

MÔN HỌC

KINH TẾ XÂY DỰNG (KC269)

GIÁO VIÊN PHỤ TRÁCH

ĐẶNG THẾ GIA

Bộ môn Kỹ Thuật Xây Dựng
Khoa Công Nghệ, Trường Đại Học Cần Thơ

Chương 2:

Thời gian & Lãi suất *Time & Interest*

Phần 1

NỘI DUNG

1. Hệ số F/P & P/F
2. Hệ số P/A & A/P
3. Hệ số F/A & A/F
4. Nội suy

2-3

Ký hiệu tiêu chuẩn ANSI cho các hệ số (ANSI: American National Standards Institute)

- Ký hiệu tiêu chuẩn đã được áp dụng để đại diện cho các hệ số lãi suất khác nhau
- Gồm hai biểu tượng dòng tiền: lãi suất và số khoảng thời gian
- Dạng chung: $(X/Y, i\%, n)$
 - ❖ X đại diện cho giá trị chưa biết
 - ❖ Y đại diện cho giá trị đã biết
 - ❖ i và n đại diện cho các thông số đầu vào; có thể được biết hoặc chưa biết phụ thuộc vào bài toán

2-4

Ký hiệu – tiếp theo

- Ví dụ: (F/P,6%,20) được đọc là:
 - Tìm F, biết P khi lãi suất là 6% và số thời đoạn là 20.
- Trong các công thức, ký hiệu tiêu chuẩn thường được dùng thay cho các phương trình.
- Các bảng lập sẵn cung cấp các giá trị thông dụng của $i\%$ & n .

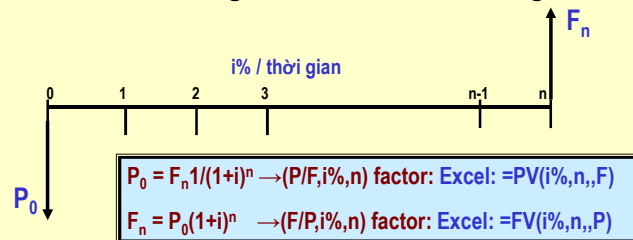
2-5

Hệ số (thanh toán) đơn F/P & P/F

Single-Amount/Single-Payment Factors

Khái niệm

- Mục tiêu:
 - Xác định giá trị hiện tại hoặc giá trị tương lai của dòng tiền
- Lược đồ dòng tiền mặt - định dạng cơ bản



2-7

Thành lập công thức

$$F_1 = P + Pi$$

$$= P(1 + i)$$

$$F_3 = F_2 + F_2 i$$

$$F_3 = P(1 + i)^3$$

$$F_2 = F_1 + F_1 i$$

$$= P(1 + i) + P(1 + i)i$$

$$F_2 = P(1 + i + i + i^2)$$

$$= P(1 + 2i + i^2)$$

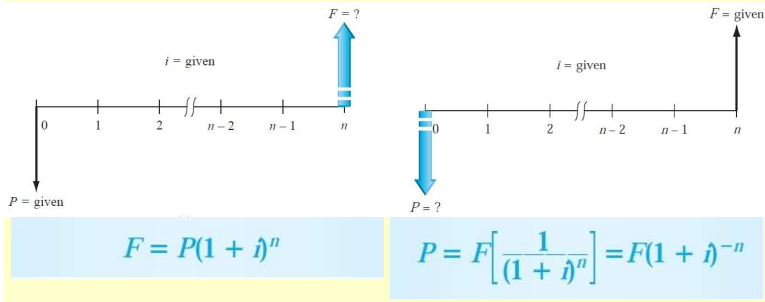
$$= P(1 + i)^2$$

$$F = P(1 + i)^n$$

$$P = F \left[\frac{1}{(1 + i)^n} \right] = F(1 + i)^{-n}$$

2-8

Dạng bài toán



2-9

Ví dụ

$$F = P(1 + i)^n$$

$$P = F\left[\frac{1}{(1 + i)^n}\right] = F(1 + i)^{-n}$$

Tìm hệ số P/F

$$(P/F, 5\%, 10) = \frac{1}{(1 + i)^n}$$

$$= \frac{1}{(1.05)^{10}}$$

$$= \frac{1}{1.6289} = 0.6139$$

2-10

EXAMPLE 2.1

Sandy, a manufacturing engineer, just received a year-end bonus of \$10,000 that will be invested immediately. With the expectation of earning at the rate of 8% per year, Sandy hopes to take the entire amount out in exactly 20 years to pay for a family vacation when the oldest daughter is due to graduate from college. Find the amount of funds that will be available in 20 years by using (a) hand solution by applying the factor formula and tabulated value and (b) a spreadsheet function.

EXAMPLE 2.2 The Cement Factory Case

As discussed in the introduction to this chapter, the Houston American Cement factory will require an investment of \$200 million to construct. Delays beyond the anticipated implementation year of 2012 will require additional money to construct the factory. Assuming that the cost of money is 10% per year, compound interest, use both **tabulated factor values** and **spreadsheet functions** to determine the following for the board of directors of the Brazilian company that plans to develop the plant.

- (a) The equivalent investment needed if the plant is built in 2015.
- (b) The equivalent investment needed had the plant been constructed in the year 2008.

Solution

Figure 2-2 is a cash flow diagram showing the expected investment of \$200 million (\$200 M) in 2012, which we will identify as time $t = 0$. The required investments 3 years in the future and 4 years in the past are indicated by $F_3 = ?$ and $P_4 = ?$, respectively.

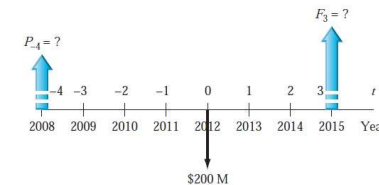


Figure 2-2
Cash flow diagram for
Example 2.2a and b.

Ví dụ

(b) The year 2008 is 4 years prior to the planned construction date of 2012. To determine the equivalent cost 4 years earlier, consider the \$200 M in 2012 ($t = 0$) as the future value F and apply the P/F factor for $n = 4$ to find P_{-4} . (Refer to Figure 2-2.) Table 15 supplies the tabulated value.

$$P_{-4} = F(P/F, i, n) = 200(P/F, 10\%, 4) = 200(0.6830) \\ = \$136.6 \quad (\$136,600,000)$$

This equivalence analysis indicates that at \$136.6 M in 2008, the plant would have cost about 68% as much as in 2012, and that waiting until 2015 will cause the price tag to increase about 33% to \$266 M.

2-13

Tóm tắt

TABLE 2-1 F/P and P/F Factors: Notation and Equations

Notation	Factor Name	Find/Given	Standard Notation Equation	Equation with Factor	Excel Function
$(F/P, i, n)$	Single-payment compound amount	F/P	$F = P(F/P, i, n)$	$F = P(1 + i)^n$	$= FV(i\%, n, P)$
$(P/F, i, n)$	Single-payment present worth	P/F	$P = F(P/F, i, n)$	$P = F(1 + i)^{-n}$	$= PV(i\%, n, F)$

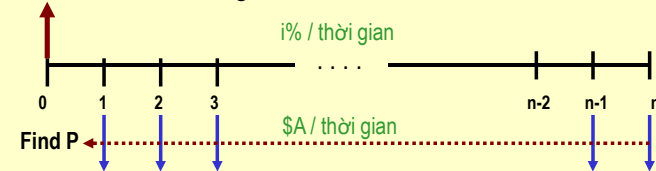
2-14

Hệ số giá trị hiện tại của chuỗi đều (P/A)
& Hệ số thu hồi vốn (A/P)

Uniform Series Present Worth Factor (P/A)
& Capital Recovery Factor (A/P)

Khái niệm

□ Cấu hình dòng tiền mặt của hệ số P/A



Yêu cầu: Tìm P, biết A

Dòng tiền mặt bằng nhau, không bị gián đoạn và trải dài đến cuối thời đoạn tính lãi suất

2-16

Thành lập công thức

- Ta có phương trình:

$$P = A \left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n} \right] \quad (1)$$

- Nhân hai vế cho $\frac{1}{(1+i)}$

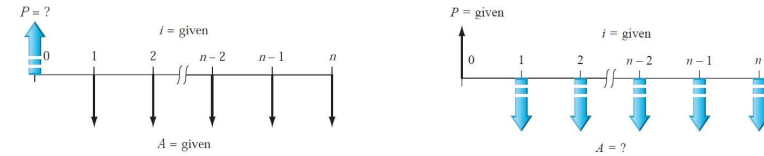
$$\frac{P}{1+i} = A \left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}} \right] \quad (2)$$

- Lấy (2) trừ (1)

$$\frac{-i}{1+i} P = A \left[\frac{1}{(1+i)^{n+1}} - \frac{1}{(1+i)} \right] \quad (3)$$

2-17

Dạng bài toán



$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \quad i \neq 0$$

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

2-18

Ví dụ

EXAMPLE 2.3

How much money should you be willing to pay now for a guaranteed \$600 per year for 9 years starting next year, at a rate of return of 16% per year?

Solution

The cash flows follow the pattern of Figure 2-4a, with $A = \$600$, $i = 16\%$, and $n = 9$. The present worth is

$$P = 600(P/A, 16\%, 9) = 600(4.6065) = \$2763.90$$

The PV function = PV(16%,9,600) entered into a single spreadsheet cell will display the answer $P = (\$2763.93)$.

2-19

Ví dụ

EXAMPLE 2.4 The Cement Factory Case

As mentioned in the chapter introduction of this case, the Houston American Cement plant may generate a revenue base of \$50 million per year. The president of the Brazilian parent company Votorantim Cimentos may have reason to be quite pleased with this projection for the simple reason that over the 5-year planning horizon, the expected revenue would total \$250 million, which is \$50 million more than the initial investment. With money worth 10% per year, address the following question from the president: Will the initial investment be recovered over the 5-year horizon with the time value of money considered? If so, by how much extra in present worth funds? If not, what is the equivalent annual revenue base required for the recovery plus the 10% return on money? Use both tabulated factor values and spreadsheet functions.

2-20

Ví dụ

Solution

Tabulated value: Use the P/A factor to determine whether $A = \$50$ million per year for $n = 5$ years starting 1 year after the plant's completion ($t = 0$) at $i = 10\%$ per year is equivalently less or greater than $\$200$ M. The cash flow diagram is similar to Figure 2-4a, where the first A value occurs 1 year after P . Using $\$1$ million units and Table 15 values,

$$P = 50(P/A, 10\%, 5) = 50(3.7908) \\ = \$189.54 \quad (\$189,540,000)$$

The present worth value is less than the investment plus a 10% per year return, so the president should not be satisfied with the projected annual revenue.

To determine the minimum required to realize a 10% per year return, use the A/P factor. The cash flow diagram is the same as Figure 2-4b, where A starts 1 year after P at $t = 0$ and $n = 5$.

$$A = 200(A/P, 10\%, 5) = 200(0.26380) \\ = \$52.76 \text{ per year}$$

The plant needs to generate $\$52,760,000$ per year to realize a 10% per year return over 5 years.

2-21

Tóm tắt

TABLE 2-2 P/A and A/P Factors: Notation and Equations

Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Function
$(P/A, i, n)$	Uniform series present worth	P/A	$\frac{(1 + i)^n - 1}{i(1 + i)^n}$	$P = A(P/A, i, n)$	$= PV(i\%, n, A)$
$(A/P, i, n)$	Capital recovery	A/P	$\frac{i(1 + i)^n}{(1 + i)^n - 1}$	$A = P(A/P, i, n)$	$= PMT(i\%, n, P)$

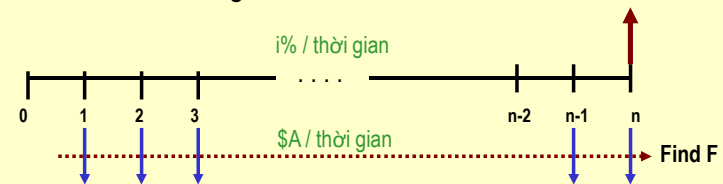
2-22

Hệ số giảm A/F &
Hệ số phức của chuỗi đều F/A

Sinking Fund Factor (A/F) &
Uniform Series Compound Amount Factor (F/A)

Khái niệm

□ Cấu hình dòng tiền mặt của hệ số F/A



Yêu cầu: Tìm F , biết A

Dòng tiền mặt bằng nhau, không bị gián đoạn và trải dài đến cuối thời đoạn tính lãi suất

2-24

Thành lập công thức

$$P = F \left[\frac{1}{(1+i)^n} \right] = F(1+i)^{-n}$$

$$A = F \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

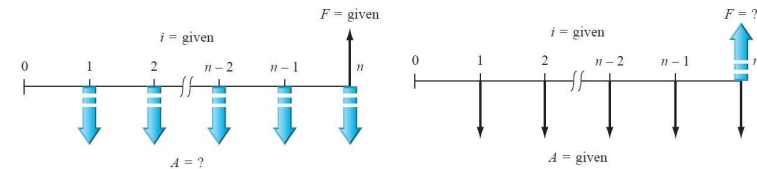
$$A = F \left[\frac{1}{(1+i)^n} \right] \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

2-25

Dạng bài toán



$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

2-26

Ví dụ

EXAMPLE 2.5

The president of Ford Motor Company wants to know the equivalent future worth of a \$1 million capital investment each year for 8 years, starting 1 year from now. Ford capital earns at a rate of 14% per year.

Solution

The cash flow diagram (Figure 2-7) shows the annual investments starting at the end of year 1 and ending in the year the future worth is desired. In \$1000 units, the F value in year 8 is found by using the F/A factor.

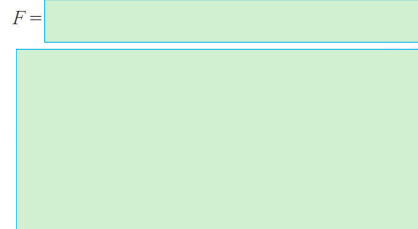


Figure 2-7

Diagram to find F for a uniform series, Example 2.5.

Ví dụ

EXAMPLE 2.6 The Cement Factory Case

Once again, consider the HAC case presented at the outset of this chapter, in which a projected \$200 million investment can generate \$50 million per year in revenue for 5 years starting 1 year after start-up. A 10% per year time value of money has been used previously to determine P , F , and A values. Now the president would like the answers to a couple of new questions about the estimated annual revenues. Use tabulated values, factor formulas, or spreadsheet functions to provide the answers.

- What is the equivalent future worth of the estimated revenues after 5 years at 10% per year?
- Assume that, due to the economic downturn, the president predicts that the corporation will earn only 4.5% per year on its money, not the previously anticipated 10% per year. What is the required amount of the annual revenue series over the 5-year period to be economically equivalent to the amount calculated in (a)?

Solution

- Figure 2-6b is the cash flow diagram with $A = \$50$ million. Note that the last A value and $F = ?$ both occur at the end of year $n = 5$. We use tabulated values and the spreadsheet function to find F in year 5.

Tabulated value: Use the F/A factor and 10% interest factor table. In \$1 million units, the future worth of the revenue series is

$$\begin{aligned} F &= 50(F/A, 10\%, 5) = 50(6.1051) \\ &= \$305.255 \quad (\$305,255,000) \end{aligned}$$

Ví dụ

If the rate of return on the annual revenues were 0%, the total amount after 5 years would be \$250,000,000. The 10% per year return is projected to grow this value by 22%.

- (b) The president of the Brazilian company planning to develop the cement plant in Georgia is getting worried about the international economy. He wants the revenue stream to generate the equivalent that it would at a 10% per year return, that is, \$305.255 million, but thinks that only a 4.5% per year return is achievable.

Factor formula: The A/F factor will determine the required A for 5 years. Since the factor tables do not include 4.5%, use the formula to answer the question. In \$1 million units,

$$A = 305.255(A/F, 4.5\%, 5) = 305.255 \left[\frac{0.045}{(1.045)^5 - 1} \right] = 305.255(0.18279) = \$55.798$$

The annual revenue requirement grows from \$50 million to nearly \$55,800,000. This is a significant increase of 11.6% each year.

2-29

Tóm tắt

TABLE 2-3 F/A and A/F Factors: Notation and Equations

Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Functions
$(F/A, i, n)$	Uniform series compound amount	F/A	$\frac{(1+i)^n - 1}{i}$	$F = A(F/A, i, n)$	$= FV(i\%, n, A)$
$(A/F, i, n)$	Sinking fund	A/F	$\frac{i}{(1+i)^n - 1}$	$A = F(A/F, i, n)$	$= PMT(i\%, n, F)$

Factor	To Do This	Excel Function
P/F	Find P , given F	$= -PV(i\%, n, 1)$
F/P	Find F , given P	$= -FV(i\%, n, 1)$
P/A	Find P , given A	$= -PV(i\%, n, 1)$
A/P	Find A , given P	$= -PMT(i\%, n, 1)$
F/A	Find F , given A	$= -FV(i\%, n, 1)$
A/F	Find A , given F	$= -PMT(i\%, n, 1)$

2-30

Nội suy

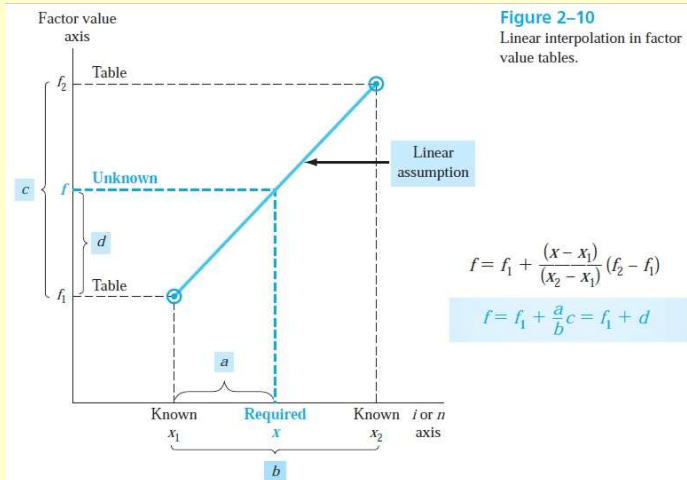
Interpolation

Giới thiệu

- Khi sử dụng các bảng tra lãi suất, chúng ta thường phải lấy gần đúng một giá trị không có trong bảng
- Có thể dùng nội suy tuyến tính để tính gần đúng
 - Các giá trị trong bảng thuộc hàm phi tuyến, do vậy nội suy tuyến tính thường cho sai số khoảng 2-4%
 - Dùng bảng tính mẫu để tính chính xác các giá trị

2-32

Nội suy tuyến tính



EXAMPLE 2.7

Determine the P/A factor value for $i = 7.75\%$ and $n = 10$ years, using the three methods described previously.

Solution

Factor formula: Apply the formula from inside the front cover of the book for the P/A factor. Showing 5-decimal accuracy,

$$(P/A, 7.75\%, 10) = \frac{(1 + i)^n - 1}{i(1 + i)^n} = \frac{(1.0775)^{10} - 1}{0.0775(1.0775)^{10}} = \frac{1.10947}{0.16348} = 6.78641$$

Spreadsheet: Utilize the spreadsheet function in Figure 2-9, that is, $= -PV(7.75\%, 10, 1)$, to display 6.78641.

Linear interpolation: Use Figure 2-10 as a reference for this solution. Apply the Equation [2.16] and [2.17] sequence, where x is the interest rate i , the bounding interest rates are $i_1 = 7\%$ and $i_2 = 8\%$, and the corresponding P/A factor values are $f_1 = (P/A, 7\%, 10) = 7.0236$ and $f_2 = (P/A, 8\%, 10) = 6.7101$. With 4-place accuracy,

$$\begin{aligned} f &= f_1 + \frac{(i - i_1)}{(i_2 - i_1)} (f_2 - f_1) = 7.0236 + \frac{(7.75 - 7)}{(8 - 7)} (6.7101 - 7.0236) \\ &= 7.0236 + (0.75)(-0.3135) = 7.0236 - 0.2351 \\ &= 6.7885 \end{aligned}$$

Comment

Note that since the P/A factor value decreases as i increases, the linear adjustment is negative at -0.2351 . As is apparent, linear interpolation provides an approximation to the correct factor value for 7.75% and 10 years, plus it takes more calculations than using the formula or spreadsheet function. It is possible to perform two-way linear interpolation for untabulated i and n values; however, the use of a spreadsheet or factor formula is recommended.

