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RESEARCH ON SOLVING MATHEMATICS PROBLEMS OF SECONDARY SCHOOL STUDENTS

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Abstract. Mathematics education in the 21st century is facing new problems in the real world, cultivating creative thinking skills and effective learning. In an effort to innovate teaching and learning methods to prepare for future generations with the requirements of the new era which is the ability to solve problems. *Objective of the research*: research on the process of solving mathematical problems and ability level in solving mathematical problems. *Research methods*: theoretical research, the author relies on relevant domestic and foreign research documents to analyze and give personal opinions; Practical research: deliver learning cards with practical and math problems to 9th grade students. *Research results*: The author proposes 6 steps for solving mathematical problems, then the author studies, tests and gives 8 specific stages in the process of solving mathematical problems of students. At each stage the author shows its characteristic manifestations. This research is the initial basis for ability to detect and solve problems for students in teaching Maths at Secondary schools.

***Keywords*:** Capacity, solving problems, math teaching method.

1. Introduction

Krulik and Rudnick (1987) [1] conceive that problem solving is the process by which an individual uses the knowledge, skills, and knowledge he has to meet the demands of unfamiliar situations. Problem solving is a process associated with a set of skills that need to be learned. To identify the components of the problem-solving process, Polya came up with the following four-step diagram. This diagram is a graphical representation of student's problem-solving process. It identifies the stages that learners go through when solving problems and also points out the skills to accelerate the search for alternatives:

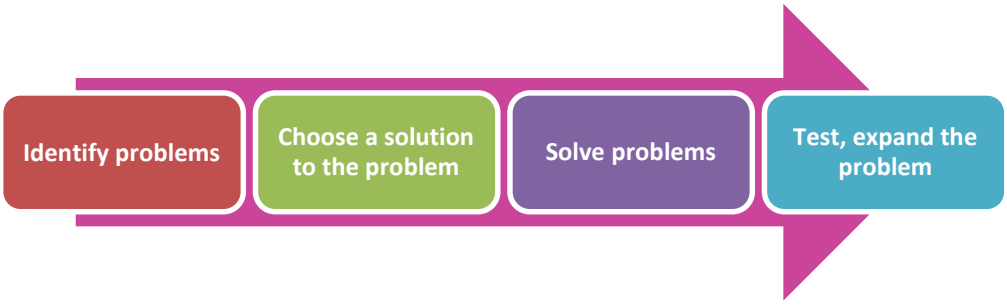

Figure 1. The problem-solving process (Polya)

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- ***Step 1: Identify problem***

Educators believe that learners will not have the opportunity to solve a problem if they do not understand it. The first step when students solve problems is to understand and explore the problem by reading the problem carefully to find all the clues, identify what problems are posed, analyze, search for key words for the direction to find a suitable solution. Observation can help students make some initial discoveries, discover patterns and rules, give themselves more chances to get higher results if there are some good comments. These findings, comments are only a holistic view, conjecture and not substantiated. When solving problems, individuals ask and answer questions by themselves such as: What to look for? What was there? What are conditions?

- ***Step 2: Choose a solution to the problem***

After reading the problem, students look for a solution to the problem based on the information gained from step 1. They continue to analyze more deeply, clarify the relationship between what they known and what they look for. They then organize data, knowledge, use cognitive methods and techniques such as goal orientation, strange rules of familiarity, specialization, generalization, etc. to find solutions to problems. The problem-solving approach can be adjusted until a reasonable solution is found for the original problem. The result of this step is to formulate a solution to the problem.

- ***Step 3: Solve problem***

After exploring the information related to the problem and deciding on the option to solve the problem, the learner presents the problem solving solution.

- ***Step 4: Test, expand the problem***

After presenting a problem-solving solution, the learner returns to the original problem and considers whether the problem has been completely solved. Students explore the applicability of the results, propose new related problems using intellectual activities such as similarize, generalize or reverse the problem.

It can be seen that the steps in the problem-solving process proposed by Polya are different from the algorithms and rules that students learn in math classes. An algorithm always guarantees success if it is selected appropriately and applied correctly. However, the guidelines presented in the diagram above only provide a four-step approach towards developing problem-solving ability for learners. These guidelines only provide a "road map", they are considered a blueprint to guide the way to the solution of a problem. Unlike the algorithm, the steps in problem solving cannot guarantee success, but if students learn math in a pedagogical environment focusing on exploration, they will be confident and successfully solve problems encountered in the classroom and in life.

2. Content

2.1. The process of solving mathematical problems

Problem solving is not just a method of teaching mathematics but a basic activity in the process of learning maths of students as they deepen their understanding of mathematical concepts and processes by analyzing and synthesize their knowledge. However, in the process of learning math, students often carry out problem-solving tasks in a routine when trying to memorize and re-apply the methods teachers have instructed to solve previous problems (Posamentier and Krulik, 1998) [2]. Therefore, they may succeed with familiar problems that they have encountered before but often face challenges when encountering unfamiliar issues. To assist students in looking for solutions to problems, Krulik and Rudnick (1987) [1] have presented ten options that learners can use to solve problems when learning math is an upward

analysis, looking for a process, approach the problem in a new way, solve similar but simpler problems, consider special cases, illustrate by drawing, guess and try, consider all possible possibilities out, sort the data, logically infer. Different problems require different problem-solving options (Schoenfeld, 1992) [3]. When using a single solution that cannot solve a problem, learners need to combine different problem-solving options to find out how to solve the problem (NCTM, 2000) [4].

Approaching the problem-solving process in terms of awareness, PISA (2003) has divided the problem-solving process into 6 steps:

- *Understanding problem*: This includes how students understand a text, a diagram, a formula or a table and give conclusions from it; Contact information from a variety of sources, understand relevant concepts and use information from existing knowledge to understand the information given in the problem.

- *Describe problem*: This includes how students identify the variables that appear in the problem and the correlations between them; decide which variables are relevant and irrelevant, formulate hypotheses, organize, critically evaluate contextual information.

- *Perform the problem:* This includes how students build tabular, graphical, symbolic or verbal representations, and flexibly manipulate representational forms to find a solution to the problem.

- *Solve problem*: This includes making decisions, analyzing systems or designing a system to meet certain purposes, anticipating and proposing a solution.

- *Reflect on the solution to the problem*: This includes how learners test their problem-solving options and find additional information, assess solutions offered from different perspectives in an attempt to restructure the problem-solving solution to make it accepted by the learning community.

- *Perform problem-solving plans:* This includes how students choose appropriate media and performances to show and present their problem-solving options.

The problem is how to help students successfully solve the problems they encounter. Some students are always successful at solving problems while others are not. Schoenfeld (1985) [5] points out that newcomers participating in the problem-solving process are often less successful because they tend to be aware of the problem according to the external characteristics expressed in the problem statement rather than going into the nature. of the problem. While experienced people are often aware of the underlying structures of the problem and this will be useful for the process of finding a suitable solution to the problem. Once students have more experience in the process of problem solving, they are more aware of the problem and more effective in solving it.

2.2. Stages in problem-solving process

The ability to solve problems is usually approached according to the problem solving process and is considered a transformation in students' skills after conducting problem solving process. According to Kulm and Bussmann (1980) [8], each stage of problem solving requires specific competencies, which are closely related to that stage. These two researchers think that there are specific competencies related to eight specific stages in the process of solving specific mathematical problems of students as follows:

- ***Stage 1:*** Focus on understanding the problem. Students need to understand the formal structure of mathematical problems and recognize the relationships between the specific factors of the problem, including related mathematical representations to understand the mathematical content of the problem. In this stage, students need to be able to recognize formal mathematical data and information hidden under the data of a given problem, arrange this information appropriately to form complete problems math.

They will then check to see if the problem situation poses a conflict with the existing knowledge and decide if the problem can be reorganized under known rules, principles and rules. To perform this activity, students need the ability to analyze and synthesize, logically reason and draw conclusions correctly, be aware of the problem as a generalization of a known problem, decide on processing method for given information and adequately evaluate the various information hidden in the problem.

- ***Stage 2:*** Analyze problems, discover and process basic information, break old rules and ideas. Students need to continue balancing knowledge, ability and problem requirements if the information received at the first stage is not enough to successfully solve the problem. They must be able to ask questions, test what they already know and what the problem requires, point out the gaps they encounter, establish temporary rules from the given data, assess the views and ideas of their classmates ...

- ***Stage 3:*** Guess the models from the representation of the factors in the problem. The process of guessing at the end of stage 2 leads to an increase in problem-specific questions and the development of new predictive models. Models are created through the construction and activation of relationships using visual images. Hypotheses are formed and verified. To accomplish this task, learners need to be able to visualize problems, imagine, connect data, make hypotheses ... This is a leap in learners' thinking in solving problems because they have formed new ideas and relationships that are not presented in the statement of the problem.

- ***Stage 4:*** Break down old structures and make predictions about new rules to reach the crux of the problem. The learner seeks connections through intellectual activity like analogy, abstraction and generalization.

- ***Stage 5:*** Control the thought process to form new knowledge. Although new knowledge has not been clearly formed, elements of the problem begin to form in new relationships. To achieve that, learners need to be able to reflect, evaluate, make rational decisions.

- ***Stage 6:*** The process of forming new knowledge becomes complete. The abstraction of aspects of the problem not only helps learners formulate problem solving solutions but also provides solutions for similarly structured problems. In order to form abstract knowledge learners need to be able to work with different representations and symbol systems. Learners need to be able to abstract, organize and systematize.

- ***Stage 7:*** Interpreting new knowledge is formed in the context of the original problem to complete the problem-solving process. Learners need to be able to apply and interpret newly formed knowledge and principles into the original problem in order to find a solution to the problem.

- ***Stage 8:*** Reflect and evaluate the problem solving plan. Learners need to be able to screen, transform and evaluate problem-solving options, so that they can gain a higher level of awareness.

According to the stages of the problem-solving process, the output standards of students' problem-solving capacity are described in Table 1.

Table 1. Describe the competency component for students' problem solving

Component	Feature
Find out the problem	- Analyze and fully explain the information of the problem in an explicit and hidden form - Discuss and create consensus on most information about the issue
Find a solution	- Understand the nature of some models and structures that fit the problem - Select the required information from multiple sources and evaluate the

	information - Actively discuss with classmates about models, structures, processes - Assign and reasonably arrange to find solutions
Plan and implement solutions	- Apply processes, principles, and problem-solving strategies to less familiar problems - Be able to describe the approach to the problem clearly through pictures, speaking, writing - Fluently implement relatively complex solutions - Organize a number of group communication methods.
Evaluate and reflect the solution	- Assess solutions' strengths and weaknesses consciously - Begin to think, assess the value of the solution to many same problems

In the modern direction, problem solving capacity is approached according to the information processing process, emphasizing the transformation in the process of formation of learners' knowledge after participating in problem solving activities. According to the research results of Griffin and Care (2014) [9], five levels of problem solving capacity of learners approached according to the level of progress in the process of forming mathematical knowledge and problem solving skills will be describe in detail in Table 2.

Table 2. Level and characteristics of students' problem-solving competence

Level	Feature
Excellent	Students understand deeply about a problem. Students can solve a problem and raise a new problem and find new ways to solve it.
Good	Students see the causal relationship and find the right strategy to give the right solution to simple or complex problems. They can adjust the original hypotheses based on the newly acquired information, test all the alternative hypotheses, and change the approach as the complexity of the problem is raised up.
Rather good	Students start to connect information samples with each other and realize the rules that exist in the information obtained. They know how to divide the problem into small problems or simplify the original problem and find a solution.
Average	Students test the hypotheses based on the information obtained. They begin to notice a causal relationship in their actions and try to gather information to get the job done.
Weak	Students try to approach problems with familiar methods but do not understand why they must be implemented. They only pay attention to the information separately and follow the instructions of the teacher.

Example 1. Compare the area of two gardens

Mr. An needs to buy one of the two gardens as shown (Figure 2). If you were Mr. An, which garden would you choose? Explain why you chose that garden. If you think that there is not enough information to give a suggestion to Mr. An, what information do you need and how will you use it to solve this problem?

Figure 2. Area comparison

This is a problematic situation for most students in general and junior high school in particular because students often have problems in comparing the area of regular shapes such as triangles, rectangles, shapes, square or circle. With the problem of comparing the area of regular geometric shapes, the learner will orient to use the learned geometry knowledge to create accurate conclusions for the given problem such as using formulas, overlapping, drawing the complement of the shapes, etc. However, in this situation, it is difficult for students to access in the usual ways that have been learned and practiced before. To compare the areas of these two shapes, students can use some area estimation methods. Students can draw extra lines to divide the shapes A and B into small shapes as follows (Figure. 3), thereby showing the area of the pieces in two approximately shapes to conclude the area of A approximating the area of B:

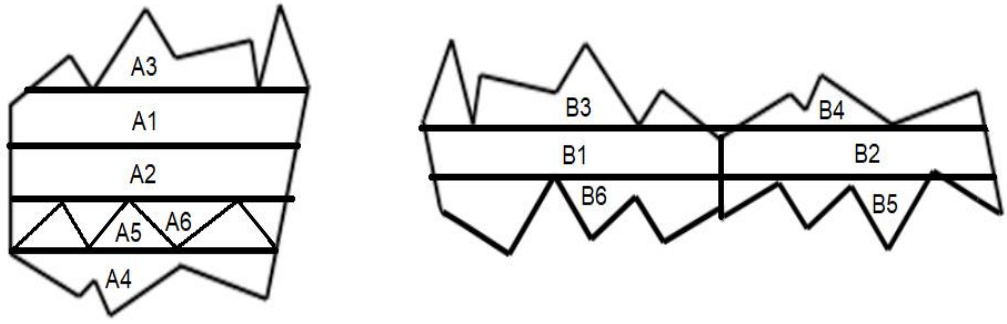

Figure 3. Divide the area of two gardens

Therefore, if you are only interested in the area of two gardens, then you can choose any of them. However, A is wider, the land is more square, so depending on the purpose of use, the buyer will consider which land to choose.

3. Conclusion

One of the goals of education in Vietnam is to teach students to think. There are several ways to promote and develop students' thinking. Research on problem solving is a new research direction in Vietnam in recent years. This study only explores the process of solving mathematical problems, levels of students' ability to solve math problems. The author proposes 6 steps for the process of solving mathematical problems and 8 specific stages in the process of solving mathematical problems of students.

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