowns for the Reissner–Mindlin model, it is necessary to introduce a new unknown for the in-plane rotation called drilling degree of freedom.

$$\begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}_e = \begin{bmatrix} l_{x'x} & l_{y'x} & l_{z'x} & 0 & 0 & 0 \\ l_{x'y} & l_{y'y} & l_{z'y} & 0 & 0 & 0 \\ l_{x'z} & l_{y'z} & l_{z'z} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{y'y} & -l_{x'y} & l_{z'y} \\ 0 & 0 & 0 & -l_{y'x} & l_{x'x} & -l_{z'x} \\ 0 & 0 & 0 & l_{y'z} & -l_{x'z} & l_{z'z} \end{bmatrix} \begin{Bmatrix} u' \\ v' \\ w' \\ \theta'_x \\ \theta'_y \\ \theta'_z \end{Bmatrix}_e \quad (15)$$

Where: $[T]$ is the transformation matrix.

l_{ij} : are the direction cosines between the global and local coordinates.

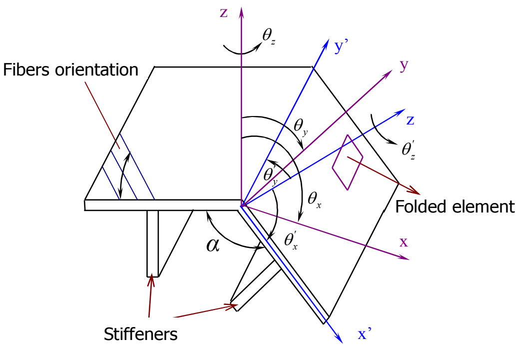


Fig.1. Global (x,y,z) and local (x',y'z') axes system for folded plate

3. NUMERICAL RESULTS

3.1 Free vibration analysis of two folded laminated plates

In this section, free vibration analysis of the unstiffened and stiffened two folded composite plate (illustrated in Fig. 2) has been carried out for various folding angle α=90^0, 120^0, 150^0. The plate made of E-glass epoxy composite material (given in Table 1) and geometry parameters given in Fig. 2.

Table 1. Material properties of E-glass Epoxy composite [19]

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{13} (GPa)	υ_{12}	ρ (kg/m^3)
60.7	24.8	12.0	12.0	0.23	1300

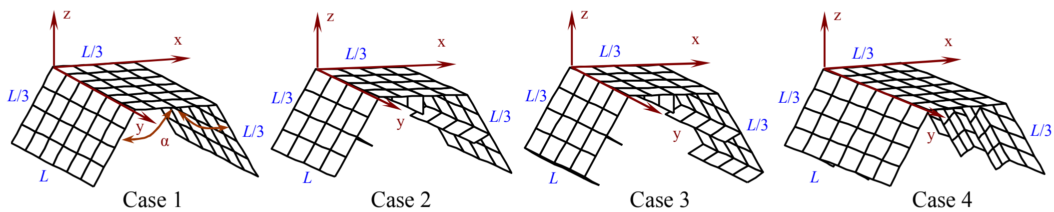


Fig.2. Geometry of two folded composite plate

Four cases are recalculated for various folding angle α = 90^0, 120^0, 150^0 of laminated plates. The geometries of studied plates are shown in Fig.2 with the fiber orientation of [90^0,90^0,90^0]. The added stiffening plates taken equal to 100mm for case 2-4, the length of the plates L = 1.5m and thickness $t = 0.02L$.

Case 1: Unstiffened two folded composite plate (Case 1 - Fig.2).

Case 2: Three stiffeners are attached below the folded plate running along the length of the cantilever (Case 2- Fig.2) with a total mass increment of 20%.

Case 3: Five stiffeners are attached below the folded plate running along the length of the cantilever (Case 3 - Fig.2) with a total mass increment of 33.33%.

Case 4: Two stiffeners are attached below the folded plate along transverse direction (Case 4- Fig.2) with a total mass increment of 11.55%.

Firstly, to observe the accuracy the presented theoretical formulation and computer code, the natural frequencies of case (1-4) are calculated and compared with the results given by [19]. The folded plate is divided by 72 eight nodded isoparametric quadrilateral elements. The stiffener running along the length of the cantilever and transverse direction are divided by 4 and 8 elements, respectively.

The results are present in Table 2, Table 3 and compared with the results given by [19] for cross ply laminate plates (in two first columns for [0^0/0^0/0^0]). The results for the unstiffened plates made of four plies angle-ply off axis and four plies cross-ply in axis are listed in four next columns of Table 2. Table 3 shown natural frequencies of stiffened plate with fiber orientation of [90^0/90^0/90^0].

The results (listed in Table 2, 3) shown that the five natural frequencies are in excellent agreement.

** Natural frequencies:*

Table 2. First five natural frequencies of two folded composite plate for folding angle α=90^0,120^0,150^0, thickness t=0.02L, L=1.5m.

α	ω_i	*[0^0/0^0/0^0]*		*Present: Angle-ply off axis*		*Present: Cross-ply in axis*	
		Present	*[19]*	*[45^0/-45^0]$_s$*	*[45^0/-45^0]$_{ns}$*	*[90^0/0^0]$_s$*	*[90^0/0^0]$_{ns}$*
90^0	1	63.3	63.6	68.7	71.49	66.4	73.5
	2	69.7	69.8	75.6	73.18	69.5	73.9
	3	150.5	152.7	155.3	157.8	149.9	146.1
	4	156.7	158.3	159.5	161.2	156.3	156.1
	5	203.9	201.9	183.5	183.6	190.8	194.6

α	ω_i	*[0⁰/0⁰/0⁰]*		*Present: Angle-ply off axis*		*Present: Cross-ply in axis*	
		Present	*[19]*	*$[45^0/-45^0]_s$*	*$[45^0/-45^0]_{ns}$*	*$[90^0/0^0]_s$*	*$[90^0/0^0]_{ns}$*
120^0	1	59.5	59.3	56.2	57.1	56.8	57.7
	2	63.1	63.4	73.3	72.7	66.1	73.1
	3	150.3	152.5	154.0	157.1	149.7	146.1
	4	153.9	155.0	156.1	158.0	153.1	152.2
	5	193.5	190.9	167.4	168.1	175.2	176.0
150^0	1	42.3	42.3	40.2	40.7	39.7	38.9
	2	60.7	60.8	66.5	66.4	62.3	67.5
	3	133.2	131.5	119.0	119.1	122.5	125.1
	4	144.9	145.6	143.0	144.2	142.9	138.7
	5	149.9	151.8	153.9	157.2	149.3	145.9

Table 3. First three natural frequencies of stiffened two folded composite plate for folding angle α=90^0,120^0,150^0, fiber orientation of [90^0/90^0/90^0].

α	ω_i	*Case 2*		*Case 3*		*Case 4*	
		Present	[19]	Present	[19]	Present	[19]
90^0	1	69.54	69.6	72.73	72.2	95.12	95.6
	2	73.98	73.9	81.55	81.1	119.36	122.5
	3	183.82	181.4	173.19	171.0	195.42	199.1
120^0	1	65.36	65.0	74.28	73.8	67.63	67.3
	2	69.80	69.9	77.04	76.2	112.11	109.6
	3	176.95	174.7	161.28	160.4	180.36	182.5
150^0	1	52.86	52.4	66.29	65.3	42.27	42.5
	2	68.54	68.5	76.27	75.7	93.15	93.5
	3	125.16	123.5	133.12	131.4	148.21	147.9

The first five mode shapes of the unstiffened and three cases of stiffened composite plate are plotted in Fig. 3 for folding angle α=120^0, fiber orientation of [45^0,-45^0/45^0].

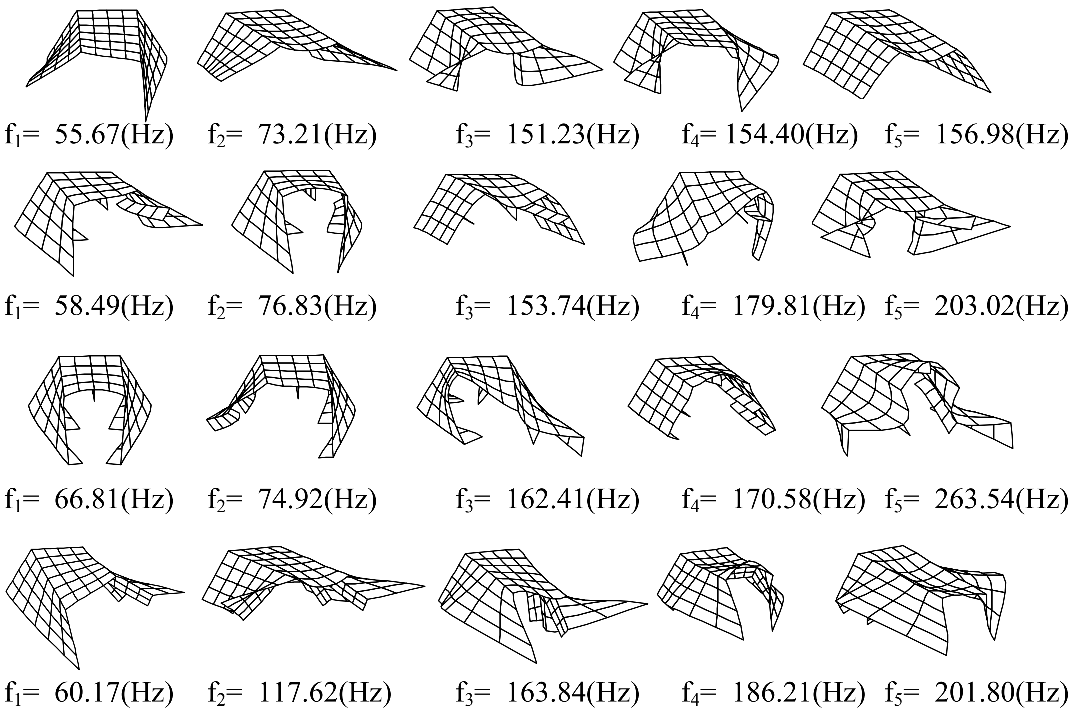
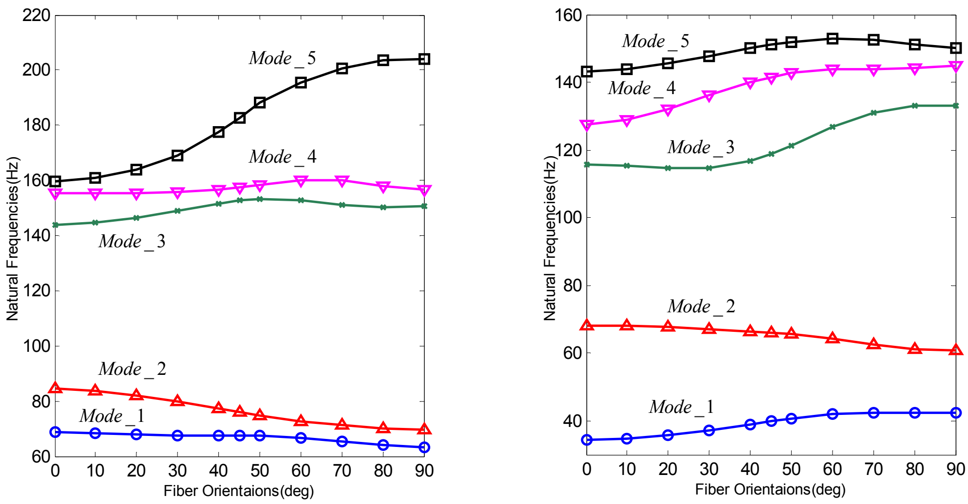


Fig.3. First five mode shapes of the unstiffened and three cases of stiffened composite plate, for folding angle $\alpha=120^0$; fiber orientation of [45^0,-45^0/45^0].



a- Folding angle α=90⁰, b- Folding angle α=150⁰

Fig.4. Effects of fiber orientation ϑ on the first five natural frequencies for folding angle $\alpha=90^0$ and $\alpha=150^0$, [ϑ^0/ϑ^0/ϑ^0], thickness t=0.02L.

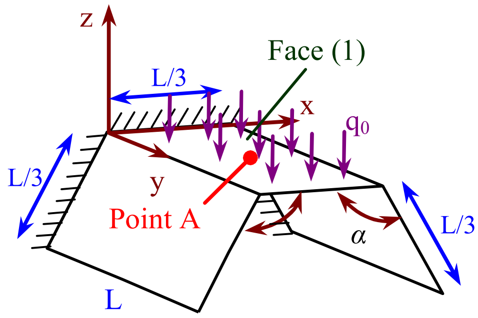
Fig.3 shows that the stiffeners do not make any change in getting mode shapes of presented plates (mode shapes make this study interesting, useful in dynamic analysis of the plates, but any generalized recommendation is very difficult without undergoing numerical experiments).

** The effects of fiber orientations on natural frequencies:*

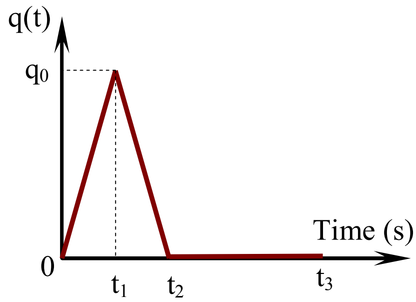
Secondly, the effects of fiber orientations on the first five natural frequencies of two folded composite plate made of $[\theta^0/\theta^0/\theta^0]$ has been carried out for various folding angle α. The results are plotted in Fig. 4a and Fig.4b for folding angle $\alpha = 90^0$ and $\alpha = 150^0$, respectively.

3.2 Transient analysis.

We consider a cantilever two folded composite plate with the same dimension and material properties of section 3.1 for unstiffened and three cases of stiffened composite plates. The folded plates subjected to a uniformly distributed step loading of intensity $q_0 = 10\text{kN/m}^2$ on face (1) for all cases. The location of point A (central point of top face) is shown in Fig.5a, analysis time step of $\Delta t = 0.0005$ ms, duration time of T = 0.025 (sec). The loading condition scheme is shown in Fig.5b with $t_1 = 1$ms, $t_2 = 2$ms, $t_3 = 25$ms.



(a)- Two folded composite plate.



(b)- Triangular step loading scheme.

Fig.5. Two folded composite plates with folding angle α subjected to uniformly step loading

Fig.6a, 6b, 6c and 6d plotted the effect of folding angle α on displacement responses measurement at point A of the plate which having the fiber orientation $[45^0/-45^0/45^0/-45^0]$ for case 1, case2, case3 and case 4, respectively.

From Fig.6, it can be observed that the displacement responses of folding angle $\alpha = 90^0$ and $\alpha = 120^0$ are closed to each other, the displacement response of $\alpha = 150^0$ is extremely higher than the others. The different become more rapidly for Case 1. The displacement amplitude and wave of Case 4 change more dramatic in the early time. Furthermore, there is a significant increase of vibration frequencies when the plates having clamped at edges.

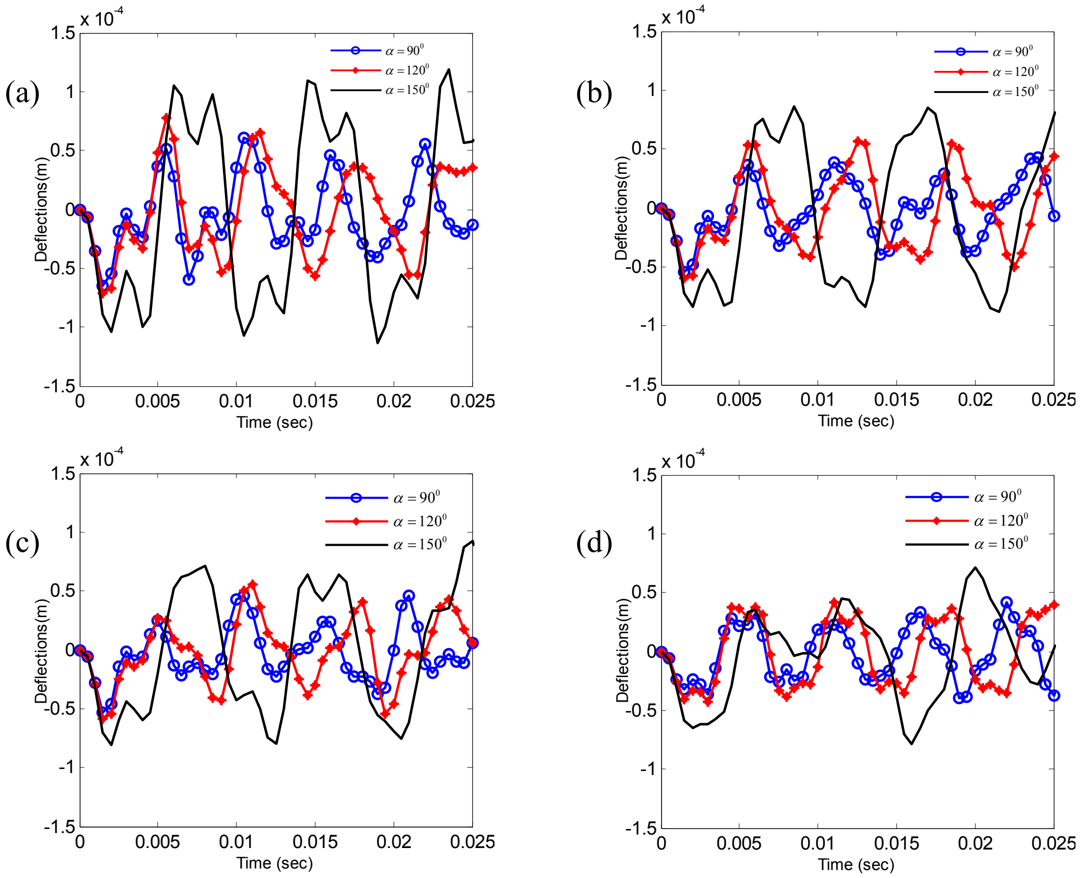
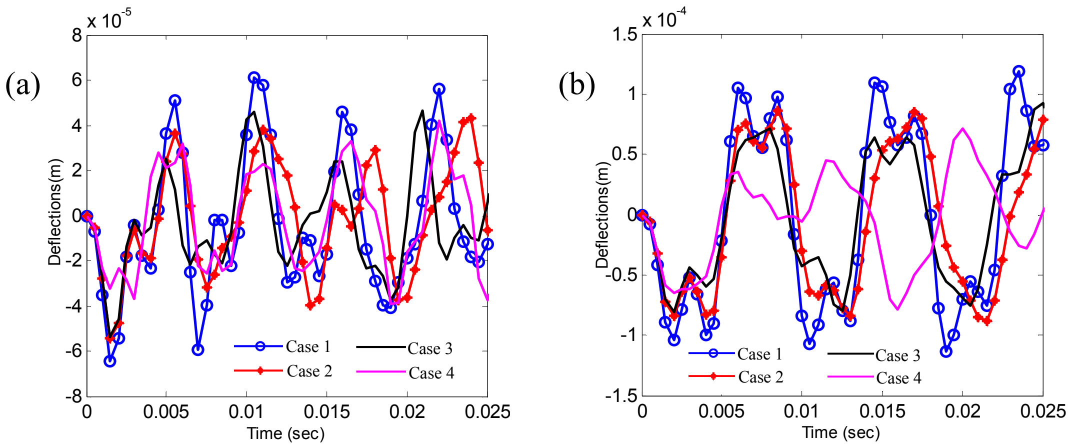


Fig.6. Effect of folding angle α on transient response, [45^0/-45^0/45^0/-45^0].



(a)- Folding angle α=90^0; (b)- Folding angle α=150^0

Fig.7.Comparision of transient response for different stiffener conditions of composite folded plate, [45^0/-45^0/45^0/-45^0]

Fig. 7a and Fig. 7b plotted comparison of transient response of the composite folded plates for different stiffener conditions for α = 90^0 and α = 150^0, respectively. It is revealed that the stiffness of the structure gradually reduces such as case1→ case2→ case3→ case4. With stiffener conditions, the deflection reduces and smallest amplitude in Case 3.

To observe effect of fiber orientation on transient response of the plates, we compared the response of two fiber orientation ([45^0/-45^0/45^0/-45^0] and [90^0/0^0/90^0/0^0]) for four cases: Case 1-Case4. The result is given in Fig. 8. In which: Fig.11a, 11b, 11c and 11d plotted the displacement responses measurement at point A of the plates (which have the folding angle α = 120^0 for: Case 1, Case2, Case3 and Case 4, respectively.

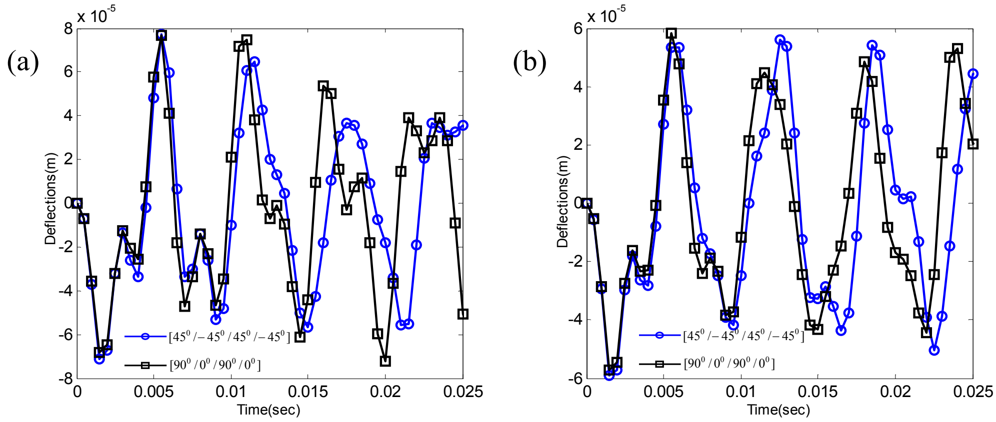


Fig.8 (a, b). Comparing effect of fiber orientation on transient response of the plate for different stiffener condition: Case 1 and Case 2, folding angle α =120^0

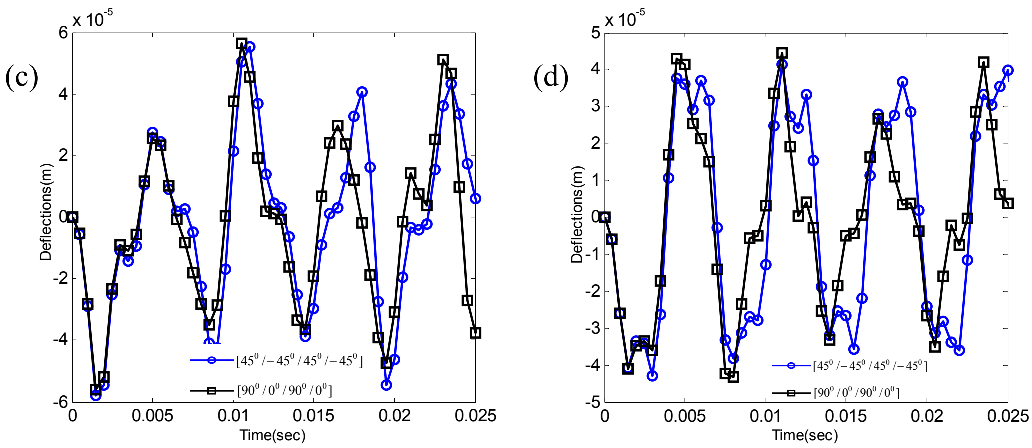


Fig.8 (c, d). Comparing effect of fiber orientation on transient response of the plate for different stiffener condition: Case 3 and Case 4, folding angle α =120^0

Fig.8 shows that the transient response of the laminate plates does not change in significant for angle-ply off axis and cross-ply in axis fiber orientation.

4. CONCLUSION

In the present study, a finite element method using an eight nodded isoparametric plate elements, based on the first order shear deformation theory were investigated for analysis of free vibration and the transient response of the unstiffened and stiffened folded laminate composite plate.

Good agreement is found between the results of this technique and other published results available in the literature.

The effects of various parameters as folding angle, fiber orientation on natural frequencies, dynamic responses and mode shapes of unstiffened; stiffened folded laminate composite plates were indicated by some numerical results.

The applicability of the present approach covers a wide range of forced vibration problems, geometric features, and boundary conditions.

The results of this study will serve as a benchmark for future research for designing folded composite structures and sandwich structures made of composite materials, as it was extremely quick and reliable in producing design results.

REFERENCE

[1] Turkmen, H.S., Mecitoglu, Z, Dynamic response of a stiffened laminated composite plates subjected to blast load. Journal of Sound and Vibration 221: 371–389, 1999.

[2] Zhao, X., Liew, K.M., Ng, T.Y, Vibrations of rotating cross-ply laminated circular cylindrical shells with stringer and rings stiffeners. Journal of Solids and Structures 39: 529–545, 2002.

[3] Sadek, E.A., Tawfik, S.A, A finite element model for the analysis of stiffened laminated plates. Computers and Structures 75: 369–383, 2000.

[4] Kumar, S.Y.V., Mukhopadhyay, M., A new triangular stiffened plate element for laminate analysis. Composites Science and Technology 60, 935–943, 2000.

[5] Olson, M. D. and Hazell, C. R, Vibration studies on some integral rib-stiffened plates. J. Sound Vibration. 50(I),43 61, 1977.

[6] Kolli, M. & Chandrashekharat, K, Finite element analysis of stiffened laminated plates under transverse loading. Composites Science and Technology 56:1355-1361, 1996.

[7] Biswal, KC., Ghosh, AK, Finite element analysis for stiffened laminated plates using higher order shear deformation theory. Computers and Structures; 53:161-171, 1994.

[8] Gangadhara Prusty, Linear static analysis of hat-stiffened laminated shells using finite elements. Finite element in analysis and design. 39: 1125-1138, 2003.

[9] Goldberg, J. E and Leve, H.L, Theory of prismatic folded structures. Int. Assoc. Bridge and Structural Engng 17: 58-86, 1957.

[10] Bar-Yoseph, P and Herscovitz, I, Analysis of folded plate structures. Thin-Walled Structures 7: 139-158, 1989.

[11] Cheung, Y. K, Finite strip method of elastic slabs. Proc.ASCE 94, 1365-1378, 1968.

[12] Cheung, Y.K, Folded-plate structures by finite strip method. J.Struct. Div., ASCE12: 2963-2979, 1969.

[13] Maleki, S, Compound strip method for Box Girders and folded plates, Comput Struct. 40: 527-538, 1991.

[14] Irie, T, Yamada, G, Kobayashi, Y, Free vibration of a cantilever folded plate, J Acoust. Soc. Am. 76(6): 1743-1748, 1984.

[15] Perry, B, Bar-Yoseph, P, Rosenhouse, G, Rectangular hybrid shell element for analysing folded plate structures. Computers and Structures 44:177-83, 1992.

[16] Kutlu Darılmaz and Nahit Kumbasar, An 8-node assumed stress hybrid element for analysis of shells. Computers and Structures 84: 1990-2000, 2006.

[17] Haldar, S, Sheikh, A.H, Free vibration analysis of isotropic and composite folded plates using a shear flexible element. Finite Elem. Anal. Des. 42: 208–226, 2005.

[18] Suresh, R., Malhotra, S.K, Vibration and damping analysis of thin-walled box beams. J. Sound Vib. 215: 201-210, 1998.

[19] Sreyashi, Pal., and Guha Niyogi, Application of folded formulation in analyzing stiffened laminated composite and sandwich folded plate vibration. Journal of Reinforced Plastics and composites, 27: 692-710, 2008.

[20] Bui van Binh, Tran Ich thinh, Tran Minh Tu, Analysis of bending folded laminated composite plate by finite element method. International conference on Science and Technology, Science and Technics Publishing House, Session 6: 711-723, 2011.

[21] Tran Ich Thinh, Bui Van Binh, Tran Minh Tu, Vibration of folded laminate composite plate. International conference on Science and Technology, Science and Technics Publishing House, Session 6: 659-670, 2011.

[22] Tran Ich Thinh, Bui Van Binh, Tran Minh Tu, Static and free vibration of laminated composite folded plate using finite element method. Journal of Science and Technology, Vol.49, No.2, 2011.

[23] Tran Ich Thinh, Bui Van Binh, Tran Minh Tu, Bending and Vibration analyses of multi-folding laminate composite plate using finite element method. Vietnam Journal of Mechanics, VAST, Vol. 34, No. 3, pp. 185 – 202, 2012.

[24] Singiresu, S.Rao, The Finite Element Method in Engineering, Elsevier, 2004.

[25] Tran Ich Thinh, Composite Materials. Viet Nam Education Publishing House, 1994 (in Vietnamese).

[26] Bathe, K-J, Finite element procedures. Prentice-Hall, Inc, 1996.

Giới thiệu tác giả:

Tác giả Bùi Văn Bình hiện đang công tác tại Khoa Công nghệ cơ khí - Trường Đại học Điện lực.

Hướng nghiên cứu chính trong 5 năm gần đây: mô hình hoá và tính toán số kết cấu composite lớp.