

Chapter 6: Entropy - a New Property

a) Defining and Evaluating Entropy

Following on the Second Law developed in [Chapter 5](#) we consider the [Clausius Inequality](#) which leads to the definition of a new property **Entropy** (S - kJ / K) as follows:

$$\text{Entropy: } dS \Leftrightarrow \frac{\delta Q}{T} \Big|_{\text{rev}}$$

A very strange definition indeed, and difficult to comprehend. It is defined in differential format as the reversible heat transfer divided by the temperature. In an attempt to try and understand it we rewrite the definition as follows:

$$\text{Heat: } \delta Q = T \cdot dS \Rightarrow Q = \int_{\text{rev}} T \cdot dS$$

It is advantageous to compare this definition with the equivalent definition of work as follows:

$$\text{Work: } \delta W = P \cdot dV \Rightarrow W = \int_{\text{rev}} P \cdot dV$$

Thus it begins to make sense. Work done requires both a driving force (pressure P) *and* movement (volume change dV). We implicitly evaluated the work done for reversible processes - always neglecting friction or any other irreversibility. Similarly we can state that heat transfer requires both a driving force (temperature T) and some equivalent form of "movement" (entropy change dS). Since temperature can be considered as represented by the vibration of the molecules, it is this transfer of vibrational energy that we define as entropy.

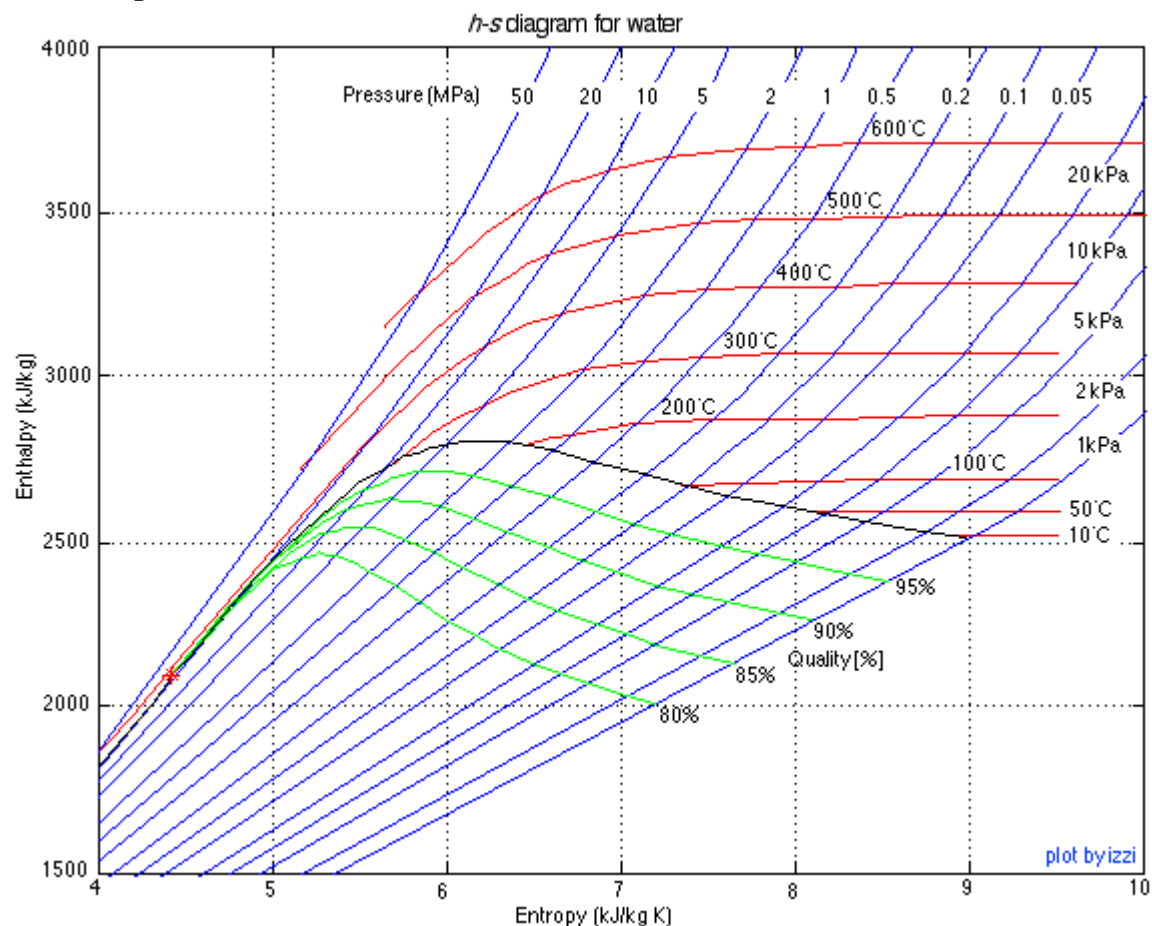
We now continue with the [Increase in Entropy Principle](#) which is also derived from the Clausius Inequality, and states that for any process, the total change in entropy of a system or control volume together with its enclosing adiabatic surroundings is always greater than or equal to zero. This total change of entropy is denoted the **Entropy Generated** during the process (S_{gen} [kJ/K] or s_{gen} [kJ/kg.K]). For reversible processes the entropy generated will always be zero.

We use the differential form of the energy equation to derive the $T \cdot ds$ relations which can be used to [evaluate the change of entropy](#) (Δs) for processes involving 2-phase fluids (Steam, R134a, CO₂), solids or liquids, or ideal gasses.

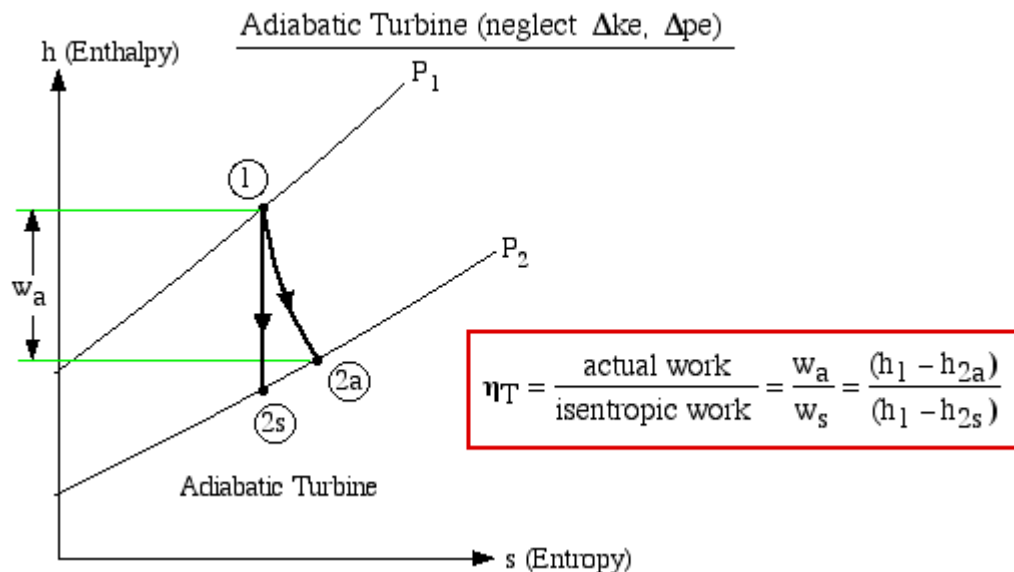
Finally we present a convenient [Entropy Equation Summary Sheet](#) which summarises the relevant relations concerning entropy generation and evaluation of entropy change Δs . The [Isentropic Processes Summary Sheet](#) extends the relations of entropy change to enable the evaluation of isentropic processes.

One of the important applications of isentropic processes is in determining the efficiency of various adiabatic components. These include turbines, compressors and aircraft jet nozzles. Thus we have made the statement that steam turbines are designed to be adiabatic, and that any heat loss from the turbine will result in a reduction in output power, however only now can we make the statement that the ideal turbine is isentropic. This enables us to evaluate the **Adiabatic Efficiency** (sometimes referred to as isentropic efficiency) of these components, and we extend the isentropic process sheet with an **Adiabatic Efficiency Summary Sheet**.

There are two property diagrams involving entropy in common usage, the temperature-entropy ($T-s$) and enthalpy-entropy ($h-s$) "Mollier" diagrams. We will find that the $h-s$ diagram is extremely useful for evaluating adiabatic turbines and compressors, and complements the $P-h$ diagram which we used in Chapter 4 to evaluate entire steam power plants or refrigerator systems. The $h-s$ diagram for steam is presented below:



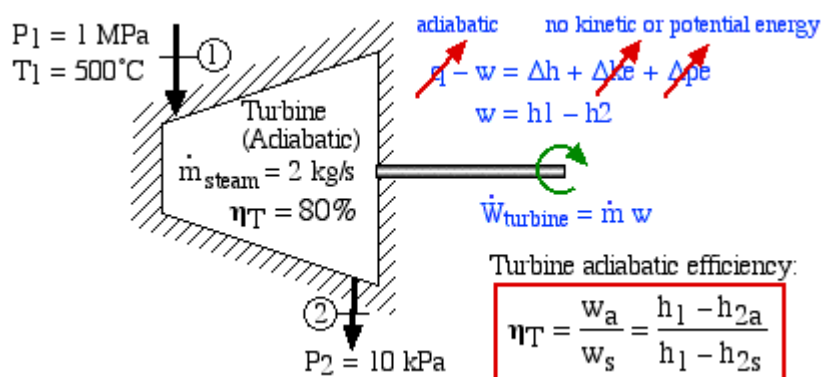
The important characteristic of the $h-s$ diagram is that the ideal adiabatic turbine can be conveniently plotted as a vertical line, allowing an intuitive visual appreciation of the turbine performance. We define the turbine adiabatic efficiency as follows:



Notice that for the actual turbine there will always be an increase in entropy, which means that the turbine adiabatic efficiency will always be less than 100%.

An Adiabatic Steam Turbine Example

Consider an adiabatic steam turbine having a turbine adiabatic efficiency $\eta_T = 80\%$, operating under the conditions shown in the following diagram:



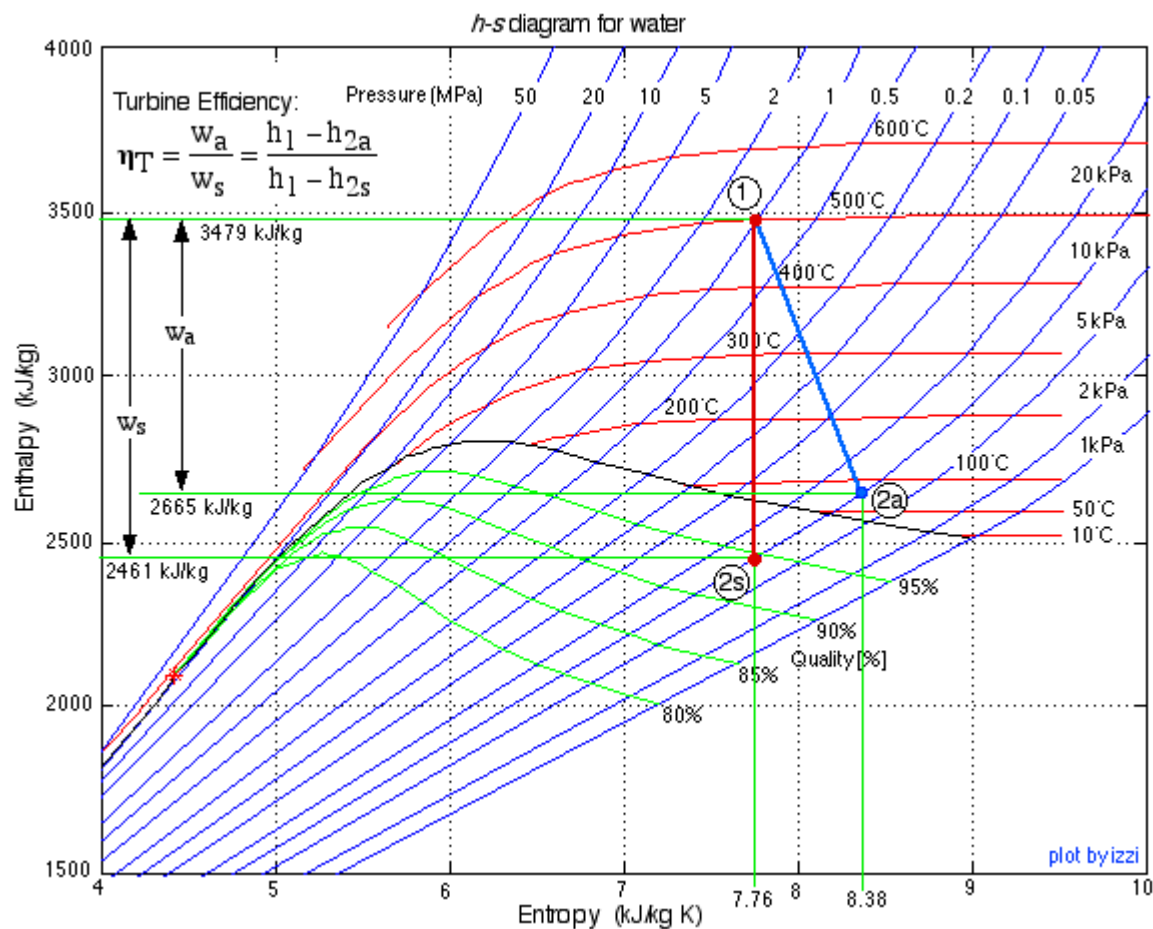
Using [steam tables](#), determine the enthalpy and entropy values at station (1) and station (2s) assuming that the turbine is isentropic. [$h_1 = 3479 \text{ kJ/kg}$, $s_1 = 7.764 \text{ kJ/kg.K}$; $h_{2s} = 2461 \text{ kJ/kg}$, $s_{2s} = s_1$]

From the definition of turbine adiabatic efficiency (shown on the diagram), and given that $\eta_T = 80\%$, determine the actual enthalpy and entropy values as well as the temperature at station (2a). [$h_{2a} = 2665 \text{ kJ/kg}$, $s_{2a} = 8.38 \text{ kJ/kg.K}$, $T_{2a} = 88^\circ\text{C}$]

Plot the actual and isentropic turbine processes (Stations (1)-(2a) and (1)-(2s)) on the enthalpy-entropy h - s "Mollier" diagram, and indicate the actual turbine specific work (w_a) as well as the isentropic turbine specific work (w_s) on the diagram.

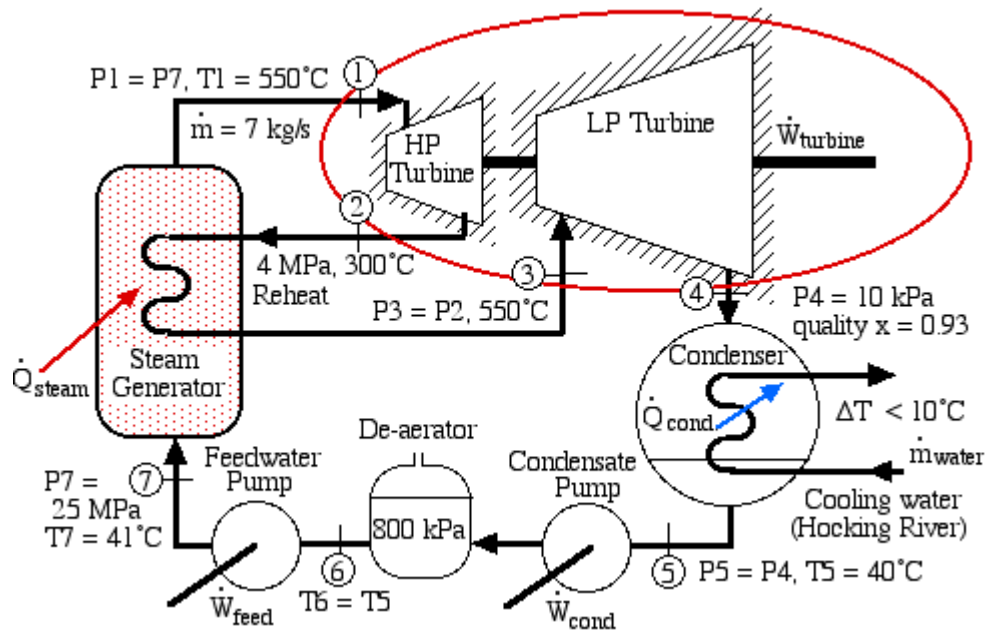
Determine the actual power output of the turbine (kW). [1629 kW]

The h - s diagram plot follows. Notice that we have indicated all the enthalpy and entropy values (which we determined from the steam tables) on the plot. This allows a check on the feasibility of our results.



Solved Problem 6.1 - A Supercritical Steam Power Plant for Athens, Ohio

Consider the supercritical steam power plant with reheat for Athens, Ohio, which we evaluated in [Solved Problem 4.1](#). The system diagram is repeated here for convenience:



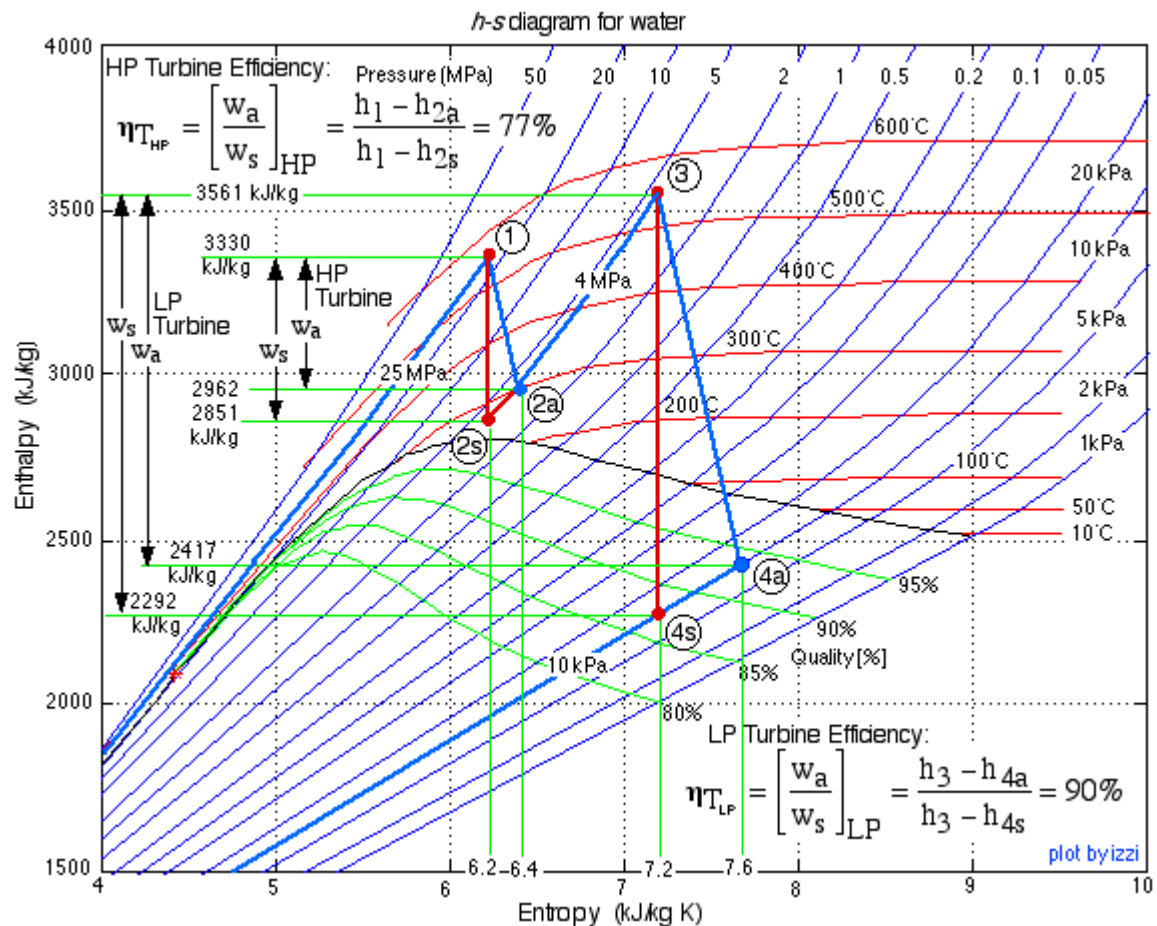
In this exercise we wish to evaluate the high pressure (HP) and low pressure (LP) turbines of this system (circled in red), both of which are assumed to be adiabatic.

- 1) Plot the two turbine processes (Stations (1)-(2) and (3)-(4)) on the [enthalpy-entropy \$h-s\$ "Mollier" diagram](#). Plot also the equivalent isentropic turbine processes on the diagram, and indicate the actual turbine specific work as well as the isentropic turbine specific work for both turbines on the $h-s$ diagram.
- 2) Using [steam tables](#), determine the turbine adiabatic efficiency η_T of both turbines.
- 3) Discuss your results as well as the feasibility of the turbine set.

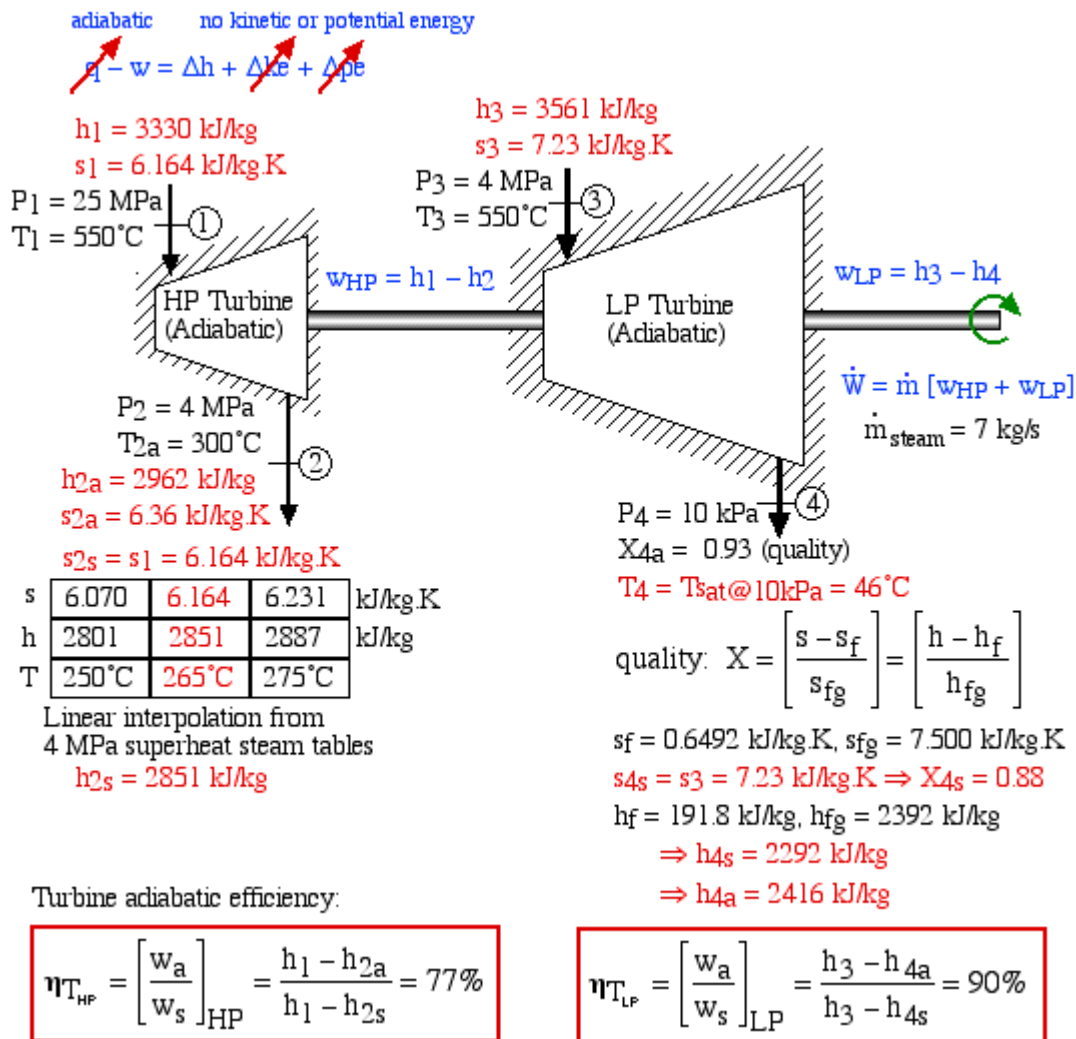
Justify all values used and *derive* all equations used starting from the basic energy equation for a flow system, the basic definition of turbine adiabatic efficiency η_T .

Solution Approach:

- 1) Plot the two turbine processes (Stations (1)-(2) and (3)-(4)) on the enthalpy-entropy $h-s$ "Mollier" diagram below. Plot also the equivalent isentropic turbine processes on the diagram, and indicate the actual turbine specific work as well as the isentropic turbine specific work for both turbines on the $h-s$ diagram. [refer $h-s$ diagram below]



2) Using [steam tables](#), determine the turbine adiabatic efficiency η_T of both turbines. [enthalpy and entropy data derived from steam tables shown in red on schematic diagram below. State (2s) required linear interpolation of the [superheat table](#) values and both states (4a) and (4s) required the [saturation properties \(pressure\) table](#) and use of the quality relation X indicated below. These values were then indicated on the above *h-s* diagram plot in order to validate their feasibility.]



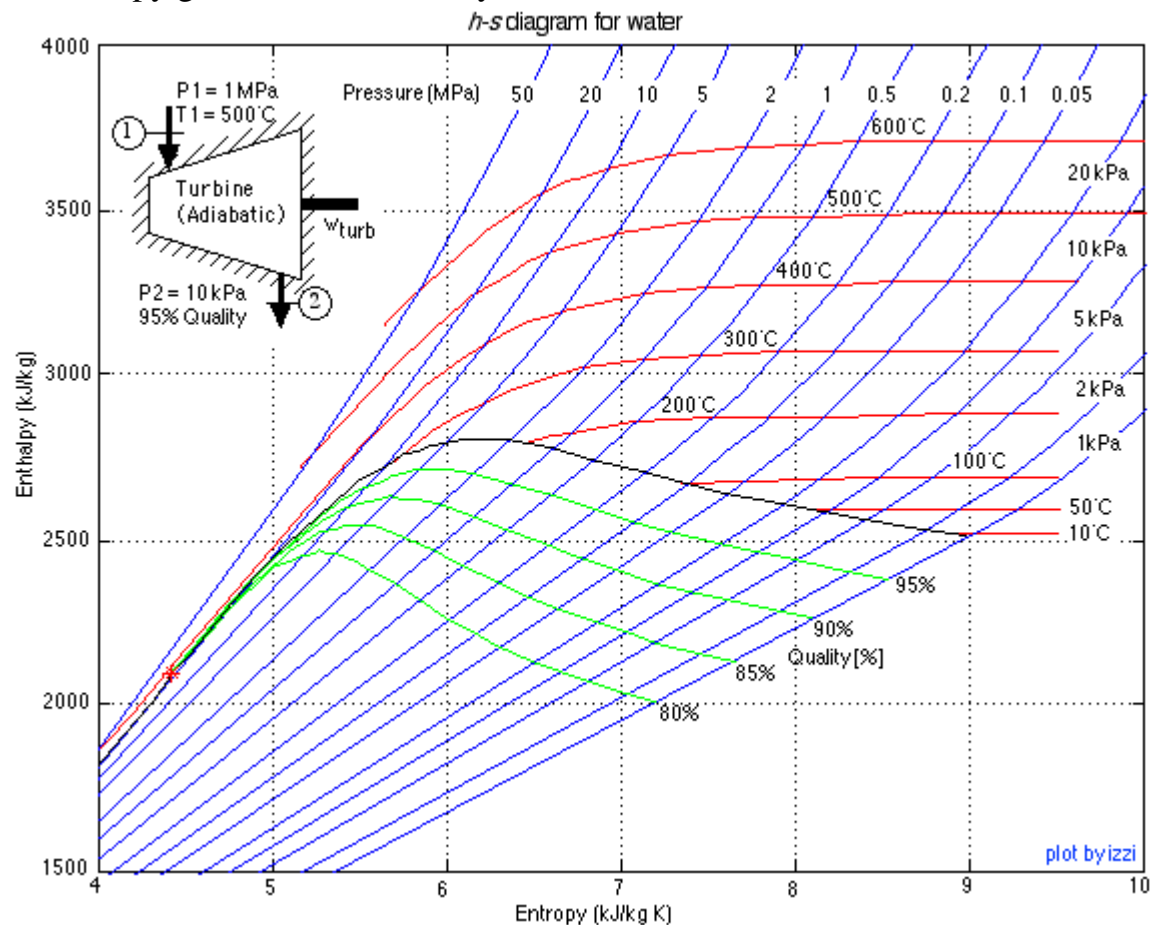
3) Discuss your results as well as the feasibility of the turbine set. [Both adiabatic efficiencies seem to be feasible and do not violate the second law. We need to question why the LP turbine efficiency (90%) is so much higher than the HP turbine efficiency (77%) - it may be due to the ability of higher relative accuracy in the manufacture of a much larger turbine however this large difference is suspect and needs more investigation.]

Problem 6.2 - Turbine Adiabatic Efficiency

Consider the adiabatic steam turbine shown below.

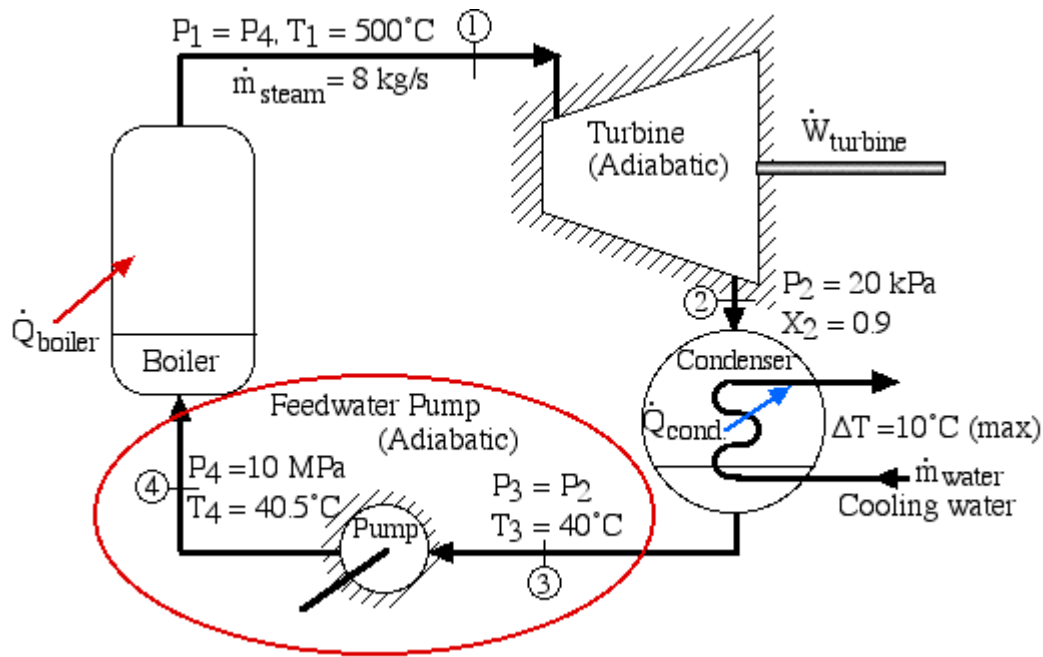
- Carefully plot the turbine process on the h-s diagram and indicate the turbine work done on the plot.
- Using steam tables determine the specific work output of the turbine [1015 kJ/kg].
- Using steam tables determine the entropy generated by this process. Assume that the surroundings temperature is at 27°C. [0.01 kJ/kg.K]

d. Discuss these results and determine if this is a feasible turbine design.
Derive all equations used starting from the basic energy equation and the equation for entropy generated in a flow system.



Problem 6.3 - Adiabatic Efficiency of a Power Plant Feedwater Pump

Recall the steam power plant of Example 4.1 in [Chapter 4b](#). Assume that the pump is adiabatic and that the temperature of the water increases by 0.5°C due to friction as it passes through the pump, as in the following figure:



Under these conditions (10 MPa, 40°C) we normally treat liquid water as an incompressible liquid. Determine: the pump adiabatic efficiency (η_p) defined as the isentropic pump work divided by the actual pump work required to drive the pump. [$\eta_p = 83\%$]

Note: In solving this problem we recall the following relations for an incompressible fluid:

enthalpy: $\Delta h = C_{H_2O} \cdot \Delta T + v \cdot \Delta P$

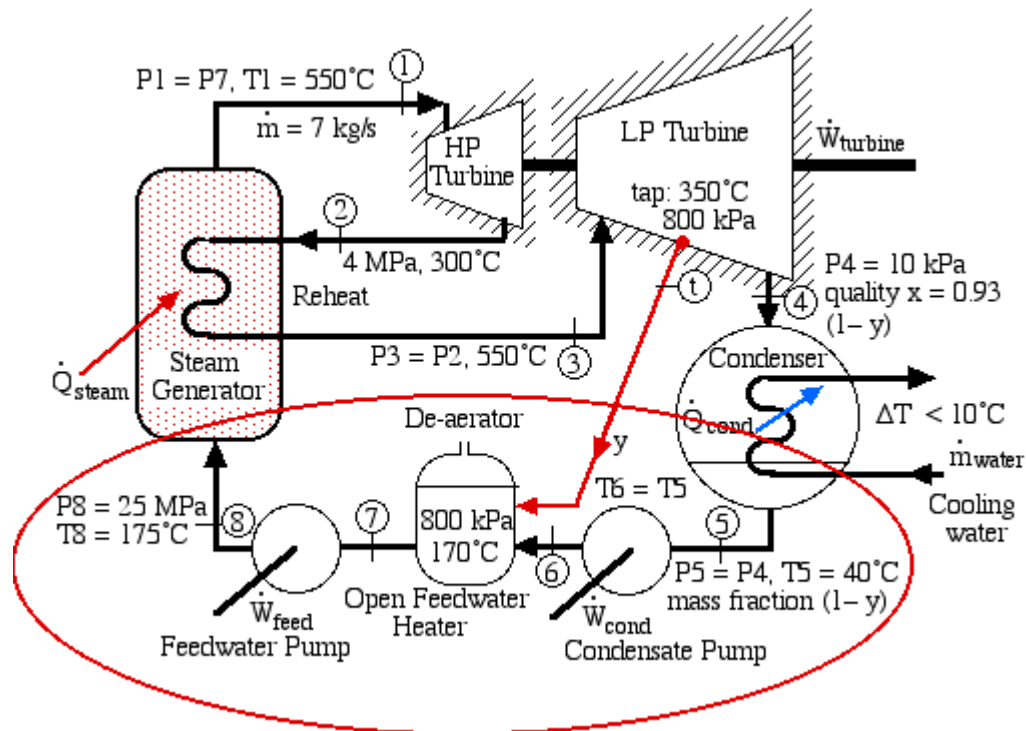
entropy: $\Delta s = C_{H_2O} \cdot \ln\left(\frac{T_2}{T_1}\right)$

where $C_{H_2O} = 4.18 \text{ kJ/kg.K}$ and specific volume $v = 0.001 \text{ m}^3/\text{kg}$.

Thus for the isentropic process: ($\Delta s = 0$) $\Rightarrow T_2 = T_1$ ($\Delta T = 0$).

Problem 6.4 - Adiabatic Efficiency of a Supercritical Power Plant Feedwater Pump

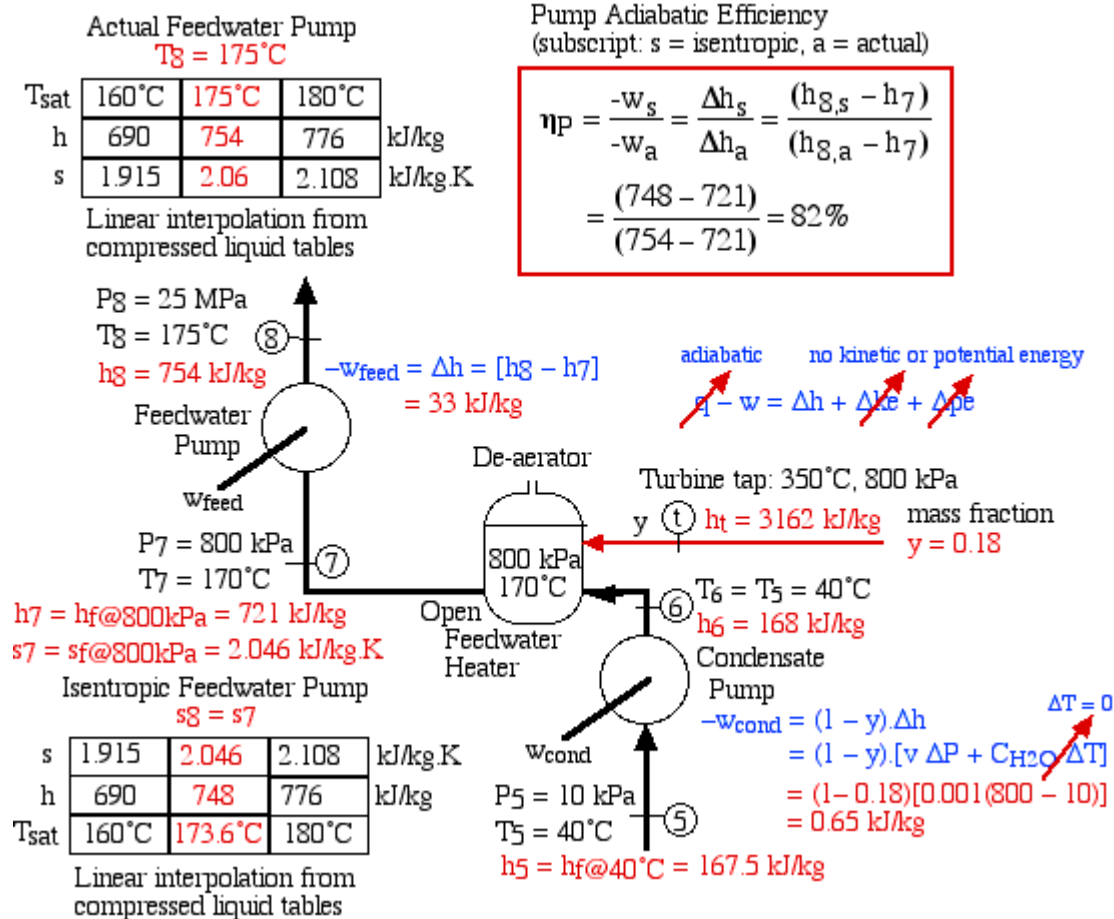
Recall the supercritical steam power plant of [Solved Problem 4.2](#) in [Chapter 4b](#) in which the Feedwater pump is required to pump the liquid from 800kPa, 170°C to 25 MPa, as in the following figure:



In this problem we wish to determine the Adiabatic Efficiency of the feedwater pump (η_p) defined as the isentropic pump work divided by the actual pump work required to drive the pump. [$\eta_p = 82\%$]

Solution Approach:

Again we are surprised to see that the feedwater pump causes the liquid to undergo a significant temperature rise of 5°C . We usually consider liquid water to be incompressible, thus pumping it to a higher pressure does not result in an increase of its temperature. However on a recent visit to the Gavin Power Plant we discovered that at 25MPa pressure and more than 100°C water is no longer incompressible, and compression will always result in a temperature increase. We cannot use the simple incompressible liquid formula to determine pump work, however need to evaluate the various values of enthalpy from the [Compressed Liquid Water](#) tables, leading to the following results:



Notice that the isentropic compression of the liquid to 25 MPa resulted in a temperature increase of 3.6°C , thus the actual temperature rise of 5°C lead to a pump adiabatic efficiency of 82%. As an exercise, assuming that the liquid water behaves as an incompressible fluid, determine the pump efficiency for the given temperature rise from 170°C to 175°C [$\eta_P = 55\%$]

Note: Recall from [Problem 6.3](#) that in solving this exercise you will need to use the following relations for an incompressible fluid:

enthalpy: $\Delta h = C_{\text{H}_2\text{O}} \cdot \Delta T + v \cdot \Delta P$

entropy: $\Delta s = C_{\text{H}_2\text{O}} \cdot \ln\left(\frac{T_2}{T_1}\right)$

where $C_{\text{H}_2\text{O}} = 4.18 \text{ kJ/kg}\cdot\text{K}$ and specific volume $v = 0.001 \text{ m}^3/\text{kg}$.

Thus for the isentropic process: $(\Delta s = 0) \Rightarrow T_2 = T_1$ ($\Delta T = 0$).

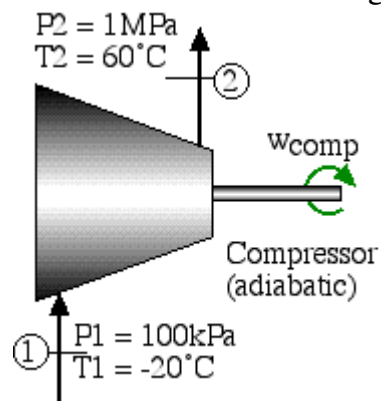
In the [Gavin Power Plant](#), each generating system has a single-stage feedwater pump driven by a steam turbine, as shown below. Its purpose is to pump the condensed saturated-liquid water at 800 kPa, 170°C to a high pressure of 30 MPa at the inlet to the boiler. The pump is powered by a dedicated 48.49 MW (65,000 hp) turbine. On a recent visit to the Gavin Power Plant we were informed that the liquid water will normally experience a temperature rise of around 5°C during this

process. The inlet pressure to the steam generator is 30 MPa, however the water has to pass through 350 miles of piping and undergoes a 5 MPa pressure drop while being heated to 550°C, thus the inlet pressure to the high pressure turbine is 25 MPa.



Solved Problem 6.5 - Adiabatic Efficiency of a R134a Compressor

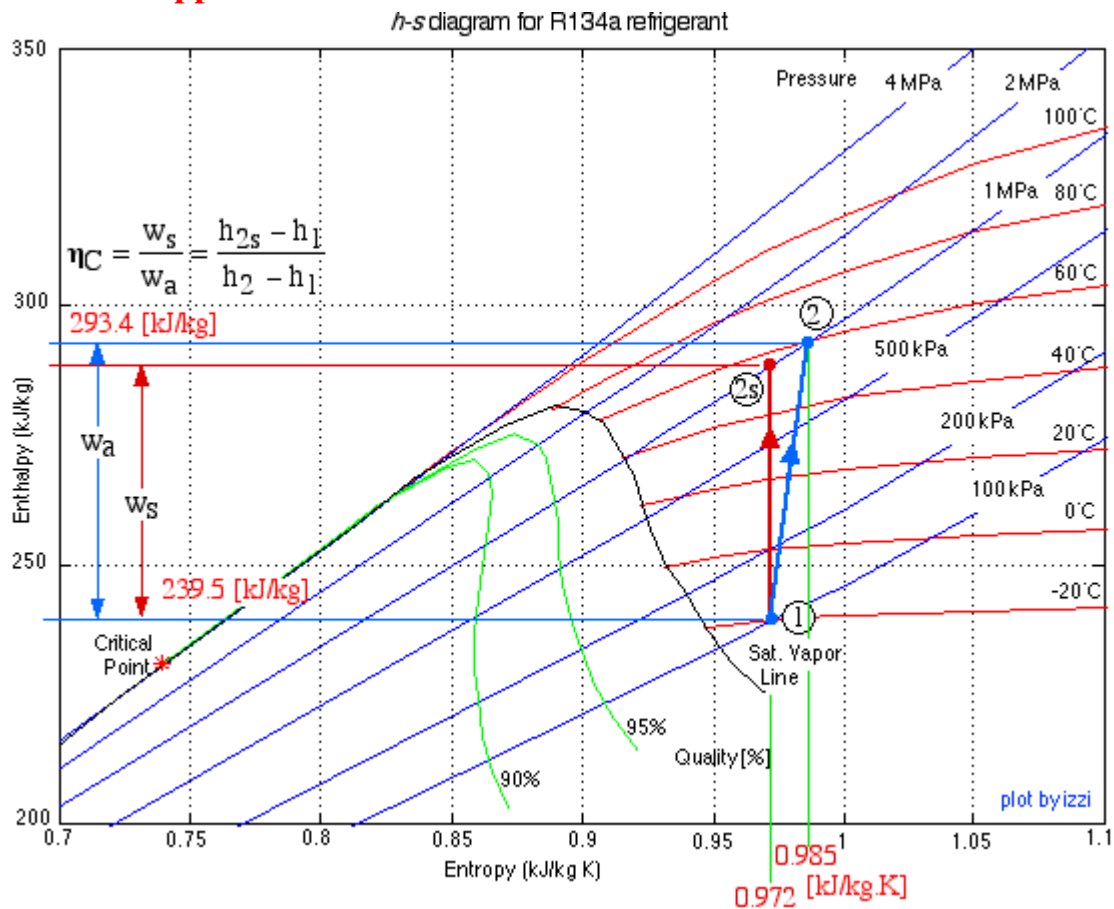
Consider the R134a refrigerator compressor shown below.

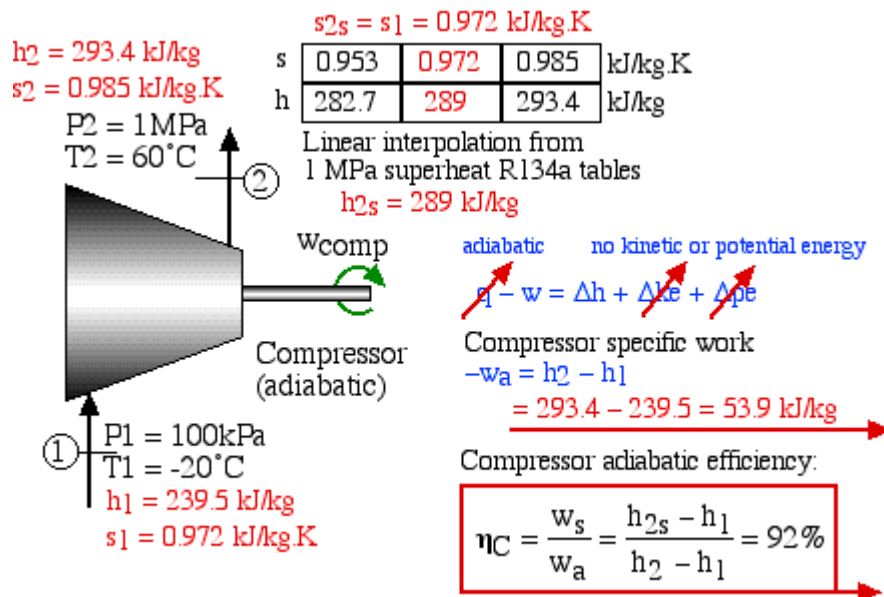


- Carefully plot the actual and isentropic compression processes on the h - s diagram and indicate the actual and isentropic specific work done to drive the compressor on the plot.
- Using R134a refrigerant tables determine the specific work required to drive the compressor [53.9 kJ/kg].
- Using R134a refrigerant tables determine the adiabatic efficiency of the compressor [$\eta_c = 92\%$].
- Discuss these results and determine if this is a feasible compressor design.

Derive all equations used starting from the basic energy equation for a flow system and the equation for adiabatic efficiency of a compressor.

Solution Approach:





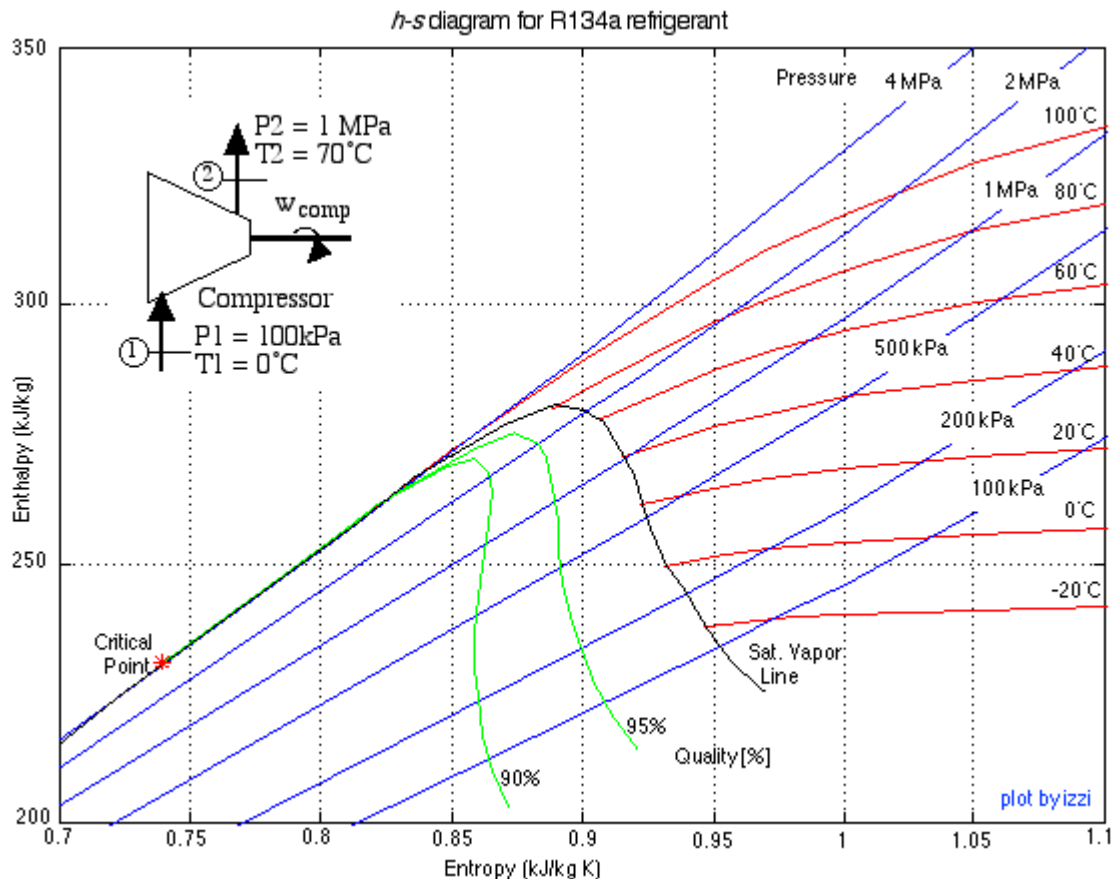
Discussion: This is a feasible compressor design - the adiabatic efficiency is higher than we expected however within an acceptable range (less than around 95%),

Problem 6.6 - Adiabatic Evaluation of a R134a Compressor

A young engineer was assigned to evaluate the compressor of a proposed R-134a refrigeration system shown below, and assumed it to be adiabatic.

- Under this assumption, and ignoring kinetic and potential energy effects, determine the specific work input to the compressor [48.3 kJ/kg].
- His supervisor checked the results and told him that his assumption was incorrect and not physically feasible. On the h - s diagram below draw the actual compression process (as assumed) as well as the equivalent isentropic compression process. Using this diagram, as well as relevant data evaluated at the inlet and outlet of the compressor, explain how she arrived at that conclusion.

Derive all equations used starting from the basic energy equation and the equation for entropy generated.



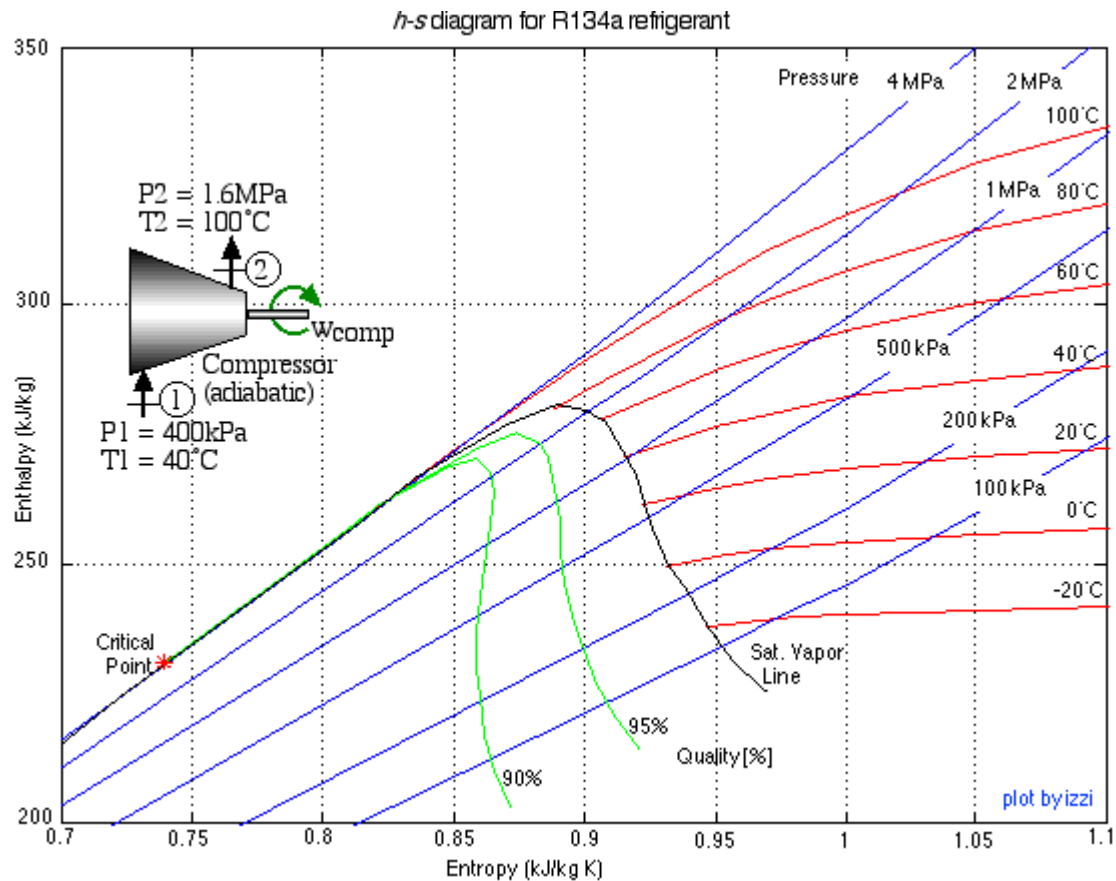
Problem 6.7 - Recall in [Chapter 4c](#) that we provided [Problem 4.7](#) concerning a home refrigerator, and examining its performance before and after adding an internal heat exchanger.

- o a) Plot the actual and the isentropic compressor processes on the [enthalpy-entropy \(\$h-s\$ \) diagram](#) provided, for both cases - with and without the internal heat exchanger.
- o b) Using the R134a tables determine the actual compressor adiabatic efficiency (η_c) for both cases [75%, 76%]

Problem 6.8 - Recall in [Chapter 4c](#) that we provided [Problem 4.9](#) concerning an innovative home air conditioner and hot water heating system, in which we determined the COP for both the air conditioning and the water heating systems. Do sections a) and b) specified in Problem 6.7 above on the compressor of this home air conditioning system. [$\eta_c = 76\%$]

Problem 6.9 - Adiabatic Efficiency of a High Pressure R134a Compressor

Recall [Problem 4.12](#), in which we evaluated a R134a heat pump system using a compressor under the conditions shown below.



- Carefully plot the actual and isentropic compression processes on the *h-s* diagram and indicate the actual and isentropic specific work done to drive the compressor on the plot.
- Using R134a refrigerant tables determine the specific work required to drive the compressor [43.5 kJ/kg].
- Using R134a refrigerant tables determine the adiabatic efficiency of the compressor [$\eta_c = 78\%$].
- Discuss these results and determine if this is a feasible compressor design.

Derive all equations used starting from the basic energy equation for a flow system and the equation for adiabatic efficiency of a compressor.