GENERATION OF MAXIMALLY ENTANGLED STATES IN PUMPED NONLINEAR COUPLERS INDUCED BY BROADBAND LASER LIGHT

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Abstract: *In this paper, model of optical state truncation of two cavity modes are analysed. The Kerr nonlinear coupler with two of them pumped by external classical fields, which is assumed to be decomposed into two parts: a coherent part and a white noise. We can see that the quantum evolution of the pumped couplers is closed in a Hilbert space of two-qubit spanned by single-photon and vacuum states only. Hence, the pumped couplers can treat as a system of two-qubit. Analysis of time evolution of the quantum entanglement shows that maximally entangled states can be generated and compare these results with that obtained previously in the literature.*

Keywords: Kerr nonlinear coupler, Quantum entanglement, Bell states, White noise.

1. INTRODUCTION

An important research area in quantum optics is methods for manipulation of nonclassical states of light, especially in relation to possible optical implementations of systems for quantum communication and quantum computers and quantum cryptography [1]. Among the various schemes for the generation of optical-qubit, the device of quantum scissors of Pegg *et al*. [2] produces a superposition of single-photon and vacuum states, by optical-state truncation of an input single-mode coherent light. The quantum scissors device was considered in numerous papers [3–8]. In all above-mentioned schemes are restricted to the single-mode optical truncation and have supposed that the laser light is perfectly monochrome. Nevertheless, a real laser is never completely monochrome and it is usually modeled by Gaussian process. Furthermore, exactly analytical averaging of stochastic equations with Gaussian process is a not easy task. Nearly only the extreme case of white noise has been well studied. Even for the case of white noise, we can achieve some interesting results [9-12]. Here we should expand the formalism given in [13] to the more realistic case, when the laser width should be taken into account. By generalising former scheme in [14], we display a realization of nonlinear quantum scissors for optical-state truncation of two cavity modes by means of a pumped nonlinear coupler. We analyze Kerr-like nonlinear couplers that can be modelled by systems composed of two quantum nonlinear oscillators linearly coupled to each other and these oscillators are excited by the external fields, which are assumed to be decomposed into two parts: a coherent part and white noise. We consider scheme based on the coupler with an external excitation of the coupler with two modes pumped. We demonstrate that the states generated in the excited nonlinear couplers under appropriate conditions can be limited to a superposition of only single-photon and vacuum states. We compare the possibilities of generation of Bell states by the couplers excited in two modes when chaotic parameter is present and absent.

2. COUPLER PUMPED IN TWO MODES

This section is devoted to the general scheme of Kerr-like nonlinear coupler, which includes two nonlinear oscillators linearly coupled to each other and linearly coupled to external excitations. We suppose here that both modes of the coupler are excited by external classical fields, whereas for the case considered previously we supposed that only one of the modes was coupled to the external classical field [12]. The Hamiltonian describing such system is of the form

$$\hat{H} = \omega_a \hat{a}^+ \hat{a} + \omega_b \hat{b}^+ \hat{b} + \frac{\chi_a}{2}\left(\hat{a}^+\right)^2 \hat{a}^2 + \frac{\chi_b}{2}\left(\hat{b}^+\right)^2 \hat{b}^2 + \varepsilon \hat{a}^+ \hat{b} + \varepsilon^* \hat{a} \hat{b}^+ + \alpha \hat{a}^+ + \alpha^* \hat{a} + \beta \hat{b}^+ + \beta^* \hat{b} \text{ , (1)}$$

where $\hat{a}\left(\hat{b}\right)$ and $\hat{a}^+\left(\hat{b}\right)^+$ are boson annihilation and creation operators, respectively; the parameters χ_a and χ_b are constants of the nonlinearity of the oscillators a and b, respectively; ε is the strength of the oscillator-oscillator coupling; α and β are the strength of the external excitation of the oscillators a and b, respectively.

We assume that amplitude of the external classical field includes two parts: a deterministic coherent part and a randomly fluctuating chaotic part (white noise) and shall restrict ourselves to the case of real $\varepsilon = \alpha = \beta$. In addition, we also presume that for the time $t = 0$, two oscillators are in vacuum states, so we can write the wave function of our model as

$$\left|\psi^{(00)}(t)\right\rangle = c_{00}(t)\left|0\right\rangle_a\left|0\right\rangle_b + c_{10}(t)\left|1\right\rangle_a\left|0\right\rangle_b + c_{01}(t)\left|0\right\rangle_a\left|1\right\rangle_b + c_{11}(t)\left|1\right\rangle_a\left|1\right\rangle_b . \qquad (2)$$

Hence, we obtain the solutions of stochastic averages equations of the variables for the probability amplitudes $c_{mn}(t)$ $(\mathrm{m,n} = 0,1)$ as (we shall focus here only on physical aspects of the problem because the volume of this paper is limited, whereas the mathematical details of this procedure will be discussed elsewhere [15]):

$$\begin{aligned}
c_{00}(t) &= -\frac{1}{2} e^{-\frac{i(3a_0-2\alpha_0)t}{4}} \left[\frac{i(a_0+2\alpha_0)}{\theta_1}\sin\left(\frac{\theta_1 t}{4}\right) - \cos\left(\frac{\theta_1 t}{4}\right)\right] + \frac{1}{2} e^{-\frac{i(7a_0+2\alpha_0)t}{4}} \left[\frac{i(3a_0+2\alpha_0)}{\theta_2}\sin\left(\frac{\theta_2 t}{4}\right) + \cos\left(\frac{\theta_2 t}{4}\right)\right], \\
c_{01}(t) &= -i\left[\frac{a_0}{\theta_1} e^{-\frac{i(3a_0-2\alpha_0)t}{4}} \sin\left(\frac{\theta_1 t}{4}\right) + \frac{(a_0+4\alpha_0)}{\theta_2} e^{-\frac{i(7a_0+2\alpha_0)t}{4}} \sin\left(\frac{\theta_2 t}{4}\right)\right], \\
c_{10}(t) &= i\left[\frac{a_0}{\theta_1} e^{-\frac{i(3a_0-2\alpha_0)t}{4}} \sin\left(\frac{\theta_1 t}{4}\right) - \frac{(a_0+4\alpha_0)}{\theta_2} e^{-\frac{i(7a_0+2\alpha_0)t}{4}} \sin\left(\frac{\theta_2 t}{4}\right)\right], \\
c_{11}(t) &= \frac{1}{2} e^{-\frac{i(3a_0-2\alpha_0)t}{4}} \left[\frac{i(a_0+2\alpha_0)}{\theta_1}\sin\left(\frac{\theta_1 t}{4}\right) - \cos\left(\frac{\theta_1 t}{4}\right)\right] + \frac{1}{2} e^{-\frac{i(7a_0+2\alpha_0)t}{4}} \left[\frac{i(3a_0+2\alpha_0)}{\theta_2}\sin\left(\frac{\theta_2 t}{4}\right) + \cos\left(\frac{\theta_2 t}{4}\right)\right].
\end{aligned} \qquad (3)$$

Where: α_0 is a coherent component of the external classical field, a_0 is chaotic parameter, $\theta_1 = \sqrt{5a_0^2 + 4a_0\alpha_0 + 4\alpha_0^2}$, and $\theta_2 = \sqrt{13a_0^2 + 44a_0\alpha_0 + 68\alpha_0^2}$.

We are going to express the derived wave function in the Bell basis

$$\left|\psi\right\rangle = b_1\left|B_1\right\rangle + b_2\left|B_2\right\rangle + b_3\left|B_3\right\rangle + b_4\left|B_4\right\rangle, \qquad (4)$$

Where: $\left|B_i\right\rangle$, $i = 1,2,3,4$ are Bell-like states, which can be expressed as functions of the n-photon states discussed here:

$$\left|B_1\right\rangle = \frac{\left|11\right\rangle + i\left|00\right\rangle}{\sqrt{2}},\ \left|B_2\right\rangle = \frac{\left|00\right\rangle + i\left|11\right\rangle}{\sqrt{2}},\ \left|B_3\right\rangle = \frac{\left|01\right\rangle - i\left|10\right\rangle}{\sqrt{2}},\ \left|B_4\right\rangle = \frac{\left|10\right\rangle - i\left|01\right\rangle}{\sqrt{2}}, \qquad (5)$$

these states are maximally entangled states.

The entanglement degree of the system is defined as in [16]:

$$E(t) = -p.\log_2 p - (1-p).\log_2(1-p), \tag{6}$$

in which $p = \dfrac{1+\sqrt{1-C^2}}{2}$ and $C = 2\left|c_{00}(t)c_{11}(t) - c_{01}(t)c_{10}(t)\right|$. These results show that in the absence of chaotic component, our result becomes exactly the same as that obtained by Miranowicz *et al.* [13]. The entanglement degree of states (2) is shown in figure 1. For the case $a_0 = 0$, entanglement degree of states (2) varies over period of time. The maximum values of them gradually reduce after each period $T \approx 2\pi/|\alpha|$. The entanglement can maximally achieve at the time of $t(n) = (n - 1/2)T$. At the time of $t(1) = (1/2)T$, the entanglement can maximally achieve 0.995 ebits. When the chaotic component is present, entanglement degree of states (2) also varies over period of time but the maximum values of them gradually increase and the entanglement can also maximally achieves 0.995 ebits but the maximum position change in comparison to the case when $a_0 = 0$.

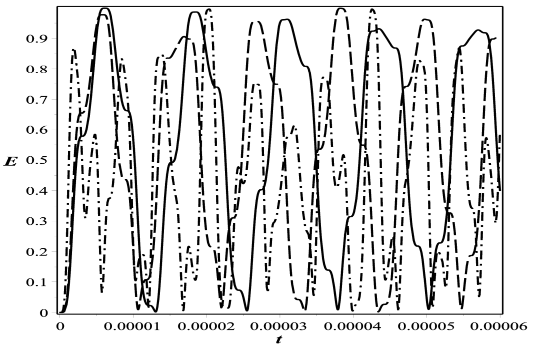

***Figure 1**. Entanglements degree of the generated states by the coupler pumped in two modes with $\alpha_0 = 5\times10^5$ rad/s. Solid line: $a_0 = 0$. Dotted line: $a_0 = 5\times10^4$ rad/s. Dashed-dotted line: $a_0 = 5\times10^5$ rad/s. The time unit is $1/\chi$.*

When we express the states (2) in the basis of the Bell states, the probabilities of finding the system in these states are shown in figure 2. We can see that when $a_0 = 0$, the probabilities to the system exists in Bell states B_1 and B_2 vary with time corresponds to the modulation of the oscillations with frequencies greater according to the oscillations with frequencies smaller. Maximum entanglement of the states B_1 achieves 0.992 ebits and B_2 achieves 0.997 ebits. When $a_0 \neq 0$, the probabilities to the system exists in Bell states B_1 and B_2 also vary with time but the maximum position of them change and maximum entanglement of these states decrease in comparison to the case when the chaotic component is absent.

On the other hand, when $a_0 = 0$, the probabilities to the system exists in Bell states B_3 and B_4 are equal. However, the maximum values of these states are only about 0.235 ebits. Especially when $a_0 \neq 0$, the probabilities to system exists in the Bell states B_3 and B_4 are different on the maximum position and have much larger value than when $a_0 = 0$.

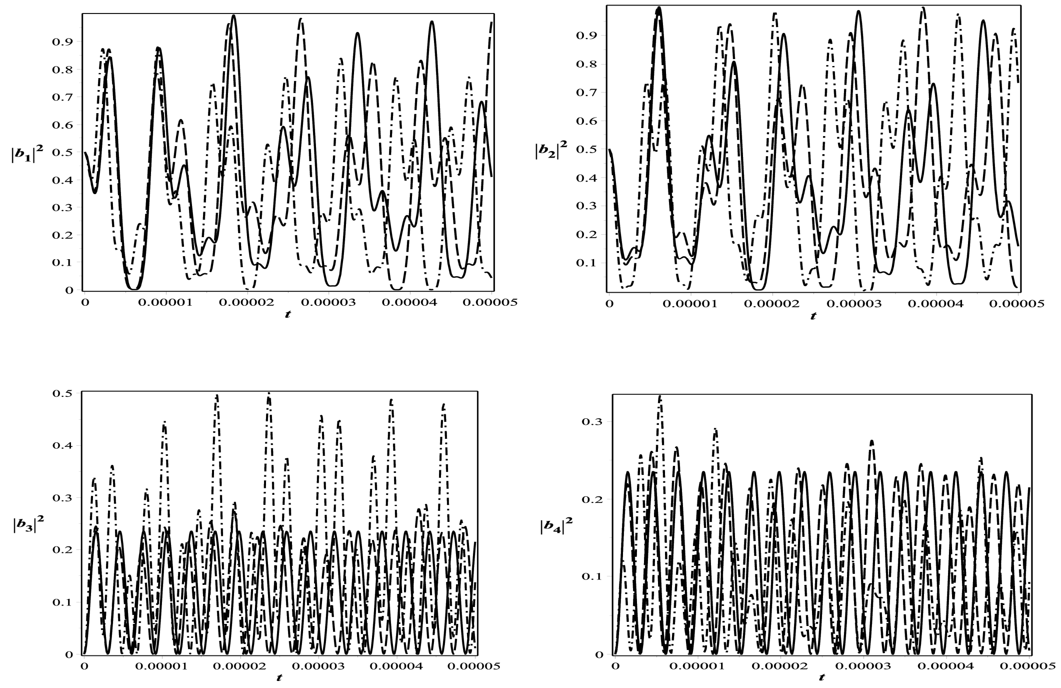

Figure 2. *Probabilities* $|b_i|^2$ *to the system exist in the Bell states with* $\alpha_0 = 5\times10^5$ *rad/s. Solid line:* $a_0 = 0$. *Dotted line:* $a_0 = 5\times10^4$ *rad/s. Dashed-dotted line:* $a_0 = 5\times10^5$ *rad/s. The time unit is* $1/\chi$.

3. CONCLUSION

In this paper, we discussed the model of Kerr-like nonlinear coupler comprising two nonlinear oscillators linearly coupled to each other. These two nonlinear oscillators are coupled to external classical fields, which are modelled by chaotic processes. Furthermore, we studied the dependence of maximally entangled states on noise parameter and compared maximally entangled states when chaotic parameter is present and absent. We found that in the presence of chaotic part, the location and the magnitude of the maximum change, especially maximally entangled of Bell states B_3 and B_4 increase in comparison to the case when chaotic component is absent. Consequently, the parameter a_0 related to the chaotic component is an important parameter that controls the maxima entanglement of Bell states.

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TÓM TẮT

SỰ TẠO RA TRẠNG THÁI ĐAN RỐI CỰC ĐẠI TRONG BỘ NỐI PHI TUYẾN ĐƯỢC BƠM BỞI ÁNH SÁNG LASER BĂNG RỘNG

Trong bài báo này, mô hình cắt trạng thái quang học của buồng cộng hưởng hai mode được phân tích. Bộ nối phi tuyến kiểu Kerr với hai mode được kích thích bởi trường cổ điển ngoài được giả thiết tách thành hai phần: kết hợp và nhiễu trắng. Chúng ta có thể thấy rằng sự tiến triển lượng tử của các bộ nối khép kín trong không gian Hilbert hai qubit chỉ mở rộng ra bởi các trạng thái đơn photon và chân không. Do đó, các bộ nối có thể nghiên cứu như hệ hai qubit. Phân tích sự tiến triển theo thời gian của sự đan rối lượng tử chỉ ra rằng các trạng thái đan rối cực đại có *thể được tạo ra và so sánh những kết quả này với những kết quả tìm được trong các tài liệu trước đó.*

Từ khóa: Bộ nối phi tuyến Kerr, Sự đan rối lượng tử, Các trạng thái Bell, Nhiễu trắng.

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