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EXPERIENCES OF IMPROVING MATHEMATICAL COMMUNICATION COMPETENCE FOR VIETNAMESE SECONDARY SCHOOL THROUGH THEME "SOLVING PROBLEM BY SETTING UP SYSTEM OF EQUATIONS"

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Abstract. This paper aims to discuss our experiences of promoting mathematical communication competence for students at secondary school in Vietnam. In this research, we applied the qualitative research that consists of the designed experiment and the participant's observation method. From result experiment, we show out detail about *Vietnamese* students not only skills solving productivity problem but also mathematical communication competence. Besides, we offer solutions to enhance students' effective learning activity.

Keywords: Mathematical communication competence, Vietnamese educational program, enhance students' effective learning activity.

1. Introduction

Mathematical communication has been much interested by researchers and countries: In international Symposium 2008 Innovative Teaching Mathematics through Lesson Study III focused on mathematical communication, Isoda (2008), Lim (2008), Vui (2009) and Tuong (2014) interested that "Mathematical communication itself is necessary to develop mathematical thinking". Programme for International Student Assessment (PISA, 2003) talked about mathematical communication in some core principles of their test design. Mathematical communication is an important key idea not only for improving mathematics but also for developing necessary ability for sustainable development on the knowledge based society.

Views on the role of mathematical communication competence in teaching and learning mathematics have been studied: Understand the comprehension of the mathematical language, such as symbols, terms, tables, graphs and informal deductions (Mónica, 2007). The new Mathematics Curriculum in Vietnam is emphasized that Mathematical communication competence is one of core competences training to students.

The form of the paper is including: First, we refer teaching mathematics to develop competence for students. Second, we define mathematical communication competence in this research. Third, we analyze the characters of teaching mathematics to develop mathematical communication competence for students. Finally, we evaluate students' mathematical communication competence through qualitative collecting data.

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2. Content

2.1. Literature Review

2.2.1. Mathematical communication competence

Mathematical communication is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps to build meaning and to permanence ideas and to make them public (Lim, 2008).

Mathematical communication competence is students' own opinions about the math problems, understand people's ideas when they present the matter, express their own ideas crisply and clearly, use mathematical language, symbols and conventions (Đuc Pham Gia and Quang Pham Đuc, 2002; Tuong, 2014).

2.2.2. Forms of mathematical representations

Bruner focused on the study of children's mathematical awareness as well as on representation thinking, he pointed out that it is possible to divide the representation into three categories from low to high as following (Vui, 2009):

- Reality: the actual representations of the lowest level, and by hand;
- Imagery: visual representations using images, graphs, charts, tables ...;
- Symbols: include language and symbol representations.

Tadao (2007) classifies representations in math education into five more specific forms as following:

- Realistic representation: Representations based on the actual state of the object. This type of representation can be directly, specific and natural effects.
- Manipurative representation: they are teaching aids tools, replacement or imitation of objects that students can affect directly. This type of representation can be very specific and artificial.
- Visual representation: Representation using illustrations, diagrams, graphs, charts. This is a kind of visual and lively representation.
- Language representation: These representations use pure language to express (say or write). This type of representation is governed by conventions, but lacking in succinctness; On the other hand, this representation is descriptive and can create a sense of familiarity.
- Represented by algebraic symbols: Representations using mathematical symbols such as numbers, letters, symbols.

2.2.3. The scale levels of mathematical communication competence

Phat (2019) give the components and standards of mathematical communication competence.

Table 1. Components and standards of mathematical communication competence

Components	*Standards*
1. Listening comprehension, reading comprehension and recording necessary mathematical information presented in written form or spoken or written by others.	1.1. Students can listen comprehension, read comprehension and summarize basic and main mathematical information from spoken or written text. 1.2. Students know how to analyze, select, extract essential mathematical information from spoken or written text. 1.3. Students know how to connect, link, and synthesize mathematical information from different documents.

2. Presenting, expressing (speaking or writing) the mathematical contents, mathematical ideas, and arithmetic to make mathematics in interaction with others	2.1. Students present fully, accurately and logically the contents and ideas of mathematics. 2.2. Students participate in discussions and debates about mathematical content and ideas with others. 2.3. Students explain coherently, clear their thoughts about solutions and they know how to argument mathematics exactly.
3. Effective use of mathematical language (numbers, symbols, charts, graphs, logical links,...) combined with ordinary language or body language when presented or explained and evaluate mathematical ideas in interaction (discussion, debate) with others.	3.1. Students use mathematical language suitably and combine common language to express ways of thinking, arguing and proving mathematical assertions. 3.2. Students analyze, compare, evaluate and select suitable mathematical ideas.
4. Demonstrate confidence when presenting, expressing, posing questions, discussing and debating content and ideas related to mathematics.	4.1. Students present and express mathematical content confidently. 4.2. When students participate in discussions and debates, they should explain mathematical content clearly, make a strong argument to affirm or reject a mathematical proposition.

In our research, we give the scale levels of mathematical communication competence Tuong (2014).

Level 1. Expressing initial idea

Level 1.1. Students describe and present methods or algorithms to solve the given problems (the mentioned method can be right or wrong).

Level 1.2. Students know how to use mathematical concepts, terminologies, symbols and conventions formally.

Level 2. Explaining

Level 2.1. Students explain the validity of the method and present reasons why they choose that method.

Level 2.2. Students use mathematical concepts, terminologies, symbols and conventions to support their logical and efficient ideas.

Level 3. Argumentation

Level 3.1. Students argue the validity of either the method or the algorithm. Students can use examples or counter-examples to test the validity of them.

Level 3.2. Students can argue mathematical concepts, terminologies, symbols and conventions which are suitable.

Level 4. Proving

Level 4.1. Students use mathematical concepts, mathematical logic to prove the given result.

Level 4.2. Students use mathematical language to present the mathematical result.

2.2. Research Methodology

We applied the qualitative research that consists of the designed experiment and the participant's observation method. Experimental teaching was conducted in the year 2016–2017

at: Saigon High School, District 5, Ho Chi Minh city; Tran Mai Ninh, Dien Bien, Nguyen Chinh, Nhu Ba Sy, Le Loi, Thanh Hoa province; Nguyen Tat Thanh, Ha Noi city.

There are 1020 students including 14 classes 9. The data are presented here, as evidence of students' arguments, students' mathematical reasoning and students' writting. Data analysis is qualitative.

2.3. Finding and the research question

2.3.1. Finding

In this section, we analyze Problem 1 that was hint by Vietnam textbook and has made opportunity for students to show mathematical communication competence but teacher don't know how to encourages students to express standards and scale levels of mathematical communication competence.

Problem 1. Two teams of workers together complete a road in 24 days. Each day, the work of team A is 1.5 times as many as that of team B. How long does it take each team alone to complete the road? How to solve. Solving problem by setting up system of equations.

i. Remark

a) Problem 1 was hint by Vietnam textbook.

From the assumption that both teams complete the road in 24 days (similar to completing 1 job), it gives that in 1 day both teams do $\frac{1}{24}$. *Similarly, the part of work each team does in 1 day, and the number of days for that team to complete the work are inverse variations (in the problem, the number of day is not always integer).*

Thus, we can solve the problem as follows:

Let x be numbers of days for team A to complete all work alone; y be number of days for team B to complete all work alone. Condition of variables is that x and y are positive numbers.

Each day, team A completes $\frac{1}{x}$ *(work), team B completes* $\frac{1}{y}$ *(work).*

Since each day, the work of team A is 1.5 times as many as that of team B, they have equation $\frac{1}{x} = \frac{3}{2}\text{x}\frac{1}{y}$ (1).

If two teams work together in 24 days, then the work is completed. Therefore, each day two teams complete $\frac{1}{24}$ *(work). We have equation:* $\frac{1}{x}+\frac{1}{y}=\frac{1}{24}$ (2).

From (1) and (2), we have system of equations: $\begin{cases} \frac{1}{x} = \frac{3}{2}\text{x}\frac{1}{y} \ (1) \\ \frac{1}{x}+\frac{1}{y}=\frac{1}{24} \ (2) \end{cases}$ (I)

Solve system of equations (I) by setting new variable ($u=\frac{1}{x};v=\frac{1}{y}$*) and then give answer to the given problem.*

b) According to this teaching by Vietnam textbook, students have flexible skill of solving system of equations and give answer to the given problem.

c) In my opinion, the above problem demand students:

- Students realize that productivity and time quantities are inversely proportional.
- Students know how to choose the variables and make the condition of variables.
- Students realize that the relationships in the problem are more or less difficult for them in making equations.
- Apply the rule of solving system of equations.

ii. We analyze the content to find opportunities for students to represent mathematical communication competence.

Let x be numbers of days for team A to complete all work alone; y be number of days for team B to complete all work alone. Since each day, the work of team A is 1.5 times as many as that of team B. If two teams work together in 24 days, then the work is completed.

With the above setting:

- The first, students need to calculate the amount of work that each team can do in a day.

Each day, team A completes $\frac{1}{x}$ *(work), team B completes* $\frac{1}{y}$ *(work).*

- Then, students make the equation $\frac{1}{x}=\frac{3}{2}\text{x}\frac{1}{y}\ (1)$. Setting up the equation (1) also requires students to apply knowledge about ratio of two numbers.
- Setting up the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{24}\ (2)$ also requires students to understand the relationship between productivity and time.

If two teams work together in 24 days, then the work is completed. Therefore, each day two teams complete $\frac{1}{24}$ *(work).*

2.3.2. The research question

The research question: 'How do the teacher encourage students to express their mathematical communication competence when they solve productivity problem by making system of equations?'

2.4. Discussion

To find the data to answer the research question, we design Problem 2 and Problem 3. Through problem 2, we propose solutions to enhance students' effective learning activity. Through problem 3, we want to test these solutions to enhance students' effective learning activity that develop students' mathematical communication competence.

2.4.1. Pre-Test

Problem 2 (Experimental teaching was conducted on 520 students).

If two water taps flow together into an empty pool, the pool is filled in $4\frac{4}{5}$ hours. If at the beginning, the first tap is turned, and 9 hours later the second tap is turned on, then it takes $\frac{6}{5}$ hours more to make the pool full. How long does it take to make the pool full if at the beginning only the second tap?

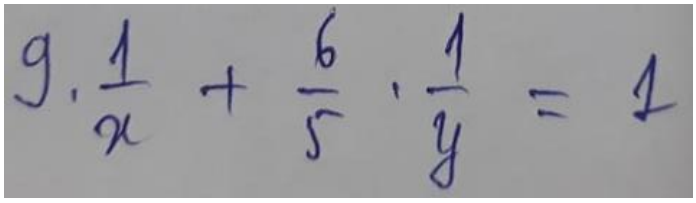
We have result.

Let x be numbers of hours for the first tap to make the pool full alone; y be numbers of hours for the second tap to make the pool full alone. Condition of variables is that x and y are positive numbers.

Each hour, the first tap flows $\frac{1}{x}$ *(pool), the second tap flows* $\frac{1}{y}$ *(pool).*

If two water taps flow together into an empty pool, the pool is filled in $4\frac{4}{5}$ *hours then we have equation* $\frac{1}{x}+\frac{1}{y}=\frac{1}{4\frac{4}{5}}$ *or* $\frac{1}{x}+\frac{1}{y}=\frac{5}{24}$

If at the beginning, the first tap is turned, and 9 hours later the second tap is turned on, then it takes $\frac{6}{5}$ *hours more to make the pool full. Some of students can not give equation or they have a wrong equation (there is 30% students).*



In this experiment, we collect the following information:

Step 1. Students decide the algorithm of solving productivity problem by making system of equations;

Step 2. Students have reading skills and find out the main information;

Step 3. Students can know how to choose the variables and make the condition of variables;

Step 4. Students can translate main information into equation;

Step 5. Finally, apply the rule of solving system of equations and give answer to the given problem.

We recognize that students have difficulty in the step 4. Students often find difficulty to hide an analysis of the relationship between given quantities and variables.

2.4.2. Solutions to enhance students' effective learning activity

2.4.2.1. Teacher use multiple representation to help students understand problem and use mathematical language effectively.

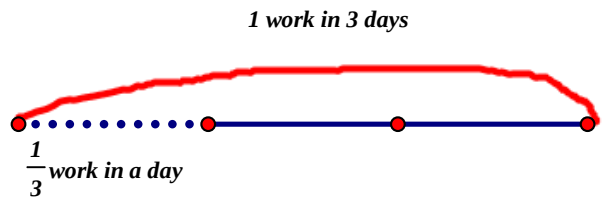
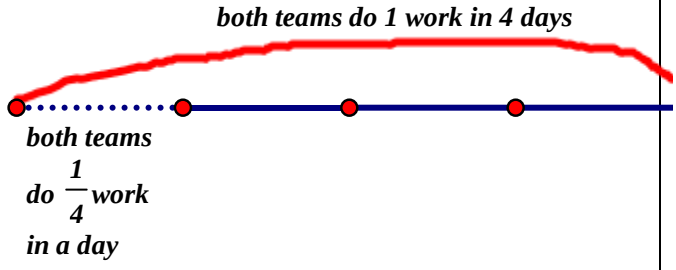
- Teacher encourage students:

- Give a detail representation which is corresponding to teacher' language representation.

- Try to make different representations that are corresponding to teacher' language representation.

- We illustrate the content clearly (we respectively note T and S be teacher and student)

Language representation	**Visual representation**
T: Suppose that team *A* to complete all work for 3 days and the effective of the work is the same. One day, what work does team *A* complete?	**S:** Mathematical expression: $\frac{1}{3}$ **T:** Illustrate Mathematical diagram

	
T changes *3 days* into *5 days or 7 days or 13 days*, S has the answer immediately.	**S:** Mathematical expression: $\frac{1}{5};\frac{1}{7};\frac{1}{13}$
T changes *3 days* into *x days*, S has the right answer because he used to do this question.	**S:** Mathematical expression: $\frac{1}{x}$
T: Suppose that team A and team B to complete all work alone for *x* days and *x* days respectively and the effective of the work is the same. One day, what work do team A or team B or two teams complete?	**S:** Mathematical expression. One day: the work team A completed: $\frac{1}{x}$; the work team B completed: $\frac{1}{y}$; the work both team A and team B completed: $\frac{1}{x}+\frac{1}{y}$
T: If two teams work together in 4 days, then the work is completed. Which equation do you have?	**S:** Mathematical expression: $\frac{1}{x}+\frac{1}{y}=\frac{1}{4}$ **T:** Illustrate Mathematical diagram 
T: Each day, the work of team A is 1.5 times as many as that of team B. What does it mean? Which equation do you have?	**S:** Mathematical terminology. Each day, the work of team A is 1.5 times as many as that of team B. It means that one day, the completed work of team A equals 1.5 times the completed work of team B. **S:** Mathematical expression $\frac{1}{x}=\frac{3}{2}\text{x}\frac{1}{y}$;

2.4.2.2. Teacher have to use not only effective teaching methods but also effective teaching strategies to support students express initial ideas and explain, discuss, argue about problem given by teacher

- Teacher encourage students:

- Express initial ideas by answering teacher' question.
- Decide main information that is help students to make system equations.
- Explain, discuss, argue about problem given by teacher or classmates.

- We illustrate the content clearly:

If at the beginning, the first tap is turned, and 9 hours later the second tap is turned on, then it takes $\frac{6}{5}$ *hours more to make the pool full.*

T: Which main information do you notice?

S: Time for each tap turned.

	The first tap	The second tap
Times	Flows alone: 9 hour	Flows alone: 0 hour
	Flows together: $\frac{6}{5}$ hours	Flows together: $\frac{6}{5}$ hours

S: The water was flew by each tap maked the pool full.

T: Can you design table represented for quantities by teacher's construction?

Teacher's construction	**Students' answering**	
	The first tap	The second tap
Call the variable represented for quantities and make the condition for called variable.	Let x be numbers of hours for the first tap to make the pool full alone; $x>0$	Let y be numbers of hours for the second tap to make the pool full alone; $y>0$
One hour, how much pool do each tap or two taps flow?	Each hour, the first tap flows $\frac{1}{x}$ (pool)	Each hour, the second tap flows $\frac{1}{y}$ (pool)
	Each hour, two taps flows $\frac{1}{x}+\frac{1}{y}$ (pool)	
If at the beginning, the first tap is turned, and 9 hours later the second tap is turned on, then it takes $\frac{6}{5}$ hours more to make the pool full. What does it mean?	The first tap flows alone in 9 hours and the second tap flows alone in 0 hour. Two taps flows together in $\frac{6}{5}$ hours. The water of the first tap flows alone in 9 hours is $9x\frac{1}{x}$ The water of the two taps flows together in $\frac{6}{5}$ hours is $\frac{6}{5}x\left(\frac{1}{x}+\frac{1}{y}\right)$	
Which equation do you have?	The water of the first tap flows alone in 9 hours is $9x\frac{1}{x}$	

	The water of the two taps flows together in $\frac{6}{5}$ hours is $\frac{6}{5}\text{x}\left(\frac{1}{x}+\frac{1}{y}\right)$ The water was flew by each tap maked the pool full. So we obtained $9\text{x}\frac{1}{x}+\frac{6}{5}\text{x}\left(\frac{1}{x}+\frac{1}{y}\right)=1$

2.4.3. Post - Test

To test the effect of above discussion, we have another experimental teaching was conducted on 500 students.

This exercise was experimental to test students can solve problem by setting up system of equations after they have studied method by teacher construction.

- **Problem 3:** It takes two workers 16 hours to complete a work. If the first worker does for 3 hours and the second worker does for 6 hours then they finish 25% of the work. How long does it take each worker alone to complete the work?

- We have result:

Method	Standard	Level	Content
Call the variable represented for quantities and make the condition for called variable.	1.1 and 1.2	1.1 and 1.2	Let x, y respectively be numbers of hours for the first worker and the second worker to complete the work alone; $x>0$ and $y>0$
One hour, how much work do each workers or two workers complete?	2.1	2.2	Each hour, the first worker completed $\frac{1}{x}$ *(work)* the second worker completed $\frac{1}{y}$ *(work)* the both worker completed $\frac{1}{x}+\frac{1}{y}$ *(work)*
Decide main information that is help students to make system equations.	2.1	2.1	It takes two workers 16 hours to complete a work so we obtain $\frac{1}{x}+\frac{1}{y}=\frac{1}{16}(1)$ If the first worker does for 3 hours and the second worker does for 6 hours then they finish 25% of the work so we obtain $3\text{x}\frac{1}{x}+6\text{x}\frac{1}{y}=25\%(2)$ From (1) and (2) we have system equations $\begin{cases}\frac{1}{x}+\frac{1}{y}=\frac{1}{16}(1)\\ 3\text{x}\frac{1}{x}+6\text{x}\frac{1}{y}=25\%(2)\end{cases}$

Can set up new variables they transform the given system into the system of two linear equations in two variables.	1.3 and 2.1	1.2 and 2.1	set $a=\frac{1}{x}$ and $b=\frac{1}{y}$ then $\begin{cases}\frac{1}{x}+\frac{1}{y}=\frac{1}{16}(1)\\ 3x\frac{1}{x}+6x\frac{1}{y}=25\%(2)\end{cases}$ becomes $\begin{cases}a+b=\frac{1}{16}\\ 3a+6b=\frac{1}{4}\end{cases}$
Students solved the given systems by algebraic addition method.	3.2, 2.1, 1.2 and 4.1	3.2, 2.1, 1.1 and 4.1	Multiply both sides of $a+b=\frac{1}{16}$ by –3 and add each side of two equations of system $\begin{cases}-3a-3b=\frac{-3}{16}\\ 3a+6b=\frac{1}{4}\end{cases}$ to make an equation with only b variable. $\begin{cases}a+b=\frac{1}{16}\\ 3a+6b=\frac{1}{4}\end{cases}\Leftrightarrow\begin{cases}-3a-3b=\frac{-3}{16}\\ 3a+6b=\frac{1}{4}\end{cases}$ $\Leftrightarrow\begin{cases}3b=\frac{1}{16}\\ a+b=\frac{1}{16}\end{cases}\Leftrightarrow\begin{cases}b=\frac{1}{48}\\ a=\frac{1}{24}\end{cases}$
Students solved the given systems by substitution method.	3.2, 2.3, 2.1, 1.2 and 4.1	1.1, 2.1 and 4.1	The coefficient of variable a in the equation $a+b=\frac{1}{16}$ was simple. Represented a in terms of b we have $a=\frac{1}{16}-b$ and substitute $a=\frac{1}{16}-b$ in the equation $3a+6b=\frac{1}{4}$ we have $3\left(\frac{1}{16}-b\right)+6b=\frac{1}{4}$ so we have $\begin{cases}b=\frac{1}{48}\\ a=\frac{1}{24}\end{cases}$

Students give answer	4.1	4.1	We have $\begin{cases} b = \frac{1}{48} \\ a = \frac{1}{24} \end{cases}$ so $\begin{cases} \frac{1}{x} = \frac{1}{24} \\ \frac{1}{y} = \frac{1}{48} \end{cases} \Leftrightarrow \begin{cases} x = 24 \\ y = 48 \end{cases}$ (satisfied condition)

From experimental teaching in Problem 3, we have result represented in Table 2.

Table 2. The percentage of students having standards and levels of mathematical communication competence in Problem 3

		Ho Chi Minh city	**Central of Viet Nam**	**Ha Noi city**
Standard	1.1	97	96	98
	1.2	96	95	96
	1.3	96	92	94
	2.1	89	85	90
	2.2	0	0	0
	2.3	81	80	83
	3.1	0	0	0
	3.2	88	86	88
	4.1	94	92	94
	4.2	0	0	0
Level	1.1	96	96	97
	1.2	94	92	95
	2.1	87	89	89
	2.2	85	86	88
	3.1	0	0	0
	3.2	86	85	88
	4.1	95	96	97
	4.2	0	0	0

3. Conclusion

- From above experiment, we can have result that student can solve productivity mathematics by setting up system of equations by the following steps:

o Students have reading skills and noting main information which is helped them to make equations.

o Students can know how to choose the variables and make the condition of variables.

o Students decide the algorithm of solving productivity problem:

✓ Call the variable represented for quantities and make the condition for called variable.

✓ Calculate the completed work of each object and both object in one unit time.

✓ Find the time for each object when that object completes alone or together.

✓ Calculate the completed work of each object in the found time.

✓ Use main information to make equations.

✓ Apply algebraic addition method or substitution method or setting up new variables to solve the system of two linear equations.

✓ Answer the given problem.

- Given solutions not only can enhance students' effective learning activity but also help them translate main information into equation confidently.

- Given solutions encourage students to express their mathematical communication competence.

o Students show standards of mathematical communication competence: 1.1; 1.2; 1.3; 2.1; 2.3; 3.2 and 4.1.

o Students show levels of mathematical communication competence: 1.1; 1.2; 2.1; 2.2; 3.2 and 4.1.

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