HNUE JOURNAL OF SCIENCE DOI: 10.18173/2354-1075.2019-0146
Educational Sciences, 2019, Volume 64, Issue 12, pp. 191-199
This paper is available online at http://stdb.hnue.edu.vn

DEVELOPING MODELING COMPETENCY FOR PRIMARY STUDENTS THROUGH REAL-LIFE CONTEXT IN NATURAL NUMBER TOPIC COMPARISON IN GERMANY, SINGAPORE AND VIETNAM

Chu Cam Tho[1] and Nguyen Tien Dat[2]
[1]*Vietnam National Institute of Educational Sciences*
[2]*Hanoi National University of Education*

Abstract. Mathematical modeling is one of the competencies that have been focused in the curriculum of many countries in the world such as Australia, Germany, Singapore and USA. In Vietnam, the Mathematics curriculum promulgated in 12/2018 clarified that modeling competency would be one of the domain-specific competencies that contribute to the formation and development of the mathematical competency in particular, and numeracy in general. In the new mathematical curriculum, the topic of natural number remains central and throughout accounting for more than 50% of the knowledge in the primary curriculum and the first half of grade 6 as the foundation to build new sets of numbers which are not mentioned in the primary curriculum. In this paper, we present a study on modeling competencies and transformational pedagogical analysis of specific learning tasks based on real-life context included in the German, Singapore and Vietnamese curricula, in order to make recommendations in teaching to develop the model competency for the topic of natural numbers by model method.

***Keywords*:** Mathematical competency, mathematical modeling, modeling process, natural number topic, model method.

1. Introduction

In Germany, according to lower secondary education standards [1] and primary school level [2], mathematical modeling competency is one of the compulsory educational goals that the Minister of Culture conference has agreed on. Recently, mathematical modeling competency is considered one of the measurement standards for higher education entrance qualification [3]. Singapore's education program identifies modeling as a skill in the process of identifying and applying mathematical knowledge to solve problems; Through modeling, students learn how to solve uncertain data, create connections, select and apply mathematical concepts and skills appropriately, identify assumptions and reflect on solutions to real-world problems and make decisions based on data given or collected [4]. In the Mathematics curriculum of Vietnam, the modeling competency in Mathematics is one of the five components of mathematical competence

Received November 28, 2019. Revised December 17, 2019. Accepted December 24, 2019.
Contact Chu Cam Tho, e-mail address: chucamtho1911@gmail.com

(including thinking and mathematical argument competency, mathematical modeling competency, problem–solving competency, mathematical communication competency, using tools and means in learning mathematics [5]). In this article, we present the research on the modeling competency and pedagogical analysis of the specific type of task based on real-life context in Germany, Singapore and Vietnam to bring out recommendations in teaching to develop modeling competency in natural number topic through model method. In particular, the textbooks that we have chosen to compare are Primary Mathematics (US Edition) of Singapore [6], Book of Numbers [7] of Ernst Keltt Verlag in Stuttgart Leipzig and current Math textbooks in Vietnam [8] respectively.

2. Content

2.1. Mathematical modeling competency in German, Singaporean and Vietnamese curricula

According to German curriculum [3], modeling competency includes:

K3.1: translate the domain or situation to be modeled into mathematical concepts, structures and relations;

K3.2: work in the respective mathematical model;

*K3.3***:** interpret and test results in the appropriate area or situation.

Modeling competency is expressed at different levels in a situation that needs modeling. Table 1 shows the behavioral indicator of the modeling capacity by each level.

Table 1. Behavior indicator description for each level of modeling competency

Reproduce	Establish relationships	Generalize and reflect
- Apply familiar and directly apparent models - Transfer real situations directly into mathematical models - Validate mathematical results with regard to the real situation.	- Carry out modeling processes consisting of several steps and with few and not clearly formulated restrictions - Interpret results of such modeling processes - Adjust mathematical models to varying facts.	- Model complex real situations whereby variables and conditions have to be determined - Check, compare, and evaluate mathematical models considering the real situation.

According to Singapore's education syllabus [4], modeling competency is represented by the elements given by Table 2.

Table 2. Behavior indicator description for modeling competency

Component	Behavior indicator
Formulating	- Understand the problem - Make assumptions to simplify the problem - Represent the problem mathematically
Solving	- Select and use appropriate mathematical methods and tools (including ICT) - Solve the problem and present the solution
Interpreting	- Interpret the mathematical solution in the context of the real–world problem - Present the solution of the real–world problem
Reflecting	- Reflect on the real–world solution - Improve the model

In the Mathematics curriculum of Vietnam [5], the mathematical modeling competency is expressed through the elements and levels described in Table 3.

Table 3. Behavior indicator description for each level of modeling competency

Component	Primary	Lower Secondary	High School
Identify mathematical models (including formulas, equations, tables, graphs) for situations that appear in real-world problems.	Select operations, arithmetic formulas, diagrams, tables, drawings to present, express (speak or write) the contents and ideas of situations that appear in simple real–world problems.	Use mathematical models (including mathematical formulas, diagrams, tables, figures and equations, representations) to describe situations that appear in some real-world problems which are not too complicated.	Establish mathematical models (including formulas, equations, diagrams, drawings and tables, graphs) to describe the situation set out in some real-world problems.
Solve mathematical problems in the established model.	Solve problems that appear from the above choices.	Solve mathematical problems in the established model.	Solve mathematical problems in the established model.
Demonstrate and evaluate the solution in real context and improve the model if the solution is not appropriate.	Bring out the answer to the situation appearing in the real-world problem.	Demonstrate mathematical solutions in real-world context and familiarize yourself with verifying the correctness of the solution.	Explaining the correctness of the solution (the conclusions obtained from calculations are meaningful, consistent with real–world or not). In particular, recognizing how to simplify, adjust real - world requirements (approximation, adding assumptions).

We found out the similarities in the way to define and construct elements of process modeling competency including the following steps: establishing the model (transforming real situations into problem to be solved), solving the problem (give the solution in accordance with the problem after setting up the model), evaluating the solution (show that the solution is suitable for the real-world model and adjust the solution to match the model or adjust the original model). In German and Singaporean curricula, the levels of behavioral indicators are divided according to the perceived ability of the students. In the Vietnamese Mathematics curriculum, the levels of behavioral indicators are divided according to school level that leads to difficulties in the teaching process to develop modeling competency. Because a low-level student can achieve higher-level behavioral indicators on the contrary at the same element, a high-level student may only achieve lower-level behavioral indicators. Following German and Singaporean research, we apply the behavioral division modeling competency according to cognitive levels: reconstruction, relationship setting, reflection and evaluation in this study.

Therefore, the question is how to teach to develop modeling competency through a step-by-step process.

According to Werner Blum and Leiβ [9], the process of modeling involves seven steps.

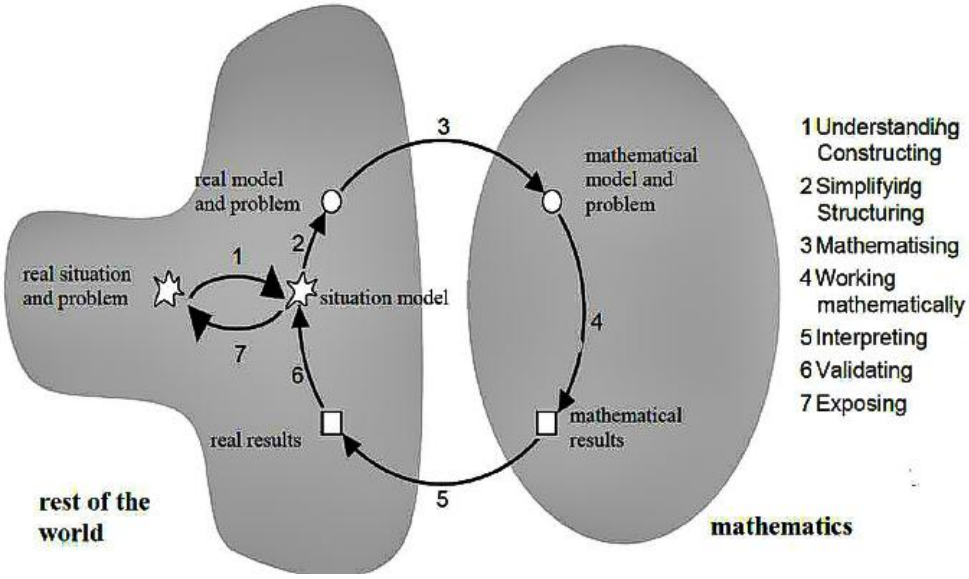


Figure 1. Modeling process of Blum and Leiβ

According to the above modeling process, Ferri [10] points out those students often ignore certain steps or do things in the wrong order. Factors affecting teaching to develop modeling competency that are the synthesis of individual psychological and cognitive characteristics, teacher support, class regulation, learning tasks or situations that need to be applied to modeling competency. According to Blum [11], in order to enhance modeling competency, students need to develop both beliefs and attitudes simultaneously. In addition, teachers need to have appropriate support to promote students' modeling competency. Specifically, Borromeo Ferri and Blum [12] identified 5 elements of teaching modeling competency for teachers, including: Theory-oriented competency, Task-related competency, Teaching competency, Diagnostic competency and Assessment competency. Finally, designing tasks or modeling exercises with appropriate context is the key to motivate students to develop modeling competency [13].

In Singapore's curriculum [4], the modeling process consists of 4 steps described in Figure 2.

The modeling teaching in the process (Figure 2) is recommended by Singapore's Math curriculum to help students construct their own mathematics knowledge and find solutions to solve problems from reality and develop good habits as well as necessary skills in the model such as: arranging meaningful information, recording feedback for evaluation, improving results or building models. At the same time, to develop modeling competency to solve problem, teachers should follow the principle of connecting learning to the real world, harness ICT tools and giving students an understanding and importance of practical applications of mathematics through real-world models and problems. From there, students will see the meaning and relevance of mathematics.

According Nguyen Danh Nam [14], students mainly have difficulty in formulating real-world situations into mathematical models because the system of exercises in textbooks do not present clearly enough and only focus on solving the mathematical model to bring out mathematical solution.

Therefore, in this paper, we focus on analyzing pedagogical transformation of some typical teaching situations in math textbook to clarify the process of transforming real-world situations

into mathematical situations is included in the content in Singaporean textbooks and German textbooks, helping students develop modeling competency for students.

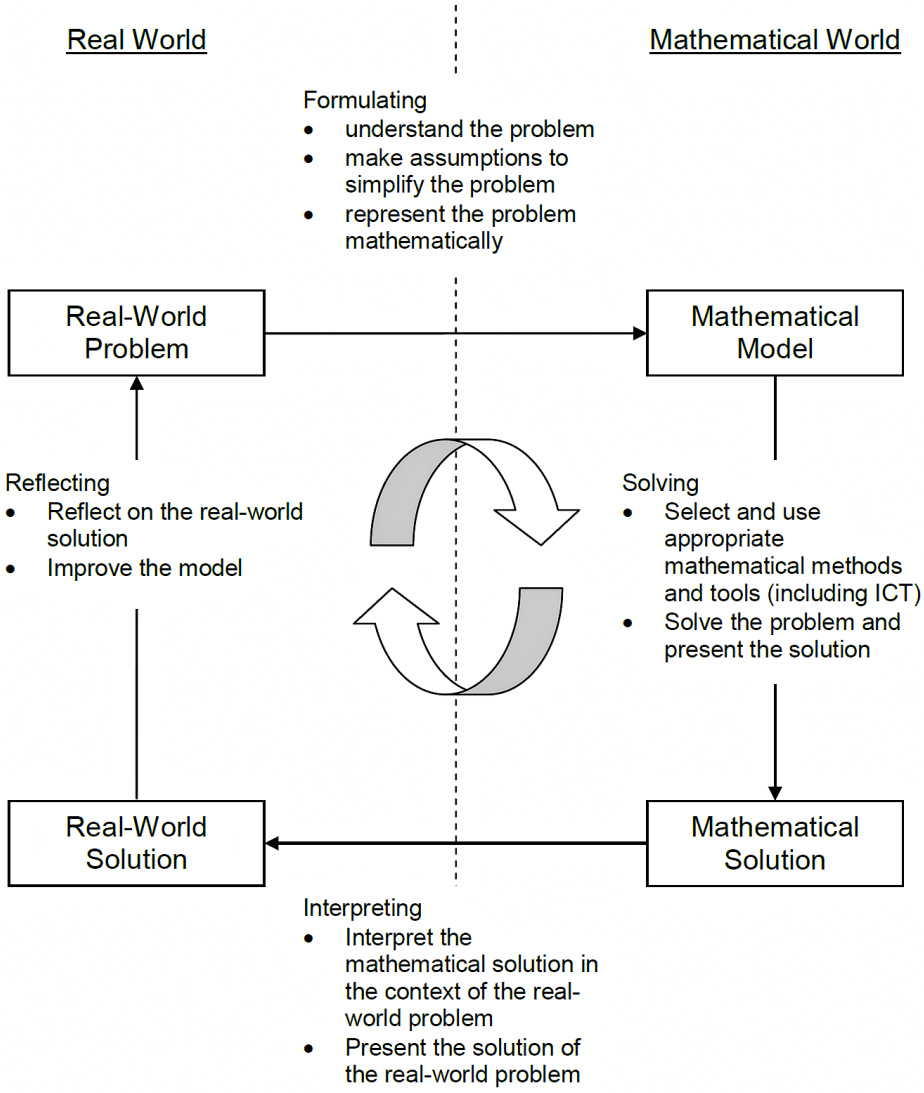


Figure 2. Mathematical modeling process

2.2. Natural number topic in German, Singaporean and Vietnamese curricula

Natural number topic in German curriculum [15] and Singaporean curriculum [3], and in the new Mathematics general curriculum of Vietnam [5] in particular holds a central position throughout the Primary Mathematics, mainly including contents of:

- Read, write, compare and order natural numbers;

- Performing calculations of addition, subtraction, multiplication and division of natural numbers;

- Applying properties of calculations with natural numbers for mental arithmetic and rationality;

- Estimate and round out numbers in simple calculations;

- Solve the problem associated with solving problems step by step related to calculations of natural numbers; related to direct and simple dependency relations.

In the following part of research, we will analyze the didactic transposition in three problems in natural number topics in Singaporean, German and Vietnamese textbooks, thereby making recommendations in teaching to develop modeling competency through model method.

2.3. Analysis of the specific type of tasks based on real-life context included in Germany, Singapore and Vietnam

2.3.1. Million and million period task

In mathematics grade 4 [8], the million and million period task have been shown as million and million period.

We call 10 hundred thousand as *1 million*, write as: 1 000 000.

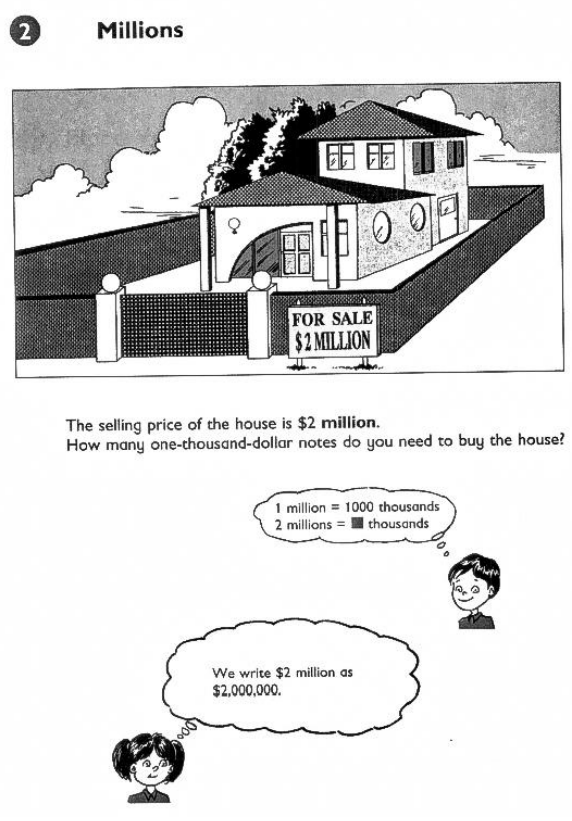
We call 10 million thousand *as 1 ten million*, write as: 10 000 000

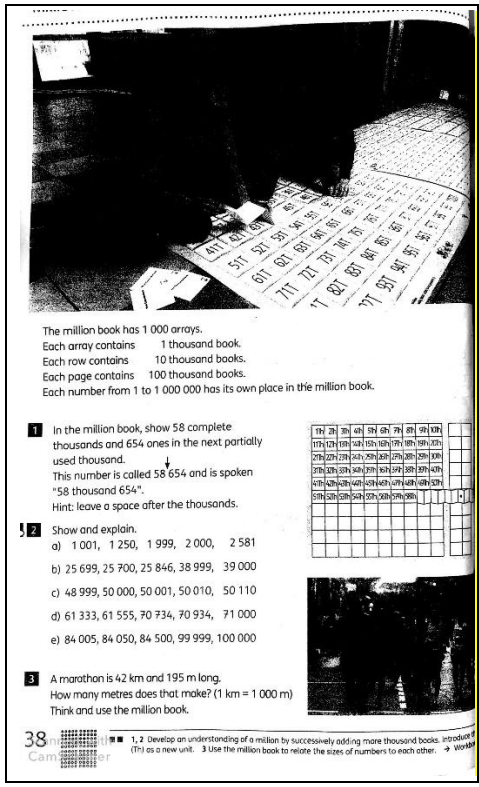
We call 10 ten millions as *1 hundred million*, write as: 100 000 000

Million period includes: million place, ten million place, hundred million place

Thus, in the current textbooks, million and million period are presented in a purely introductory manner to mathematical knowledge of reading and writing number in million place. Singapore's textbook, in contrast, presents the context of millions in real life.

Table 4. Context analysis in Singaporean and German textbooks

The context is taken here: A house is for sale for $ 2 million. From there arise two questions: 1. What does this two-million number mean? How to write the number in figure form? 2. How to quantify that amount in detail? If you know that 100 million equals 1,000 thousand. How much money would it cost to buy this house if I used $ 1,000 dollars?	



The million book has 1 000 arrays.
Each array contains 1 thousand book.
Each row contains 10 thousand books.
Each page contains 100 thousand books.
Each number from 1 to 1 000 000 has its own place in the million book.

1 In the million book, show 58 complete thousands and 654 ones in the next partially used thousand.
This number is called 58 654 and is spoken "58 thousand 654".
Hint: leave a space after the thousands.

2 Show and explain.
a) 1 001, 1 250, 1 999, 2 000, 2 581
b) 25 699, 25 700, 25 846, 38 999, 39 000
c) 48 999, 50 000, 50 001, 50 010, 50 110
d) 61 333, 61 555, 70 734, 70 934, 71 000
e) 84 005, 84 050, 84 500, 99 999, 100 000

3 A marathon is 42 km and 195 m long.
How many metres does that make? (1 km = 1 000 m)
Think and use the million book.

38

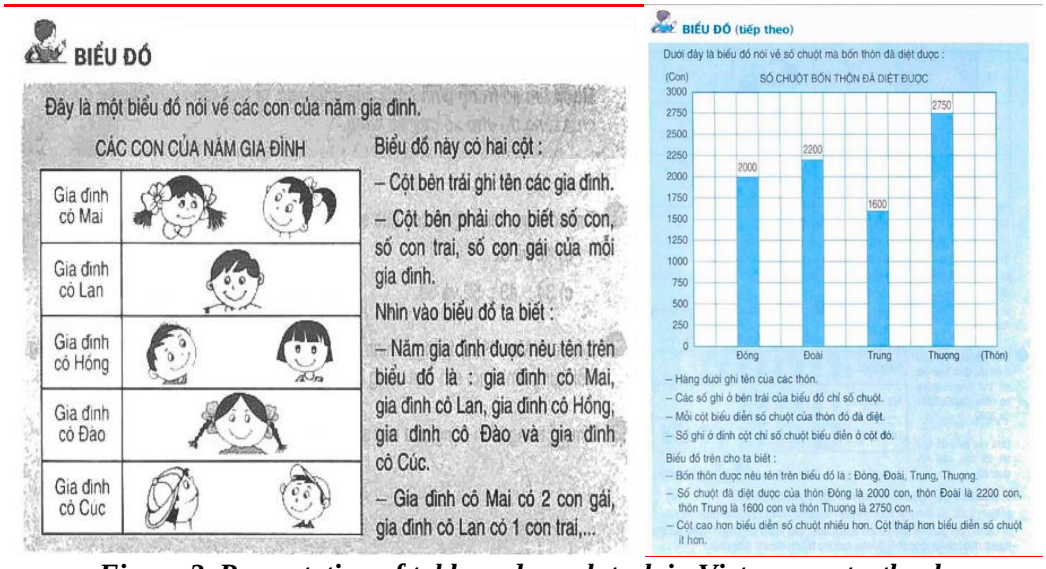
1, 2 Develop an understanding of a million by successively adding more thousand books. Introduce it (Tr) as a new unit. 3 Use the million book to relate the sizes of numbers to each other. → Workbook

The context taken here is the experiential activity on a number place (million book) with the goal of introducing 1 thousand as a new unit from which the student expands the place and period of numbers to millions through this million book.

Subsequently, reading numbers, performing calculations or comparing numbers are presenting throughout million book.

2.3.2. Table and graph task

In Vietnamese textbooks, the task of graphs and bar charts to represent data is shown in a discrete way, without creating a connection with each other, the process of transforming from real situations into models is ignored and only the mathematical model is presented. Real situations do not have a relationship with the learners themselves.



BIỂU ĐỒ

Đây là một biểu đồ nói về các con của năm gia đình.

CÁC CON CỦA NĂM GIA ĐÌNH

Gia đình cô Mai
Gia đình cô Lan
Gia đình cô Hồng
Gia đình cô Đào
Gia đình cô Cúc

Biểu đồ này có hai cột :
– Cột bên trái ghi tên các gia đình.
– Cột bên phải cho biết số con, số con trai, số con gái của mỗi gia đình.
Nhìn vào biểu đồ ta biết :
– Năm gia đình được nêu tên trên biểu đồ là : gia đình cô Mai, gia đình cô Lan, gia đình cô Hồng, gia đình cô Đào và gia đình cô Cúc.
– Gia đình cô Mai có 2 con gái, gia đình cô Lan có 1 con trai,...

BIỂU ĐỒ (tiếp theo)

Dưới đây là biểu đồ nói về số chuột mà bốn thôn đã diệt được :

SỐ CHUỘT BỐN THÔN ĐÃ DIỆT ĐƯỢC

(Con) 3000 2750 2500 2250 2000 1750 1500 1250 1000 750 500 250 0

2000 2200 1600 2750

Đông Đoài Trung Thượng (Thôn)

– Hàng dưới ghi tên của các thôn.
– Các số ghi ở bên trái của biểu đồ chỉ số chuột.
– Mỗi cột biểu diễn số chuột của thôn đó đã diệt.
– Số ghi ở đỉnh cột chỉ số chuột biểu diễn ở cột đó.

Biểu đồ trên cho ta biết :
– Bốn thôn được nêu tên trên biểu đồ là : Đông, Đoài, Trung, Thượng.
– Số chuột đã diệt được của thôn Đông là 2000 con, thôn Đoài là 2200 con, thôn Trung là 1600 con và thôn Thượng là 2750 con.
– Cột cao hơn biểu diễn số chuột nhiều hơn. Cột thấp hơn biểu diễn số chuột ít hơn.

Figure 3. Presentation of table and graph task in Vietnamese textbook

In Singaporean textbooks and German textbooks, the relationship between the data sheet and the graph is related. Real-life situations are clearly presented and closely related to learners. In addition, the guiding question system that will help learners discover information from the table, how to use collected data to model in form of tables or bar charts.

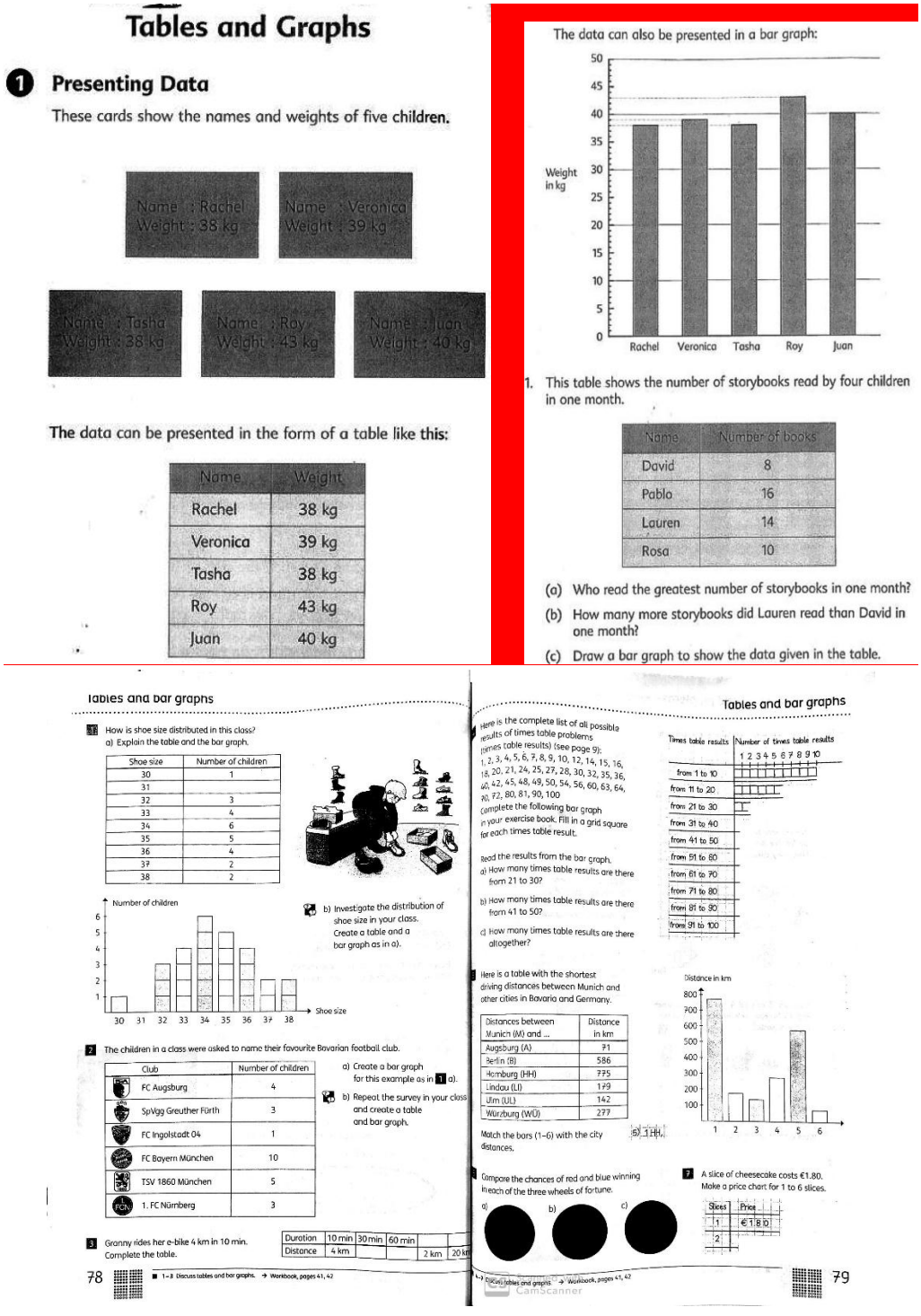


Figure 4. Presentation of table and graph task in Singaporean and German textbooks

3. Conclusions

Developing the mathematical modeling competency for students is one of the important goals of mathematic education in Vietnam in the renovation of the upcoming curriculum. Along with the change of the curriculum, content and teaching techniques also need improving. The problem content will need to be improved so that students have the opportunity to practise component competency especially that is translating a definite situation into mathematical models and have condition to validate conclusion which is suitable for the corresponding situations that require problematic situation and associated with the reality.

REFERENCES

[1] KMK, 2003. Educational standards in mathematics for middle school. Secretariat of the Standing Conference of Ministers for Education and Cultural Affairs in the Federal Republic of Germany (in German).

[2] KMK, 2004. Educational standards in mathematics for primary education. Secretariat of the Standing Conference of Ministers for Education and Cultural Affairs in the Federal Republic of Germany (in German).

[3] KMK, 2012. Educational standards in mathematics General higher education entrance qualification. Decision of the Conference of Ministers of Education (in German).

[4] MOE, 2013. MATHEMATICS SYLLABUS Primary One to Five. Singapore. https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/primary_mathematics_syllabus_pri1_to_pri5.pdf.

[5] MOET, 2018. Maths curriculum. Vietnam Ministry of Education and Training.

[6] Primary Math Textbook 4A U.S. EDITION, 2003. Singapore: Marshall Cavendish Education Pte Ltd.

[7] Wittmann, E. C., & Müller., G. N., 2018. Book of number 3. Stuttgart; Leipzig: Ernst Klett Verlag.

[8] Hoan, D. D., Chung, V. Q., & Thuy, V. D., 2011. Mathematic Grade 4. Vietnam Education Publishing house Litmited Company.

[9] Blum, W., & Leiβ, D., 2007. *How do students and teachers deal with mathematical modelling problem?* Mathematical modelling (ICTMA 12): Education, engineering and economics, 222-231.

[10] Borromeo Ferri, R., 2011. *Paths to the inner world of mathematical modeling: cognitive. Wiesbaden*: Vieweg + Teubner.

[11] Blum, W., 2015. *Quality teaching of mathematical modelling: What do we know, what can we*. The Proceedings of the 12th International Congress on Mathematical (p.73-96). Cham: Springer International Publishing.

[12] Borromeo Ferri, R., 2010. *Mathematical Modelling in teacher education* -. CERME 6. Proceedings of the sixth congress of the European Society for Research in Mathematics Education (p. 2046-2055). Lyon: National Institute for Educational Research (in France).

[13] Kaiser, G., 1995. *Reality references in math lessons: An overview of the current and historical discussion*. Series from the Istron Group: Materials for Realistic Mathematics Lessons, 66-81 (in German).

[14] Nam, N.D., 2015. *Design modeling activities in teaching mathematics*. HNUE Journal of Science, Educational Sci., 2015, Vol. 60, No. 8A, pp. 152-160.

[15] *Deutscher bidungs server,* 2019. https://www.bildungsserver.de/Lehrplaene-400-de.html.

[16] Bryant, D. P., 2000. *Characteristic behaviors of students with LD who have teacher-identified math weaknesses*. Journal of Learning Disabilities, 168-199.